PHYSICAL REVIEW B VOLUME 43, NUMBER 4 1 FEBRUARY 1991

Electrodynamics of moving superconductors and superconductors under the influence of external forces

H. Peng and D. G. Torr
Center for Space Plasma and Aeronomic Research and Department of Physics, The University of Alabama in Huntsville, Huntsville, Alabama 35899

E. K. Hu
Center for Space Plasma and Aeronomic Research and Department of Physics, The University of Alabama in Huntsville, Huntsville, Alabama 35899
and Physics Department, Zhongshan University, Guangzhou, People's Republic of China

B. Peng
Department of Mathematics, University of Arizona, Tucson, Arizona 85721
(Received 20 March 1990; revised manuscript received 13 August 1990)

We carry out a systematic and phenomenological approach to investigate the electrodynamics of moving superconductors and of superconductors under the influence of external forces. In applying the approach to various situations, we predict the presence of detectable electromagnetic fields in the interior. We argue that the detection, particularly of the induced electric fields in a variety of situations, will not only test the approach that we develop in the paper but also validate the principle for a gravitational-wave antenna.

I. INTRODUCTION

A theoretical derivation of the London moment[1] by Becker, Sauter, and Haller[2] was an early attempt at formulating the electrodynamics of a moving superconductor. Although the results of this effort have proved to be of great experimental value in that the effect could be applied to precise determinations of the ratio h/m_e,[3] Cooper-pair mass,[4] and absolute rotation, and to the readout system of the Stanford gyroscope experiment[5] which is designed to detect the effect of vector gravitomagnetic fields,[6] the theory remains incomplete and is applicable under certain limited conditions. Recently, adopting London's method, we have studied the electromagnetic properties of vibrating superconducting cylinders driven by a gravitational wave[7] and by a mechanical exciter. What is needed now is a more general and systematic treatment, based on the same phenomenological approach, but which will provide a comprehensive description of the electrodynamics of moving superconductors and of superconductors under the influence of external fields. This new approach will also provide a basis for further theoretical development and practical application.

Based on the assumption that, in the interior of a superconductor, the current density is directly proportional to the velocity of the superconductor, Anandan[8] proposed the covariant equation

$$-[\hbar\partial_\mu\phi+(2e/c)A_\mu]=(2/c)\zeta U_\mu \ , \qquad (1)$$

where U_μ is the four-velocity of the superconductor, the function ζ is determined by the requirement that the equation must be in agreement with the Josephson equation. He demonstrated that the Josephson equation, the London moment, the magnetic-flux quantization for a closed curve in the interior, and the expression of the electric field in the interior can be obtained from Eq. (1). However, there was no physical explanation for the assumption. As will be shown, our results differ from this assumption.

In this paper we begin with the Ginzburg-Landau theory (GL) (Ref. 9) and, then, derive a set of covariant equations which can be used to describe the electromagnetic properties of both a moving superconductor and a superconductor under the influence of external forces. We shall demonstrate that when subjected to certain kinds of motion some new affects arise in a superconductor which we predict should be experimentally detectable.

II. COVARIANT EQUATIONS FOR MOVING SUPERCONDUCTOR

In the GL theory, the current flowing in superconductor characterized by order parameter $\psi(\mathbf{r})$ in the presence of a magnetic field is given by

$$\mathbf{j}=\frac{e^*\hbar}{2im^*}(\psi^*\nabla\psi-\psi\nabla\psi^*)=\frac{e^{*2}}{m^*}\psi^*\psi\,\mathbf{A} \ , \qquad (2)$$

where m^* and e^* are, respectively, the mass and charge of a Cooper pair $\psi=|\psi|e^{i\theta}$. In this paper we only consider the very simplest situations in which the perturbing fields and currents are so weak that

$$|\psi|^2=n=\text{const} \ , \tag{3}$$

where n, is the number density of Cooper pairs, i.e., n does not vary spatially and the nonlinear effects in fields are not strong enough to change n. For these situations, Eq. (2) reduces to the London equation

$$\mathbf{j}=-\frac{ne\hbar}{m_e}\nabla\theta-\frac{2ne^2}{m_e}\mathbf{A} \ , \tag{4}$$

or

$$\frac{\partial \mathbf{v}_e}{\partial t}=\frac{e}{m_e}\mathbf{E} \ , \tag{5}$$

$$\nabla\times\mathbf{v}_e=-\frac{e}{m_e}\mathbf{B} \ , \tag{6}$$

where $\mathbf{v}_e$, $e(<0)$, and m_e are, respectively, the velocity, charge, and mass of a superelectron.

The London equations (5) and (6) can be combined into a single covariant expression[1]

$$\frac{\partial p_\mu}{\partial x^\nu}-\frac{\partial p_\nu}{\partial x^\mu}=0 \quad (\mu,\nu,=1,2,3,4) \tag{7}$$

with $x_4=it$ and

$$p_\mu=mu_\mu+eA_\mu \ , \tag{8}$$

where u_μ is the four-velocity of superelectrons, $\mathbf{u}=\mathbf{v}_e/(1-\beta_e^2)^{1/2}$, and $\beta_e^2=(\mathbf{v}_e/c)^2$.

When there is an external nonelectromagnetic force f_e acting on superelectrons, a natural procedure is to replace Eq. (8) by

$$p_\mu=mu_\mu+eA_\mu-\int K_\mu\,d\tau \ , \tag{9}$$

where $d\tau=dt\sqrt{(1-\beta_e^2)}$, K_μ is the Minkowski force, and

$$\mathbf{f}_e\equiv K\sqrt{(1-B_e^2)} \ .$$

In deriving the London moment, London[1] assumed that the London equations, Eqs. (5) and (6), apply for a moving superconductor as well as for a static one. The London equations, however, are the equations of motion for *superelectrons* in a superconductor and, thus, can only describe the electrodynamics of superconductors at *rest*, i.e., where there is no motion of the ions. The London equations are not adequate for investigating the electrodynamics of *moving* superconductors. Thus London introduced the concept that the net current should be the sum of the supercurrent and the current due to the motion of the ions, and gave the equations of motion for *ions* in a rotating superconducting sphere. For an arbitrarily moving superconductor following London's method, we have to introduce the equation of motion of the ions. At this juncture, however, it is not necessary to introduce any specific equation of motion, since it suffices to maintain generality and to simply work with the four-velocity of ions, U_μ, where $\mathbf{U}=\mathbf{v}_i/\sqrt{(1-\beta_i^2)}$, $\beta_1^2=(\mathbf{v}_i/c)^2$, and $\mathbf{v}_i$ is the velocity of ions.

The net four-current density is

$$J_\mu=J_{e\mu}+J_{i\mu}=2ne(u_\mu-U_\mu) \ . \tag{10}$$

Equations (7), (9), and (10) with the Maxwell equations

$$\frac{\partial F_{\mu\nu}}{\partial x_\nu}=\mu_0 J_\mu \ , \tag{11}$$

$$F^{\mu\nu,\lambda}+F^{\nu\lambda,\mu}+F^{\lambda\mu,\nu}=0 \ , \tag{12}$$

where

$$F_{\mu\nu}=\frac{\partial A_\nu}{\partial x_\mu}-\frac{\partial A_\mu}{\partial x_\nu} \ ,$$

form a complete set of covariant equations which describe the electrodynamics of an arbitrarily moving superconductor and of a superconductor under the influence of nonelectromagnetic external forces.

In the limit of low velocity, $\beta\simeq 0$, Eqs. (7) and (9) yield

$$\frac{\partial \mathbf{v}_e}{\partial t}=\frac{e}{m_e}E+\frac{1}{m_e}\mathbf{f}_e \ , \tag{13}$$

$$\nabla\times\mathbf{v}_e=-\frac{e}{m_e}B+\frac{1}{m_e}\int\nabla\times\mathbf{f}_e\,dt \ , \tag{14}$$

Equations (7), (10), and (11) yield

$$\nabla^2 J=\frac{1}{\lambda^2}\left[J+\frac{m_e}{\mu_0 e}\left[\nabla\times\nabla\times\mathbf{v}_i+\frac{\partial^2\mathbf{v}_i}{\partial t^2}\right]-\frac{1}{\mu_0 e}\left[\int\nabla\times\nabla\times\mathbf{f}_e\,dt+\frac{\partial\mathbf{f}_e}{\partial t}\right]\right] \ , \tag{15}$$

and Eqs. (7) and (9)–(12) yield

$$\nabla^2 E=\frac{1}{\lambda^2}\left[E-\frac{m_e}{e}\frac{\partial\mathbf{v}_i}{\partial t}+\frac{1}{e}\mathbf{f}_e\right] \ , \tag{16}$$

$$\nabla^2 B=\frac{1}{\lambda^2}\left[B+\frac{m_e}{e}\nabla\times\mathbf{v}_i-\frac{1}{e}\int\nabla\times\mathbf{f}_e dt\right] \ , \tag{17}$$

where

$$\lambda=\left[\frac{m_e}{2\mu_0 ne^2}\right]^{1/2} \ ,$$

is the London penetration depth, the terms involving the second derivatives of J, E, and B with respect to time have been ignored, since for $\omega\ll 10^{15}$ these terms are negligibly small, where ω is the frequency of J, E, and B. Equations (15)–(17) show that the motion of ions and the forces acting on the superelectrons determine the electromagnetic currents and fields in the interior of a moving superconductor. The forces acting on the ions are also important. These will appear in the explicit expression for v_i which will be case dependent.

III. EXAMPLES

Solving Eqs. (15)–(17) with the equations for both ions and external forces acting on superelectrons, one can describe the electrodynamics of a moving superconductor or a superconductor under the influence of arbitrary external forces. The solutions of Eqs. (15)–(17) have the following form:

$$J = J_{\text{screen}} + J_{\text{interior}} , \tag{18}$$

$$E = E_{\text{screen}} + E_{\text{interior}} , \tag{19}$$

$$B = B_{\text{screen}} + B_{\text{interior}} . \tag{20}$$

In this paper we only consider some simple cases

$$\nabla^2 J_{\text{int}} = \nabla^2 E_{\text{int}} = \nabla^2 B_{\text{int}} = \nabla \times \mathbf{f}_e = \mathbf{0} , \tag{21}$$

and we restrict our interest to the electromagnetic properties in the interior.

The solutions for the interior parts of the fields are, respectively,

$$J_{\text{int}} = \frac{1}{\mu_0 e}\frac{\partial \mathbf{f}_e}{\partial t} - \frac{m_e}{\mu_0 e}\frac{\partial^2 \mathbf{v}_i}{\partial t^2} - \frac{m_e}{\mu_0 e}\nabla \times \nabla \times \mathbf{v}_i , \tag{22}$$

$$E_{\text{int}} = \frac{m_e}{e}\frac{\partial \mathbf{v}_i}{\partial t} - \frac{\mathbf{f}_e}{e} , \tag{23}$$

$$B_{\text{int}} = -\frac{m_e}{e}\nabla \times \mathbf{v}_i . \tag{24}$$

The self-consistency of Eqs. (22)–(24) can be checked by substituting them into one of Maxwell equations

$$\nabla \times B_{\text{int}} = \mu_0 J_{\text{int}} + \partial E_{\text{int}} / \partial t .$$

Therefore, for the simple cases, we do not need to solve Eqs. (15)–(17) with equations of motion for ions and of forces acting on superelectrons. The electromagnetic properties in the interior for these cases can be readily found by substituting $\mathbf{v}_i$ and $\mathbf{f}_e$ into Eqs. (22)–(24).

Comparing Eqs. (22)–(24) with the results of Ref. 8 we find that Eqs. (23) and (24) are similar to the Anandan results. Equation (22), however, indicates that, in the interior, the current is not proportional to the four-velocity of a moving superconductor.

We now consider some simple cases.

(A) A uniformly moving superconductor: $\mathbf{f}_e = 0$ and $\mathbf{v}_i = \text{const}$. Equations (22)–(24) give us

$$J_{\text{int}} = E_{\text{int}} = B_{\text{int}} = \mathbf{0} ,$$

i.e., there are no currents, electric and magnetic fields in the interior. This fact implies that the electrodynamics in the interior of an uniformly moving superconductor is the same as that of a superconductor at rest, i.e., detection of the electromagnetic fields or currents in a superconductor cannot be used to distinguish between inertial reference frames. Newton's first law is valid for superconductivity.

(B) A superconductor bar uniformly accelerating along the axis: $\mathbf{f}_e = 0$, $\nabla \times \mathbf{v}_i = 0$,

$$\frac{\partial \mathbf{v}_i}{\partial t} = \text{const} \equiv \mathbf{a} \quad \text{and} \quad \frac{\partial^2 \mathbf{v}_i}{\partial t^2} = 0 .$$

Then we have, from Eqs. (22)–(24),

$$J_{\text{int}} = 0, \quad B_{\text{int}} = 0, \quad E_{\text{int}} = \frac{m_e}{e}\mathbf{a} = -\frac{m_e}{|e|}\mathbf{a} . \tag{25}$$

There will be an electric field in the interior which will drive superelectrons to move with ions. The voltage between two ends is

$$V = \frac{m_e L}{|e|} a , \tag{26}$$

where L is the length of the bar. For $L = 1$ m, $a = 10$ m/s^2, we have $V \simeq 5 \times 10^{-11}$ V, which is measurable.

(C) The London moment, e.g., a uniformly rotating superconductor sphere or cylinder: $f_e = 0$,

$$\mathbf{v}_i = \boldsymbol{\omega} \times \mathbf{r}, \quad \nabla \times \mathbf{v}_i = 2\omega, \quad \nabla \times \nabla \times \mathbf{v}_i = 0 , \tag{27}$$

$$\frac{\partial \mathbf{v}_i}{\partial t} = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) , \tag{28}$$

and

$$\frac{\partial^2 \mathbf{v}_i}{\partial t^2} = \boldsymbol{\omega} \times [\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})] . \tag{29}$$

Equations (22)–(24) yield

$$J_{\text{int}} = \frac{m_e}{\mu_0 |e|}\boldsymbol{\omega} \times [\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})] , \tag{30}$$

$$E_{\text{int}} = -\frac{m_e}{|e|}\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) , \tag{31}$$

$$B_{\text{int}} = \frac{2m_e}{|e|}\boldsymbol{\omega} . \tag{32}$$

Equations (32) and (31) show that there exist not only the London moment but an induced radial electric field. London only considered the curl of the velocity of ions, Eq. (27), so he only obtained the London moment. The first and second time derivatives of the velocity of ions, Eqs. (28) and (29), give rise to an electric field and current in the interior, respectively. The electric field is necessary to balance the centrifugal force on superelectrons. On the axis the E field vanishes. At the equator the E field reaches the maximum value. The potential difference between the equator and axis is measurable. For $\omega = 10^3$, $r = 0.1$ m, we have

$$E \simeq 5 \times 10^{-7} \text{ V/m} .$$

Actually a radial electric potential gradient has been observed[10] to exist across a rotating conductor.

Equation (31) agrees with the result of Rystephanick.[11] He pointed out, based on a proposed principle that there must be zero net force on a moving superelectron, that the London moment results from the Coriolis force on a charged particle in a rotating frame and that there exists an induced electric field directed outward from the axis of rotation. He also argued that the origin of the electric field is the separation of charges resulting from the rotation, leading to a build up of negative charge on the surface of the superconductor.

Here we give a different argument, namely, that the E field is induced by the change of the vector potential A as shown below. It is convenient to write the current in the form of

$$J = -\frac{ne^2}{m_e}A - ne\mathbf{v}_i . \tag{33}$$

Then substituting Eq. (33) into one of Maxwell's equations we obtain

$$\nabla^2 A = \frac{\mu_0 n e^2}{m_e}\left[A + \frac{m_e}{e}\mathbf{v}_i\right] , \tag{34}$$

where $\nabla \cdot A = 0$ has been used. Solving Eq. (34) gives

$$A_{\text{int}} = -(m_e/e)\mathbf{v}_i \tag{35}$$

and

$$E_{\text{int}} = -\frac{\partial A}{\partial t} = \frac{m_e}{e}\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) ,$$

which is just Eq. (31).

Equation (30) shows there is also a current in the interior but negligibly small, for $\omega = 10^3$ and $r = 0.1$ m, $J_{\text{int}} \simeq 10^{-15}$ A/m^2.

(D) A free vibrating superconductor bar: $\mathbf{f}_e = \mathbf{0}$, $\nabla \times \mathbf{v}_i = 0$,

$$\frac{\partial \mathbf{v}_i}{\partial t} \neq \mathbf{0} \quad \text{and} \quad \frac{\partial^2 \mathbf{v}_i}{\partial t^2} \neq \mathbf{0} .$$

Equations (22)–(24) yield

$$J_{\text{int}} = \frac{m_e}{\mu_0 e}\frac{\partial^2 v_i}{\partial t^2} , \tag{36}$$

$$E_{\text{int}} = \frac{m_e}{e}\frac{\partial v_i}{\partial t}, \quad B_{\text{int}} = \mathbf{0} . \tag{37}$$

Now we need the equation of motion for the ions. In this case we consider a superconductor bar which is mechanically excited and then disconnected from the driving force. The superconductor bar will then vibrate freely. From the theory of elasticity we obtain the equation for the vibration of a mass element in a solid cylinder. We assume that this equation can also describe the vibrations of bulk ions in a superconductor bar. When excitation of the superconductor bar commences, the ions will vibrate first without the superelectrons following suit. Then the ion motion creates an electric field which will cause superelectrons to vibrate. The different vibrations of ions and superelectrons create a current which in turn induces an electric field which reacts on both ions and superelectrons.

For simplicity we will restrict our attention to the case of a thin superconductor bar in which we need only consider longitudinal vibrations propagating along the bar axis, i.e., the z axis. The displacement of the bulk ions x_i along the z direction satisfies the following equation:

$$\frac{\partial^2 x_i}{\partial t^2} + \frac{1}{\tau_0}\frac{\partial x_i}{\partial t} - v_s^2\frac{\partial^2 x_i}{\partial z^2} = -\frac{eE}{m_i} , \tag{38}$$

where v_s is the speed of sound in the superconductor bar, $\mathbf{E}$ is the induced electric field. Solving Eqs. (37) and (38) one obtains for the fundamental frequency ω,

$$E_{\text{int}} \simeq -\frac{m_e}{e}X_0\omega^2 \sin\left[\frac{\pi}{L}z\right]\cos(\omega t)e^{-t/(2\tau_0')} , \tag{39}$$

where X_0 is the amplitude, $\tau_0' = (1 + m_e/m_i)\tau_0$. The induced electric field in the interior vibrates as a standing wave. The voltage between the center and one end is

$$V \simeq (X_0 \omega v_s m_e/e)\cos(\omega t)\exp[-t/(2\tau_0)] . \tag{40}$$

If we choose a high $Q(=\omega\tau_0)$ bar, the vibration and induced E field will decay very slowly with time. For a niobium bar with $L = 0.4$ m, $Q \simeq 10^8$, and $X_0 \simeq 10^{-5}$ m, the voltage is

$$V \simeq 10^{-9}\ \text{V} ,$$

which is measurable. This effect, to our knowledge, has not been tested experimentally.

Note that for this case the E_{int} given by Eq. (39) does not satisfy the conditions given by Eq. (21). It can be shown, however, that Eq. (39) is approximately correct by substituting it into Eq. (16) and ignoring small terms.

Next we consider two different cases in which external forces exist, e.g., gravitational forces which will penetrate a superconductor.[7]

(E) A superconductor at rest in a Newtonian gravitational field g: $f_e = m_e g$, $\partial f_e/\partial t = 0$, and $v_i = \mathbf{0}$.

Equations (22)–(24) become

$$J_{\text{int}} = 0, \quad B_{\text{int}} = 0, \quad \mathbf{E}_{\text{int}} = -(m_e/e)\mathbf{g} . \tag{41}$$

There is an electric field in the interior to prevent a drift of superelectrons due to the gravitational field g. The same result for conductor has been obtained and verified experimentally.[12] The comparison between Eqs. (25) and (41) shows that there is no distribution between a superconductor at rest in a Newtonian gravitational field g and one uniformly accelerating with $\mathbf{a} = -\mathbf{g}$, i.e., the weak equivalence principle is valid for superconductivity.

(F) A vibrating superconductor bar under the influence of gravitational waves (GW): $f_e = m_e a_{\text{GW}}$, $\partial f_e/\partial t \neq \mathbf{0}$, $\partial v_i/\partial t \neq \mathbf{0}$, $\partial^2 v_i/\partial t^2 \neq \mathbf{0}$, and $\nabla \times v_i = \mathbf{0}$, a_{GW} is the gravitational driving acceleration that results from projecting the tidal gravitational force due to a GW onto the superconductor bar. Actually the superconductor bar is a GW antenna. For details, see Ref. 7. It is well known that GW's will penetrate a superconductor and exert a force on the ions and superelectrons not only on the surface but in the interior also.

Equations (22)–(24) become

$$J_{\text{int}} = \frac{1}{\mu_0 e}\left[\frac{\partial \mathbf{f}_e}{\partial t} - m_e\frac{\partial^2 \mathbf{v}_i}{\partial t^2}\right] , \tag{42}$$

$$E_{\text{int}} = \frac{m_e}{e}\left[\frac{\partial \mathbf{v}_i}{\partial t} - \mathbf{a}_{\text{GW}}\right], \quad B_{\text{int}} = 0 . \tag{43}$$

In a superconductor antenna, ions vibrate as forced harmonic oscillators and satisfy the following equation:[7]

$$\frac{d^2 x_i}{dt^2} + \frac{1}{\tau_0}\frac{dx_i}{dt} + \omega_0^2 x_i = \mathbf{a}_{\text{GW}} - \frac{e}{m_i}E . \tag{44}$$

Solving Eqs. (43) and (44) we obtain

$$E_{\text{int}} = \frac{m_e}{e}\frac{\omega_0^2 - i\omega/\tau_0}{(1+d)\omega^2 - \omega_0^2 + i\omega/\tau_0}\mathbf{a}_{\text{GW}} , \tag{45}$$

which is exactly the result of Ref. 7. Based on this effect a new GW antenna has been proposed.[7]

The effects (D) and (F) are, in principle, similar in the sense that there is an induced electric field in the interior of a vibrating superconductor. Thus detecting the induced electric field of case (D) will validate case (F), i.e., the principle of a new GW antenna.

In summary, the electrodynamics of an arbitrarily moving superconductor is determined by the motion of not only superelectrons but also the ions. The induced electric fields in a uniformly accelerating superconductor rod, in a rotating superconducting sphere, and in a freely vibrating superconductor bar are detectable. We argue that it is important to detect these predicted induced electric fields, because it will test not only the generalized London equations derived in this paper but will in addition further validate a principle for a new GW antenna.

Note added in proof. It has been verified experimentally that there is an induced electric field in the interior of a vibrating conductor bar.[13] This effect is very similar to case (D).

ACKNOWLEDGMENTS

This work was supported by Alabama EPSCor program and incentive funding from the UAH Physics Department.

[1]F. London, *Superfluids* (Dover, New York, 1960).
[2]R. Becker, F. Sauter, and C. Haller, Z. Phys. **85**, 772 (1933).
[3]S. B. Felch, J. Tate, B. Cabrera, and J. T. Anderson, Phys. Rev. B **31**, 7006 (1985).
[4]B. Cabrera and M. E. Peskin, Phys. Rev. B **39**, 6425 (1989).
[5]The papers in *Near Zero: New Frontiers of Physics,* edited by J. D. Fairbank, B. S. Deaver, Jr., C. W. F. Everitt, and P. F. Michelson (Freeman, New York, 1988).
[6]K. S. Thorne, in *Quantum Optics, Experimental Gravitation, and Measurement Theory,* edited by P. Meystre and M. O. Scully (Plenum, New York, 1983); H. Peng, Gen. Relativ. Gravit. **15**, 725 (1983).
[7]H. Peng, J. Appl. Phys. **67**(6), 3204 (1990); H. Peng, Gen. Relativ. Gravit. **22**, 33 (1990); H. Peng and B. Peng, *ibid.* **22**, 45 (1990); H. Peng and D. G. Torr, *ibid.* **22**, 53 (1990).
[8]J. Anandan, Phys. Lett. **105A**, 280 (1984); J. Anandan, in *Quantum Concepts in Space and Time,* edited by R. Penrose and C. J. Isham (Oxford Science Publications, Oxford, 1986).
[9]V. L. Ginzburg and L. D. Landau, Z. Eksp. Teor. Fiz. **20**, 1064 (1950).
[10]J. W. Beams, Phys. Rev. Lett. **21**, 1093 (1968).
[11]R. G. Rystephanick, Amer. J. Phys. **44**, 647 (1976); B. Cabrera, H. Gutfreund, and W. A. Little, Phys. Rev. B **25**, 6644 (1982).
[12]L. I. Schiff and M. V. Barnhill, Phys. Rev. **151**, 1067 (1966).
[13]T. Davis and G. Opat, Classical Quantum Gravity **5**, 1011 (1988).