Magnetic-field-induced resonant tunneling across a thick square barrier

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The spatial part of the wave function is calculated exactly for tunneling in a magnetic field, confined within a thick square barrier. Due to the presence of the field perpendicular to the current flow, transmission is heavily reduced and a threshold is found, based on very general kinematical arguments. However, magnetic-field-induced scattering states could be present, which have a high probability density inside the barrier giving rise to resonant tunneling. They are responsible for a strong dependence of the transmission on the energy and the angle of incidence of the incoming beam. These resonances are analyzed as a function of an applied external bias and the possibility of their appearance in the I-V characteristic is discussed. They certainly affect any typical time for tunneling, as shown in the limiting case of a totally reflecting magnetic barrier.

I. INTRODUCTION

Model calculations on single-electron tunneling across a junction in the presence of a magnetic field were first discussed in the context of the so-called "Larmor clock," to give an operative definition of the tunneling traversal time.[1-3] There have been great disputes on the subject, starting from the evaluation of the expected value of the spin under the influence of a magnetic field, restricted to the barrier. Of course the magnetic field also affects the spatial part of the wave function mainly when the barrier is thick or the field is strong.

On the other hand, the production of high-quality heterostructures allows for reliable experimental tests of the theory.[4,5] The applications of these devices are growing wider and wider, together with double-barrier heterostructures or superlattices, in which resonant tunneling across the gate can give rise to sharp nonlinearities in the I-V characteristics.[6-8]

The effect of the magnetic field on these structures is currently under study: most of the experimental results refer to thin insulating barriers.

We are concerned with magnetic fields parallel to the barrier (in the z direction) and perpendicular to the current flow (y direction).

Because the magnetic field extends everywhere, the single-particle stationary solutions of the Schrödinger equation vanish asymptotically on both sides of the barrier. These correspond to Landau level wave functions centered at some point on the y axis which is fixed by $k_{\parallel}=k_x$ in the Landau gauge. Deep in the bulk each eigenvalue is degenerate because it does not depend on k_x. Getting closer to the barrier the breaking of the translational invariance removes the degeneracy and each level gives rise to bands of states, which are still localized in space and labeled by k_x. Transport in the presence of an external bias needs scattering against impurities which allows for the hopping between these states.

In discussing resonant tunneling across double barrier structures, Helm *et al.*[9] relate features in the I-V characteristic to states within the double well (*electric* subbands), which are sensitive to an increasing magnetic field. These features eventually disappear and are superseded by *magnetic* ones related to Landau levels localized at the barrier with an equal probability amplitude on both sides of it. Often accumulation layers can be formed on the side of the incoming current (lhs) at the interface,[10,4] what enhances the structures in the voltage dependence of the current. In fact, the layer traps electrons at the interface, so that a two-dimensional gas is formed with subbands at discrete energy and little dispersion. Also in this case the conduction is due to an hopping from one of the states in the accumulation layer to a deformed Landau level corresponding to classical skipping orbits.

In this work we are concerned with magnetotunneling across much thicker square barriers. The effects quoted above are a consequence of the presence of the magnetic field outside the barrier. Our aim is to show that in our case the magnetic field inside the barrier itself can induce extra features in addition to the ones just mentioned. They mostly resemble to proper resonant states centered inside the insulating layer.

Tunneling currents in thick barrier diodes have been measured recently in good agreement with WKB calculations.[11] This approximation allows for a direct estimate of the traversal time of tunneling, according to the definition given by Buttiker and Landauer.[2]

When the barrier is thick, the broadening of the Landau levels on both sides of the barrier is so large that they can be viewed as a continuum of incoming and outgoing states. However they are still localized states, so that current continuity can only be assured by means of some scattering process.

We have studied the extreme case of a magnetic field localized strictly within the barrier, so that the Landau levels outside of it are absent. This means that asymptotic states are free waves in our picture and use can be made of the usual definition of the transmission coefficient, within the conventional transfer Hamiltonian formalism for the tunneling current.[12]

Our full calculation shows that, if the barrier is thick enough, resonances appear centered within the barrier and the tunneling acquires a strong dependence on the energy and on the angle of incidence of the incoming flux. We also give arguments to show how the resonances influence all kinds of times that could be defined in the scattering process.

To characterize their effect on the incoming flux further, we discuss also the limiting case of an infinitely thick magnetic barrier. The discussion is essentially illustrative: because the beam is totally reflected, no ambiguity can arise in defining a peculiar time associated with the scattering. We show that the phase delay time and Smith's dwell time[13] coincide and they are found to depend in a characteristic way on the resonances. In particular, when the energy of the incoming flux matches one of the values for a resonance, they become longer or shorter than the time associated with the classical trajectory, depending on the parity of the Landau level from which the resonance originates.

Landau gauge is particularly suitable to describe the magnetic field: the vector potential is a constant at the right of the barrier, shifting the k vector perpendicular to the barrier. It is interesting to note that for any given energy there is a corresponding angle of incidence which allows for a description of the scattering as if it were one-dimensional with an effective $k_{\parallel}$ dependent scattering potential.

In the next section we report the details of the calculation and show that the kinematics of the scattering imply the occurrence of a voltage threshold for the conductance.

In Sec. III we describe the resonances induced by the magnetic field and classify them according to their symmetry, having care for a realistic choice of the values of the parameters which refer to junctions like GaAs/$Al_xGa_{1-x}As$/GaAs. Because our potential has an inversion center, even and odd parities of the scattering amplitudes can be decoupled and resonances can be classified according to their parity.[14] We also give qualitative arguments inferring what happens to the resonances in case the magnetic field is not strictly confined to the barrier. In presence of a voltage bias we expect that the peaks of the transmission are still present in the most favorable condition, although they do no longer saturate to 1.

In Sec. IV the semi-infinite barrier case is analyzed together with the change of the reflection time when the energy of the incoming beam is close to that of a resonance.

Our results are collected and discussed in the last section on the basis of the calculation of the I-V characteristic that we are currently undertaking.

II. THE CURRENT DENSITY AND SCATTERING KINEMATICS

In this section we derive the current density of a beam of charged particles of spin-$\frac{1}{2}$ impinging on a magnetic barrier. We call magnetic barrier a confined magnetic field between the planes $y=0$ and $y=l$, and added to it an external potential that is constant for $y>l$ and $y<l$.

The results presented here are well known and can be found in textbooks. However we collect them here, both to fix notations and to stress the three-dimensional nature of the problem studied. Because the magnetic field is confined and the barrier potential has finite width, it is straightforward to define the transmission and reflection coefficients for the particle beam, which is asymptotically plane-wave-like.

We consider an uniform magnetic field directed along the z axis. In the Landau gauge the vector potential has an x component only, which is given by

$$\begin{aligned} &A_x=0, \quad y<0 \\ &A_x=-yB, \quad 0\le y\le l \\ &A_x=-lB, \quad y>l \end{aligned} \tag{1}$$

where B is the intensity of magnetic field. The Hamiltonian of a particle of charge $-e$ (with $e>0$) has the form

$$\begin{aligned} H&=\frac{1}{2m^*}(p_x^2+p_y^2+p_z^2), \quad y<0 \\ H&=\frac{1}{2m^*}(p_x-yBe/c)^2+\frac{1}{2m^*}(p_y^2+p_z^2) \\ &\quad +U(y)+g^*\mu_B S_z B, \quad 0\le y\le l \\ H&=\frac{1}{2m^*}(p_x-lBe/c)^2+\frac{1}{2m^*}(p_y^2+p_z^2)+W, \quad y>l \end{aligned}$$

where m^* is a suitable effective mass of the electron, g^* is the effective g factor, μ_B is the Bohr magneton, and $S_z=\pm\frac{1}{2}$. A constant energy shift W has been allowed for at the rhs of the barrier, to mimic a difference in the chemical potentials of two metals, when thinking of a real heterojunction with an applied voltage. The choice of the Landau gauge permits separation of the motion along the three axis. The eigenfunction of H belonging to the eigenvalue E of the continuous part of the spectrum has the form

$$\psi=\begin{bmatrix}\chi_+(y)\\ \chi_-(y)\end{bmatrix}\exp i(k_x x+k_z z)\ . \tag{2}$$

The x and z components of the momentum are constants of motion. On the left and right sides of the barrier we have

$$\begin{aligned} \begin{bmatrix}\chi_+(y)\\ \chi_-(y)\end{bmatrix}&=\begin{bmatrix}\alpha\\ \beta\end{bmatrix}\exp(ik_y y)+\begin{bmatrix}\alpha A_+\\ \beta A_-\end{bmatrix}\exp(-ik_y y), \quad y<0 \\ \begin{bmatrix}\chi_+(y)\\ \chi_-(y)\end{bmatrix}&=\begin{bmatrix}\alpha D_+\\ \beta D_-\end{bmatrix}\exp(ik_y' y), \quad y>l \end{aligned} \tag{3}$$

where α and β fix the spin polarization of the incoming particles. The wave vector k_y' satisfies the equation

$$\begin{aligned} E&=\frac{\hbar^2}{2m^*}(k_x^2+k_y^2+k_z^2) \\ &=\frac{1}{2m^*}\left[\left[\hbar k_x-\frac{eBl}{c}\right]^2+\hbar^2(k_y'^2+k_z^2)\right]+W\ . \end{aligned} \tag{4}$$

This equation has consequences on the kinematics, as we discuss later in this section. The constants A and D are fixed by the specific behavior of $U(y)$ in the inside of the barrier. The coupling of the spin to the magnetic field rises and lowers the barrier by the value $g^*\mu_B B/2$ and we label by $\pm$ the coefficients depending on the spin polarization.

The expression of current density $\mathbf{J}$ in a magnetic field is[15]

$$\mathbf{J}=\frac{1}{m^*}\mathcal{R}\left[\psi^*\left[\frac{\hbar}{i}\nabla+\frac{e}{c}\mathbf{A}\right]\psi\right]+\frac{\hbar g^*}{2m^*}\frac{m}{m^*}\nabla\times(\psi^*\mathbf{S}\psi) ,$$

where $\mathbf{S}=\sigma/2$ and $\sigma_x,\sigma_y,\sigma_z$ are the three Pauli's matrices. If the particle moves in the $x-y$ plane ($k_z=0$) we obtain for $y<0$

$$\begin{aligned}J_x=&\frac{\hbar k_x}{m^*}\{|\alpha|^2(1+|A_+|^2)+|\beta|^2(1+|A_-|^2)\\&+2\mathcal{R}[(|\alpha|^2A_+^*+|\beta|^2A_-^*)\exp(2ik_yy)]\}\\&-\frac{\hbar k_y}{m^*}\frac{g^*}{2}\frac{m^*}{m}2\mathcal{I}[(|\alpha|^2A_+^*-|\beta|^2A_-^*)\exp(2ik_yy)] ,\\J_y=&\frac{\hbar k_y}{m^*}[|\alpha|^2(1-|A_+|^2)+|\beta|^2(1-|A_-|^2)] ,\\J_z=&\frac{\hbar k_y}{m^*}\frac{g^*}{2}\frac{m^*}{m}2\mathcal{I}[(\alpha^*\beta A_+^*+\beta^*\alpha A_-^*)\exp(2ik_yy)] .\end{aligned}$$

On the left-hand side of the barrier, the x and z components of $\mathbf{J}$ depend on the y coordinate, due to the interference between the incoming and the reflected wave. On the other hand, at the right-hand side of the barrier ($y>l$) we have to distinguish between two cases. If k_y' is real and the wave propagates, then is $\nabla\times(\psi^*\mathbf{S}\psi)=\mathbf{0}$ so that

$$\begin{aligned}J_x&=\frac{\hbar k_x-eBl/c}{m^*}(|\alpha|^2|D_+|^2+|\beta|^2|D_-|^2) ,\\J_y&=\frac{\hbar k_y'}{m^*}(|\alpha|^2|D_+|^2+|\beta|^2|D_-|^2) ,\\J_z&=0 .\end{aligned}$$

If $k_y'(=i\xi)$ is purely imaginary the wave function decays to the right and

$$\begin{aligned}J_x=&\frac{\hbar k_x-eBl/c}{m^*}(|\alpha|^2|D_+|^2+|\beta|^2|D_-|^2)\exp(-2\xi y)\\&-\frac{\hbar\xi}{m^*}\frac{g^*}{m}\frac{m^*}{m}(|\alpha|^2|D_+|^2-|\beta|^2|D_-|^2)\exp(-2\xi y) ,\\J_y=&0 ,\\J_z=&-\frac{\hbar\xi}{m^*}\frac{g^*}{2}\frac{m^*}{m}2\mathcal{R}(\alpha^*\beta D_-D_+^*)\exp(-2\xi y) .\end{aligned}$$

In this case $\mathbf{J}$ is directed perpendicularly to the y axis and J_x and J_z have an exponential decay. If we take into account a wave packet that is totally reflected, this current parallel to the barrier describes the lateral shift of the packet during the reflection and its penetration in the forbidden region. It makes sense to define the transmission and the reflection coefficients t and r only with respect to the motion along the y axis. In particular, denoting by $J_y^{(i)}$ the y component of the incoming flux:

$$J_y^{(i)}=\frac{\hbar k_y}{m^*}(|\alpha|^2+|\beta|^2) .$$

r is defined as the ratio of $J_y^{(i)}-J_y$ for $y<0$ and $J_y^{(i)}$ itself, while the transmission t is the ratio of J_y for $y>l$ with respect to $J_y^{(i)}$. That is,

$$\begin{aligned}r&=\frac{|\alpha|^2|A_+|^2+|\beta|^2|A_-|^2}{|\alpha|^2+|\beta|^2} ,\\t&=\frac{k_y'}{k_y}\frac{|\alpha|^2|D_+|^2+|\beta|^2|D_-|^2}{|\alpha|^2+|\beta|^2}\end{aligned}\qquad(5)$$

when k_y' is real. For a purely imaginary k_y' we have $t=0$ and $r=1$. The flux conservation $t+r=1$ requires that

$$\begin{aligned}&\frac{k_y'}{k_y}|D_\pm|^2=|A_\pm|^2\quad\text{for } k_y' \text{ real} ,\\&|A_\pm|^2=1\quad\text{for } k_y' \text{ imaginary} .\end{aligned}$$

These conditions are satisfied by the proper solution of Schrödinger equation and are implied by the Wronskian theorem.[16] This scheme is independent of the actual shape of the potential and the variable separation permits us to discuss an effectively one-dimensional scattering problem parametrized by k_x.

We now discuss how Eq. (4) determines the kinematic of scattering process. Here and in the following we use magnetic units, measuring the energies in units of $\hbar\omega_c$ and the lengths in units of $\lambda=(\hbar/2m^*\omega_c)^{1/2}$ ($\omega_c=eB/m^*c$ is the cyclotron frequency).

For a given value E of the energy we have

$$-\sqrt{E}\le k_z\le\sqrt{E} ,\quad -k\le k_x\le k ,$$

where $k=\sqrt{E-k_z^2}$ is the modulus of the momentum of the incoming wave in the $x-y$ plane. Then

$$\begin{aligned}k_y&=(k^2-k_x^2)^{1/2} ,\\k_y'&=[k^2-k_z^2+L(k_x-L/4)-W]^{1/2} .\end{aligned}\qquad(6)$$

Here L is the width of the barrier l in units of λ. The reality of k_y' defines the region in the $k-k_x$ plane in which the solution is propagative at the rhs of the barrier. For $-W<L^2/4$ this occurs when

$$-\sqrt{k^2-W}+L/2\le k_x\le k\quad\text{for } k\ge L/4+W/L ,\qquad(7)$$

while if $-W>L^2/4$ the range is

$$-k\le k_x\le k\quad\text{for } 0\le k\le -L/4-W/L\qquad(8)$$

and

$$-\sqrt{k^2-W}+L/2\le k_x\le k\quad\text{for } k\ge -L/4-W/L .\qquad(9)$$

We note that, no matter what W is, it is found that $k_y = k_y'$ when $k_x = k_0 = L/4 + W/L$ and for $k_x > k_0$ ($k_x < k_0$) is $k_y' > k_y$ ($k_y' < k_y$). Figure 1 shows these regions in the $k - k_x$ plane. The points (k, k_x) laying below the curve $k_x = -(k^2 - W)^{1/2} + L/2$ and over $k_x = -k$ correspond to an imaginary k_y', and there is $r = 1$.

Our aim is to discuss the influence of a transverse magnetic field on tunneling through $GaAs/Al_xGa_{1-x}As/GaAs$ heterostructures,[11] for which it is $m^*/m = 0.067$ and $g^* = -0.44$. The height of the barrier is of the order of 100 meV. At the maximum field intensity of the experiment described in Ref. 11 ($B = 4$ T), $\hbar\omega_c$ is of 10 meV. The particles with spin up or down see a barrier which is 0.1 meV lower or higher, respectively. Because we do not focus on the effect of the magnetic field on the particle spin in this work, we take advantage of the expected smallness of the spin-dependent part of the current and we neglect it.

In the theory of tunneling the link between the calculated transmission coefficient $t(k, k_x)$ and the measured current is given within the transfer Hamiltonian formalism[12] by the equation

$$I = 2I_0 \int_0^\infty dE[f(E) - f(E + ev)] \times \int \int_{\mathcal{D}} dk_x dk_z t(\sqrt{E - k_z^2}, k_x) , \qquad (10)$$

where I is the tunneling current for unit area, v is the applied voltage, $I_0 = e\omega_c / 8\pi^3\lambda^2$ is a reference current, and f is the Fermi distribution. The domain $\mathcal{D}$, on which k_x, k_z must be integrated, is given by Eqs. (7) and (9) with $W = -ev = -V$. If the temperature is so low that $E_F \gg k_B T$ (e.g., compare with Ref. 11, in which $E_F \sim 12$ meV and $T \simeq 4.2$ K), then

$$I = 2I_0 \int_{E_F - V}^{E_F} dE \int \int_{\mathcal{D}} dk_x dk_z t(\sqrt{E - k_z^2}, k_x) . \qquad (11)$$

The shape of $\mathcal{D}$, discussed before, implies that if

$$E_F < \left[\frac{L}{4} - \frac{V}{L} \right]^2 ,$$

then $t = 0$ and $I = 0$. Thus, the application of a magnetic field to the barrier introduces a threshold voltage V_s in the I-V characteristic of the device. When the barrier is so thick or the magnetic field so strong that

$$E_F < \frac{L^2}{16} , \qquad (12)$$

this threshold is given by

$$V_s = \frac{L^2}{4} - L\sqrt{E_F} .$$

The dependence on the intensity of the magnetic field is more explicit if we leave the magnetic units for the moment

$$V_s = \frac{l^2 e^2 B^2}{2m^* c^2} - \frac{leB\sqrt{2E_F}}{\sqrt{m^*} c} \quad \text{for } E_F < \frac{l^2 e^2 B^2}{8m^* c^2} .$$

In the samples taken into account by Gueret et al.[11] condition (12) is not fulfilled and no threshold is found. At $B = 4$ T and $E_F = 12$ meV the threshold would appear when the insulating layer thickness l is larger than 480 Å (note that the maximum value of Ref. 11 is $l = 430$ Å). We stress that the shape of the potential barrier $U(y)$ for $0 < y < l$ does not play any role in the argument. The domain $\mathcal{D}$ in which t is nonzero remains the same when changing $U(y)$ while the values of t vary.

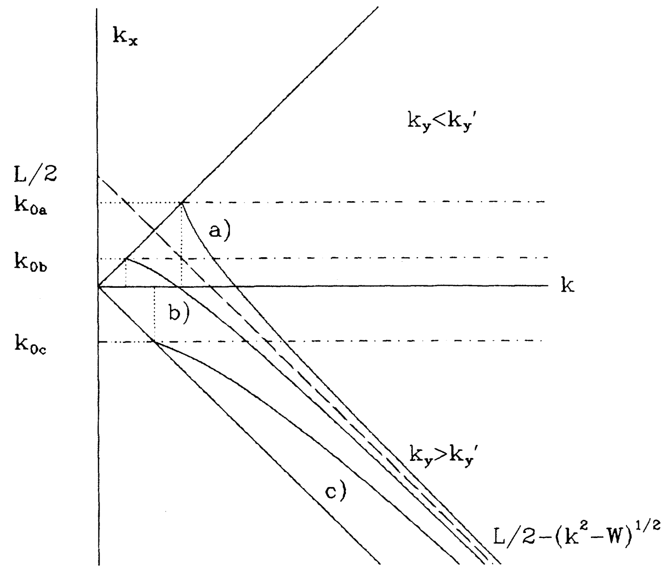

FIG. 1. The domain of definition of the transmission coefficient t as a function of k and k_x. The three cases (a)–(c) refer to the three different ranges of values of W (the constant external potential on the right-hand side of both the barrier and the magnetic field): (a) $W > 0$, (b) $-L^2/4 < W < 0$, (c) $W < -L^2/4$.

III. LANDAU LEVELS APPEARING AS RESONANCES IN THE TRANSMISSION

Here we analyze the transmission coefficient of a single barrier

$$U(y) = U_0 - \frac{Vy}{L}, \quad 0 < y < L . \qquad (13)$$

We have included, in addition to the magnetic field, an electric field V/L, parallel to direction of current flow, biasing the junction ($U_0 > 0$). If we neglect the effect of the spin $\chi_\pm(y)$ becomes indistinguishable. They satisfy the Schrödinger equation for $0 \le y \le L$:

$$-\chi''(y) + [\tfrac{1}{4}(y - y_0)^2 + a]\chi = 0 , \qquad (14)$$

in which

$$y_0 = 2(k_x + V/L), \quad a = U_0 - k^2 - V^2/L^2 - 2k_x V/L .$$

Even and odd solutions of this differential equation are the parabolic cylinder functions[17] $Y_1(a, y - y_0)$, $Y_2(a, y - y_0)$, respectively, whose power series are

$$Y_1(a,y)=1+\frac{ay^2}{2!}+\left(a^2+\frac{1}{2}\right)\frac{y^4}{4!}+\left(a^3+\frac{7}{2}a\right)\frac{y^6}{6!}+\cdots ,$$

$$Y_2(a,y)=y+\frac{ay^3}{3!}+\left(a^2+\frac{3}{2}\right)\frac{y^5}{5!}+\left(a^3+\frac{13}{2}a\right)\frac{y^7}{7!}+\cdots ,$$

in which the prefactors of $y^n/n!$, appearing in both the series, a_n, which are nonzero, are connected by

$$a_{n+1}=a\cdot a_n+\tfrac{1}{4}n(n-1)a_{n-1} .$$

In terms of these independent solutions, whose Wronskian is 1, we have

$$\chi(y)=BY_1(a,y-y_0)+CY_2(a,y-y_0) .$$

The matching of χ and χ' at $y=0$ and l gives a linear system for A,B,C,D. We have

$$A=-\frac{w_1-k_yk_y'w_4-i(k_yw_2+k_y'w_3)}{w_1+k_yk_y'w_4+i(k_yw_2-k_y'w_3)} ,$$
$$D=\exp(-ik_y'L)\frac{2ik_y}{w_1+k_yk_y'w_4+i(k_yw_2-k_y'w_3)} , \tag{15}$$

in which

$$w_1=Y_1'(a,-y_0)Y_2'(a,L-y_0)-Y_2'(a,-y_0)Y_1'(a,L-y_0) ,$$
$$w_2=Y_1(a,-y_0)Y_2'(a,L-y_0)-Y_2(a,-y_0)Y_1'(a,L-y_0) ,$$
$$w_3=Y_1'(a,-y_0)Y_2(a,L-y_0)-Y_2'(a,-y_0)Y_1(a,L-y_0) ,$$
$$w_4=Y_1(a,-y_0)Y_2(a,L-y_0)-Y_2(a,-y_0)Y_1(a,L-y_0) .$$

The transmission coefficient is given by

$$t=\frac{4k_yk_y'}{w_1^2+k_y^2w_2^2+k_y'^2w_3^2+k_y^2k_y'^2w_4^2+2k_yk_y'} . \tag{16}$$

The behavior of t as a function of U_0,L,V,E,k_x,k_z can be analyzed more easily by observing that it is the same as that arising from an equivalent one-dimensional scattering problem. In fact, the differential equation (14) for $\chi(y)$ can be rewritten as

$$-\chi''+u(y)\chi=k_y^2\chi ,$$

in which a particle of energy k_y^2 is scattered by the potential $u(y)$ given by

$$u(y)=0,\quad y<0$$
$$u(y)=\frac{y}{4}(y-L)-y(k_x-k_0)+U_0,\quad 0\le y\le L$$
$$u(y)=-L(k_x-k_0),\quad y>L .$$

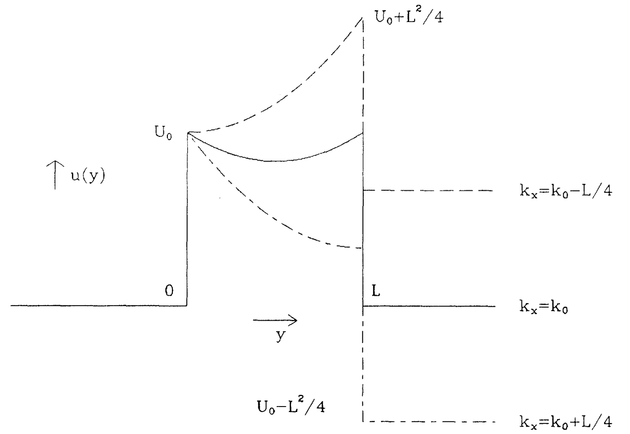

FIG. 2. The effective one-dimensional potential $u(y)$ along the y axis, for $k_x=k_0$ (the most favorable condition for the resonances to appear) and for $k_x=k_0\pm L/4$. Note that the resonances can only be found inside this k_x-vector interval.

The equivalent potential $u(y)$ is an explicit function of U_0,L,k_x and depends on V via

$$k_0=L/4-V/L . \tag{17}$$

The values of E and k_z fix the range of values of k_x according to what was discussed in the preceding section. The effect of the magnetic and the electric fields appears as a deformation of the barrier, whose shape changes with k_x. When $k_x=k_0$ the barrier becomes symmetric around $y=L/2$, and a parabolic attractive potential adds to U_0, for $0\le y\le L$, giving a minimum at $L/2$, in which $u(y)=U_0-L^2/16$. This minimum appears at $y=0$ when $k_x=k_0-L/4$ and disappears at $y=L$ when $k_x=k_0+L/4$. Figure 2 shows this behavior. Therefore, when U_0 and L are large enough, the effective potential allows for resonances which appear around $k_x=k_0$. At this value of k_x, in fact, the potential is attractive at its center. Because the shape of the potential is parabolic, these resonances fall roughly at k_y values given by

$$k_y^2=U_0-\frac{L^2}{16}+n+\frac{1}{2} \tag{18}$$

and there is a finite number of them because the nonnegative integer n must satisfy the condition

$$n<\frac{L^2}{16}-\frac{1}{2} ,$$

which follows from the requirement that is $k_y^2<U_0$. These resonances show up in the direct calculation of t given by Eq. (16) which is plotted in Fig. 3 as a function of k^2 at $k_x=k_0$. Here and in the following we have chosen $k_z=0$. The lowest resonance is roughly located at

$$k^2\sim U_0-V/2+(V/L)^2+\tfrac{1}{2} . \tag{19}$$

It is very sharp and saturates to 1, as it is expected due to the parity symmetry of the effective potential. There are four resonances appearing in Fig. 3 up to the energy $k^2=U_0+k_0^2$. The resonances which are higher in energy become broader and broader the more the confining of

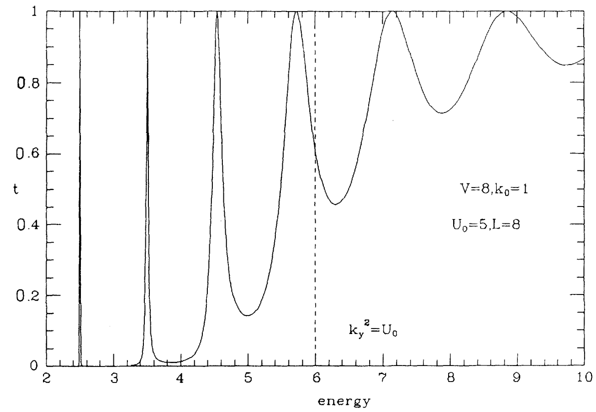

FIG. 3. Peaks of total transmission due to resonant tunneling as a function of the energy (units of $\hbar\omega_c$) for $k_x = k_0$ and $k_z = 0$. The dashed line indicates the edge of the repulsive barrier, i.e., the beginning of the classical transmission ($k_y^2 = U_0$).

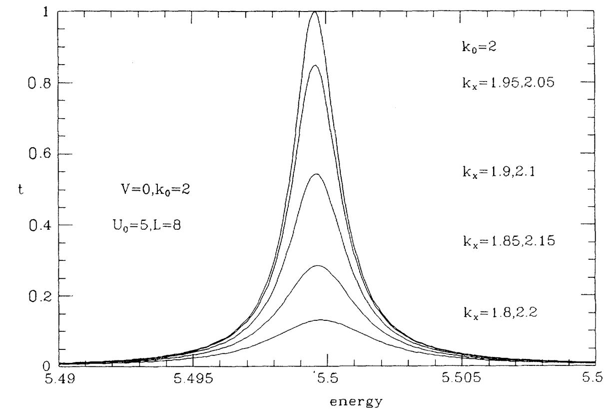

FIG. 5. The strong reduction of the resonance peak in the transmission when we move away from $k_x = k_0$. Because voltage is zero the maximum of the resonance is always centered at the same energy (units of $\hbar\omega_c$).

the effective potential is reduced. At higher energies transmission is also allowed classically and the oscillations, which appear in the picture, are due to the usual quantum interference. The presence of the magnetic field does not change their shape, but their position. The value of L has been chosen in Fig. 3 large enough to show various resonances, for the sake of demonstration. The plot of the transmission using the parameters given in Ref. 11 is reported in Fig. 4. Here L is smaller; just one resonance can be found, and this is rather broad.

According to Eq. (19) the effect of increasing the voltage is to shift the position of the peaks. When $k_x = k_0$ the effective potential $u(g)$ no longer depends on k_0 and the value of k_y^2 for which the resonance exists is independent of k_0 too. The total energy, being the sum of k_y^2 and k_x^2, reaches its minimum value when $k_0 = 0$, that is, when the voltage $V = L^2/4$ [see Eq. (17)]. This minimum is expected to be found close to the value $U_0 + \frac{1}{2} - L^2/16$ which corresponds to the untruncated harmonic potential. Due to the presence of the repulsive barrier the true value of k_y^2 for the resonance is just a bit higher than that.

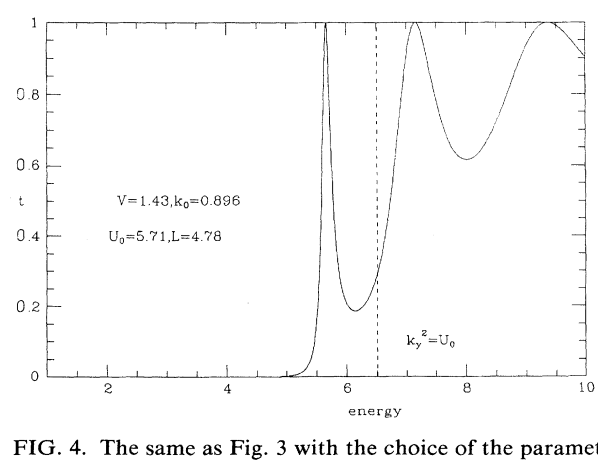

FIG. 4. The same as Fig. 3 with the choice of the parameters reported in Ref. 11.

This is relevant because a nonlinearity in the I-V characteristic could appear when one of the resonances is near the Fermi level. The device taken into account in Ref. 11 is $E_F = 1.7$. On the other hand, the minimum is at 4.8 magnetic units for a voltage of $V = 5.8$ in that case, so that these effects are absent, no matter how large the voltage is. To let them appear, higher Fermi levels and thicker barriers are needed. The minimum required value of L is $4\sqrt{U_0 - E_F + \frac{1}{2}}$. For the values of parameters quoted previously the first resonance falls at the Fermi energy when $L = 8.5$ and $V = 18$. Of course the condition is even less favorable when incidence is not in the plane, because one has to add k_z^2 to the energy of the resonance.

The transmission coefficient integrated over k_x and k_z is needed for the evaluation of the current. Therefore let us now discuss the k_x dependence of these resonances. At zero voltage, Fig. 5 shows that the height of the peaks in the resonant transmission is lowered dramatically in moving off the value of k_0, while their location in k^2 is

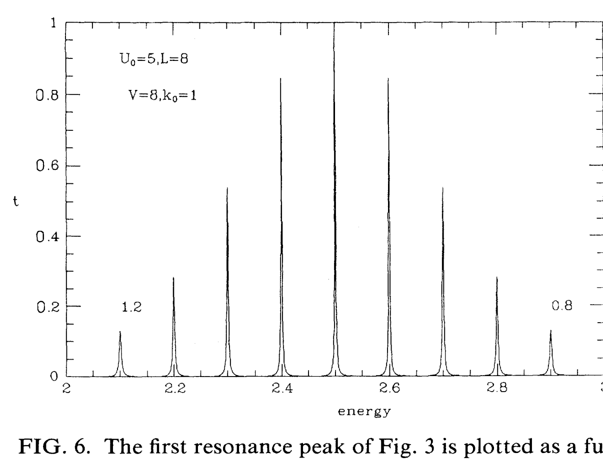

FIG. 6. The first resonance peak of Fig. 3 is plotted as a function of energy (units of $\hbar\omega_c$) for various values of k_x ranging from 1.2 to 0.8 with a step of 0.05. The peak is moving to higher energies due to the presence of the voltage bias.

unaffected. On the contrary their location is also moved when a voltage is applied, as appears in Fig. 6. Their width remains rather small when moving k_x away from k_0. These features imply that resonant transmission, induced by the magnetic field, is strongly directional, in the sense that only particles impinging with definite values of k_x and k_y given by Eqs. (17) and (18) are totally transmitted, while t drops heavily outside of this direction. As already stated in Sec. I this is just the k_x value at which the plane wave emerging from the barrier keeps the same k_y vector as the incoming one. This does not mean, however, that the particle beam crosses the barrier without being deflected. In fact, the components of the velocity are what matters to fix the direction of the outcoming beam semiclassically.

To better characterize the properties of the transmission and the nature of its resonances we move on to discuss the phases of the amplitudes of transmitted and reflected waves D and A given by Eq. (15) which we label with the subscripts T and R, respectively:

$$\Phi_T=\frac{\pi}{2}-\arctan\frac{k_y w_2-k_y' w_3}{w_1+k_y k_y' w_4} \quad (\mathrm{mod}\ \pi)$$
$$\Phi_R=\frac{\pi}{2}+\Phi_T-\arctan\frac{k_y w_2+k_y' w_3}{w_1+k_y k_y' w_4} \quad (\mathrm{mod}\ \pi)\ . \tag{20}$$

In Figs. 7(a) and 7(b) the phases are plotted versus energy for the two cases of k_x equal to or different from k_0, respectively. They show a jump at the resonances each time the transmission goes through a maximum. As seen from the picture, when $k_x=k_0$ they differ for a constant. In fact, in this case $w_3=-w_2$, so that the last contribution in Φ_R vanishes, and we have

$$\Phi_R=\frac{\pi}{2}=\Phi_T \quad (\mathrm{mod}\ \pi) \quad (k_x=k_0)\ . \tag{21}$$

Because, in this case, the effective potential is even for parity around $L/2$, this and other features of the scattering can be easily justified if one uses eigenfunctions of definite parity.[14] Out of the potential barrier an S matrix can be defined which is labeled by the index l referring to even ($l=0$) and odd parity ($l=1$) in terms of scattering amplitudes. Using the definitions given in Ref. 14 we find that

$$S^{0,1}=e^{-ik_yL}(t^{1/2}e^{i\Phi_T}\pm r^{1/2}e^{i\Phi_R})\ , \tag{22}$$

where the upper (lower) sign refers to even (odd) parity. It is immediately seen that, due to Eq. (21), both S^0 and S^1 are of unitary modulus, and the flux is conserved. This was expected because, being the effective potential of a given well-defined parity when $k_x=k_0$, the parities of scattering eigenfunctions do not mix. The determination of the inverse tangent, which fixes the relationship of Φ_R with respect to Φ_T in Eq. (21) has to be chosen carefully, in such a way that S^0,S^1 are continuous when the energy passes across a value of total transmission.

It follows that it is useful to define phase shifts δ^l for the two parities according to

$$S^l=\exp(2i\delta^l) \quad (k_x=k_0)\ . \tag{23}$$

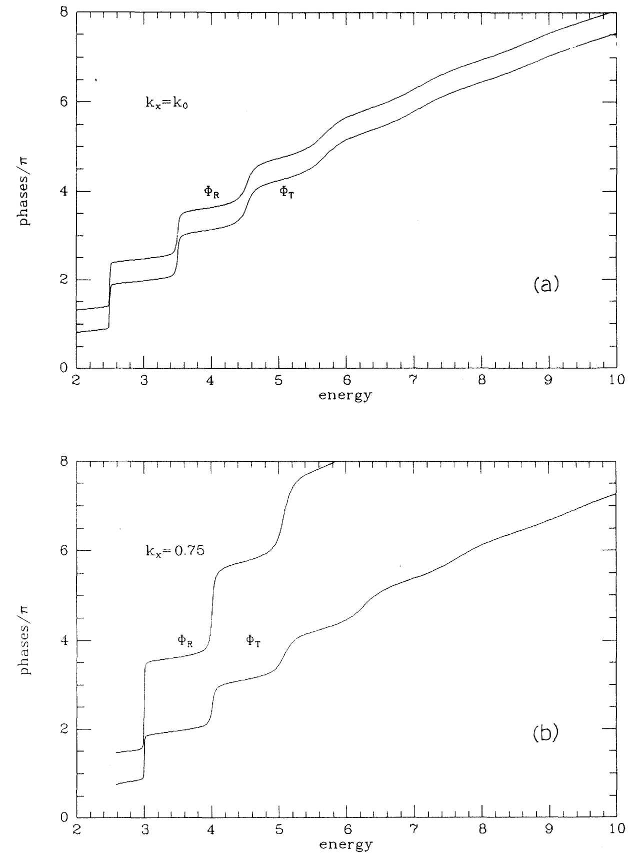

FIG. 7. (a) Phases of the reflected and the transmitted beam corresponding to the transmission of Fig. 3 at $k_x=k_0=1.0$ as a function of energy (units of $\hbar\omega_c$). They differ exactly by $\pi/2$. (b) The same as (a) for $k_x=0.75$. Because in this case is $k_x\neq k_0$, transmission is no longer total at the resonances. The effective barrier is not symmetric and the difference between the phases increases with the energy.

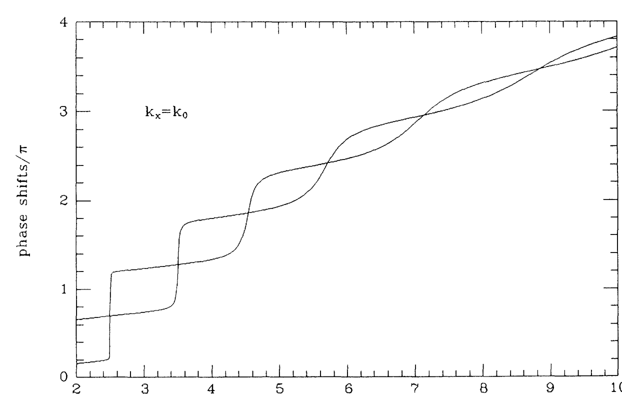

FIG. 8. Even and odd phase shifts as defined by Eq. (23) for $k_x=k_0$, as a function of the energy (units of $\hbar\omega_c$). They cross at the resonance values: the parity of the resonant state is that of the phase shift that undergoes the larger variation around a multiple of $\pi/2$. The parameters are those of Fig. 3 and the first resonance is an even parity one.

Phase shifts cross at the energy values for which the total transmission takes place. Only one of the two varies strongly in energy in a resonance, jumping by π, thus allowing for the identification of the parity of the resonant state. This can be seen in Fig. 8, which reports four resonances under the classical transmission threshold. The jumps in the phases are alternatively in the even and odd channels, corresponding to even and odd metastable levels in the parabolic well within the barrier.

We close this section with a brief discussion of what is going to happen if we relax the assumption that the magnetic field is contained entirely into the square barrier. The magnetic field spans a region of width L larger than that of the barrier L_0. If the latter is located just in the middle of the field region the effective potential $u(y)$ for the motion along the y axis becomes

$$u(y)=\begin{cases} 0, \quad y<0 \\ y^2/4-yk_x, \quad 0<y<\frac{L}{2}-\frac{L_0}{2} \\ y^2/4-yk_x+U_0-\frac{v}{2}\left[y-\frac{L}{2}+\frac{L_0}{2}\right]; \quad \frac{L}{2}-\frac{L_0}{2}<y<\frac{L}{2}+\frac{L_0}{2} \\ y^2/4-yk_x-V, \quad y>\frac{L}{2}+\frac{L_0}{2} \\ L^2/4-Lk_x-V, \quad y>L\ . \end{cases}$$

We have again included a voltage bias V across the barrier. Due to the presence of the magnetic field this potential depends on k_x as before. The barrier is symmetrically deformed around its center at $k_x=k_0=L/4-V/L_0$. The parabolic potential $y^2/4-yk_x$ has the effect of shifting down the barrier and digging a well in the middle of it. The minimum value of the potential, which is located at $L/2$ when $k_x=k_0$, is

$$u\left[\frac{L}{2}\right]=U_0\frac{L^2}{16}+\frac{V}{2}\left[\frac{L}{L_0}-1\right],$$

while at the edges of the square barrier we have

$$u\left[\frac{L}{2}-\frac{L_0}{2}\right]=u\left[\frac{L}{2}+\frac{L_0}{2}\right]=u\left[\frac{L}{2}\right]+\frac{L_0^2}{16}\ .$$

Bound states or resonances can appear in this well. Limiting ourselves to a qualitative discussion, we neglect the effect of the attractive wells that the magnetic field produces on the two sides of the barrier. We also approximate the levels (bound states or resonances) in the barrier with the corresponding values given by an untruncated harmonic potential $k_y^2=u(L/2)+n+\frac{1}{2}$. The condition for a maximum in the transmission at these energies is

$$V\left[\frac{L}{L_0}-1\right]<u\left[\frac{L}{2}\right]+n+\frac{1}{2}<u\left[\frac{L}{2}\pm\frac{L_0}{2}\right]\ .$$

It is apparent from this inequality that if the number of levels to be present is N, the width of the barrier L_0 has to be

$$L_0>2\sqrt{4N-2}\ .$$

Thus the only quantity that fixes the total number of levels is the width of the barrier independent of the extension of the magnetic field. Above we identify the maxima in the transmission with true resonances of the potential. This is strictly true only at zero bias, when $k_x=k_0$ is $k_y'=k_y$ and the maxima of transmission saturate to 1. Applying a bias, a step of height $V(L/L_0-1)$ arises at the right-hand side of the barrier. Changing k_x the step can be eliminated ($k_x=k_0'=L/4-V/L$). This makes the barrier unsymmetrical, however, and the transmission is never 1. We expect our considerations to hold and the maxima to t to be found for $k_0'\le k_x\le k_0$. Within this interpretation at any value of L and L_0 the nth level is a resonance when

$$U_0>\frac{L^2}{16}+\frac{V}{2}\left[\frac{L}{L_0}-1\right]-n-\tfrac{1}{2}\ .$$

Therefore a magnetic field leaking out of the barrier requires the latter to be higher if structures have to be found in the transmission. The calculation in this work is only concerned in the case of $L=L_0$. This is just for the sake of simplicity. On the other hand a confined magnetic field is needed in this picture to take advantage of the gauge chosen. In fact, the gauge used here [Eq. (2)] allows for separation of the coordinates and makes the analytical discussion of the scattering feasible. Only free waves in the asymptotic regions guarantee a well-defined transmission coefficient, in the direction of the current flow. However, within the gauge chosen this only happens if the magnetic field is limited to a finite region of space.

IV. SEMI-INFINITE BARRIER: REFLECTION TIME

It is interesting to discuss the case when a particle beam impinges on a semi-infinite magnetic barrier occupying the region $y\ge 0$. In this case the flux is totally reflected no matter what its incoming k vector is. We consider the simplest situation assuming that $U(y)$ is a potential step U_0 in the $y\ge 0$ half-space. We show that the Landau levels also play a role in the backscattering.

We now put in Eq. (14) y_0 equal to $2k_x$, a equal to U_0-k^2, and $\chi(y)$ is the standard solution of the parabolic cylinder equation $Y(a,y)$ going to zero when $y\to\infty$:[17]

$$Y(a,y)=\cos\pi\left[\frac{1}{4}+\frac{a}{2}\right]Y_1-\sin\pi\left[\frac{1}{4}+\frac{a}{2}\right]Y_2 .$$

Its asymptotic behavior for $y\gg|a|$ is

$$Y(a,y)\sim\exp(-y^2/4)y^{-a-1/2} .$$

Because $r=1$, the amplitude of the reflected wave is of unitary modulus and is given by

$$A=-\frac{Y'(U_0-k^2,-2k_x)-i\sqrt{k^2-k_x^2}\,Y(U_0-k^2,-2k_x)}{Y'(U_0-k^2,-2k_x)+i\sqrt{k^2-k_x^2}\,Y(U_0-k^2,-2k_x)}=e^{i\Phi_R} ,$$

so that its phase is

$$\Phi_R=\pi-2\arctan\sqrt{k^2-k_x^2}\,\frac{Y(U_0-k^2,-2k_x)}{Y'(U_0-k^2,-2k_x)} . \tag{24}$$

All the physical information about the scattering is embodied in this phase and in its energy dependence. The phase delay time τ is obtained by deriving Φ_R with respect to the energy $\varepsilon=k^2$, that is,

$$\tau\omega_c=-\frac{d\Phi_R}{d\varepsilon} . \tag{25}$$

We analyze τ in the case of normal incidence ($k_x=0$), when Y can be expressed in terms of the Γ function[17]

$$Y(a,0)=\frac{\sqrt{\pi}}{2^{a/2+1/4}\Gamma\left[\frac{3}{4}+\frac{a}{2}\right]}$$

$$Y'(a,0)=-\frac{\sqrt{\pi}}{2^{a/2-1/4}\Gamma\left[\frac{1}{4}+\frac{a}{2}\right]} ,$$

which yields

$$\tau\omega_c=-\frac{1}{\sqrt{2\varepsilon}}\,\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma^2(\beta)+\frac{\varepsilon}{2}\Gamma^2(\alpha)}[\varepsilon(\Psi(\alpha)-\Psi(\beta))-1] , \tag{26}$$

where

$$\alpha=\frac{1}{4}+\frac{U_0}{2}-\varepsilon,\quad \beta=\tfrac{1}{2}+\alpha$$

and Ψ is the logarithmic derivative of the Γ function ($\digamma$ function).[17] Increasing the energy, the arguments of the Γ and $\digamma$ functions go through values which are negative integers and diverge there. This happens when the energy crosses a Landau level shifted by the value of the step U_0:

$$\varepsilon=n+\tfrac{1}{2}+U_0 . \tag{27}$$

Odd (even) n implies $\alpha(\beta)$ being a negative integer. For these values of the energy the limit of Eq. (26) has to be handled analytically.

Let us study the case $U_0=0$ first. We indicate the phase delay times corresponding to the energies with n even (odd) as even (odd) times and label them with 0(1). If $n=2m$ then $\alpha=-m$ and the even time is given by

$$\tau_m^0=\frac{2^{m+1}m!\sqrt{\pi}}{(2m-1)!!\sqrt{4m+1}}\,\frac{1}{\omega_c} \tag{28}$$

because of the identity

$$\frac{\Psi(-m)}{\Gamma(-m)}=(-1)^m m! .$$

If $n=2m+1$ the singularity arises from $\beta=-m$ and the odd time is

$$\tau_m^1=\frac{\sqrt{4m+3}\,2^m m!\sqrt{\pi}}{(2m-1)!!(2m+1)}\,\frac{1}{\omega_c} . \tag{29}$$

The odd and even times are related by the equations

$$\frac{\tau_m^1}{\tau_m^0}=\left[1-\frac{1}{4(2m+1)^2}\right]^{1/2}<1 ,$$

$$\frac{\tau_m^1}{\tau_{m+1}^0}=\left[1-\frac{1}{4(2m+2)^2}\right]^{1/2}<1 .$$

The recurrence relation

$$\tau_{m+1}^0=\tau_m^0\,\frac{m+1}{m+\frac{1}{2}}\left[\frac{4m+1}{4m+5}\right]^{1/2}$$

gives the sequence of the even times, which decreases, starting from

$$\tau_0^0\omega_c=2\sqrt{\pi} .$$

Figure 9 shows the values of these sequences. We note that any even time is greater than all the odd ones. The even and odd times converge to the same limit from above and below, respectively. This limit can be evaluated making use of Wallis's formula[17]

$$\frac{2^{m+1}m!}{(2m-1)!!}\Gamma(\tfrac{1}{2})\overset{m\to\infty}{\sim}2\sqrt{m}\,\pi ,$$

which implies

$$\lim_{m\to\infty}\tau_m^0\omega_c=\lim_{m\to\infty}\tau_m^1\omega_c=\pi .$$

This is just the time spent by a classical charged particle in the semi-infinite magnetic field, whatever its incoming momentum is. When the energy goes to infinity, the classical description of the motion is recovered, as expected.

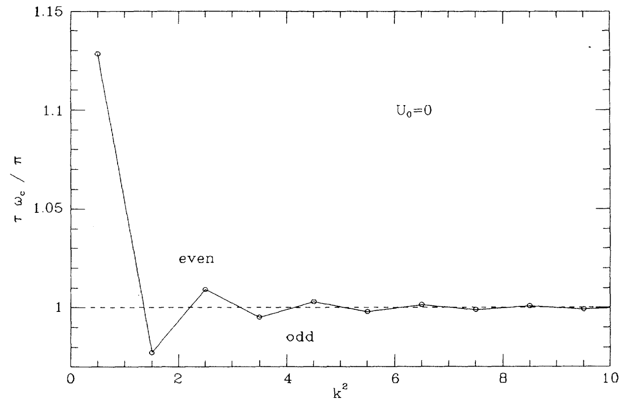

FIG. 9. The ratio between the reflection time and the classical time vs the energy (k^2) (units of $\hbar\omega_c$). Here the particle beam impinges normally on a magnetic field occupying the right-hand side half-space.

When a potential step U_0 is superimposed to the magnetic field the time sequences rescale becoming

$$t_m^0 = \left[\frac{4m+1}{4m+1+2U_0} \right]^{1/2} \tau_m^0 = s_m^0 \tau_m^0 ,$$

$$t_m^1 = \left[\frac{4m+3+2U_0}{4m+3} \right]^{1/2} \tau_m^1 = s_m^1 \tau_m^1 .$$

When $U_0 > 0$ we have

$$s_m^0 < 1, \quad s_m^1 > 1 .$$

For U_0 large enough the role of even and odd channels interchange. The even channels become the fast ones, while the odd channels become slower and slower the higher the barrier grows. This is shown in Figs. 10(a) and 10(b). When $U_0 < 0$ the scattering states require

$$n \geq |U_0| - \tfrac{1}{2} ,$$

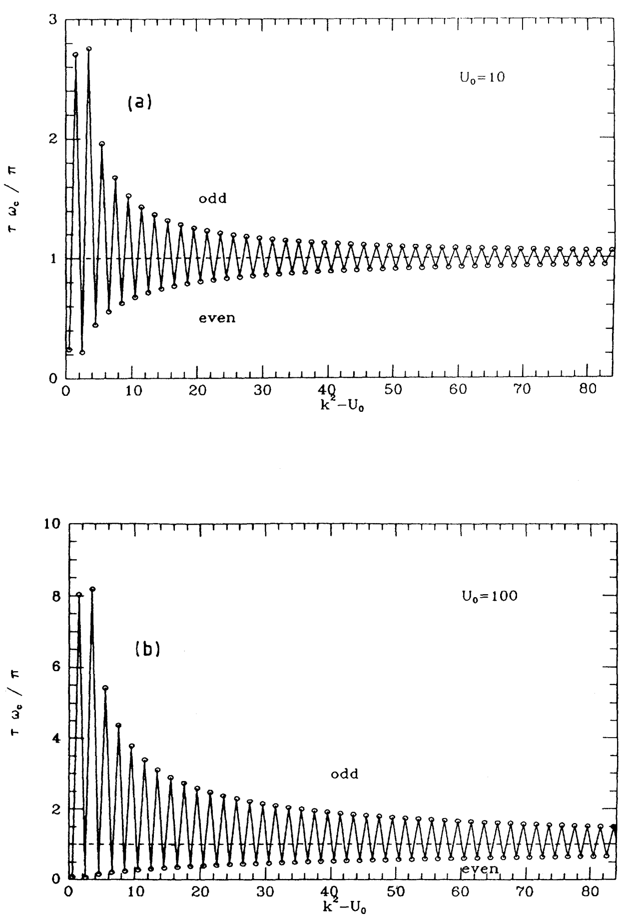

FIG. 10. The ratio between the reflection time and the classical time vs the energy (k^2) (units of $\hbar\omega_c$) when a repulsive step barrier is added on top of the magnetic field: (a) $U_0 = 10$, (b) $U_0 = 100$.

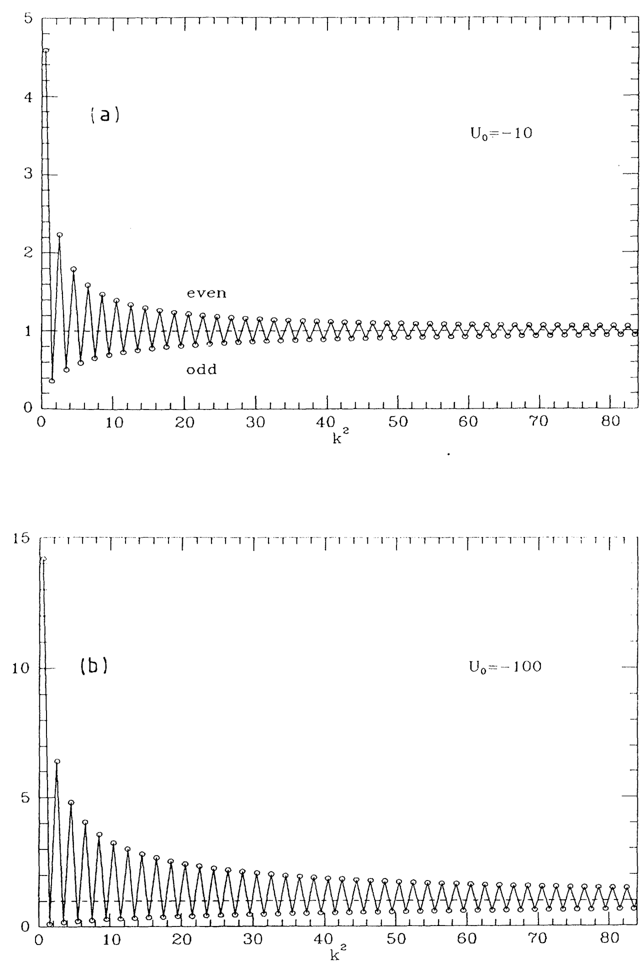

FIG. 11. The same as Fig. 10 except for the step barrier which is attractive here: (a) $U_0 = -10$, (b) $U_0 = -100$.

that is, their energy has to be positive. Denoting by m_0 the integer part of $|U_0|/2-\frac{1}{4}$, we have, for $m \geq m_0$,

$$s_m^0 > 1, \quad s_m^1 < 1 .$$

In other words, an attractive potential step makes the even channels faster and the odd ones slower than the corresponding channels when only the magnetic field is present. The increasing of the well depth enhances this effect as can be seen in Figs. 11(a) and 11(b).

In conclusion the analysis of the phase delay time emphasizes that in correspondence of the energies of the Landau levels the dilation or shortening of the time of backscattering is at a maximum, with respect to the classical traversal time. In this sense, due to the quantum resonance, the energies of the Landau levels are the fastest or slowest channels for a particle to be backscattered, depending on their parity. On the contrary classical particles employ the same time to emerge from the barrier whatever the incoming energy is.

The same results can be found when calculating the dwell time introduced by Smith[13] as the ratio of the number of particles in the barrier to the incoming flux of the particles:

$$\tau^D = \frac{m}{\hbar k_y} \int_0^\infty |\psi(y)|^2 dy .$$

In our case we have

$$\tau^D \omega_c = \frac{2\sqrt{k^2-k_x^2}}{Y'^2(U_0-k^2,-2k_x)+(k^2-k_x^2)Y^2(U_0-k^2,-2k_x)} \int_0^\infty Y^2(U_0-k^2, y-2k_x)dy .$$

As before, we restrict ourselves to the normal incidence case, that is $k_x=0$. When the energy $\varepsilon=k^2$ is equal to $U_0+n+\frac{1}{2}$, the Y becomes a Hermite polynomial:

$$Y(-n-\tfrac{1}{2},y)=2^{-n/2}e^{-y^2/4}H_n(y/\sqrt{2})$$

so that the dwell time becomes

$$\tau_n^D = \frac{1}{\omega_c} \frac{\sqrt{2\pi}\, 2^{n+1} n! \sqrt{U_0+n+\frac{1}{2}}}{H_n'^2(0)+2(U_0+n+\frac{1}{2})H_n^2(0)} .$$

Because[18]

$$H_{2m}(0)=(-1)^m 2^m (2m-1)!!, \quad H_{2m+1}(0)=0 ,$$

$$H'_{2m}(0)=0, \quad H'_{2m+1}(0)=2(2m+1)H_{2m}(0) ,$$

choosing n even or odd we get

$$\tau_{2m}^D = \frac{\sqrt{2\pi}(2m)!}{\sqrt{U_0+2m+\frac{1}{2}}[(2m-1)!!]^2} \equiv \tau_m^0 ,$$

$$\tau_{2m+1}^D = \frac{\sqrt{2\pi}(2m+1)!\sqrt{U_0+2m+\frac{3}{2}}}{(2m+1)^2[(2m+1)!!]^2} \equiv \tau_m^1$$

so that the dwell time and the phase delay time exactly coincide.

V. CONCLUSIONS

When magnetic fields are involved, the three-dimensional nature of the tunneling has to be retained if one wants to give a sensible estimate of the tunneling current. The theoretical description of the tunneling in the presence of a magnetic field immediately comes across the difficulty, how the incoming and the outgoing flux of particles can be defined properly. In the case of thin barriers, the dynamics of the tunneling process can be described by means of localized wave packets, whose motion is numerically simulated. It has been shown,[19,20] that quantum interference effects constitute just a small correction to the results obtained by application of the Ehrenfest theorem and can be interpreted in terms of classical concepts as trajectories, skipping orbits and so on. When thicker barriers are considered, this method is unsatisfactory, because the wave packet undergoes a strong deformation in tunneling across the barrier and numerical results are less transparent. On the other hand square barriers which are thick and low can be easily described by means of a one-dimensional WKB approximation for the transmission and reflection coefficients and this picture should be reliable in describing a steady-state tunneling current. This amounts to confining the magnetic field inside a finite region of space and taking free waves as the asymptotics of the scattering states.

Here we solve the Schrödinger problem of tunneling across a square barrier with uniform magnetic field localized just in the barrier region and orthogonal to the current flow, possibly in the presence of an applied voltage. This is to show that extra features can be contained in the full transmission, which are lost when a semiclassical picture is adopted. At the end of Sec. III the conditions have been discussed under which the peaks in the transmission can survive when the magnetic field extends outside the barrier. Nonetheless effects such as band bending, inversion layers, and tunneling from and to Landau levels are not included in our model. These are often invoked as the cause for structures in the I-V characteristics.

In the case of a thick barrier the tunneling current is expected to be heavily reduced by the presence of a magnetic field. However careful investigation of the transmission as a function of the energy, k vector, and applied voltage, shows that the simultaneous presence of both the repulsive barrier of the insulating layer and the magnetic field can give rise to resonances in the spectrum which are ignored in the semiclassical picture. These are remnants of Landau levels localized in the barrier and show up as very sharp peaks with strong dependence on $k_\parallel$ (the wave vector parallel to the barrier which is conserved in the tunneling). Their location in energy is not far from that of Landau levels of an uniform magnetic field shifted by the height of the barrier according to Eq. (19). The effects of the resonances on the transmission are striking only in a very small $(E,k_{\parallel})$ domain. Therefore the total current as a function of the applied voltage, being an integrated quantity, is most likely insensitive to these, except when their energy is close to the Fermi level. Obviously, they can be moved to lower energies by increasing the voltage. But, as we have shown in Sec. III, the values of the parameters corresponding to real devices require a very high bias to make the lowest resonance close to the Fermi level.

We have undertaken the full calculation of the current relative to this geometry, to check whether these effects can originate special features in the conductivity. Our results show that this can happen, as we will report in a forthcoming publication. Anyway, these resonances are not expected to be able to give rise to negative differential resistance in the characteristic. The latter is the better signature of tunneling via lower-energy resonant states, which would be present if double-barrier heterostructures could localize states in the insulating region.

The full three-dimensional problem reduces a one-dimensional scattering by means of an effective potential parametrized by $k_{\parallel}$. This allows us to recover some peculiar features of scattering in one dimension. It can be seen that, for a particular bias-dependent angle of incidence, the effective potential becomes symmetric around its center. In this case the resonance can be classified according to its parity using the odd or even phase shifts along the lines proposed in Ref. 14.

In Sec. IV we discuss the case of an uniform magnetic field located in the halfspace on the rhs with a particle flux incoming from the lhs. Although the beam of particles is totally reflected, some effects of the Landau resonances are still present. In the particular configuration studied, no ambiguity arises in defining a time for the scattering because all possible definitions reduce to the reflection time. For instance, we have checked that Smith's dwell time[13] and the phase delay time coincide. The time spent in the barrier by the particles oscillates around the classical value: the maxima and the minima are located just at the Landau resonances. With no barrier present, the maxima correspond to the even levels and the odd ones to the minima. The addition of an attractive step barrier on top of the magnetic field does not change the picture. On the contrary, if the barrier is repulsive the even and odd resonances interchange their role of fast and slow channels. In both cases the higher is the barrier the larger are the deviations from the classical time.

When the barrier has a finite width the definition of a tunneling time is a subtle question.[22,21] However, we infer that in a thick barrier the deformation due to the magnetic field allows resonant tunneling across it and the Landau resonances localized at the barrier could have some influence on the traversal time of tunneling, whatever its definition is.

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