Magnetoplasmon polaritons in finite *n-i-p-i* superlattices

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We investigate the effects of an external magnetic field on plasmon polaritons propagating in truncated *n-i-p-i* superlattices, using a local theory with retardation. We use a transfer-matrix method to simplify the algebraic expressions, which are otherwise quite involved. The external magnetic field is taken to be parallel to the surface and perpendicular to the direction of propagation of the plasmon polaritons. We also present numerical results pointing out the differences of the spectra with and without the presence of the external magnetic field. Our results generalize previous work on this subject.

I. INTRODUCTION

Recently a number of papers have appeared dealing with the propagation of bulk and surface plasmons in semiconductor superlattices of various types. Binary superlattices consisting of alternating layers of materials A and B with or without a two-dimensional electron (hole) gas at the interfaces were studied by many authors.[1-6] A particular superlattice structure, the so-called *n-i-p-i* superlattices, has also been investigated and its particular features discussed.[7,8]

The *n-i-p-i* structure is a different superlattice composed of doped semiconductors that presents many interesting properties. It is formed by a periodic array of n- and p-doped semiconductor layers separated by insulators as it was proposed by Döhler.[9,10] A certain fraction of the donors and acceptors is ionized, producing doped layers with positive charge in each n layer and negative charge in each p layer.

In this paper we present a full theory of the bulk and surface-plasmon excitation spectrum of a finite *n-i-p-i* superlattice. We have included the effects of both retardation and an external magnetic field, and we have obtained the dispersion relation for surface magnetoplasmon polaritons in this structure. In some ways, this paper can be considered as an extension of the work of Wallis *et al.*,[11] which investigated these phenomena for semi-infinite binary superlattices. On the other hand, it completes the work done previously by Farias *et al.*[7] where the spectra of bulk and surface-plasmon polaritons in an *n-i-p-i* superlattice without an applied magnetic field were extensively discussed. Although the treatment presented here generalizes those papers, it is not just a mathematical extension since new physical features are encountered.

Our model is based on a transfer-matrix treatment, already presented earlier,[12] to simplify the algebra, which is otherwise quite involved. Since the quantization of the electronic states into subbands is quite negligible due to our basic assumption that the layer thicknesses are sufficiently large, we can describe the properties of the layers by macroscopic dielectric functions. Thus the electromagnetic fields in each layer are described by solving Maxwell's equations subject to the appropriate boundary conditions.

The plan of this paper is as follows. Section II presents the full theoretical derivation of the magnetoplasmon-polariton dispersion relation, assuming that the electromagnetic mode is p polarized. Section III is devoted to the presentation of some numerical results. The conclusions are given in Sec. IV. We also present (in the Appendix), for completeness, our theory for the case where the electromagnetic mode in the superlattice is s polarized.

II. GENERAL THEORY

The *n-i-p-i* semiconductor superlattice that is considered in this paper is depicted in Fig. 1. Materials A and C are n- and p-doped semiconductors with dielectric tensor $\epsilon_a(\omega)$ and $\epsilon_c(\omega)$ and thickness a and c, respectively. Materials B and D are intrinsic semiconductors with frequency-independent dielectric tensor ϵ_b and ϵ_d and thickness b and d, respectively. The unit cell has length $L=a+b+c+d$ and is designated by the index n, as is shown in Fig. 1. In the nth unit cell, at the interfaces $z=nL$ and $z=nL+a$, there is a two-dimensional electron gas (2DEG), while at $z=nL+a+b$ and $z=nL+a+b+c$ there is a two-dimensional hole gas (2 DHG). We assume that a uniform external magnetic field is imposed in the y direction and that surface magnetoplasmon polaritons are allowed to propagate in the x direction (parallel to the interfaces) with wave vector $\mathbf{Q}$

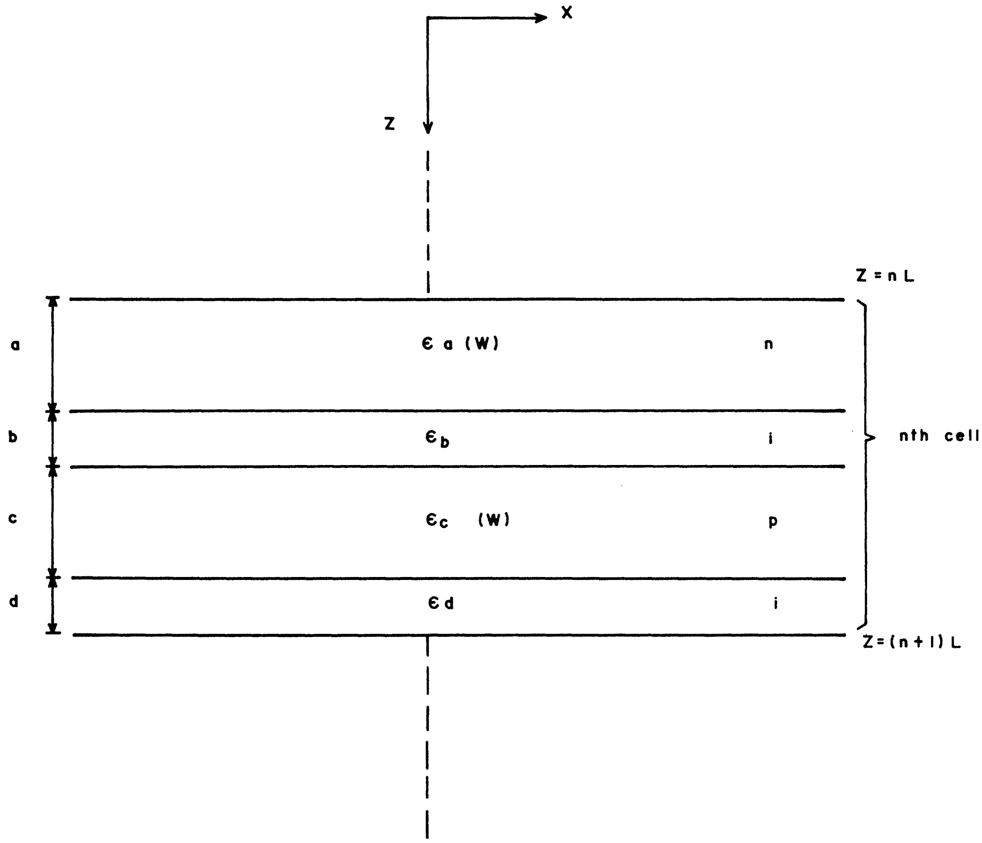

FIG. 1. Schematic representation of an infinite *n-i-p-i* superlattice.

and frequency ω.

We start our theoretical development by deriving first the dispersion relation for magnetoplasmon polaritons in an infinite superlattice and then for the finite one. In both cases the field amplitudes are assumed to be localized at each interface.

A. Bulk modes

In this section, we follow the lines of Ref. 7, hereafter referred to as I (its Sec. II A). We assume p polarization for the electromagnetic mode with the external magnetic field along the y axis. Thus the dielectric function in layers A and C has the form[13]

$$\overleftrightarrow{\epsilon}(\omega)=\begin{bmatrix}\epsilon_1 & 0 & -i\epsilon_2\\ 0 & \epsilon_3 & 0\\ i\epsilon_2 & 0 & \epsilon_1\end{bmatrix}, \tag{2.1}$$

where

$$\begin{aligned}\epsilon_1(\omega)&=\epsilon_\infty[1+\omega_p^2/(\omega_c^2-\omega^2)]\;,\\ \epsilon_2(\omega)&=\epsilon_\infty\omega_c\omega_p^2/\omega(\omega_c^2-\omega^2)\;,\\ \epsilon_3(\omega)&=\epsilon_\infty(1-\omega_p^2/\omega^2)\;.\end{aligned} \tag{2.2}$$

Here ω_p, ω_c, and ϵ_∞ are the plasmon frequency, cyclotron frequency, and background dielectric constant, respectively, in the layer under consideration.

The solutions of Maxwell's equations at the nth cell are

$$E_{xj}^{(n)}(z|k\omega)=A_{1j}^{(n)}\exp(-\alpha_j z)+A_{2j}^{(n)}\exp(\alpha_j z)\;, \tag{2.3}$$

$$E_{zj}^{(n)}(z|k\omega)=i[\lambda_{1j}A_{1j}^{(n)}\exp(-\alpha_j z)+\lambda_{2j}A_{2j}^{(n)}\exp(\alpha_j z)]\;, \tag{2.4}$$

$$H_{yj}^{(n)}(z|k\omega)=-i\omega\epsilon_0[\epsilon'_{1j}A_{1j}^{(n)}\exp(-\alpha_j z)-\epsilon'_{2j}A_{2j}^{(n)}\exp(\alpha_j z)]\;. \tag{2.5}$$

Here,

$$\alpha_j=\begin{cases}[k_x^2-\epsilon_j(\omega^2/c^2)]^{1/2} & \text{if } k_x>\epsilon_j\omega/c\\ i[\epsilon_j(\omega^2/c^2)-k_x^2]^{1/2} & \text{if } k_x<\epsilon_j\omega/c\;,\end{cases} \tag{2.6}$$

$$\epsilon_j=\epsilon_{1j}-\epsilon_{2j}^2/\epsilon_{1j}\;, \tag{2.7}$$

$$\lambda_{rj}=\frac{k_x\epsilon_{1j}\pm\alpha_j\epsilon_{2j}}{k_x\epsilon_{2j}\pm\alpha_j\epsilon_{1j}}\;, \tag{2.8}$$

$$\epsilon'_{rj}=\frac{\epsilon_{2j}[k_x^2(\epsilon_{1j}^2+\epsilon_{2j}^2)-2\alpha_j^2\epsilon_{1j}]\pm\alpha_j k_x\epsilon_{1j}(\epsilon_{2j}^2-\epsilon_{1j}^2)}{k_x(k_x^2\epsilon_{2j}^2-\alpha_j^2\epsilon_{1j}^2)}\;, \tag{2.9}$$

where the $+$ ($-$) sign is associated with the subscript $r=1$ (2).

The boundary conditions for the electromagnetic fields at the interfaces are that the component $E_{xj}^{(n)}(z|k\omega)$ is continuous across an interface, and that the magnetic field $H_{yj}^{(n)}(z|k\omega)$ is discontinuous across the interface, the discontinuity being due to the presence of a current density at the interface given by

$$J_{xy}^{(n)}(z|k\omega)=i\omega\epsilon_0\sigma_p E_{xj}^{(n)}(z|k\omega) , \quad (2.10)$$

where the subscript p reads e for electrons or h for holes. Here

$$\sigma_p=\frac{n_p e^2}{m_p^* \omega^2 \epsilon_0}, \quad p=e,h . \quad (2.11)$$

n_p is the carrier concentration per unit area and m_p^* is the effective mass for the charge carriers. Using these boundary conditions in the same way as in I we find, after some algebra, the following dispersion relation for the bulk-plasmon polariton of wave vector Q:

$$\cos QL=\tfrac{1}{2}\mathrm{Tr}\overleftrightarrow{\mathrm{T}} . \quad (2.12)$$

Here $\overleftrightarrow{\mathrm{T}}$ is a unimodular matrix which relates the coefficients of the electromagnetic fields in one cell to those in the preceding one. It is defined in the same way as in I provided we replace ϵ_j' by ϵ_{rj}' in the M_s' and N_s' matrices with $r=1$ (2) for the matrix elements M_{12} and N_{12} (M_{22} and N_{22}).

This dispersion relation is the generalization of the dispersion relation for bulk-plasmon polaritons found in I to include the presence of an external magnetic field. Another special case of interest is the dispersion relation

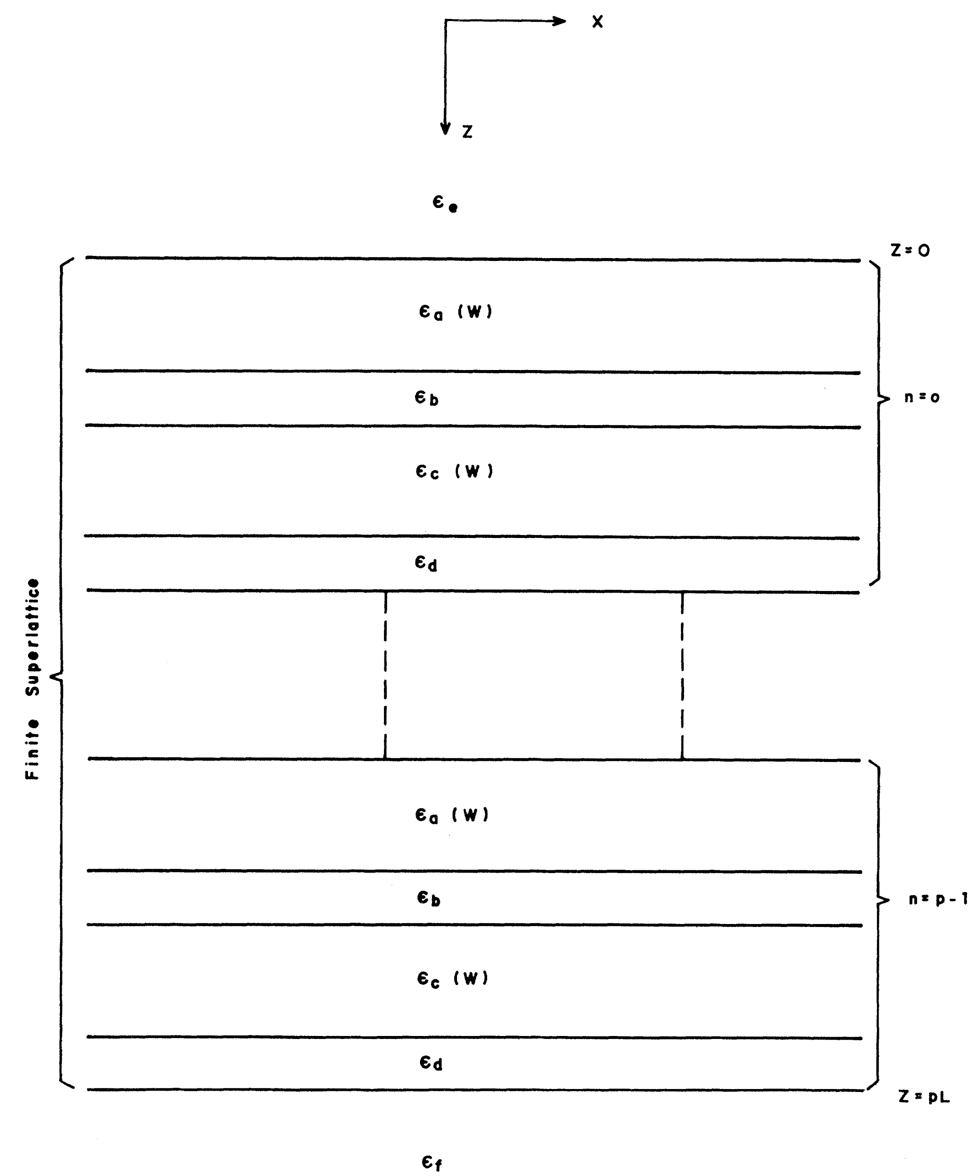

FIG. 2. Finite n-i-p-i superlattice whose length is pL, where p is an arbitrary integer and L the size of the unit cell.

obtained by Wallis *et al.*[11] for a periodic heterostructure consisting of two different types of dielectric slabs. Our dispersion relation reduces to their result for the limiting case $\sigma_e = \sigma_h \to 0$, $a \to c$, and $b \to d$.

B. Surface modes

In order to study surface modes we consider the structure shown in Fig. 2, where we replace the parts of the superlattice lying in the regions $-\infty \le z \le 0$ and $pL \le z \le \infty$ (p being an integer) by isotropic media E and F unaffected by the external field. So media E and F have dielectric tensors that are diagonal, with the diagonal elements all equal to ϵ_e and ϵ_f, respectively.

Obviously, the periodicity in the z direction is now destroyed and we can no longer use Bloch's theorem which relates the amplitude in one film to that in another through the envelope function $\exp(iQmL)$, m being the differences of the layers involved. Instead, we have the envelope functions $\exp(-\beta mL)$ and $\exp[-\beta(p-m)L]$ which correspond to localization at the top and bottom surface of the superlattice, respectively. Observe that for bulk waves one should replace β by $-iQ$.

Let us assume that the coefficients A_{1j} and A_{2j} ($j=a$, b, c, or d) which are related to the envelope function $\exp(-\beta mL)$ are independent of the coefficients A'_{1j} and A'_{2j} which are associated with the envelope function $\exp[-\beta(p-m)L]$. This assumption enables us to relate these coefficients in the eigenvalue equation of $\overleftrightarrow{T}$ (see I) provided we replace iQ by $-\beta$ and β, respectively. Thus

$$[\overleftrightarrow{T} - \exp(-\beta L)\overleftrightarrow{I}]\,|A_j^{(n)}\rangle = 0 \tag{2.13}$$

and

$$[\overleftrightarrow{T} - \exp(\beta L)\overleftrightarrow{I}]\,|A_j'^{(n)}\rangle = 0 \tag{2.14}$$

yielding

$$A_{2a} = KA_{1a} \quad \text{and} \quad A'_{2a} = K'A'_{1a} \tag{2.15}$$

with

$$K = \frac{\exp(-\beta L) - T_{11}}{T_{12}} \tag{2.16}$$

and K' identical to K providing we replace $\exp(-\beta L)$ by $\exp(\beta L)$. T_{11} and T_{12} are elements of the matrix $\overleftrightarrow{T}$.

Since we should know the coefficients A_{1d}, A_{2d}, A'_{1d}, and A'_{2d} in order to match the electromagnetic fields at $z=pL$, the boundary conditions at the A-D interface yield

$$\begin{aligned} &A_{1d}f_d + A_{2d}\bar{f}_d = \exp(-\beta L)(A_{1a} + A_{2a}) , \\ &A'_{1d}f_d + A'_{2d}\bar{f}_d = \exp(\beta L)(A'_{1a} + A'_{2a}) , \\ &\epsilon'_{1d}A_{1d}f_d - \epsilon'_{2d}A_{2d}\bar{f}_d \\ &\qquad = \exp(-\beta L)[(\epsilon'_{1a} - \sigma_p)A_{1a} - (\epsilon'_{2a} + \sigma_p)A_{2a}] , \\ &\epsilon'_{1d}A'_{1d}f_d - \epsilon'_{2d}A'_{2d}\bar{f}_d \\ &\qquad = \exp(\beta L)[(\epsilon'_{1a} - \sigma_p)A'_{1a} - (\epsilon'_{2a} + \sigma_p)A'_{2a}] . \end{aligned} \tag{2.17}$$

The electric and magnetic field in the region $z<0$ can be written as

$$\begin{aligned} E_x^{(e)}(z|k\omega) &= E_e \exp(\alpha_e z) , \\ H_y^{(e)}(z|k\omega) &= i\omega\epsilon_0\epsilon'_e E_e \exp(\alpha_e z) , \end{aligned} \tag{2.18}$$

while for $z>pL$

$$\begin{aligned} E_x^{f}(z|k\omega) &= E_f \exp(-\alpha_f z) , \\ H_y^{(f)}(z|k\omega) &= -i\omega\epsilon_0\epsilon'_f E_f \exp(-\alpha_f z) . \end{aligned} \tag{2.19}$$

Here

$$\alpha_i = (k_x^2 - \epsilon_i\omega^2/c^2)^{1/2}, \quad k_x > \epsilon_i\omega/c \text{ and } i=e \text{ or } f \tag{2.20}$$

and

$$\epsilon'_i = \epsilon_i/\alpha_i \quad \text{with } i=e \text{ or } f . \tag{2.21}$$

Imposing the boundary conditions at the E-A interface (see Fig. 2), $z=0$, we obtain

$$E_e = A_{1a} + A_{2a} + A'_{1a} + A'_{2a} , \tag{2.22}$$

$$\epsilon'_e E_e = -\epsilon'_{1a}(A_{1a} + A'_{1a}) + \epsilon'_{2a}(A_{2a} + A'_{2a}) . \tag{2.23}$$

The boundary conditions at the D-F interface, $z=pL$, yield

$$E_f \exp(-\alpha_f pL) = \exp[-\beta L(p-1)](A_{1d}f_d + A_d\bar{f}_d) + \exp[\beta L(p-1)](A'_{1d}f_d + A'_{2d}\bar{f}_d) , \tag{2.24}$$

$$\begin{aligned} \epsilon'_f E_f \exp(-\alpha_f pL) = {}& \epsilon'_{1d}f_d\{A_{1d}\exp[-\beta L(p-1)] + A'_{1d}\exp[\beta L(p-1)]\} \\ & - \epsilon'_{2d}\bar{f}_d\{A_{2d}\exp[-\beta L(p-1)] + A'_{2d}\exp[\beta L(p-1)]\} . \end{aligned} \tag{2.25}$$

Now using (2.25) and (2.27) we can reduce (2.32), (2.33), (2.34), and (2.35) to the convenient matrix equation shown below:

$$\begin{bmatrix} 1+k & 1+k' & -1 & 0 \\ \epsilon'_{1a} - K\epsilon'_{2a} & \epsilon'_{1a} - K'\epsilon'_{2a} & \epsilon'_e & 0 \\ (1+K)\exp(-\beta Lp) & (1+K')\exp(\beta Lp) & 0 & -\exp(-\alpha_f pL) \\ \lambda_a \exp(-\beta Lp) & \lambda'_a \exp(\beta Lp) & 0 & -\epsilon'_f \exp(-\alpha_f pL) \end{bmatrix} \begin{bmatrix} A_{1a} \\ A'_{1a} \\ E_e \\ E_f \end{bmatrix} = 0 . \tag{2.26}$$

Here

$$\lambda_a=\epsilon'_{1a}-K\epsilon'_{2a}-\sigma_p(1+K) \tag{2.27}$$

and λ'_a is equal to λ_a provided we replace K by K'. The condition that (2.36) has a nontrivial solution gives us the desired dispersion relation for the propagation of magnetoplasmon polaritons in a finite *n-i-p-i* superlattice. The result is

$$\exp(2\beta pL)=\frac{[\lambda_a-\epsilon'_f(1+K)][\epsilon'_{1a}-K'\epsilon'_{2a}+\epsilon'_e(1+K')]}{[\lambda'_a-\epsilon'_f(1+K')][\epsilon'_{1a}-K\epsilon'_{2a}+\epsilon'_e(1+K)]} . \tag{2.28}$$

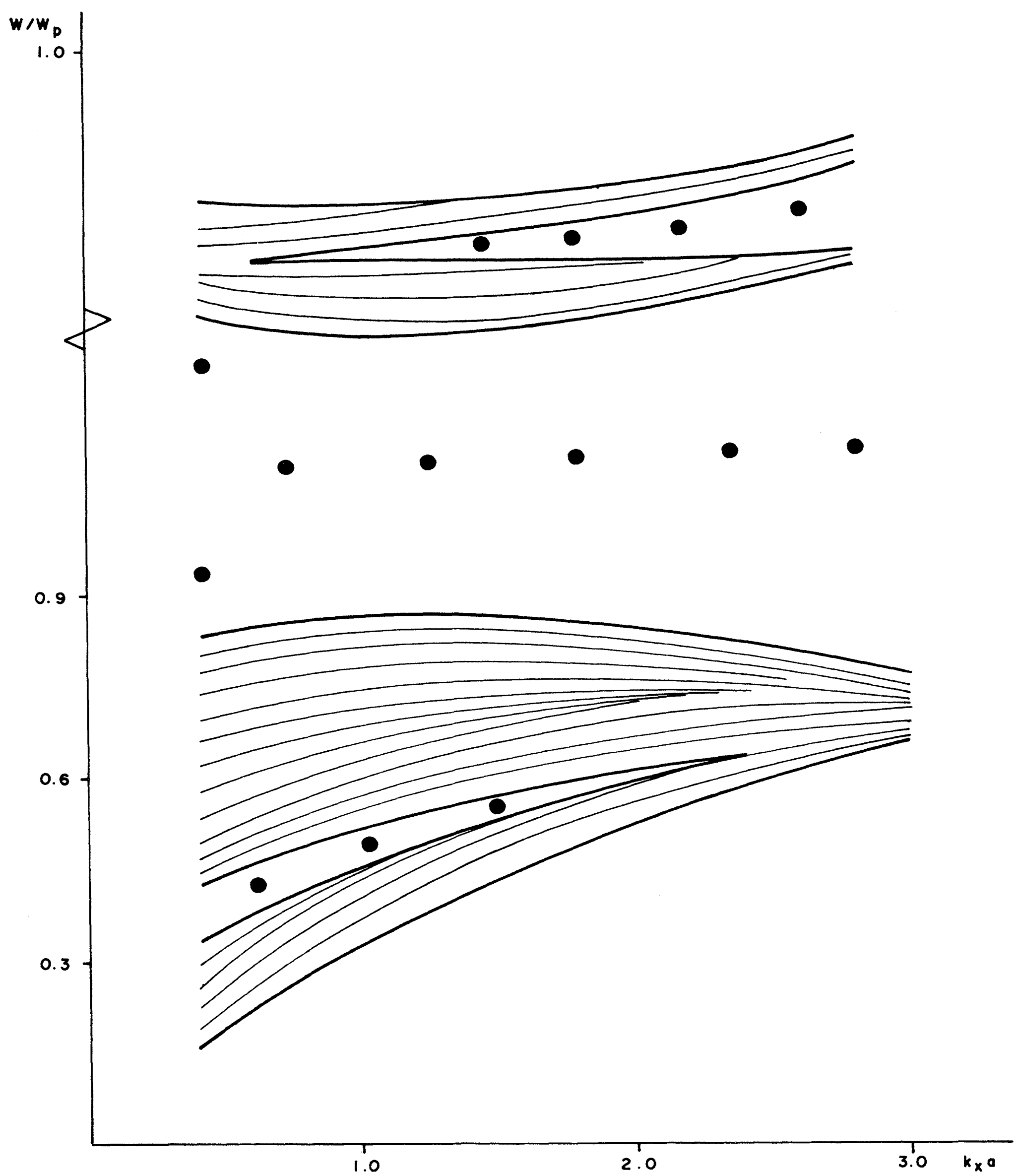

FIG. 3. Plasmon-polariton dispersion curves for a finite *n-i-p-i* superlattice in the absence of a magnetic field. The surface modes are here represented by dots. The thicknesses are $a=c=40$ nm and $b=d=20$ nm. Observe the change in the vertical scale at the position $\omega/\omega_p=0.9$.

III. NUMERICAL RESULTS

In this section we present some numerical studies of the surface magnetoplasmon-polariton dispersion relation for a finite *n-i-p-i* superlattice. The dielectric materials *A* and *C* are considered to be Si doped with *n* and *p* impurities.

We consider a lightly doped Si *n-i-p-i* structure which is referred to as a "conventional" *n-i-p-i* structure. Because of the light doping, the band structure of the Si crystal can be used to predict the electronic properties of the *n-i-p-i* sample and extremely long lifetimes for the carriers can be obtained.[14] Since we do not use highly doped semiconductors, we assume $\epsilon_a(\omega)=\epsilon_c(\omega)$ defined by

$$\epsilon(\omega)=\epsilon_L(1-\omega^2/\omega_p^2) , \tag{3.1}$$

where $\epsilon_L=11.7$ is the background dielectric constant and $\omega_p=7.65\times10^{13}$ s^{-1} is the electronic plasma frequency. The assumed doping level (electrons and holes) at the interfaces is equal to 2×10^{16} carriers/m^2. The dielectric materials *B* and *D* are taken as SiO_2 with dielectric constant $\epsilon_b=\epsilon_d=3.7$. Also it is considered that the finite superlattice is surrounded on all sides by vacuum, so that $\epsilon_e=\epsilon_f=1$. The applied magnetic field is measured through the cyclotron frequency ω_c which is taken as half of the value of the plasma frequency ω_p.

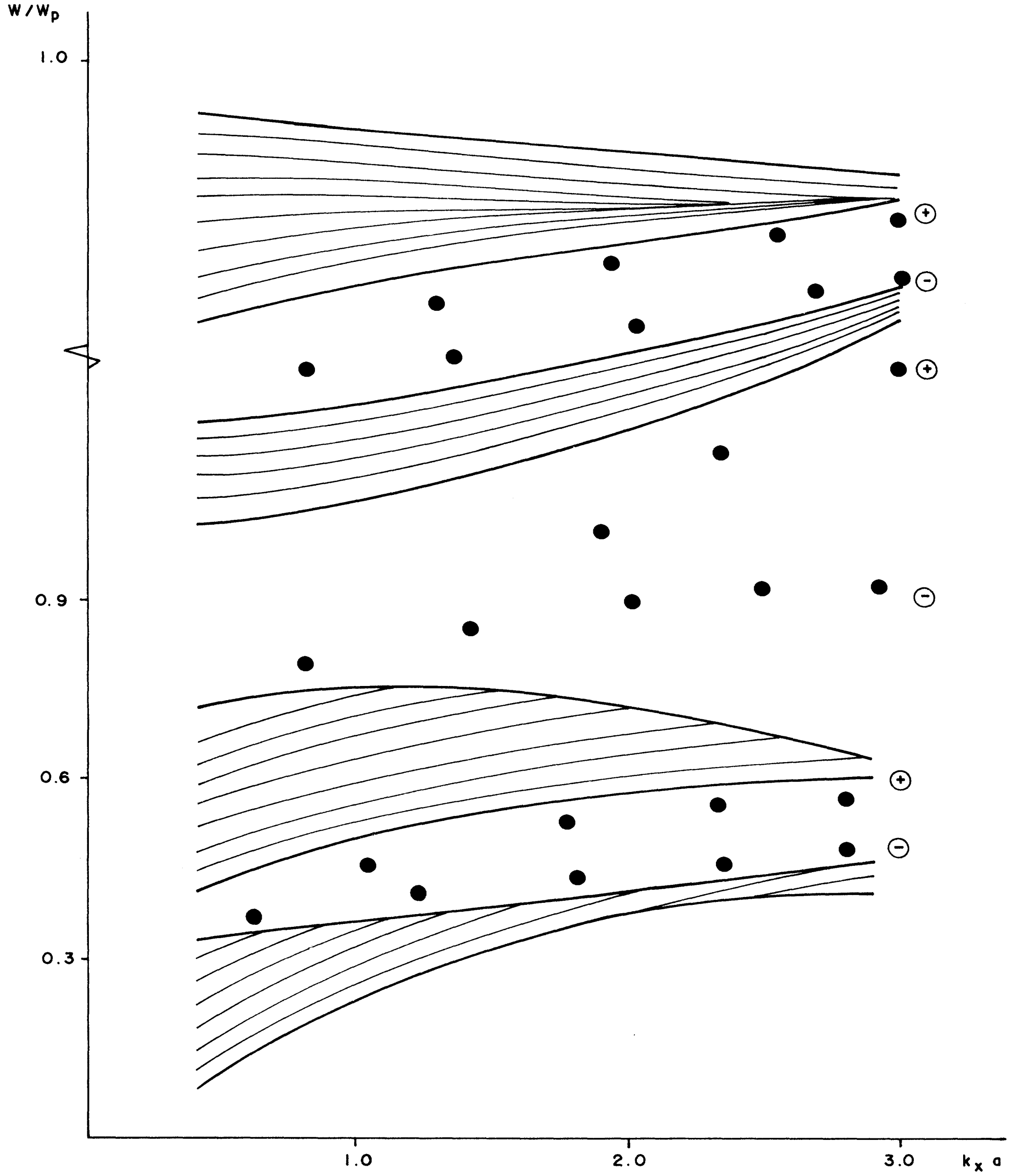

FIG. 4. The same as Fig. 3 when an external magnetic field of magnitude such that $\omega_c=\frac{1}{2}\omega_p$ is present.

The truncation of the superlattice yields surface-plasmon polaritons located at the surface but still affected by the layered structure of the bulk constituents. These modes occur in the gaps between the bulk bands as well as above and below the bands. For truncated superlattices of finite sizes, the boundaries of the bulk bands and the frequencies of the surface modes are in good agreement with those values found for the semi-infinite superlattice with the same parameters, for as few as 20 unit cells.[15] Therefore we assume that our superlattice has a size p equal to $20L$, L being the size of the unit cell. We have investigated two representative values of the thicknesses a, b, c, and d. The first case is $a=c=40$ nm and $b=d=20$ nm and Fig. 3 presents the plasmon-polariton curve in the absence of the magnetic field. We notice in Fig. 3 the quantization of the bulk continuum due to the finite thickness of the specimen. This quantization, as is known, becomes more widely spaced as p becomes smaller. There are four bulk bands with surface-plasmon modes (represented by dots) localized inside the gaps between them. Observe that for $k_x a \simeq 0.7$ the surface-plasmon mode, localized between the top and bottom bulk branches, splits into two modes. The reason is that the wavelength of the surface plasmon is then larger than the finite size of the superlattice. This effect is not observed in a semi-infinite superlattice.

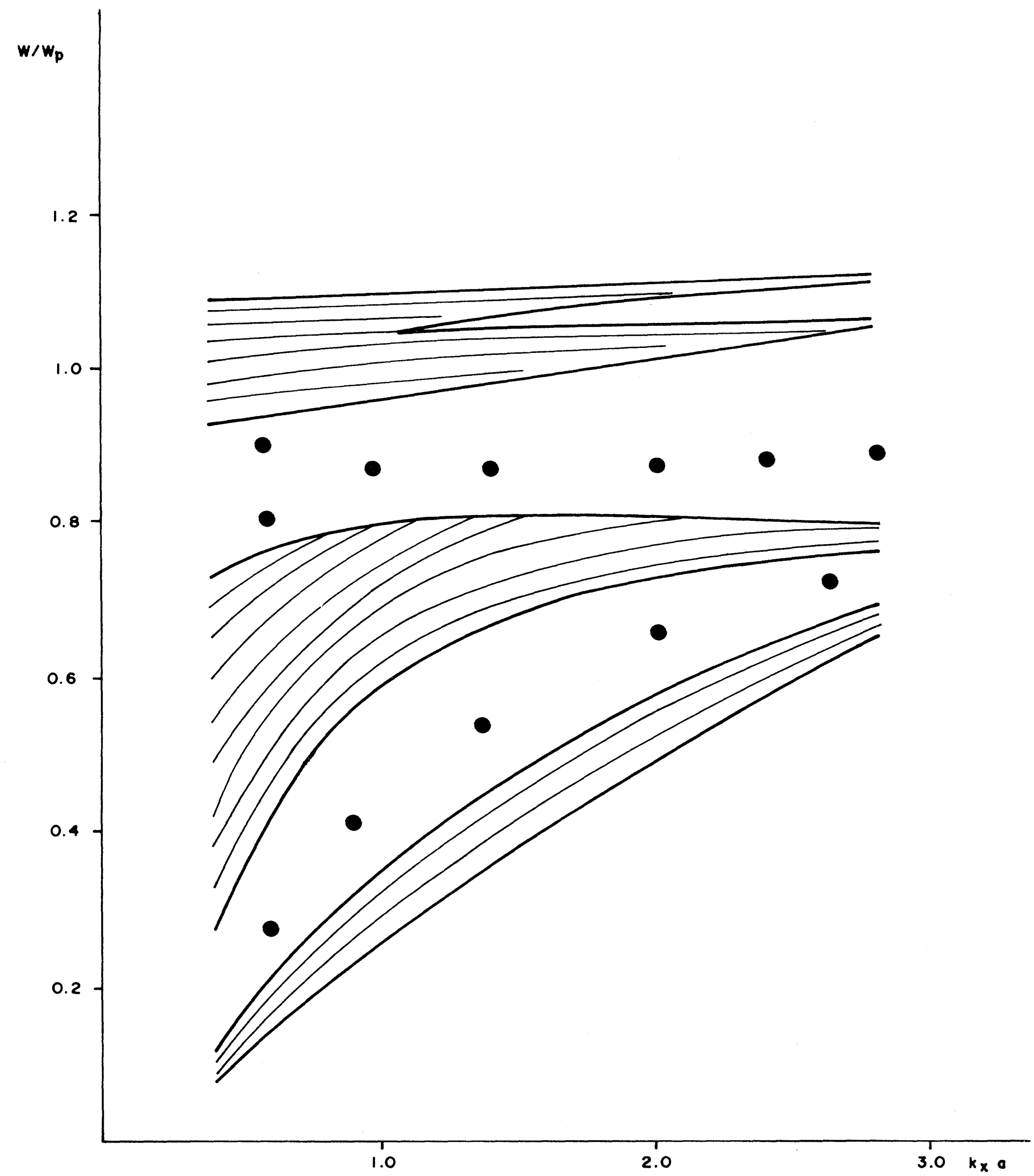

FIG. 5. Plasmon-polariton dispersion curves for a finite n-i-p-i superlattice in the absence of a magnetic field whose parameters are $a=b=c=d=20$ nm. The surface modes are again denoted by dots.

The surface plasmons that exist in the gap between the bulk bands are doubly degenerate corresponding to propagation in opposite directions for the wave vector **k**. When the magnetic field is switched on, this degeneracy is removed, and the surface mode split, giving rise to six surface magnetoplasmon-polariton branches indicated by the plus and minus signs in Fig. 4. Two of these modes are localized between the upper bulk bands (in the range $\omega/\omega_p \simeq 0.93$) and two more modes are localized between the lower bulk bands (in the range ω/ω_p between 0.3 and 0.5). All these four modes are associated, generally speaking, with $\beta L = i\pi + \chi$ with χ positive. These modes terminate when they enter the bulk bands at $k_x a \simeq 3.2$. The other remaining two modes are localized between the upper and lower bulk bands, corresponding to βL purely real and positive. They exist in the range $0.7 < \omega/\omega_p < 0.95$. The upper (+) branch enters the upper bulk bands at $k_x a \simeq 3.5$ while the lower one (−) terminates when it enters the lower bulk bands at $k_x a \simeq 4.2$.

The other case studied here corresponds to $a = b = c = d = 20$ nm, with the remaining parameters taken to be the same as in the preceding case. Figure 5 shows the spectrum for zero magnetic field, while the inclusion of a magnetic field is considered in the spectrum shown in Fig. 6. Since the surface modes of the plasmon are limited to frequencies lower than ω_p, there are only two surface modes in the spectrum in the absence of the magnetic field (see Fig. 5). Besides, as in the preceding case and for the same reason, the surface mode localized between the top and bottom bulk branches, which corre-

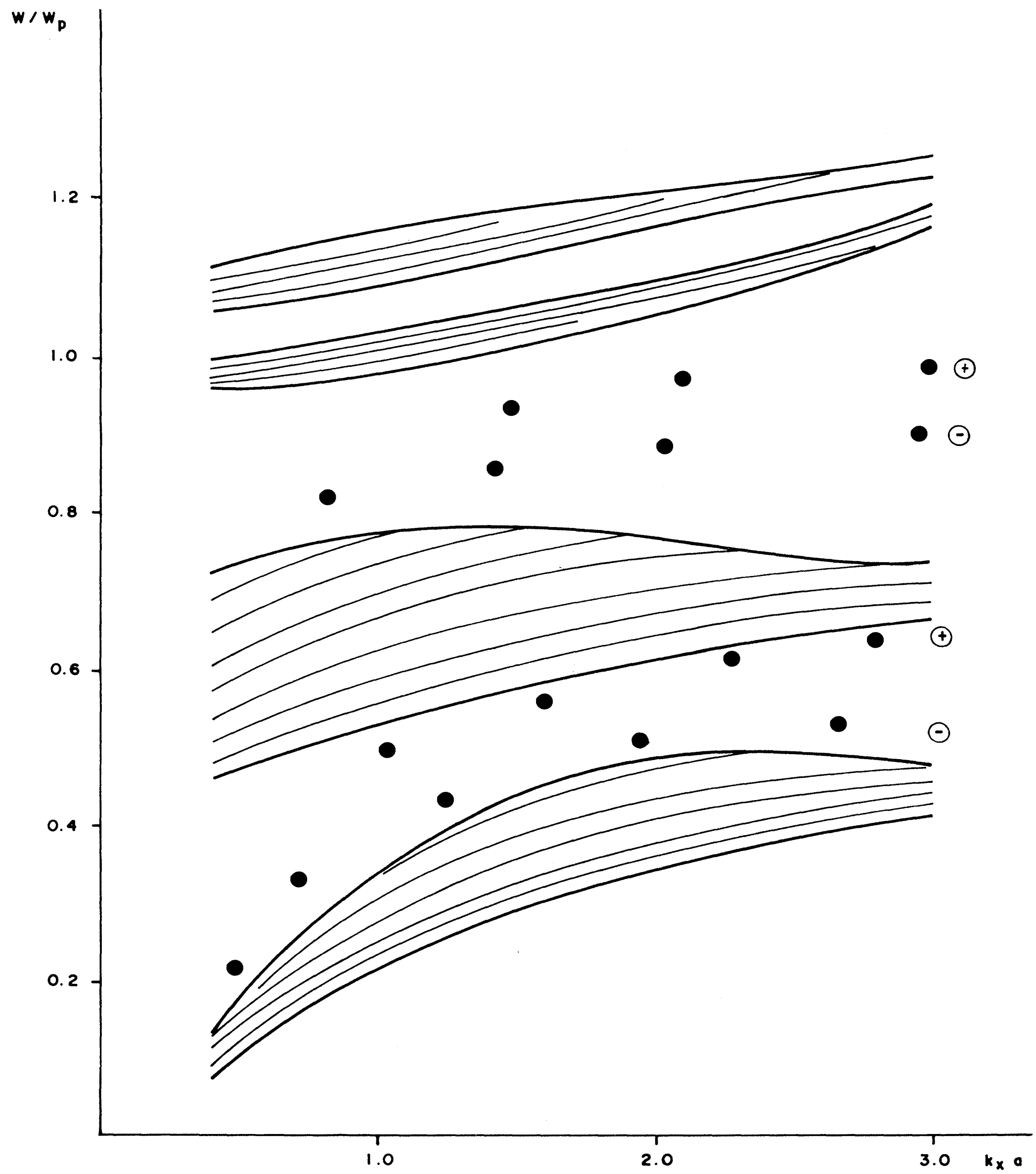

FIG. 6. The same as Fig. 5 for the case where an external magnetic field of magnitude such that $\omega_c = \frac{1}{2}\omega_p$ is present.

spond to βL purely real and positive, splits also into two branches for $k_x a \simeq 0.98$. With the presence of the external magnetic field (Fig. 6) there are four surface magnetoplasmon-polariton modes. Two of them are observed in the region $0.2 < \omega/\omega_p < 0.6$ and they terminate at $k_x a \simeq 3{,}0$, while the other two modes, which exist at $0.7 < \omega/\omega_p < 0.9$, enter the upper bulk bands for $k_x a$ approximately equal to 12. In all cases considered here, retardation effects are important only in the region of small values of $\mathbf{k}$.

IV. DISCUSSION

The results presented here generalize those of I in two major ways. First, the dispersion equation (2.38) applies to a specimen of finite thickness pL, rather than the semi-infinite geometry of I. Second, a static magnetic field is included, to give characteristic magnetoplasmon effects.

The effects of finite thickness, which are well known for polaritons in slabs of optically active materials,[16] are seen in Figs. 3 and 5. First the bulk continuum of the semi-infinite specimen is replaced by the discrete spectrum of guided waves. These correspond to a real Bloch wave vector within the superlattice which is quantized by the boundary conditions at the upper and lower surfaces of the superlattice. Second, for a thick specimen the surface polaritons at the top and bottom surfaces are essentially degenerate in frequency. As the thickness is decreased, the decaying-exponential envelopes of these degenerate modes begin to overlap, and the frequencies split into an antibonding and bonding pair. This effect is clearly seen in the intermediate-frequency mode of Figs. 3 and 5. Another important point can be inferred from (2.18) that can be written as $\exp(2\beta pL)=N/D$. As $p\to\infty$, i.e., the semi-infinite limit, $\exp(2\beta pL)$ tends to ∞ for $\beta>0$ and tends to zero for $\beta<0$. The different signs correspond to the surface modes localized on the upper and lower surfaces, so $N=0$ correspond to the dispersion relation of one of these modes while $D=0$ correspond to the other.

The application of an external magnetic field in thin films and multilayered heterostructures has been shown to cause interesting qualitative change in the behavior characteristic of the interface excitations.[17] In particular, the case studied here, where the field is parallel to the surface and propagation is perpendicular to the field, has as one important effect the introduction of nonreciprocal propagation,[8,18] in which the surface modes traveling in $+k_x$ and $-k_x$ directions have different frequencies. For the data used here, the differences between $\omega(k)$ and $\omega(-k)$ are too close to be resolved in the scale used in Figs. 4 and 6. This effect is better noted if the *n-i-p-i* superlattice is resting on a substrate, which serves to further lower the symmetry. In addition to introducing nonreciprocity of the surface modes, the magnetic field produces substantial shifts in the boundaries of the guided-wave regions, as can easily be seen by comparing the figures with and without the applied magnetic field.

The most promising experimental technique for the study of the magnetoplasmon modes is Raman scattering. Up to now, the majority of applications of Raman scattering have been to phonon-related properties,[19] and only a small proportion of the results has been concerned with plasma effects. Our analytical and numerical results give an indication of the rich spectrum of modes to be observed. Observe that so far no surface-plasmon mode has been observed experimentally for values of $k_x a$ up to approximately 1.38.[20] Also the Raman experiment of Olego *et al.*[21] has shown that the surface mode exists only for $k_x a > 1.7$.

Two other important techniques may be mentioned. First, the deposition of a grating on the specimen surface enables coupling of incident far-infrared radiation to surface and guided modes; in general the increase of wave number k_x by $2n\pi/d$ due to the grating period d is relatively large, so that retardation effects are unimportant. Second, attenuated total reflection (ATR) may be used to give a small advance of k_x, so that retarded surface modes are detected. A substantial body of results on phonon-related surface modes of superlattices,[22] and preliminary results on plasmon-related modes in a δ-doped sample,[23] show the potential power of this method.

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APPENDIX: *s* POLARIZATION

We have concentrated our results on p polarization, since this is expected to give the most striking effects. However, the derivation of the formal results for s polarization is very similar, so they are given here for completeness.

We have now

$$\mathbf{E}^{(n)}(\mathbf{x},t)=(0,E_y^{(n)}(z|k\omega),0)\exp[i(k_x x-\omega t)]\ , \tag{A1}$$

$$\mathbf{H}^{(n)}(\mathbf{x},t)=(H_x^{(n)}(z|k\omega),0,H_z^{(n)}(z|k\omega))\exp[i(k_x x-\omega t)]\ , \tag{A2}$$

with

$$E_{yj}^{(n)}(z|k\omega)=B_{1j}^{(n)}\exp(-\alpha_j z)+B_{2j}^{(n)}\exp(\alpha_j z)\ . \tag{A3}$$

The in-plane component of $\mathbf{H}^{(n)}$ is

$$H_{xj}^{(n)}(z|k\omega)=-(i\alpha_j/\mu_0\omega)[B_{1j}^{(n)}\exp(-\alpha_j z)-B_{2j}^{(n)}\exp(\alpha_j z)]\ . \tag{A4}$$

The field components are characterized by the vector

$$|B_j^{(n)}\rangle=\begin{bmatrix}B_{1j}\\ B_{2j}\end{bmatrix}\ . \tag{A5}$$

The transfer matrix can still be written as in I with matrices $\overleftrightarrow{\mathrm{M}}_a$, $\overleftrightarrow{\mathrm{N}}_b$, etc., given by

$$\overleftrightarrow{\mathbf{M}}_a = \begin{bmatrix} \exp(-\alpha_a a) & \exp(\alpha_a a) \\ (\alpha_a/\mu_0)\exp(-\alpha_a a) & -(\alpha_a/\mu_0)\exp(\alpha_a a) \end{bmatrix}, \tag{A6}$$

$$\overleftrightarrow{\mathbf{N}}_b = \begin{bmatrix} \exp(-\alpha_b b) & \exp(\alpha_b b) \\ \left[\dfrac{\alpha_b}{\mu_0} - \dfrac{n_h e^2}{m_p^*}\right]\exp(-\alpha_b b) & -\left[\dfrac{\alpha_b}{\mu_0} + \dfrac{n_h e^2}{m_p^*}\right]\exp(\alpha_b b)] \end{bmatrix}, \tag{A7}$$

and so on. With these changes, the dispersion equation for bulk modes is still the general form (2.12).

The analysis of surface modes is parallel to that given for p polarization. The result may be written in the form (2.38), provided the following changes are made: (a) $\epsilon'_{ra}(r=1,2)$ is replaced by α_a both in (2.28) and in the definitions of λ_a and λ'_a; and (b) K and K' are given by (2.16), but the T-matrix elements for s polarization are used.

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