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Analysis of guided electron waves in coupled quantum wells

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The propagation of guided electron waves in two coupled quantum wells is analyzed with use of coupled-mode equations. The conditions for energy transfer from one well to the other are examined and expressed in terms of coupling constants and propagation constants. The model developed in this work allows for the accurate determination of key structure parameters, such as the transfer length, and remains valid under strong-coupling conditions where the first-order model becomes inadequate. The analysis presented here should provide a useful basis for the design and evaluation of mesoscopic devices operating with coupled electron waves.

I. INTRODUCTION

Owing to the recent advances of semiconductor technologies, it has been possible to reduce the critical dimensions of devices to below the mean free path of the electrons and to increase the mean free path itself (on the order of a micrometer at low temperature) by suppressing scattering and using a small-electron-effective-mass material. Particularly, mesoscopic devices in which electrons can retain phase coherence while traveling through the active region have been demonstrated experimentally and have stimulated much theoretical interest in exploiting the true wave nature of electrons.[1-9] Like light, electron waves can be guided in quantum-well channels. The guided electron waves in adjacent channels can be coupled strongly or weakly depending on the thickness and/or the barrier height of the coupling layer. Such an electron-wave-coupling function opens up interesting and potentially important opportunities for device applications in quantum electronics and optoelectronics. For instance, a quantum transistor[4] and a quantum-directional-coupler switch[5] have been proposed. The operation of such devices differs intrinsically from that of classical transistors. Instead of turning the current on and off, one modulates the relative phase or the degree of coupling or the direction of the propagating waves. Very high switching speeds and transconductance could therefore be expected since the modulation involves little (if any) capacitive charging or current injection. Take the quantum-field-effect directional coupler proposed in Ref. 5 as an example. By biasing the gate clad on the coupling region and consequently tuning the coupling barrier height, one can change the coupling strength between two channels in close proximity and thereby change the transfer length (or coupling length) at which a complete transfer of electron wave from one channel to the other occurs. Since the physical dimension of the device's active region is fixed and the transfer length varies with the gate voltage, the portion of the electron wave transferred from the input channel to the output channel can be modulated by the gate voltage.

It is conceivable that such an entirely different operating principle may become the basis for the development of a new class of transistors—quantum-wave transistors. Clearly, accurate determination of the transfer length is crucial to the design and analysis of such devices. Naively, the approaches developed for optical-wave guiding could be adopted to calculate electron-wave coupling, recognizing the similarities between electron waves and light. Indeed, a recent attempt of calculating the transfer length based on a first-order model valid for weakly coupled waveguides[10] was described in Ref. 5. We will show, however, that under strong-coupling conditions the results of such calculations deviate significantly fom that obtained by a more rigorous model presented in this work.

Similar to most anlaytic coupled-mode models in optics,[10,11] the model presented here also only considers the first two normal modes. This approximation is introduced purely for the sake of simplicity in order to obtain closed-form equations. However, unlike most analytic models in optics, no other approximations are made in our model. Particularly, the coupling is not treated as a perturbation, and the guide does not have to be weakly guiding. In this sense it is an exact treatment of electron-wave coupling, and is applicable to the general problem of two-coupled waveguides (not necessarily symmetric or weakly coupled).

Although in carrying out the analysis one may take advantage of the similarities between light and electron waves, it should also be realized that there are some important differences. Particularly, photons and electrons have different dispersion relations reflecting the fact that the former has a zero mass and the latter has a finite mass. In the analysis, we will examine some of the consequences of wave coupling resulting from such differences. Also, light is a vector field while the electron wave is a scalar field. Typically, by approximations such as weak guiding,[11] the vector wave equations in optics are reduced to scalar equations. The fact that photons are bosons while electrons are fermions gives rise to an additional constraint for electron-wave coupling, that is, the transfer from one guide to the other has to obey the Pauli principle. In this work, however, we will confine the discussion

to a single electron system, and do no more than just mention this difference here.

II. GENERAL FORMULATION OF GUIDED WAVES IN TWO COUPLED QUANTUM WELLS

A coupled electron waveguide system comprising two quantum-well channels a and b formed by the confinement potential $V_c(x)$ is shown in Fig. 1. Also shown in Fig. 1 are the potentials, $V_a(x)$ and $V_b(x)$, corresponding to the individual uncoupled channel a and channel b. Not explicitly shown are the Ohmic contacts to the channels acting as electron reservoirs. In such a system an electron wave packet initially launched into one channel can be coupled to the other while propagating in the z direction. The motion of the electron wave is governed by the Schrödinger equation:

$$\left[\frac{-\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]+V_c(x)\right]\psi(x,y,z) = E\psi(x,y,z) , \quad (1)$$

where $\psi(x,y,z)$ is the electron wave function corresponding to the electron energy E. The electron waves guided by the quantum wells are assumed to be bounded in the following discussion. The rotation symmetry in the y-z plane warrants a simplification in notation. Namely, we may formally omit the variable y, and rewrite the wave function as $\psi(x,z)$ and the Schrödinger equation as

$$\left[\frac{-\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right]+V_c(x)\right]\psi(x,z)=E\psi(x,z) . \quad (1')$$

Further, the electron wave $\psi(x,z)$ propagating in the z direction can be expressed as

$$\psi(x,z)=\psi(x)e^{ik_z z} , \quad (2)$$

where k_z denotes the wave vector.

Substituting Eq. (2) into Eq. (1'), we have

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+V_c(x)\psi(x)=\left[E-\frac{\hbar^2k_z^2}{2m}\right]\psi(x) = E_x\psi(x) \quad (3)$$

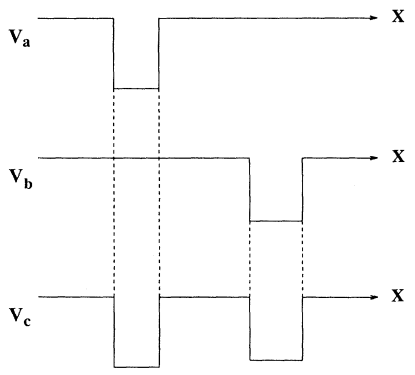

FIG. 1. Potential profile of quantum-well electron waveguides.

where $E_x=E-(\hbar^2k_z^2/2m)$ is the eigenenergy in the x direction which is quantized to a series of separated levels $E_{x1},E_{x2},\ldots$, etc. As mentioned in the Introduction, for simplicity, only the two lowest energy levels E_{x1},E_{x2} will be considered in the following. In other words, we consider the situation where the initial isolated mode (i.e., the input wave packet) launched into one channel excites the first two normal modes of the coupled system characterized by wave vectors k_1 and k_2:

$$\frac{\hbar^2k_1^2}{2m}=E-E_{x1} ,$$
$$\frac{\hbar^2k_2^2}{2m}=E-E_{x2} .$$

Thus the electron wave propagating in the z direction is a linear combination of the two normal modes:

$$\psi(x,z)=A_1\psi_1(x)e^{ik_1z}+A_2\psi_2(x)e^{ik_2z} , \quad (4)$$

where $\psi_1(x)$ and $\psi_2(x)$ are the eigenfunctions of Eq. (3) corresponding to the eigenvalues E_{x1} and E_{x2}, respectively, and A_1 and A_2 are arbitrary constants.

We can also express the electron wave function $\psi(x)$ of the coupled system in terms of the electron wave functions, $\psi_a(x)$ and $\psi_b(x)$, of the individual uncoupled quantum-well channels represented by $V_a(x)$ and $V_b(x)$ shown in Fig. 1. $\psi_a(x)$ and $\psi_b(x)$ satisfy the following equations:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_a(x)}{dx^2}+V_a(x)\psi_a(x)=\left[E-\frac{\hbar^2k_a^2}{2m}\right]\psi_a(x) = E_{xa}\psi_a(x) , \quad (5)$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_b(x)}{dx^2}+V_b(x)\psi_b(x)=\left[E-\frac{\hbar^2k_b^2}{2m}\right]\psi_b(x) = Ex_b\psi_b(x) . \quad (6)$$

Multiplying Eq. (5) by $\psi_b(x)$, Eq. (6) by $\psi_a(x)$, and subtracting one from the other, then integrating the resultant equation along x from $-\infty$ to $+\infty$, we have

$$(E_{xa}-E_{xb})\int_{-\infty}^{\infty}\psi_a(x)\psi_b(x)dx = \int_{-\infty}^{\infty}[V_a(x)-V_b(x)]\psi_a(x)\psi_b(x)dx - \frac{\hbar^2}{2m}\int_{-\infty}^{\infty}\left[\psi_b\frac{d^2\psi_a}{dx^2}-\psi_a\frac{d^2\psi_b}{dx^2}\right]dx . \quad (7)$$

The last term on the right-hand side of Eq. (7) equals zero, because $\psi_a=0=\psi_b$ at $x=\pm\infty$. Therefore we obtain

$$E_{xa}-E_{xb}=\frac{\hbar^2k_b^2}{2m}-\frac{\hbar^2k_a^2}{2m} = \frac{1}{\alpha}\int(V_a-V_b)\psi_a(x)\psi_b(x)dx = \frac{\alpha_{ab}-\alpha_{ba}}{\alpha} , \quad (8)$$

where $\alpha=\int_{-\infty}^{\infty}\psi_a(x)\psi_b(x)dx$ is the overlap integral of

the wave functions, and

$$\alpha_{ab}=\int (V_c-V_b)\psi_a(x)\psi_b(x)dx \ , \tag{9}$$

$$\alpha_{ba}=\int (V_c-V_a)\psi_a(x)\psi_b(x)dx \ . \tag{10}$$

Equation (8) defines the relation between the wave vectors, k_a and k_b, in the z direction of the individual quantum wells a and b.

Similarly, we can obtain the following two equations:

$$(E_x-E_{xa})\int \psi(x)\psi_a(x)dx = \int [V_c(x)-V_a(x)]\psi(x)\psi_a(x)dx \ , \tag{11}$$

$$(E_x-E_{xb})\int \psi(x)\psi_b(x)dx = \int [V_c(x)-V_b(x)]\psi(x)\psi_b(x)dx \ . \tag{12}$$

The electron wave function $\psi(x)$ of the coupled system can then be expressed as a linear combination of the eigenfunctions of the individual quantum wells:

$$\psi(x)=\sum_i A_{ai}\psi_{ai}(x)+\sum_j A_{bj}\psi_{bj}(x) \ , \tag{13}$$

where $\psi_{ai}(x)$ and $\psi_{bj}(x)$ are the eigenfunctions of Eqs. (5) and (6) corresponding to the eigenenergies $(E_{xa})_i$ and $(E_{xb})_j$ of the quantum wells a and b. For propagation in uncoupled individual wells, the total electron energy E should be greater than the corresponding quantum levels, that is,

$$E>(E_{xa})_i \ , \tag{14a}$$

$$E>(E_{xb})_j \ . \tag{14b}$$

As before, if we neglect the higher modes $(i,j>1)$, namely, assume that only the lowest energy level in each individual quantum level is occupied, we can simplify Eq. (13) to

$$\psi(x)=A_a\psi_a(x)+A_b\psi_b(x) \ , \tag{15}$$

where A_a and A_b are coefficients to be determined. The normalization of the wave function $\int \psi(x)\psi(x)dx=1$ gives

$$A_a^2+A_b^2+2A_aA_b\alpha=1 \ . \tag{16}$$

Substituting Eq. (15) into Eqs. (11) and (12), we have

$$(E_x-E_{xa})(A_a+A_b\alpha)=A_a\alpha_a+A_b\alpha_{ba} \ , \tag{17}$$

$$(E_x-E_{xb})(A_b+A_a\alpha)=A_b\alpha_b+A_a\alpha_{ab} \ , \tag{18}$$

where

$$\alpha_a=\int (V_c-V_a)\psi_a(x)\psi_a(x)dx \ , \tag{19}$$

$$\alpha_b=\int (V_c-V_b)\psi_b(x)\psi_b(x)dx \ . \tag{20}$$

The three unknown parameters A_a, A_b, E_x are thus determined completely by three equations (16), (17), and (18), and we obtain two sets of solutions as follows:

$$A_{a_1}=\left\{\frac{1}{2(1-\alpha^2)}\left[1-\left[1-\frac{4\alpha^2P_a^2(1-\alpha^2)}{1-2P_a(1-2\alpha^2)+P_a^2}\right]^{1/2}\right]\right\}^{1/2},$$
$$A_{b_1}=\left\{\frac{1}{2(1-\alpha^2)}\left[1-\left[1-\frac{4\alpha^2P_b^2(1-\alpha^2)}{1-2P_b(1-2\alpha^2)+P_b^2}\right]^{1/2}\right]\right\}^{1/2}, \tag{21}$$

for the lower energy E_{x1} of the coupled system:

$$E_{x1}=E_{xa}+(A_{a_1}\alpha_a+A_{b_1}\alpha_{ba})/(A_{a_1}+A_{b_1}\alpha) \ , \tag{22}$$

and

$$A_{a_2}=\left\{\frac{1}{2(1-\alpha^2)}\left[1+\left[1-\frac{4\alpha^2P_a^2(1-\alpha^2)}{1-2P_a(1-2\alpha^2)+P_a^2}\right]^{1/2}\right]\right\}^{1/2},$$
$$A_{b_2}=-\left\{\frac{1}{2(1-\alpha^2)}\left[1+\left[1-\frac{4\alpha^2P_b^2(1-\alpha^2)}{1-2P_b(1-2\alpha^2)+P_b^2}\right]^{1/2}\right]\right\}^{1/2}, \tag{23}$$

for the high energy E_{x2} of the coupled system:

$$E_{x2}=E_{xa}+(A_{a_2}\alpha_a+A_{b_2}\alpha_{ba})/(A_{a_2}+A_{b_2}\alpha) \ , \tag{24}$$

where

$$P_a=(\alpha_{ab}-\alpha\alpha_b)/(\alpha_{ba}-\alpha\alpha_a)=P_b^{-1} \ . \tag{25}$$

The two eigenfunctions $\psi_1(x)$ and $\psi_2(x)$ corresponding to the two solutions are orthogonal

$$\int \psi_1(x)\psi_2(x)dx=\int [A_{a_1}\psi_a(x)+A_{b_1}\psi_{(b)}(x)]\times[A_{a_2}\psi_a(x)+A_{b_2}\psi_b(x)]dx = A_{a_1}A_{a_2}+A_{b_2}A_{b_2}+\alpha(A_{a_1}A_{b_2}+A_{a_2}A_{b_1})=0 \ . \tag{26}$$

For a special case where the two quantum wells are identical, i.e., $V_a=V_b$ and $P_a=P_b=1$, we can easily see

what these two eigenfunctions (normal modes) are,

$$A_{a_1}=A_{b_1}=[2(1+\alpha)]^{-1/2}\ , \tag{27}$$

which means that $\psi_1(x)$ is a symmetric state (even function) with the lower energy

$$E_{x1}=E_{xa}+\frac{\alpha_a+\alpha_{ba}}{1+\alpha}\ , \tag{28}$$

which is smaller than E_{xa} due to $\alpha_{ba}<\alpha_a<0$. Therefore, in the coupled system propagation modes exist even if the total electron energy E is smaller than E_{xa} but greater than E_{x1}. This is different from the conditions given by Eq. (14) which applies to uncoupled wells.

For the higher normal mode, we have

$$A_{a_2}=-A_{b_2}=[2(1-\alpha)]^{-1/2}\ , \tag{29}$$

which indicates that $\psi_2(x)$ is an asymmetric state (odd function) with a higher energy

$$E_{x2}=E_{xa}+\frac{\alpha_a-\alpha_{ba}}{1-\alpha}>E_{xa}\ . \tag{30}$$

The difference between the E_{x1} and E_{x2} is

$$E_{x2}-E_{x1}=\frac{2}{1-\alpha^2}(\alpha\alpha_a-\alpha_{ba})>0\ . \tag{31}$$

Therefore the two wave vectors for the two normal modes in the coupled system are related:

$$k_1^2-k_2^2=\frac{4m}{(1-\alpha^2)\hbar^2}(\alpha\alpha_a-\alpha_{ba})>0\ . \tag{32}$$

Following a standard approach in optical-wave-guiding analysis,[10] we rewrite the wave $\psi(x,z)$ in Eq. (4) of the coupled system in terms of the waves of the uncoupled wells:

$$\begin{aligned}\psi(x,z)&=A_1[A_{a_1}\psi_a(x)+A_{b_1}\psi_b(x)]e^{ik_1z}\\&\quad+A_2[A_{a_2}\psi_a(x)+A_{b_2}\psi_b(x)]e^{ik_2z}\\&=L(z)\psi_a(x)+R(z)\psi_b(x)\ ,\end{aligned} \tag{33}$$

where

$$L(z)=A_1A_{a_1}e^{ik_1z}+A_2A_{a_2}e^{ik_2z}\ , \tag{34}$$

$$R(z)=A_1A_{b_1}e^{ik_1z}+A_2A_{b_2}e^{ik_2z}\ , \tag{35}$$

which describe the variation of the amplitudes of the wave components in wells a and b, respectively (Fig. 2). Furthermore, one can always write two general linear differential equations for $L(z)$ and $R(z)$:

$$\frac{dL}{dz}=iM_aL+i\kappa_{ab}R\ , \tag{36}$$

$$\frac{dR}{dz}=iM_bR+i\kappa_{ba}L\ . \tag{37}$$

The coupling constant κ_{ab} stands for the wave-coupling coefficient per unit length from well a to well b while propagating along the z direction. Conversely, κ_{ba} represents the wave coupling from well b to well a.

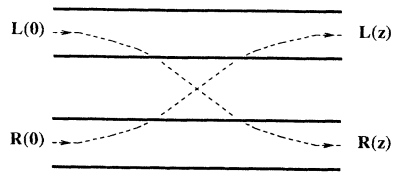

FIG. 2. Switched directional coupler consisting of two electron waveguides.

Substituting Eqs. (34) and (35) into Eqs. (36) and (37), we get

$$\begin{aligned}&(M_aA_{a_1}+\kappa_{ab}A_{b_1}-A_{a_1}k_1)A_1e^{ik_1z}\\&\quad+(M_aA_{a_2}+\kappa_{ab}A_{b_2}-A_{a_2}k_2)A_2e^{ik_2z}=0\ ,\end{aligned} \tag{38}$$

$$\begin{aligned}&(M_bA_{b_1}+\kappa_{ba}A_{a_1}-A_{b_1}k_1)A_1e^{ik_1z}\\&\quad+(M_bA_{b_2}+\kappa_{ba}A_{a_2}-A_{b_2}k_2)A_2e^{ik_2z}=0\ .\end{aligned} \tag{39}$$

Since the above equations (38) and (39) must be satisfied for any value of z, the four parentheses must be zero. The coupling constants are thus determined:

$$\kappa_{ab}=(k_1-k_2)A_{a_1}A_{a_2}/(A_{b_1}A_{a_2}-A_{a_1}A_{b_2})\ , \tag{40}$$

$$\kappa_{ba}=(k_1-k_2)(-A_{b_1}A_{b_2})/(A_{b_1}A_{a_2}-A_{a_1}A_{b_2})\ . \tag{41}$$

It can be seen that the coupling constants are directly proportional to the difference of the wave vectors of the two normal modes in the z direction. A difference in wave vectors of the modes means that as the two modes propagate, their relative phase reaches alternately between π and 2π. As a result, there will be alternating occurrences of maximum destructive interference in one well and constructive interference in the other between the two modes, as if the electron wave packet swings back and forth between the two wells while propagating along the wells. The above equations essentially state that the greater the wave-vector difference, the stronger the coupling and the sooner the wave packet transfers from one well to the other.

Together with the coupling constants we also find propagation constants:

$$M_a=(k_2A_{a_2}A_{b_1}-k_1A_{a_1}A_{b_2})/(A_{b_1}A_{a_2}-A_{a_1}A_{b_2})\ , \tag{42}$$

$$M_b=(k_1A_{a_2}A_{b_1}-k_2A_{a_1}A_{b_2})/(A_{b_1}A_{a_2}-A_{a_1}A_{b_2})\ . \tag{43}$$

From Eqs. (42) and (43), we find two useful relations

$$M_a+M_b=k_1+k_2 \tag{44}$$

and

$$\delta=M_b-M_a=(k_1-k_2)\frac{A_{b_1}A_{a_2}+A_{a_1}A_{b_2}}{A_{b_1}A_{a_2}-A_{a_1}A_{b_2}}\ . \tag{45}$$

The above general analysis establishes the basic rela-

tions between the normal modes of the coupled well system and the isolated modes of the individual wells. It allows us to calculate the coupling constants and propagation constants from basic physical quantities of the uncoupled modes. Furthermore, we note that so far only one approximation has been made for the sake of simplicity, that is, the inclusion of the first two modes only. Also, it can be verified that the above coupled-mode equations satisfy the basic physical laws such as particle conservation and particle flux conservation (see the Appendix). With these results we are now ready to address the main issue of this work, that is, under what conditions an electron wave can be transferred from one well to the other while propagating along the channel.

III. RESULTS FOR DIRECTIONAL COUPLING

Define $L'(z)$ and $R'(z)$ as

$$L(z)=L'(z)e^{iM_a z}, \tag{46}$$

$$R(z)=R'(z)e^{iM_b z}, \tag{47}$$

and from Eq. (33), we have

$$\psi(x,z)=L'(z)e^{iM_a z}\psi_a(x)+R'(z)e^{iM_b z}\psi_b(x) \tag{48}$$

and the coupled equations (36) and (37) become

$$\frac{dL'}{dz}=i\kappa_{ab}R'e^{i2\delta z}, \tag{49}$$

$$\frac{dR'}{dz}=i\kappa_{ba}L'e^{-i2\delta z}. \tag{50}$$

With the boundary conditions $L'(0)=L_0$, $R'(0)=R_0$, the solution of Eqs. (49) and (50) can be expressed as

$$\begin{bmatrix}L'(z)\\R'(z)\end{bmatrix}=\begin{bmatrix}C_{11}\gamma & iC_{12}\gamma\\ iC_{21}\gamma^* & C_{22}\gamma^*\end{bmatrix}\begin{bmatrix}L_0\\R_0\end{bmatrix}, \tag{51}$$

where $\gamma=e^{i\delta z}$ and

$$C_{11}=\cos fz-\frac{i\delta}{f}\sin fz=C_{22}^*, \tag{52}$$

$$C_{12}=\kappa_{ab}\frac{\sin fz}{f}, \tag{53}$$

$$C_{21}=\kappa_{ba}\frac{\sin fz}{f}, \tag{54}$$

with $f=(\delta^2+\kappa_{ab}\kappa_{ba})^{1/2}$.

The transfer matrix (51) is a very useful result for it describes the conditions for electron wave transfer from one well to the other as shown in Fig. 2. The matrix elements, given in Eqs. (52)–(54) and representing the interaction between the two wells, are functions of the structure parameters and the distance z in the propagation direction. For a given structure at a certain distance, an interesting situation, $C_{11}=C_{22}=0$, may happen. This corresponds to a complete energy transfer from one well to the other. Conversely, $C_{12}=C_{21}=0$ indicates that the wave packet has swung back to the well which it was originally launched into after traveling a distance. Since the matrix elements depend on the structure parameters which can be tuned by applied field, the power output from one particular channel (well) can thus be modulated by an external voltage.

To illustrate the process of energy transfer more clearly, let us consider a simple case in the following. From Eqs. (40), (41), (45), and (26), we find that two coupling constants satisfy the relation

$$\kappa_{ba}-\kappa_{ab}-2\alpha\delta=0. \tag{55}$$

Under the phase-matching condition, that is $\delta=0$, $M_a=M_b=(k_1+k_2)/2$, and we have

$$P_a=P_b=1, \tag{56}$$

consequently,

$$\alpha_{ab}-\alpha\alpha_b=\alpha_{ba}-\alpha\alpha_a \tag{57}$$

and

$$\kappa_{ab}=\kappa_{ba}=\kappa=\frac{k_1-k_2}{2}. \tag{58}$$

Corresponding to this condition is a coupled system of two identical wells or two wells having equal energy levels. The coupling constant κ can be determined by simply finding the wave vectors of the normal models of the coupled system.

The state of complete transfer from one channel to the other, i.e., $C_{11}=C_{22}=0$, occurs when

$$\kappa z=(2\nu+1)\frac{\pi}{2}, \tag{59}$$

where ν is an integer. Then the smallest length, referred to as the transfer length, at which the wave completely switches from one guide to the other, is

$$L_{\min}=\frac{\pi}{2}\Big/\kappa=\pi/(k_1-k_2)=\frac{\lambda_1\lambda_2}{2(\lambda_1-\lambda_2)}, \tag{60}$$

where $\lambda_1=2\pi/k_1$ and $\lambda_2=2\pi/k_2$ are the characteristic wavelengths of the two normal modes, respectively.

From the above equation, it is evident that after propagating a distance equal to $L_{\min}$ or its multiple along the channel, the phases of the two normal modes of the coupled system will differ by π. As a consequence, the two modes will have a constructive interference at $L_{\min}$ in one well and a destructive interference in the other. As mentioned before, subjected to an external bias the coupling constant κ and thus the transfer length $L_{\min}$ can be modulated. This is the base for the proposed quantum field-effect directional coupler.

Phenomenologically, all these are similar to optical directional coupling.[11] This comes as no surprise, considering the fact that the equation for optical-wave guiding has the same form as the Schrödinger equation for electron-wave guiding, Eq. (3), with the square of the propagation constant β^2 and the square of the product of the wave vector and the refractive index $k_0^2n^2(x)$ taking the places of the eigenvalue E_x in the former and the potential $V(x)$ in the latter.

However, it should be stressed that the two systems are intrinsically different, in addition to being different in the boundary conditions at the interfaces. The fact that the electron has a finite mass and its momentum along the channel must be conserved due to the translation symmetry indicates that a complete transfer is forbidden unless the two wells are either identical or have equal energy levels, i.e., $E_{xa}=E_{xb}$ (here and below, we assume that the effective-mass differences in the two wells can be neglected.) This can be readily verified by examining Eqs. (40), (41), (45), and (52), and it is in sharp contrast to the case of optical directional coupling where for a given asymmetric coupler there always exists a wavelength, referred to as the central wavelength, at which a complete transfer can occur.[12]

In addition to presenting the difference on physical grounds, we may also observe the difference by comparing the related terms in the two wave-guiding equations for electron and light. Whereas for electron-wave guiding the eigenvalue term E_x and the potential term $V(x)$ are fixed for a given structure, the equivalent eigenvalue term β^2 and potential term $k_0^2 n^2(x)$ for light-wave guiding depend on the structure as well as the wavelength of the input light. Therefore, for a given optical coupler of two asymmetric guides, the dispersion curve $\beta_a(\lambda)$ of the first guide may intersect with that of the second guide $\beta_b(\lambda)$ at a so-called central wavelength λ_0.[12] This means that the input light of this wavelength will be able to completely transfer from one guide to the other. For a given electron-wave coupler, however, the dispersion relation is

$$k_z=[2m(E-E_{xa})/\hbar^2]^{1/2} . \tag{61}$$

Notice that E_{xa} is a constant for a given well (independent of the electron energy). Therefore $k_{za}(E)$ and $k_{zb}(E)$ will never intersect at any energy and a complete transfer will never occur, unless E_{xa} equals E_{xb} in which case the $k_{za}(E)$ curve and $k_{zb}(E)$ curve coincide.

Moreover, the maximum transfer efficiency in the case of electron wave coupling, represented by $C_{12}=\kappa_{ab}/f$ (i.e., $\sin fz=1$), is independent of energy, as can be seen from Eqs. (40), (41), and (45), whereas the maximum transfer efficiency is a function of wavelength in the case of optical coupling.[10] The consequence is that for an asymmetric optical directional coupler there exists a narrow passband centered at the central wavelength, for an asymmetric electron-wave coupler such a filtering function does not exist.

IV. TRANSFER LENGTH CALCULATION UNDER STRONG- AND WEAK-COUPLING CONDITIONS

The above discussions and equations provide a sufficient basis for calculating the key parameters of an electron-wave coupling device. Since the operation and performance of a mesoscopic device are very sensitive to these parameters, it is critically important to determine them accurately. Although under weak-coupling conditions, the standard first-order model in optical-wave guiding[10] can be adopted as was done in Ref. 5, we will show below through an example that such a model becomes increasingly inadequate as the coupling becomes strong.

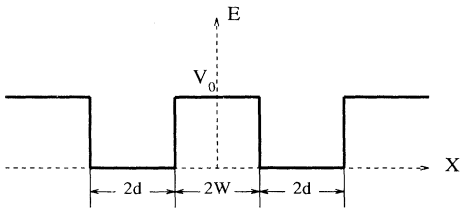

FIG. 3. A symmetric square quantum-well electron waveguide coupler.

We consider a finite square-well symmetric coupler as shown in Fig. 3. The well thickness is $2d$, the barrier height is V_0, the spatial separation of two wells is $2W$. We choose to calculate the transfer length as an example since it is a key parameter that determines the device physical dimension and performance. After some basic calculations based on the equations in Sec. II, one can obtain

$$\alpha=e^{-2k'w}\left[2k_x d\sin(k_x d)+e^{-2k'd}\cos(k_x d)+4\frac{E_{xa}}{V_0}\sin^2(k_x d)\right]\Big/(1+k'd) , \tag{62}$$

$$\alpha_{ab}=-2E_{xa}\sin^2(k_x d)e^{-2k'w}/(1+k'd) , \tag{63}$$

$$\alpha_a=-V_0 e^{-2k'(d+2W)}\cos^2(k_x d)sh(2k'd)/(1+k'd) , \tag{64}$$

where $k_x=(2mE_{xa}/\hbar^2)^{1/2}$ and $k'=[2m(V_0-E_{xa})/\hbar^2]^{1/2}=k_x tg(k_x d)$.

From Eqs. (28), (30), and (58), we have the coupling

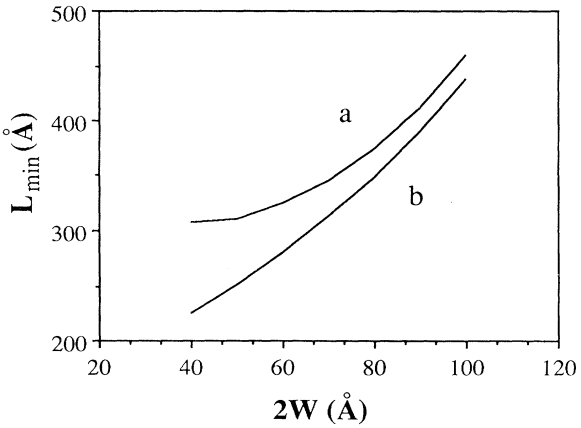

FIG. 4. Calculated transfer length L_{min} shown as a function of the barrier thickness $2W$. Curve a calculated according to the strong-coupling Eq. (66). Curve b calculated according to the weak-coupling approximation Eq. (67).

constant κ as

$$\kappa=\left[\frac{m}{2\hbar^2}\right]^{1/2}\left[\left[E-E_{xa}-\frac{\alpha_a+\alpha_{ba}}{1+\alpha}\right]^{1/2}-\left[E-E_{xa}-\frac{\alpha_a+\alpha_{ba}}{1-\alpha}\right]^{1/2}\right]$$
$$=\frac{k_z}{2}\left[\left[1-\frac{2m}{\hbar^2k_z^2}\frac{\alpha_a-\alpha_{ba}}{1+\alpha}\right]^{1/2}-\left[1-\frac{2m}{\hbar^2k_z^2}\frac{\alpha_a-\alpha_{ba}}{1-\alpha}\right]^{1/2}\right] . \quad (65)$$

Equation (65) is an exact expression for κ. Usually,

$$\frac{\hbar^2k_z^2}{2m}\gg\frac{\alpha_a+\alpha_{ba}}{1+\alpha}\quad\left[\text{or }\frac{\alpha_a-\alpha_{ba}}{1-\alpha}\right],$$

and we can have an approximate solution

$$\kappa=\frac{m}{\hbar^2k_z}\frac{\alpha\alpha_a-\alpha_{ba}}{1-\alpha^2} , \quad (66)$$

which is still valid for strong coupling. For weak coupling, i.e., $\alpha^2\ll 1$ and $\alpha\alpha_a<\alpha_{ab}$, we have

$$\kappa=-\frac{m}{\hbar^2k_z}\alpha_{ab}=\frac{2m}{\hbar^2k_z}E_{xa}\sin^2(k_xd)e^{-2k'w}/(1+k'd) , \quad (67)$$

which is proportional to the energy level of the individual quantum wells, and can be obtained directly from the first-order model.[10]

For GaAs/$Ga_{1-x}Al_xAs$ quantum-well coupler proposed in Ref. 5, $m=0.07m_e$, $k_z=8.4\times10^5$ cm^{-1}, $V_0=10$ meV and $2d=200$ Å. The transfer length $L_{\min}$ as a function of barrier thickness $2W$ is calculated and plotted in Fig. 4. Curve a corresponds to the strong-coupling result from Eq. (66) and curve b corresponds to the weak-coupling formula (67).

One can see that the difference between the rigorous mode developed in this work and the weak-coupling approximation increases rapidly as the barrier width reduces and the system becomes more strongly coupled. We also note that the projected transfer length falls within the range of typical electron mean free path in GaAs, indicating the feasibility of experimental observation.

In summary, we have established a rigorous theoretical formalism for guided electron waves in coupled quantum-well waveguides, which can be applied to the studies of the various electron-wave guiding phenomena. Some of the approaches and results may be applied to optics as well. We have examined the difference between the electron-wave coupling and optical wave coupling. We find that if the effective-mass difference between the wells can be neglected a complete transfer is forbidden unless the two wells are either identical or having equal energy levels, i.e., $E_{xa}=E_{xb}$. This is in sharp contrast to the case of optical directional coupling where for a given symmetric or asymmetric coupler there exists a so-called central wavelength at which a complete transfer can occur. The model developed in this work allows accurate determination of key structure parameters such as the coupling constants, propagation constants, and transfer length, and it remains valid under strong-coupling conditions where the first-order model widely used in literature becomes inadequate.

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APPENDIX

In order to check whether the solutions of the coupled-wave equations satisfy particle conservation and particle-flux conservation requirements, we start with Eqs. (48) and (51) and express the particle density as

$$\int dx\,\psi^*(x,z)\psi(x,z)=L_0^2+R_0^2+2\alpha L_0R_0+\frac{\sin^2fz}{f}(\kappa_{ba}-\kappa_{ab}-2\alpha\delta)(\kappa_{ba}L_0^2-\kappa_{ab}R_0^2+2\delta L_0R_0) . \quad (A1)$$

From Eq. (55), we have

$$\int dx\,\psi^*(x,z)\psi(x,z)=L_0^2+R_0^2+2\alpha L_0R_0 , \quad (A2)$$

which does not change with z. Using Eq. (55), we can show that the flux along the z direction is

$$j_z=\int\Psi^*(x,z)\left[-\frac{i\hbar}{m}\frac{\partial}{\partial_z}\right]\Psi(x,z)dz=\frac{\hbar}{m}[(M_a+\alpha\kappa_{ba})L_0^2+(M_b+\alpha\kappa_{ab})R_0^2+(\kappa_{ab}+\kappa_{ba}+\alpha M_a+\alpha M_b)L_0R_0] , \quad (A3)$$

which is indeed conserved.

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