Application of antiferromagnetic-Fermi-liquid theory to NMR experiments in $La_{1.85}Sr_{0.15}CuO_4$

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NMR experiments on $La_{1.85}Sr_{0.15}CuO_4$ by Kitaoka *et al.* and Imai *et al.* are analyzed using the phenomenological antiferromagnetic (AF) Fermi liquid theory of Millis, Monien, and Pines, and the results are compared with those previously obtained for $YBa_2Cu_3O_7$ and $YBa_2Cu_3O_{6.63}$. A one-component model, with hyperfine couplings that are unchanged from those found previously for $YBa_2Cu_3O_7$ and $YBa_2Cu_3O_{6.63}$, and parameters obtained from experiment, provide a quantitative fit to the data. At all temperatures the antiferromagnetic correlations found in $La_{1.85}Sr_{0.15}CuO_4$ are stronger than those found for the Y-Ba-Cu-O samples with the result that the characteristic energy for the antiferromagnetic paramagnons that describe the AF spin dynamics is quite low ($<kT$ and ~ 20 K at T_c). We use the deduced paramagnon energies to calculate the contribution to the electrical resistivity from quasiparticle-antiferromagnetic paramagnon scattering for $La_{1.85}Sr_{0.15}CuO_4$, $YBa_2Cu_3O_7$, and $YBa_2Cu_3O_{6.63}$, and find that it displays a linear temperature dependence for all three materials. Our results support the proposal that the properties of a nearly antiferromagnetic Fermi liquid are genuinely novel, and suggest that both the spin and charge aspects of the normal-state properties of the cuprate oxide superconductors can be quantitatively explained in terms of quasiparticles coupled to antiferromagnetic paramagnons whose characteristic energy scale is $<kT$.

I. INTRODUCTION

The phenomenological antiferromagnetic-Fermi-liquid theory developed by Millis, Monien, and Pines,[1] hereafter MMP, has been shown to provide a quantitative account of nuclear magnetic resonance relaxation rates for Cu(2), O(2), O(3), and Y nuclei in both $YBa_2Cu_3O_7$ (Ref. 1) and $YBa_2Cu_3O_{6.63}$ (Ref. 2). It is natural to inquire whether this one-component theory is equally capable of describing the results of NMR experiments on other cuprate oxide superconductors. In this paper we use the theory to analyze the NMR experiments of Kitaoka *et al.*[3] and Imai *et al.*[4] on one member of the La-Sr-Cu-O family, $La_{1.85}Sr_{0.15}CuO_4$. We find that a one-component model with hyperfine couplings of the ^{63}Cu(2) nuclei and ^{17}O(2) nuclei that are unchanged from those found previously for (1) $YBa_2Cu_3O_7$ and (2) $YBa_2Cu_3O_{6.63}$ provides a quantitative fit to the experimental data. As is the case for $YBa_2Cu_3O_{6.63}$, the temperature dependence of the ^{63}Cu relaxation rate reflects in part strong temperature-dependent antiferromagnetic correlations between the Cu^{2+} spins, and in part a temperature-dependent long-wavelength static spin susceptibility. We determine the latter from the measurement of the ^{17}O Knight shift.[3] When this temperature dependence is taken into account, at all temperatures the antiferromagnetic correlations found in $La_{1.85}Sr_{0.15}CuO_4$ are stronger than those found for the Y-Ba-Cu-O samples; moreover, over much of the measured temperature range the antiferromagnetic correlation length is found to be of the mean-field form established earlier for $YBa_2Cu_3O_7$ and $YBa_2Cu_3O_{6.63}$.

In $La_{1.85}Sr_{0.15}CuO_4$ the tendency toward antiferromagnetism is, in fact, so strong that for temperatures above 130 K the system behaves as though it were an itinerant antiferromagnet with a Néel temperature ~ 25 K. Below that temperature, however, the correlation length ceases its rapid increase, and levels off toward a constant value ~ 3 times the interparticle spacing.

A consequence of the stronger antiferromagnetic correlations is that the characteristic energy scale for the antiferromagnetic paramagnons that describe the antiferromagnetic (AF) spin dynamics is quite low. It is always less than kT, being ~ 20 K for temperatures in the vicinity of T_c and varying linearly with temperature for temperatures above 130 K. Low-energy paramagnons provide an effective scattering mechanism for quasiparticles; we calculate the spin-fluctuation resistivity for $La_{1.85}Sr_{0.15}CuO_4$ and compare it with our calculated results for $YBa_2Cu_3O_7$ and $YBa_2Cu_3O_{6.63}$. We find that for all three materials the temperature dependence of the resistivity is linear and that if one assumes that the matrix elements for quasiparticle-paramagnon scattering are similar for the three materials, the increasing tendency toward antiferromagnetic behavior as one goes from $YBa_2Cu_3O_7$ to $YBa_2Cu_3O_{6.63}$ to $La_{1.85}Sr_{0.15}CuO_4$ leads to a comparable progression in the magnitude of the resistivity. These results suggest that a resistivity that varies linearly with temperature is a natural attribute of an almost antiferromagnetic Fermi liquid and they provide support for the view that the anomalous charge and spin properties of the normal state of the cuprate oxide superconductors possess a common physical origin.

The plan of the paper is the following. In Sec. II, we introduce the hyperfine Hamiltonian for the planar nuclei and use it to determine the temperature-dependent spin susceptibility from measurements of the ^{17}O Knight shift.[3] As is the case for the Y-Ba-Cu-O samples, the transferred hyperfine coupling between the Cu^{2+} spins and the Cu and O nuclei plays a major role in determining the Knight shift of these nuclei. In Sec. III we review the MMP model for spin-lattice relaxation and use it to determine the antiferromagnetic enhancement factor from experiment in Sec. IV.

We use the above results to analyze two candidate models for antiferromagnetic behavior in Sec. V: a temperature-dependent interaction between almost localized spins and Fermi surface nesting of weakly localized itinerant quasiparticles. A comparison of our results for the antiferromagnetic behavior of $La_{1.85}Sr_{0.15}CuO_4$ with those obtained for the Y-Ba-Cu-O family leads us to conclude that only the nearly localized moment model is consistent with the NMR experiments on these systems. We present our results for spin-fluctuation resistivity in Sec. VI, and discuss the nature of the normal state of cuprate oxide superconductors, the possible connection between antiferromagnetic behavior and superconductivity, and give our conclusions in Sec. VII.

II. THE HYPERFINE HAMILTONIAN AND KNIGHT-SHIFT EXPERIMENTS

In analyzing the experimental results for the Knight shift and spin-lattice relaxation rates, we use the hyperfine Hamiltonian for the planar Cu and O nuclei adopted by Mila and Rice[5] and Millis, Monien, and Pines[1] to describe NMR experiments in $YBa_2Cu_3O_{7-\delta}$:

$$H_{\rm hf}=\sum_{i,\alpha} {}^{63}I_{i,\alpha}A_{\alpha\alpha}S_{i,\alpha}+\sum_{\langle ij\rangle} {}^{63}I_{i,\alpha}BS_{j,\alpha} + \sum_{\langle ij\rangle,\alpha} {}^{17}I_{i,\alpha}CS_{j,\alpha} \tag{2.1}$$

in which the ^{63}Cu nuclear spins, $^{63}\mathbf{I}_i$, are assumed coupled to an electron spin $\mathbf{S}_i$ (one spin per unit cell composed out of planar Cu and O spins) at the same site, by the direct hyperfine coupling tensor $A_{\alpha\alpha}$ and to electron spins $\mathbf{S}_i$, at the four nearest-neighbor sites, by the transferred hyperfine coupling constant B. In this one-component model, the ^{17}O nuclear spins $^{17}\mathbf{I}_i$, are assumed to couple only by the transferred hyperfine coupling constant C to the two nearest-neighbor electron spins S_j. The spin contribution to the Knight shifts of the planar nuclei are then given by

$$ {}^{63}K_{\parallel}^{S}=\frac{(A_{\parallel}+4B)\chi_0}{{}^{63}\gamma_n\gamma_e\hbar^2}\ , \tag{2.2a}$$

$$ {}^{63}K_{\perp}^{S}=\frac{(A_{\perp}+4B)\chi_0}{{}^{63}\gamma_n\gamma_e\hbar^2}\ , \tag{2.2b}$$

$$ {}^{17}K_{\rm iso}^{S}=\frac{2C\chi_0}{{}^{17}\gamma_n\gamma_e\hbar^2}\ , \tag{2.2c}$$

where the γ_n are the various nuclear gyromagnetic ratios, γ_e is the electron gyromagnetic ratio, and subscripts refer to shifts for magnetic fields applied parallel and perpendicular to the crystalline c axis, with corresponding values for the hyperfine couplings.

Monien, Pines, and Takigawa[2] (hereafter MPT) have shown that in $YBa_2Cu_3O_{7-\delta}$ the hyperfine couplings A, B, and C are, to within a few percent, independent of the oxygen doping level δ, as one goes from the antiferromagnetic insulator to the 60-K superconductor to the 90-K superconductor. From their fit to the NMR experimental results for $YBa_2Cu_3O_7$, and the antiferromagnetic resonance results for $YBa_2Cu_3O_6$, MMP have determined $A_\parallel$, $A_\perp$, B, and C; their results are shown in Table I. Given the insensitivity of these hyperfine couplings to the degree of oxygenation of the $YBa_2Cu_3O_{7-\delta}$ compounds it appears reasonable to assume that, to within a few percent, the planar hyperfine couplings for the Cu and O nuclei in the La_{2-x} and Sr_xCuO_4 compounds will likewise be independent of the Sr doping level x, and that to the same degree of accuracy there will be no change in these couplings from those found in the Y compounds. Two sets of experimental results support this hypothesis. First, Tsuda *et al.*[6] find that the antiferromagnetic resonance frequency for the antiferromagnetic insulator La_2CuO_4 lies within a few percent of that measured for $YBa_2Cu_3O_6$ by Yasuoka *et al.*[7] Thus the quantity

$$\mu_{\rm eff}(4B-A_\perp)=H_{\rm AF}\cong 80\ \text{kOe} \tag{2.3}$$

is essentially unchanged. If, therefore, as seems plausible, $\mu_{\rm eff}=(0.62\pm0.02)\mu_B$ (Ref. 8) is changed by at most a few percent on going from $YBa_2Cu_3O_6$ to La_2CuO_4, it follows that $A_\perp$ and B do not vary. Second, the total ^{63}Cu Knight shift, $^{63}K_\parallel$, found by Kitaoka *et al.*[3] for a field applied in the c direction is independent of temperature and is given by

$$ {}^{63}K_\parallel \cong 1.3\%\ . \tag{2.4}$$

It is therefore identical, within experimental error, to the temperature-independent value found for $^{63}K_\parallel$ in $YBa_2Cu_3O_7$ (Ref. 9) and $YBa_2Cu_3O_{6.63}$ (Ref. 10); the latter results reflect the fact that $A_\parallel\cong -4B$, so that the measured total shift is of purely orbital origin. Since one finds the same temperature-independent magnitude shift for $La_{1.85}Sr_{0.15}CuO_4$, it follows that for the latter material one also has $A_\parallel=-4B$, and ${}^{63}K_\parallel={}^{63}K_\parallel^{\rm orb}$.

If we now assume that C is likewise unchanged from the Y compounds, and take the orbital oxygen Knight shift $^{17}K_\parallel^{\rm orb}$ to be -0.0136%, the value it possesses for $YBa_2Cu_3O_{6.63}$, and subtract this value from the experimental results of Kitaoka *et al.*,[3] for the temperature-dependent total shift $^{17}K_\parallel$, we obtain the result shown in Fig. 1 for the temperature-dependent spin component of the Knight shift $^{17}K_c(T)$ and the planar spin susceptibility $\chi_0(T)$. A good fit to $\chi_0(T)$ is obtained with the expression

TABLE I. Hyperfine couplings (in kOe/μ_B) for planar Cu and O nuclei.

Hyperfine couplings	$A_\parallel$	$A_\perp$	B	C
Deduced values	-163 ± 3	34 ± 1	40.8 ± 1	69 ± 2

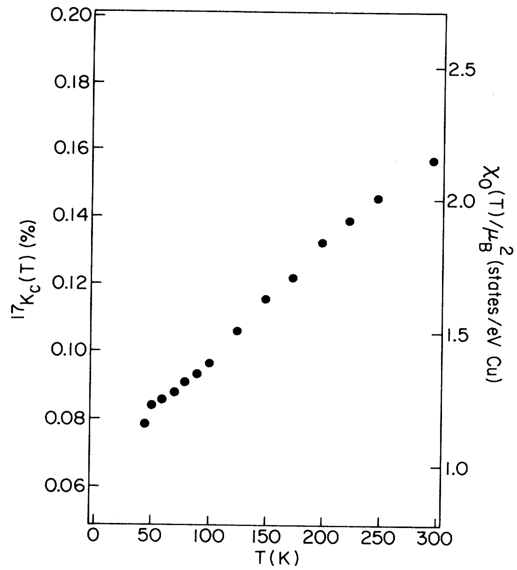


FIG. 1. The spin component, $^{17}K_s(T)$ of the planar oxygen Knight shift and the planar spin susceptibility, $\chi_0(T)$.

$$\frac{\chi_0(T)}{\mu_B^2}=[1.01+0.41(T/100)]\ \text{states/eV}\ \text{Cu}^{2+}\ . \tag{2.5}$$

As may be seen in Fig. 1, this result is quite similar to that found by MPT for $YBa_2Cu_3O_{6.63}$. We discuss its physical origin in Sec. IV and, in the Appendix, compare it with results obtained from an analysis of the experiments on $YBa_2Cu_3O_{6+\delta}$ of Alloul, Ohno, and Mendels.[11] Note that the measured temperature independence of $^{63}K_\parallel$ over a temperature range (60 K $\leq T \leq$ 120 K) (Ref. 3) in which $17K_\parallel^s$ and χ_0 change by some 40% can only come about if $^{63}K_\parallel$ is of purely orbital origin, as we have argued.

III. SPIN-LATTICE RELAXATION RATES

In a one-component model for the spin-spin correlation function the nuclear-spin-lattice relaxation rates depend on a product of the form factors, which differ from one nucleus to the other, and the dynamical structure factor $S(\mathbf{q},\omega)$. In the low-frequency regime $\omega \ll T$, in which NMR experiments are done, $S(q,\omega)$ is related to the imaginary part of the spin-spin correlation function $\chi''(q,\omega)$ by

$$S(\mathbf{q},\omega)\approx\left[\frac{T}{\omega}\right]\chi''(q,\omega)\ . \tag{3.1}$$

The MMP model for spin-lattice coupling may be thought of as an antiferromagnetic-Fermi-liquid model in which the dominant (low-frequency) elementary spin excitations are antiferromagnetic paramagnons at wave vectors near the antiferromagnetic wave vector $Q=(\pi/a,\ \pi/a)$. Thus one assumes that the system is almost, but not quite, antiferromagnetic. In their antiferromagnetic-Fermi-liquid theory, MMP model the spin-correlation function using two separate parts. One describes a quasiparticle part χ_{QP}, and the other, more important, part, χ_{AF}, describes the short-wavelength antiferromagnetic correlations,

$$\chi(\mathbf{q},\omega)=\chi_{QP}(\mathbf{q},\omega)+\chi_{AF}(\mathbf{q},\omega)\ , \tag{3.2}$$

The quasiparticle contribution may be written as

$$\chi_{QP}(\mathbf{q},\omega)=\frac{\bar{\chi}_0}{1-i\omega\pi/\Gamma}\ , \tag{3.3}$$

independent of the wave vector $\mathbf{q}$. Here Γ is the characteristic spin-fluctuation energy for the quasiparticle part, and $\bar{\chi}_0$ is the quasiparticle long-wavelength contribution to the measured long-wavelength static susceptibility χ_0.

The antiferromagnetic part of the spin-spin correlation function is modeled around the antiferromagnetic wave vector $\mathbf{Q}=(\pi/a,\ \pi/a)$ by

$$\chi_{AF}(\mathbf{q},\omega)=\frac{\chi_Q}{1+\xi^2(\mathbf{Q}-\mathbf{q})^2-i(\omega/\omega_{SF})}\ ; \tag{3.4}$$

here χ_Q is the static spin susceptibility at the antiferromagnetic wave vector $\mathbf{Q}$, which is related to $\bar{\chi}_0$ by

$$\chi_Q=\bar{\chi}_0(\xi/\xi_0)^2\ , \tag{3.5}$$

where ξ is the antiferromagnetic correlation length and $1/\xi_0$ is the wave vector at which the antiferromagnetic part of the correlation function, χ_{AF}, starts to dominant the quasiparticle contribution. We can relate $\bar{\chi}_0$ to the measured static susceptibility χ_0 using Eqs. (3.2) and (3.4),

$$\chi_0=\bar{\chi}_0\left[1+\frac{1}{2\pi^2}\left[\frac{a}{\xi_0}\right]^2\right]=\bar{\chi}_0\left[1+\frac{\beta^{1/2}}{2\pi^2}\right] \tag{3.6}$$

on introducing the parameter

$$\beta=\left[\frac{a}{\xi_0}\right]^4\ , \tag{3.7}$$

which measures, *inter alia*, the relative strength of the antiferromagnetic paramagnon contribution to the static spin susceptibility. For a typical value of $(a/\xi_0)^2\approx\pi$ we find that the quasiparticle part contributes 84% of the static susceptibility, whereas the antiferromagnetic part contributes some 16%.

$\hbar\omega_{SF}$ is a typical energy scale for the antiferromagnetic paramagnons, which describe the AF spin dynamics. It can be very small, since it is related to the energy scale of the spin dynamics of the noninteracting quasiparticle system Γ by

$$\omega_{SF}=\frac{\Gamma}{\pi}\times\left[\frac{\xi_0}{\xi}\right]^2$$

$$=\frac{\Gamma}{\pi}\frac{\bar{\chi}_0}{\chi_Q}=\frac{\Gamma}{\pi}\left[\frac{\chi_0}{\chi_Q}\right]\left[\frac{1}{1+\beta^{1/2}/2\pi^2}\right]\ . \tag{3.8}$$

On comparing the spin-lattice relaxation rate for a noninteracting Fermi gas with that calculated from Eq. (3.3), one can easily show that

$$\hbar\Gamma\cong E_F\ , \tag{3.9}$$

the quasiparticle Fermi energy. Thus when one has appreciable AF correlations (and we shall see that χ_Q/χ_0 can be as large as 130 for $La_{1.85}Sr_{0.15}CuO_4$), the paramagnon energy scale can be as low as 20 K.

The relaxation rates are determined by a $\mathbf{q}$ average over the Brillouin zone of the structure factor multiplied with the appropriate form factors. MMP have obtained the following explicit expressions for the relaxation rates:

$$^{63}W_{\parallel}=\frac{3}{4}\frac{1}{\mu_B^2\hbar}\lim_{\omega\to 0}\times\sum_q\{A_\perp-2B[\cos(q_xa)+\cos(q_ya)]\}^2S(\mathbf{q},\omega)\ , \tag{3.10a}$$

$$^{63}W_{\perp}=\frac{3}{8}\frac{1}{\mu_B^2\hbar}\lim_{\omega\to 0}\times\sum_q(\{A_\parallel-2B[\cos(q_xa)+\cos(q_ya)]\}^2+\{A_\perp-2B[\cos(q_xa)+\cos(q_ya)]\}^2)S(\mathbf{q},\omega)\ , \tag{3.10b}$$

$$^{17}W=\frac{3}{4}\frac{1}{\mu_B^2\hbar}\lim_{\omega\to 0}\sum_q\{2C^2[1-\cos(q_xa)]\}S(\mathbf{q},\omega)\ . \tag{3.10c}$$

Here $\mathbf{q}$ is measured from the antiferromagnetic wave vector $\mathbf{Q}=(\pi/a,\pi/a)$. Whereas the Cu relaxation rate picks up all the antiferromagnetic correlations, the O relaxation rate is not strongly enhanced by the AF correlations, since the hyperfine field of the Cu spins cancels at the oxygen site. We remind the reader that the connection between the above-defined Cu relaxation rates and the measured Cu relaxation times, T_1, depends on whether one is considering a nuclear magnetic resonance experiment, for which one has

$$(1/T_1)_{\rm NMR}=\tfrac{2}{3}W\ , \tag{3.11a}$$

or a nuclear-quadrupolar-resonance experiment, for which

$$(1/T_1)_{\rm NQR}=2W=3(1/T_1)_{\rm NMR}\ . \tag{3.11b}$$

Since we have seen that the hyperfine couplings for $La_{1.85}Sr_{0.15}CuO_4$ are within some 5% of those measured for $YBa_2Cu_3O_7$ and $YBa_2Cu_3O_{6.63}$, we use the Y-Ba-Cu-O result, $A_\perp=0.84B$ (Refs. 1 and 2) in determining $^{63}W(T)$ and $^{17}W(T)$. We further assume that the AF correlations peak at $Q=(\pi/a,\pi/a)$ and, hence, find, on making use of the MMP results for the integrals over q in Eqs. (3.10),

$$^{63}W_{\parallel}(T)=\frac{12\pi}{\mu_B^2\hbar^2}B^2k_BT\left[\frac{\chi_0(T)}{\Gamma(T)[1+\beta^{1/2}/2\pi^2]}\right]\left\{0.294+\frac{\beta}{\pi^2}\left[0.49\left[\frac{\xi(T)}{a}\right]^2-0.621\ln\left[\frac{\xi(T)}{a}\right]+0.0175\right]\right\}, \tag{3.12a}$$

$$^{63}W_{\perp}(T)=\frac{12\pi}{\mu_B^2\hbar^2}B^2k_BT\left[\frac{\chi_0(T)}{\Gamma(T)[1+\beta^{1/2}/2\pi^2]}\right]\left\{0.772+\frac{\beta}{\pi^2}\left[1.83\left[\frac{\xi(T)}{a}\right]^2-1.10\ln\left[\frac{\xi(T)}{a}\right]-0.297\right]\right\}, \tag{3.12b}$$

$$^{17}W(T)=\frac{3\pi}{2\mu_B^2\hbar^2}C^2k_BT\left[\frac{\chi_0(T)}{\Gamma(T)[1+(\beta^{1/2}/2\pi^2)]}\right]\left\{1+\frac{\beta}{\pi^2}\left[0.39\ln\left[\frac{\xi(T)}{a}\right]+0.17\right]\right\}. \tag{3.12c}$$

As MPT have noted, the above expressions make it easy to separate out the contribution made by antiferromagnetic paramagnons to the spin-lattice relaxation rate; the "quasiparticle" result is obtained if one takes $\beta=0$ in Eqs. (3.12), while the importance of antiferromagnetic correlations is measured by defining the quantity

$$^{63}(R_{\rm AF})_{\parallel}=\frac{^{63}W_{\parallel}(\beta)}{^{63}W_{\parallel}(0)}=\left\{1+\frac{\beta}{\pi^2}\left[\frac{5}{3}\left[\frac{\xi(T)}{a}\right]^2-2.1\ln\frac{\xi(T)}{a}+0.059\right]\right\}\Big/(1+\beta^{1/2}/2\pi^2)\ . \tag{3.13}$$

The relative importance of antiferromagnetic correlations, as one goes from ^{63}W to ^{17}W is seen clearly in Eqs. (3.12). As MMP have emphasized, for $\xi^2/a^2\gg 1$ antiferromagnetic correlations play a dominant role in determining $^{63}W(T)$, with the leading term being proportional to ξ^2, and a logarithmic contribution of antiferromagnetic origin (which is of opposite sign from the quasiparticle contribution) playing a more significant role than the latter for $(\xi/a)\geq 1.5$.

Given a knowledge of $\chi_0(T)$, and with the assumption that Γ is only weakly dependent on T, it is straightforward to use Eq. (3.12a) to obtain the temperature dependence of $\xi(T)$ from experiment, provided one is in the limit, $[\xi(T)/a]^2\gg 1$. Indeed, to the extent that $\xi(T)$ takes the mean-field form,

$$\left[\frac{\xi(T)}{a}\right]^2=\left[\frac{\xi(0)}{a}\right]^2\frac{|T_x|}{T+T_x} \tag{3.14}$$

and Γ is only weakly temperature dependent, the quantity $[^{63}W_{\parallel}(T)/T\chi_0(T)]^{-1}$ should display a temperature dependence of the form

$$[^{63}W_{\parallel}(T)/T\chi_0(T)]^{-1}\sim T_1T\chi_0(T)=a+bT\ , \tag{3.15}$$

where a and b are constants. Hence a plot of $T_1T\chi_0(T)$ as a function of T provides a test of the applicability of the MMP model with ξ^2/a^2 displaying the temperature dependence, Eq. (3.14). In Fig. 2 we give the results of Imai *et al.*[4] and Kitaoka *et al.*[3] for the relaxation rate $^{63}[1/T_1(T)]$, while in Fig. 3 we use our result for $\chi_0(T)$,

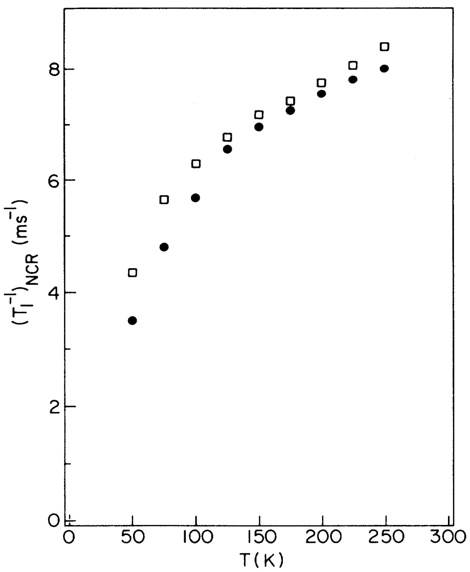


FIG. 2. The ^{63}Cu nuclear-spin-relaxation rate as measured by two different experimental groups: (●) Imai *et al.* (Ref. 4); and (□) Kitaoka *et al.* (Ref. 3).

Eq. (2.5), to plot $[T_1 T\chi_0(T)]$ as a function of temperature for the two sets of experimental measurements.

We see that over the temperature range (100 K $\leq T \leq$ 200 K) in which the results of the two groups are in good agreement; the form Eq. (3.14) is well obeyed. There are, however, significant departures at both lower and higher temperatures. At the high temperatures, the "nonleading terms" [i.e., those other than ξ^2/a^2 in Eq. (3.12a)] begin to play a significant role, while at lower temperatures, departures from the mean-field form, Eq.

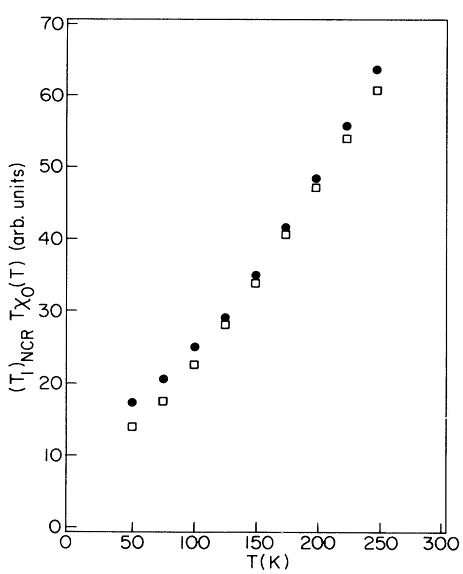


FIG. 3. The experimentally determined product $(T_1)_{NQR}T\chi_0(T)$ from the experimental measurements of Imai *et al.* (Ref. 4) (●) and Kitaoka *et al.* (Ref. 3) (□) plotted vs the temperature.

(3.14), become significant. To take into account the presence of the nonleading terms in $^{63}W_{\parallel}$ it is necessary to know, separately, Γ, β, and ξ/a, and we consider next the extent to which reliable estimates for these basic parameters of antiferromagnetic-Fermi-liquid theory may be obtained from experiment.

IV. DETERMINATION OF ANTIFERROMAGNETIC-FERMI-LIQUID PARAMETERS

We begin by considering the oxygen relation rate, which has recently been measured by Reven *et al.*[12] According to Eq. (3.12c), since $(^{17}T_1)^{-1}$ depends only logarithmically on $\xi(T)$, to the extent that Γ is independent of T, one would expect the product $^{17}[T_1 T\chi_0(T)]$ to be weakly temperature dependent, decreasing logarithmically with decreasing temperature. Moreover, since the values of $\chi_0(T)$ we have deduced here for $La_{1.85}Sr_{0.15}CuO_4$ are somewhat greater than those deduced by Monien, Pines, and Takigawa[2] for $YBa_2Cu_3O_{6.63}$ (see Fig. 11), we would expect the values of $^{17}[T_1 T\chi_0(T)]$ for the latter system to lie above those measured for $La_{1.85}Sr_{0.15}CuO_4$. Inspection of Fig. 4 shows that the latter expectation is, within the scatter of the experimental results, borne out, but that the anticipated logarithmic decrease with decreasing temperature is not found. As was the case for the $O_{6.63}$ compound, this last result then suggests that Γ is an increasing function of T, in such a way that its temperature dependence outweighs the logarithmic temperature dependence of $\xi(T)$.

To determine $\Gamma(T)$, we note that for a given choice of

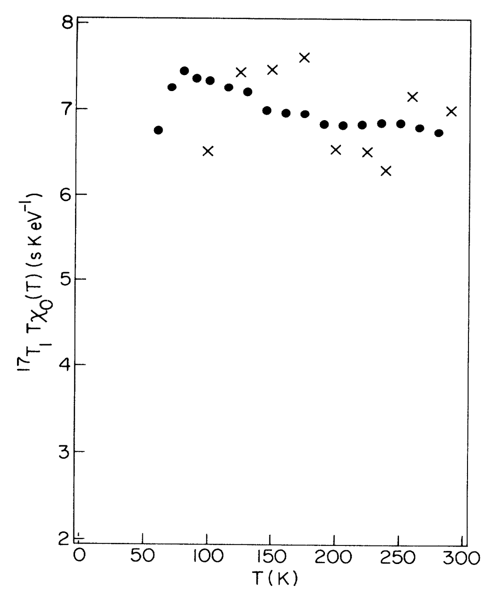


FIG. 4. The product, $^{17}T_1 T\chi_0(T)$, determined from the measurements of Reven *et al.* (Ref. 10) and Kitaoka *et al.* (Ref. 3) plotted vs the temperature (×). Also shown is the corresponding quality (●) determined by MPT for $YBa_2Cu_3O_{6.63}$ from the experiments of Takigawa *et al.* (Ref. 10).

β, the ratio of the copper and oxygen relaxation rates will determine $\xi(T)$, while either rate can then be used to determine $\Gamma(T)$. We find that with $\beta=\pi^2$, the value it takes on for $YBa_2Cu_3O_7$ and $YBa_2Cu_3O_{6.63}$, we can obtain good fits to both the copper and oxygen relaxation rates with

$$\Gamma(T)\cong(0.5-(T/2500)\ \text{eV}\ , \tag{4.1}$$

a result that is not far from that found for $O_{6.63}$,

$$\Gamma(T)\cong(0.45-T/3300)\ \text{eV}\ . \tag{4.2}$$

For a given value of β and $\Gamma(T)$, since the other quantities that enter our expression, Eq. (3.12a) for the ^{63}Cu relaxation rate, have been determined, it is straightforward to obtain $[\xi(T)/a]$ from an experimental measurements of $^{63}(T_1^{-1})$. Our results are given in Figs. 5(a)–5(c). There we see that ξ/a is considerably larger than the values deduced for O_7 and $O_{6.63}$, while the temperature

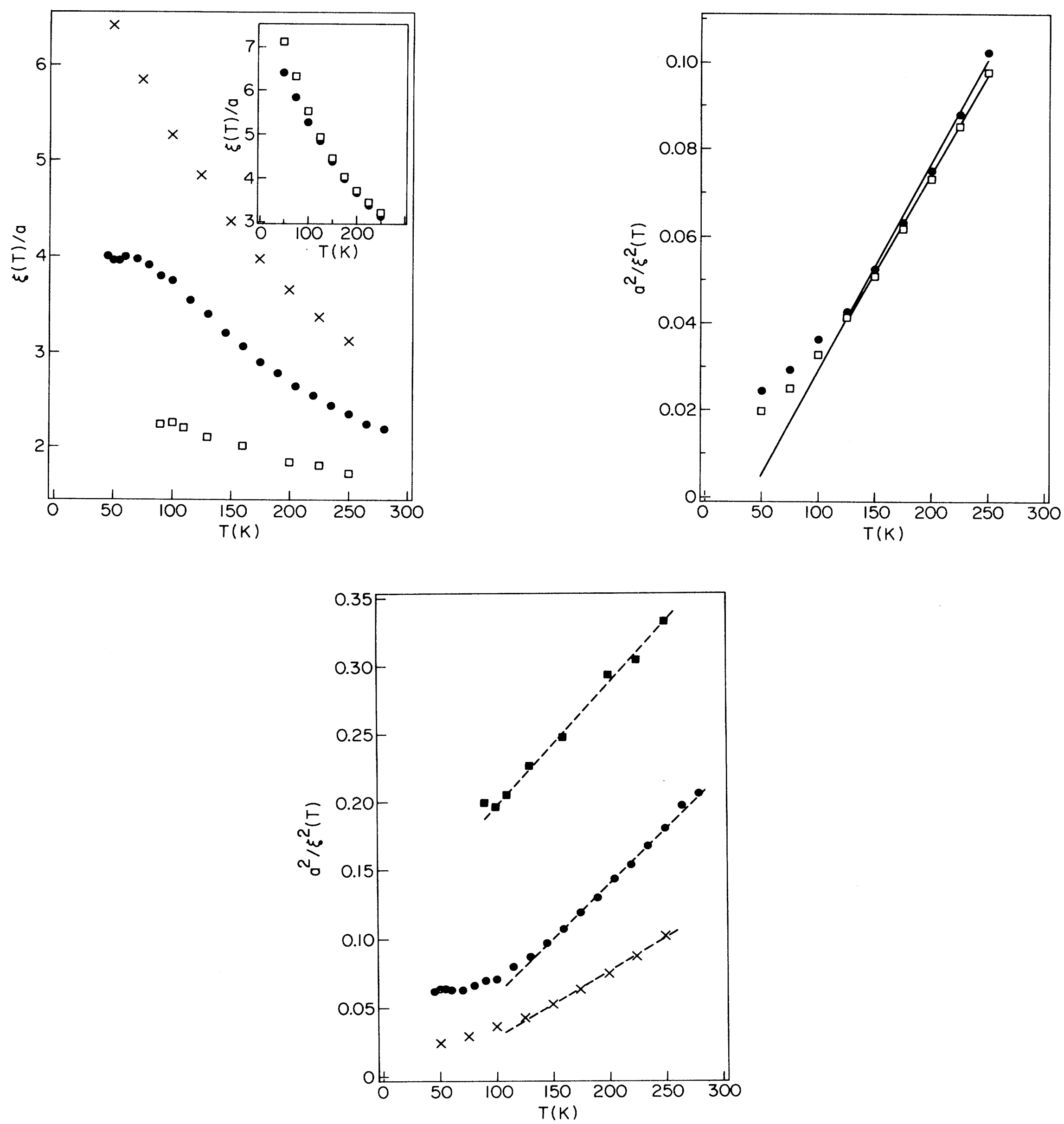


FIG. 5. (a) The antiferromagnetic correlation length as a function of temperature for $La_{1.85}Sr_{0.15}CuO_4$ (×), $YBa_2Cu_3O_{6.63}$ (●), and $YBa_2Cu_3O_7$ (□). Inset: comparison of the result for ξ/a using the experimental results of Kitaoka *et al.* (□) and Imai (●). (b) The inverse correlation length squared (a^2/ξ^2), determined from the experiments of Kitaoka *et al.* (■) and Imai *et al.* (●). (c) The inverse correlation length squared (a^2/ξ^2), determined for $\beta=\pi^2$ from the experiments of Imai *et al.* (Ref. 4) (×) compared with the corresponding values calculated by MPT for $YBa_2Cu_3O_7$ (□) and $YBa_2Cu_3O_{6.63}$ (●). The dashed lines represent the best linear fits to the data above 130 K.

dependence of (a^2/ξ^2) is quite similar to that found in $O_{6.63}$ [Fig. 5(c)]. In $La_{1.85}Sr_{0.15}CuO_4$ above 130 K, the tendency toward antiferromagnetism is so strong that $\xi(T)$ follows the approximate temperature dependence

$$\left[\frac{\xi(T)}{a}\right]^2 = 58\left[\frac{38}{T-38}\right] \quad (T \geq 130\ \text{K}) \,. \tag{4.3}$$

Thus as the temperature is lowered from 250 K, and the system appears to be on its way toward becoming an itinerant antiferromagnet with a Néel temperature ~ 31 K, but at $T \sim 130$ K this behavior begins to change, as though the system starts to become aware, even at this elevated temperature, of the fact that it is going to become a superconductor, not an antiferromagnet. As shown in Fig. 5(c), quite similar behavior is seen in $YBa_2Cu_3O_{6.63}$, where MPT note that, since ξ/a is a constant below T_c, the transition, which begins at 130 K, of ξ/a from Néel behavior to superconducting behavior may perhaps be a precursor effect. It is perhaps not an accident that this change in character of ξ/a is more striking in those materials that resemble incipient itinerant antiferromagnets for $T > 130$ K than it is for $YBa_2Cu_3O_7$, which shows much less in the way of antiferromagnetic tendencies (T_x being ~ 113 K rather than being negative and ξ/a being considerably smaller at all temperatures).

In Fig. 6 we give a fit to the $^{17}(T_1^{-1})$ measurements of Reven *et al.*[10] based on taking a somewhat smaller value of the oxygen hyperfine coupling constant ($C = 64.2$ kOe/μ_B) than that used by MPT for $O_{6.63}$ ($C = 68.6$ kOe/μ_B). The agreement between theory and experiment is seen to be satisfactory, whether one uses values of (ξ/a) deduced from the experiments of Kitaoka *et al.*[3] or those of Imai *et al.*[4]

In our calculation of $^{17}W(T)$ for $La_{1.85}Sr_{0.15}CuO_4$ we have assumed that the antiferromagnetic correlations peak at the commensurate wave vector, $Q = (\pi/a, \pi/a)$. Since a departure from commensurability introduces a term $\sim \beta(\xi^2/a^2)$ in ^{17}W, which would dominate the logarithmic term for even quite small departures from commensurability, it is evident that the measurements of $^{17}W(T)$ for this system by Reven *et al.*[10] rule out any appreciable departure from commensurability.

Our results for the quantities that provide a direct measure of the strength of the antiferromagnetic correlations in $La_{1.85}Sr_{0.15}CuO_4$, $^{63}R_{AF}$, χ_Q, and χ_Q/χ_0, are given in Figs. 7 and 8, where they are compared with the corresponding quantities for the Y-Ba-Cu-O compounds. We see that at all temperatures the antiferromagnetic correlations found in $La_{1.85}Sr_{0.15}CuO_4$ are considerably stronger than those encountered in $YBa_2Cu_3O_7$ and $YBa_2Cu_3O_{6.63}$.

Our results for the paramagnon energy, ω_{SF}, are given in Fig. 9, where they are compared with values previously obtained by MMP for O_7 and by MPT for $O_{6.63}$. Again one sees a consistent progression; the more antiferromagnetic the system is, the lower the paramagnon energy. For all three systems, the paramagnon energy increases linearly with T over a wide range of temperature as is to be expected from the proportionality of ω_{SF} and a^2/ξ^2. These results demonstrate that $\hbar\omega_{SF}(T) \leq k_B T \leq k_B T_c$ for all three materials, and that $\hbar\omega_{SF}$ is dramatically smaller than $k_B T$ in the case of $La_{1.85}Sr_{0.15}CuO_4$.

Given the fact that the relaxation rates are known to some 10% for copper and somewhat less accurately for ^{17}O, that our estimate of the magnitude and variation in

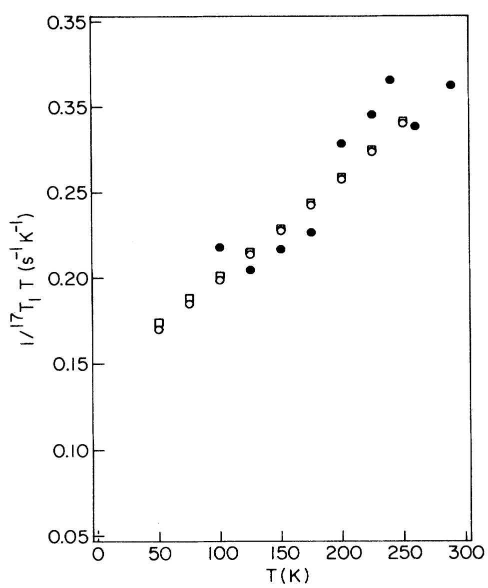


FIG. 6. Our theoretical results for the oxygen spin-lattice relaxation rate (○) are compared to the measurements of Reven *et al.* (●) for $^{17}T_1T$, to the experimental results of Kitaoka *et al.* (□) and to the experimental results of Imai *et al.* (○).

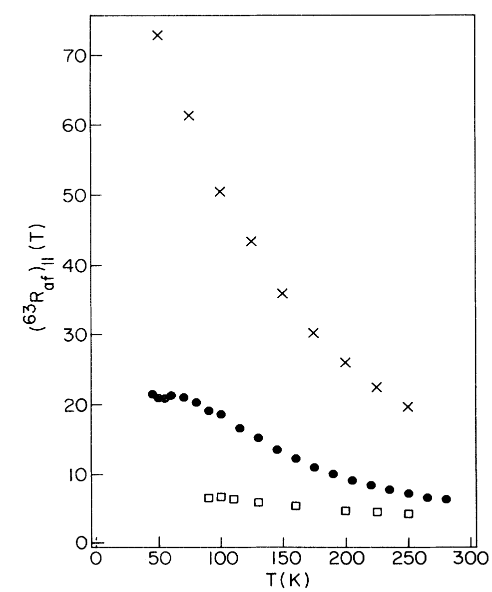
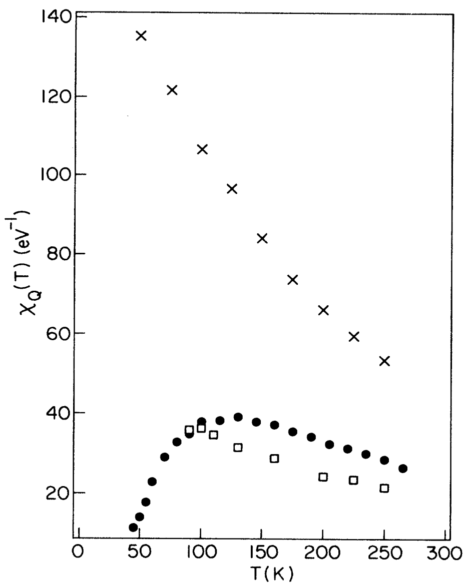


FIG. 7. The calculated values of antiferromagnetic enhancement factor, $(^{63}R_{AF})_{\parallel}$ are plotted as a function of temperature for three cuprate oxide superconductors: $La_{1.85}Sr_{0.15}CuO_4$ (×), $YBa_2Cu_3O_{6.63}$ (●), $YBa_2Cu_3O_7$ (□). The data of Imai *et al.* (Ref. 4) has been used for $La_{1.85}Sr_{0.15}CuO_4$; very nearly the same result is obtained from the data of Kitaoka *et al.* (Ref. 3).

$\Gamma(T)$ between 250 and 40 K may not be more accurate than 20%, that $\chi_0(T)$ is likely not known to be much better than 20%, and that our heuristic choice of β may not be more accurate than, say, 20%, it seems reasonable to assign an uncertainty of some ±30% to all the values we have deduced here.

V. THE PHYSICAL ORIGIN OF ANTIFERROMAGNETIC BEHAVIOR

We now consider briefly the information that our analysis of the NMR experiments on $La_{1.85}Sr_{0.15}CuO_4$ provides on the physical origin of the measured very strong antiferromagnetic correlations. For an almost localized set of spins, the correlations arise as a consequence of a temperature-dependent interaction between the (barely) itinerant quasiparticles, while for a weekly localized spin system, these would arise from Fermi-surface nesting of the (quite) itinerant quasiparticles. The MMP description of antiferromagnetic correlations makes no distinction between these alternatives, since it only assumes that the static susceptibility in the vicinity of **Q** may be written as



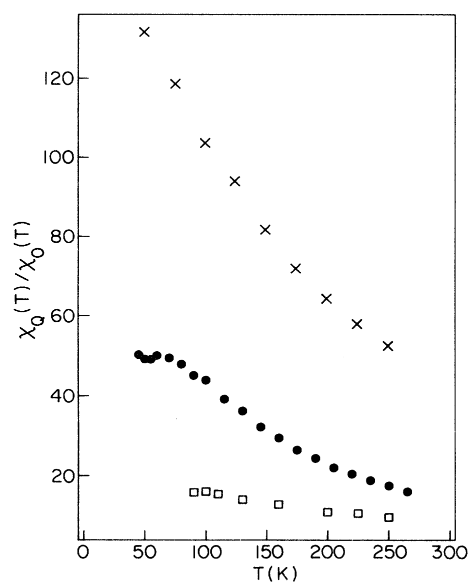


FIG. 8. (a) The calculated values of $\chi_Q(T)$ are plotted as a function of temperature for $La_{1.85}Sr_{0.15}CuO_4$ (×), $YBa_2Cu_3O_{6.63}$ (●), and $YBa_2Cu_3O_7$ (□). (b) The calculated values of $\chi_Q(T)/\chi_0(T)$ plotted as a function of temperature for $La_{1.85}Sr_{0.15}CuO_4$ (×), $YBa_2Cu_3O_{6.63}$ (●), and $YBa_2Cu_3O_7$ (□).

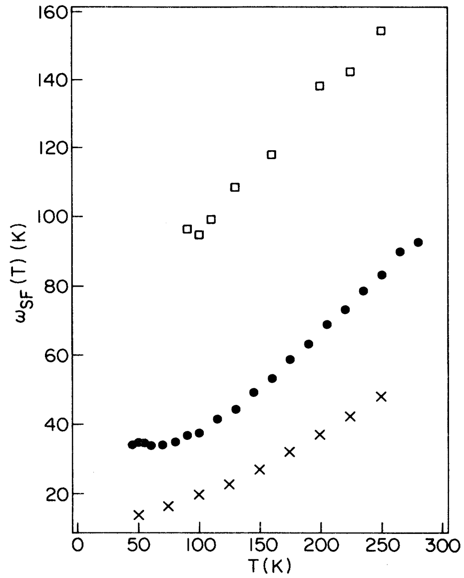


FIG. 9. The spin-fluctuation temperature, ω_{SF}, is plotted as a function of temperature for $La_{1.85}Sr_{0.15}CuO_4$ (×), and compared to results obtained previously by MPT for $YBa_2Cu_3O_{6.63}$ (●), and $YBa_2Cu_3O_7$ (□).

$$\chi_{AF}(\mathbf{Q}-\mathbf{q},0)=\frac{\chi_Q}{1+\xi^2q^2}=\frac{\bar{\chi}_0(\xi^2/\xi_0^2)}{1+\xi^2q^2} \tag{5.1}$$

without inquiring as to the physical origin of ξ_0 or $\xi(T)$. If, however, we consider the mean-field expression,

$$\chi_{AF}(\mathbf{Q}-\mathbf{q},0)=\frac{\bar{\chi}_{Q-q}}{1-F_{|\mathbf{Q}-\mathbf{q}|}}\ , \tag{5.2}$$

where χ_{Q-q} is the static susceptibility of noninteracting quasiparticles or local moments, and

$$F_{|\mathbf{Q}-\mathbf{q}|}=J(\mathbf{Q}-\mathbf{q})\bar{\chi}_{\mathbf{Q}-\mathbf{q}} \tag{5.3}$$

is a dimensionless measure of the degree by which $\bar{\chi}_{Q-q}$ is enhanced by the interaction, while $J(\mathbf{Q}-\mathbf{q})$ measures the strength of the interaction between the quasiparticles (or local moments), then we can consider separably the role played by $\bar{\chi}$ and by J in determining the antiferromagnetic enhancement. Thus we may write

$$\bar{\chi}_{Q-q}=\frac{\bar{\chi}_0}{\alpha_Q+\alpha\xi_\chi^2q^2}(\alpha=\pm 1)\ , \tag{5.4a}$$

$$F_{Q-q}=F_Q-q^2\xi_F^2 \ , \tag{5.4b}$$

$$J_{Q-q}=J_Q-q^2\xi_J^2 \ , \tag{5.4c}$$

and then combine the above expressions with Eq. (3.5) to obtain

$$\frac{\xi_0^2}{\xi^2}=\alpha_Q[1-F_Q] \tag{5.5}$$

and

$$\xi_0^2=\alpha_Q\xi_F^2+\xi_\chi^2[1-F_Q]=\alpha\xi_\chi^2+\bar{\chi}_0\xi_J^2 \ . \tag{5.6}$$

We thus see that ξ_0 is determined by the interplay between the wave-vector dependence of $\bar{\chi}_q$ in the vicinity of Q and that of J_q. The sign of α, and hence of the contribution made by of ξ_χ^2 to ξ_0^2 reflects clearly the difference between fermi surface nesting behavior and barely itinerant local moments, since to the extent that nesting is significant, we expect that $\bar{\chi}_Q > \bar{\chi}_0$, with $\bar{\chi}_q$ maximum at **Q**, so that, according to Eq. (5.4a)

$$(\alpha_Q)_{\rm nest} < 1 \ , \tag{5.7a}$$

$$(\alpha)_{\rm nest}=+1 \ , \tag{5.7b}$$

and the consequences of the wave-vector dependence of $\bar{\chi}_q$ and J_q in the vicinity of **Q** are additive. For local moment behavior, on the other hand, we have

$$(\alpha_Q)_{\rm loc} > 1 \ , \tag{5.8a}$$

$$(\alpha)_{\rm loc}=-1 \ , \tag{5.8b}$$

so that

$$(\xi_0^2)_{\rm local}=\bar{\chi}_0\xi_J^2-\xi_\chi^2 \ . \tag{5.9}$$

It follows that for nearly localized local moments significantly smaller values of ξ_0^2 are to be expected than would be the case for nearly nested Fermi liquids. From this perspective the comparatively small value, $(\xi_0/a)=0.56$, found by MMP for the Y-Ba-Cu-O family, and found here is a physically reasonable one. On the other hand, it is difficult, if not impossible, to explain the insensitivity of ξ_0 to oxygen concentration for Y-Ba-Cu-O samples from a nesting perspective, and equally difficult to understand the quite small magnitude of ξ found for both the Y-Ba-Cu-O samples and $La_{1.85}Sr_{0.15}CuO_4$.

Our conclusion that the antiferromagnetic behavior is primarily associated with barely itinerant local-moment interaction is consistent with the physical picture put forth by Monien, Pines, and Slichter[13] for $YBa_2Cu_3O_7$ and the results of MPT for $YBa_2Cu_3O_{6.63}$, since it is only in such a picture that one would find that not only is the transferred hyperfine interaction B between a ^{63}Cu nucleus and its nearest-neighbor Cu^{2+} spins unchanged by doping, but also that the dynamical consequences of this interaction are unaffected.

VI. SPIN-FLUCTUATION RESISTIVITY

We have seen that the anomalous magnetic behavior of the normal state of the cuprate oxide superconductors can be simply explained if these are nearly antiferromagnetic Fermi liquids in which the dominant excitations are very-low-frequency antiferromagnetic paramagnons. In this section we explore the possibility that a major aspect of the anomalous normal state charge behavior of these systems, a resistivity that varies linearly with temperatures for electric fields applied in the planes, is likewise a property of a nearly antiferromagnetic Fermi liquid. We do so by using our results for the temperature-dependent antiferromagnetic paramagnon energy to compute the contribution to the electrical resistivity from quasiparticle-paramagnon scattering. Moriya, Takahashi, and Ueda[14] have given an expression for the resistivity produced by quasiparticle–spin-fluctuation scattering, which is well-suited to this purpose. When one makes the translation from their notation to our own, one obtains a resistivity that can be written as

$$\rho=10^{-2}\alpha_\rho\bar{\rho}(T) \ , \tag{6.1}$$

where

$$\bar{\rho}(T)=\frac{a^2}{T}\sum_q\int_{-\infty}^{\infty}d\omega\frac{\omega e^{\omega/T}}{(e^{\omega/T}-1)^2}\chi''_{\rm AF}(q,\omega) \tag{6.2}$$

and α_ρ is a constant ($\lesssim 1$) that depends on the matrix element for quasiparticle-paramagnon scattering, the density of quasiparticles, etc. On carrying out the sum over q, one then obtains

$$\bar{\rho}(T)=\frac{\beta^{1/2}}{2\Gamma}T\int_0^\infty dx\frac{xe^x}{(e^x-1)^2}\left[\tan^{-1}\frac{xT}{\omega_{\rm SF}}-\tan^{-1}\frac{xT}{(\omega_{\rm SF}+4\Gamma/\beta^{1/2})}\right] \ . \tag{6.3}$$

In this form, we see that as long as $\omega_{\rm SF}\sim T$, one is guaranteed a linear resistivity, no matter what the magnitude of $\omega_{\rm SF}$. [Moriya *et al.* reached a similar conclusion using the linearity with temperature of (a^2/ξ^2) as their criterion.]

This condition is a sufficient but not a necessary one, as an evaluation of the integral in (6.3) reveals. In Fig. 10(a) we give our results for $\bar{\rho}(T)$ for several choices of β for $La_{1.85}Sr_{0.15}CuO_4$, and for $YBa_2Cu_3O_7$ and $YBa_2Cu_3O_{6.63}$. We see that the temperature dependence of the resistivity is remarkably linear for all three materials, despite the fact that the form for $\omega_{\rm SF}(T)$ at low temperature is far from linear in the case of both $O_{6.63}$ and $La_{1.85}Sr_{0.15}CuO_4$. These results suggest that the resistivity linearity results in part from the fact that the significant low-frequency spin excitations, the paramagnons, have energies that are less than kT.

We have examined this possibility by calculating $\bar{\rho}(T)$

for Debye paramagnons, taking two examples in which ω_{SF} is independent of temperature; $\hbar\omega_{SF}=30$ K and $\hbar\omega_{SF}=60$ K. Our results, using $\beta=\pi^2$ to facilitate comparison with those we obtained for the $O_{6.63}$ sample, are given in Fig. 10(b). We see there that at low temperatures one finds a quadratic temperature dependence which, however, for $kT \gtrsim 2\hbar\omega_{SF}$ (in the case of the Debye paramagnons) switches over to a linear temperature dependence. Comparison of the 30-K Debye paramagnon result with our results for $YBa_2Cu_3O_{6.63}$ shows that the calculated resistivities begin to differ for $T \gtrsim 100$ K, where the $O_{6.63}$ paramagnon energy begins to increase linearly with temperature. Because the latter energy is larger than 30 K, the resistivity curve falls below the Debye paramagnon result; moreover, because $(\hbar\omega_{SF})_{6.63}$ increases linearly with temperatures for $T \gtrsim 120$ K, we find $\bar{\rho}(T)=bT$, rather than an "apparent" high-temperature resistivity of the form $\bar{\rho}(T)=b'T-a$. Put another way, for $T<120$ K, the transition to $(\hbar\omega_{SF})O_{6.63}\approx$const leads to an inflection in $\bar{\rho}(T)$ and an apparently larger slope $b'>b$; however, because $T_c>\hbar\omega_{SF}$, one does not see the beginnings of a transition to quadratic behavior found in the model calculations.

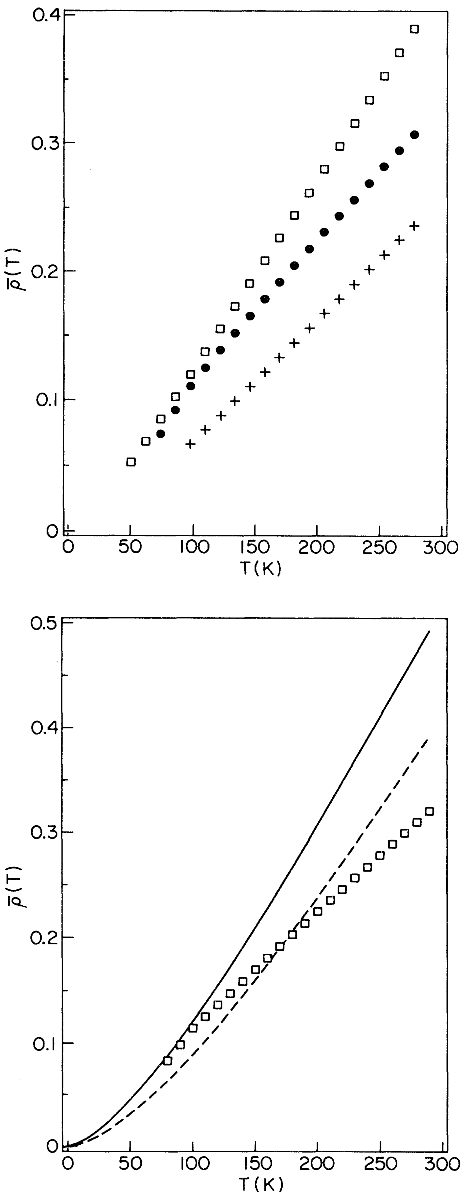


FIG. 10. (a) The reduced resistivity due to quasiparticle-antiferromagnetic scattering calculated using the formula of Moriya, Takahashi, and Ueda (Ref. 14) for $La_{1.85}Sr_{0.15}CuO_4$, using the value, $\beta=\pi^2$ (□). Also shown are comparable results for $YBa_2Cu_3O_7$ (+) and $YBa_2Cu_3O_{6.63}$ (●). (b) The temperature dependence of the reduced resistivity calculated using $\beta=\pi^2$, $\omega_{SF}=30$ K (denoted by a dashed line) and $\omega_{SF}=60$ K (denoted by a solid line). Shown for comparison is the result obtained for $O_{6.63}$ for which $\omega_{SF}\equiv 35$ K for $T\lesssim 100$ K, and $\cong(T/3)K$ for $T\gtrsim 130$ K.

Two other features of these results deserve mention. First, for a given material, as ξ_0 decreases (β increases), the slope of the linear resistivity increases. If β becomes too large, the result is a departure from linearity at high temperature, which may be traced to the contribution to the integral coming from the $\tan^{-1}[xT/(\omega_{SF}+4\Gamma/\beta^{1/2})]$ term. Second, for a fixed value of β, one sees clearly the influence of increased antiferromagnetic correlations on the slope of the resistivity; for $\beta=\pi^2$, as one goes from O_7 to $O_{6.63}$ to $La_{1.85}Sr_{0.15}CuO_4$, the slope of the resistivity curves increases. It is not, however, simply proportional to either $\beta^{1/2}$ or to $^{63}(R_{AF})_\parallel$, as a close inspection of the results depicted in Fig. 10(a) shows.

We do not here attempt a quantitative comparison with experiment for these materials, primarily because we do not yet possess a method for obtaining a reliable quantitative estimate of the constant, α_ρ, which we expect to vary from material to material, and which strong coupling (i.e., vertex) corrections may reduce considerably from the "weak coupling" estimate, $\alpha_\rho\cong 1$, given by Moriya *et al.*[14]

VII. DISCUSSION AND CONCLUSION

Because the experimental data presently available for $La_{1.85}Sr_{0.15}CuO_4$ is considerably more limited than that available, for $YBa_2Cu_3O_7$ and $YBa_2Cu_3O_{6.63}$ [for which $^{63}K^s$ and $^{89}(T_1^{-1})$ have also been measured], it is not possible to arrive at as complete a consistency check for the value, $\beta=\pi^2$, we have adopted for $La_{1.85}Sr_{0.15}CuO_4$. Thus while there can be no question that $^{63}(R_{AF})_\parallel$ for $La_{1.85}Sr_{0.15}CuO_4$ is greater at all temperatures than the corresponding quantity for $YBa_2Cu_3O_{6.63}$, the specific values we have deduced for ξ/a and $\Gamma(T)$ are not known to within $\sim$20 K.

Our results support the proposal[15] that the properties of a nearly antiferromagnetic Fermi liquid are *genuinely novel,* and suggest that both the spin and charge aspects of the normal state properties of the cuprate oxide superconductors can be quantitatively explained in terms of quasiparticles coupled to antiferromagnetic paramagnons whose characteristic energy scale is $\lesssim kT$. The low-frequency magnetic properties of a nearly antiferromagnetic liquid differ markedly from those of the usual Landau Fermi liquid, in that the dominant low-frequency excitations are not found at long wavelengths, but rather for wave vectors in the vicinity of **Q**; these paramagnons in turn give rise to a quasiparticle lifetime which, for $T > T_c$, we expect to be given by (cf. MPT)

$$\left[\frac{\hbar}{\tau}\right] \sim T$$

rather than the usual Landau result, $(\hbar/\tau) \sim T^2$. In similar fashion, for $T \gtrsim T_c$, the contribution to the resistivity from quasiparticle-quasiparticle interaction (once Umklapp processes are taken into account) will not be $\sim T^2$, as is the case for a normal Fermi liquid, but rather $\sim T$, as we have shown above.

The precise form of the resistivity depends on the temperature dependence of ω_{SF}, and hence on (a^2/ξ^2). As we have seen NMR experiments provide direct information on $a^2/\xi^2(T)$, and show that the mean-field form, Eq. (3.14), is well obeyed above $T \gtrsim 130$ K for all three cuprate oxide superconductors thus far studied in detail. The results of the preceding section show that once one gets the mean-field form for the correlation lengths, one obtains a resistivity that varies linearly with temperature.

A quite unexpected feature of nearly antiferromagnetic cuprate oxide superconductors is the strongly-temperature-dependent static magnetic susceptibility found for $YBa_2Cu_3O_{6.63}$ (Ref. 10) and here for $La_{1.85}Sr_{0.15}CuO_4$. It is natural to attribute this behavior in part to the increasingly important role of antiferromagnetic correlations as the temperature decreases,[15,16] however, there does not appear to be a "universal" correlation between the "reduced" long-wavelength static susceptibility, $[\chi_0(T)/\chi_0(300\text{ K})]$ and the "enhanced" antiferromagnetic static susceptibility, $[\chi_Q(T)/\chi_0(T)]$, which applies to all cuprate oxide systems. This explanation for $\chi_0(T)$ will, however, remain only a plausible conjecture, until born out by microscopic model calculations, which show that the presence of low-frequency antiferromagnetic paramagnons can lead to a depressed value for $\chi_0(T)$. [In passing, we note an appealing candidate for the "dangerous" diagrams responsible for this anomaly are vertex corrections to the local-field Fermi-liquid parameters, F_0^a, which might yield $F_0^a(T)$. Calculations are underway to verify this conjecture.]

It is likewise reasonable to expect that the antiferromagnetic paramagnons will give rise to a temperature-dependent quasiparticle effective mass, and the recent work of Moriya, Takahashi, and Ueda[14] supports this idea.

For systems which display a temperature-dependent long-wavelength static magnetic susceptibility, we do not expect that a straightforward local mode coupling application of Ginzburg-Landau theory to obtain, say, $a^2/\xi^2(T)$ (cf. Ref. 14) will be successful; rather a self-consistent, "strong-coupling" version will need to be developed and applied to obtain the results found by MPT for $YBa_2Cu_3O_{6.63}$ and by us for $La_{1.85}Sr_{0.15}CuO_4$.

The results we have obtained for the antiferromagnetic correlation parameters for the three materials provide a tantalizing hint of the possible connection between antiferromagnetic behavior and the transition to superconductivity. As one of us noted,[16] on the basis of a very preliminary and approximate fit of MMP theory to the NMR experiments on $O_{6.63}$ and $La_{1.85}Sr_{0.15}CuO_4$, it is evident that the closer the system is to becoming antiferromagnetic, the lower its superconducting transition temperature. The hierarchy we have established for $^{63}R_{AF}$ and for (ξ/a) is thus inverted for transition temperatures. As may be seen in Table II, one can establish a qualitative correlation between, say, T_c and a/ξ. However, the results presented there show that there does not exist a simple proportionality, $T_c \sim \hbar\omega_{SF}(T_c)$, which the preliminary analysis[16] suggested. Given the fact that the quasiparticle lifetimes are comparatively short, and that $\hbar\omega_{SF} < kT$, it is clear that any calculation of superconductivity based on an attraction produced by the Coulomb correlations responsible for the antiferromagnetic behavior will have to go well beyond taking into account the attraction between a pair of quasiparticles produced by the exchange of a single antiferromagnetic paramagnon.

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TABLE II. Antiferromagnetic Fermi liquid parameters for three cuprate oxide superconductors.

		Superconductor	
	$YBa_2Cu_3O_7$	$YBa_2Cu_3O_{6.63}$	$La_{1.85}Sr_{0.15}CuO_4$
$(^{63}R_{AF})_{\parallel}(T_c)$	8	27	73
(ξ/a) at T_c	2.25	4.0	6.9
$\omega_{SF}(T_c)$ (K)	94	36	12
T_c (K)	90	60	38

stitute from the Department of Energy and the National Science Foundation, and by the Advanced Study Program in High Temperature Superconductivity at the Los Alamos National Laboratory. He would like to thank the Santa Fe Institute for its hospitality.

APPENDIX

The experimental results of Alloul, Ohno, and Mendels[11] for the Knight shift and relaxation rate for ^{89}Y nuclei in a number of different samples of Y-Ba-Cu-O provide valuable information of the influence of oxygen content on both $\chi_0(T)$ and the antiferromagnetic-Fermi-liquid parameter, $\Gamma(T)$. In this appendix we present an analysis of the data of Alloul, Ohno, and Mendels based on taking the hyperfine Hamiltonian for the interplanar Y atoms [compare Eq. (2.1)] to be

$$H_Y=\sum_{\langle ij\rangle,\alpha} {}^{89}I_{i,\alpha}DS_{j,\alpha} , \quad (A1)$$

where $^{89}I_{i,\alpha}$ is the αth component of the ^{89}Y nuclear spin, $S_{j,\alpha}$ is the αth component of the planar Cu spin, and $\langle ij\rangle$ is the sum over nearest neighbors; the Y Knight shift is then given by

$$^{89}K^s_{\rm iso}=\frac{8D}{^{89}\gamma_n\gamma_e\hbar^2}\chi_0 , \quad (A2)$$

while the corresponding relaxation rate (see MMP) is

$$^{89}W=\frac{3}{4}\frac{1}{\mu_B^2\hbar}\lim_{\omega\to 0}\sum_q 16D^2[1-\cos(q_x a)] \times[1-\cos(q_y a)]S(q,\omega) , \quad (A3a)$$

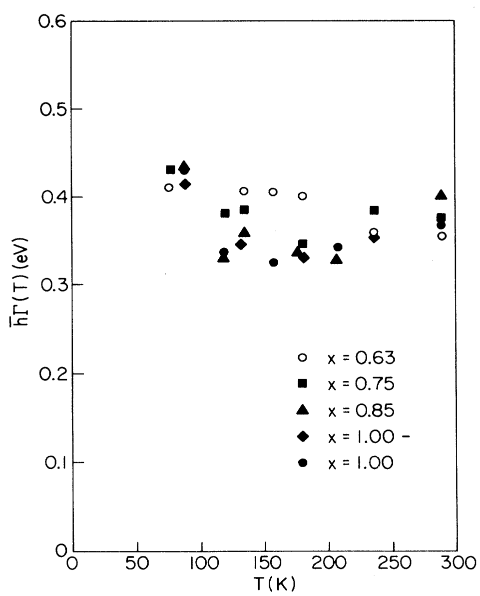

FIG. 11. A comparison of $\chi_0(T)$ for $La_{1.85}Sr_{0.15}CuO_4$ with that obtained by MPT from the data of Alloul, Ohno, and Mendels (Ref. 11) for the Y Knight shift in the Y-Ba-Cu-O family.

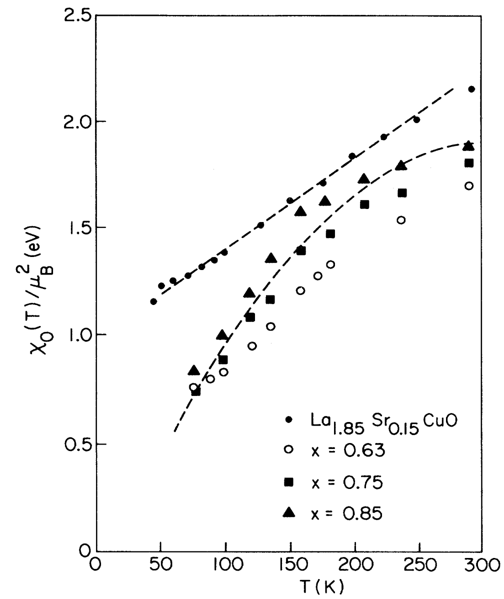

FIG. 12. Calculated values of $\Gamma(T)$ for Y-Ba-Cu-O samples obtained from the Knight shift and relaxation rate experiments on Y nuclei by Alloul, Ohno, and Mendels (Ref. 11).

which, upon carrying out the integral over q, becomes

$$^{89}W=\frac{6D^2\pi}{\mu_\beta^2\hbar}\frac{\chi_0(T)k_BT}{\hbar\Gamma(T)}\frac{[1+0.2(\beta/\pi^2)]}{(1+\beta^{1/2}/2\pi^2)} . \quad (A3b)$$

We first obtain $\chi_0(T)$ from the Y Knight shift data of Alloul, Ohno, and Mendels by assuming, as a number of authors (Walstedt *et al.*,[17] Butaud *et al.*,[18] and Imai[19]) have previously done, that the ^{89}Y chemical shift is $\sim$200 ppm, and taking the hyperfine coupling D to be that obtained by MPT,

$$D=-3.0\ \mathrm{kOe}/\mu_B . \quad (A4)$$

Our results are shown in Fig. 11, where they are compared with those for $La_{1.85}Sr_{0.15}CuO_4$ and $YBa_2Cu_3O_7$. We see that the temperature-dependent susceptibility, $\chi_0(T)$, we have derived from the Knight-shift experiments of Kitaoka *et al.*[3] falls in between the results obtained by MPT for $O_{6.63}$ and that of MMP for O_7, $\chi_0=2.62$ states/eV.

We next combine our result for $\chi_0(T)$ with the results of Alloul, Ohno, and Mendels for $^{89}W(T)$ to plot, in Fig. 12, $^{89}[T_1T\chi_0(T)]$ as a function of oxygen concentration. We use for all samples, the value, $\beta=\pi^2$ found by MPT for $O_{6.63}$, and normalize the expression [cf. Eq. (A3b)] so that we obtain agreement with the results of MPT for $\Gamma(T)$ for $O_{6.63}$. As may be seen there, for all the Alloul, Ohno, and Mendels samples with oxygen content equal or greater than $O_{6.63}$, we find

$$\Gamma(T)\cong 0.4\pm 0.04\ \mathrm{eV} , \quad (A5)$$

a value which is not far from that found here for $La_{1.85}Sr_{0.15}CuO_4$.

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