Superconductivity and dissipation in small-diameter Pb-In wires

N. Giordano
Department of Physics, Purdue University, West Lafayette, Indiana 47907-1301
(Received 20 June 1990)

Superconductivity in very small-diameter Pb-In wires has been studied through measurements of the resistance in the low-current limit, and the voltage-current characteristics. These properties were measured as functions of temperature and magnetic field, and in the presence of microwave radiation. The results are analyzed in terms of several different theoretical models. Predictions based on possible granularity and the Coulomb blockade, on the effects of mesoscopic fluctuations, and on the phenomena of thermal and quantum phase slip, are all considered. It appears that over most of the range which has been studied the behavior is best described by phase slip, and evidence for both thermally activated and quantum phase slip is found. The behavior during a phase-slip event is closely analogous to the motion of a particle in a multiwell potential. In this picture, thermally activated phase slip occurs when the particle is thermally activated over the potential barrier which separates adjacent wells, while quantum phase slip corresponds to tunneling of the particle between wells. We have also observed behavior which implies the existence of discrete energy levels of the particle within a well. While most of the results can be understood at least qualitatively with use of the phase-slip picture, certain aspects remain unexplained.

I. INTRODUCTION

The nature of dissipation in a one-dimensional superconductor below T_c is a problem which first attracted attention more than twenty years ago. Little[1] pointed out that a dissipative process, now known as thermally activated phase slip, could occur at all temperatures below T_c, with the result that a one-dimensional superconductor would have a vanishing resistance only at $T=0$. This problem was subsequently discussed by Langer and Ambegaokar (LA),[2] and McCumber and Halperin (MH),[3] who developed a quantitative theory of this process based on Ginzburg-Landau theory.

A real system will behave one dimensionally as far as superconductivity is concerned if its diameter is less than the coherence length ξ. The value of ξ is material dependent, but is typically of order 500–1000 Å or larger at $T=0$, and diverges as $T\rightarrow T_c$. It is not difficult to produce systems with diameters smaller than this, and hence it is quite feasible to study one-dimensional superconductors experimentally. A number of such experiments[4–7] were performed shortly after the theoretical work mentioned above, and the results were in reasonably good agreement with the LA-MH theory. Those experiments employed samples which were typically $\gtrsim 5000$ Å in diameter. In the past two years, we have reported studies[8–11] of systems with diameters as much as a factor of 30 smaller than this, and have obtained results which cannot be understood solely in terms of the thermal activation model of LA-MH. We have successfully interpreted the experiments by assuming that in addition to thermally activated phase slippage, quantum phase slip also takes place. This process is closely related to the phenomenon of macroscopic quantum tunneling (MQT) which has been much discussed in recent years with regards to Josephson junctions.[12–20]

In the present paper we present new results for Pb-In alloy samples which are significantly smaller than the In wires we have studied previously. The experiments reported here are an extension of our earlier work, which consisted of measurements of the resistance R in the limit of low current as a function of temperature, and the voltage-current (V-I) characteristics at different temperatures. In our latest measurements we have studied the V-I characteristics in much greater detail, and have also examined the effect of magnetic and microwave fields. In this paper we compare our results with different explanations which have been suggested, with special attention to very recent theoretical work. Explanations which have been proposed as alternatives to the quantum-phase-slip model are also considered. As we will see, the quantum-phase-slip picture is reasonably consistent with most aspects of the experiments, although certain features of our results are still not understood. A preliminary account of some of the work reported below was given in Ref. 10.

II. THEORY

The behavior a one-dimensional superconductor is, within the framework of Ginzburg-Landau theory, closely analogous to the problem of a particle moving in a multiwell potential.[2,3,21,22] A stable, current-carrying superconducting state corresponds to the particle being trapped near a metastable minimum of the potential. The different minima correspond to different values of $\Delta\phi$, where $\Delta\phi$ is the (Ginzburg-Landau) phase difference across the system. Finite voltage states are associated with a moving particle, as the voltage is proportional to the particle "velocity." Any mechanism by which the particle can escape from the vicinity of a potential minimum will lead to a finite voltage, and thus dissipation. The LA-MH model deals specifically with thermal activation of the particle over the barriers which separate potential minima. Motion from one minimum to a neighboring one changes $\Delta\phi$, and this process is therefore known as thermally activated (TA) phase slip. The LA-MH theory leads to a resistance which in the limit of low current is

$$R_{\rm TA}=\frac{\Phi_0\Omega}{I_1}\exp(-\Delta F_0/k_B T)\ . \tag{1}$$

Here $\Phi_0 \equiv hc/2e$ is the flux quantum, and ΔF_0 is the magnitude of the energy barrier, with[2]

$$\Delta F_0=\sqrt{2}H_c^2\xi\sigma/3\pi\ . \tag{2}$$

H_c is the (bulk) critical field, ξ is again the coherence length, σ is the cross-sectional area, and $I_1=k_B T/\Phi_0$. The attempt frequency Ω is given by[3]

$$\Omega=\frac{\sqrt{3}L}{2\pi^{3/2}\xi\tau_{\rm GL}}\left[\frac{\Delta F_0}{k_B T}\right]^{1/2}, \tag{3}$$

where L is the length of the system, and $\tau_{\rm GL}$ is a parameter which arises in the time dependent Ginzburg-Landau equation near T_c. In this case the time evolution of the order parameter is diffusive, with a characteristic time scale[23-25]

$$\tau_{\rm GL}=\frac{\pi\hbar}{8k_B(T_c-T)}\ . \tag{4}$$

In previous work we have found that the thermal activation prediction (1) can only account for the behavior near T_c. The experiments imply that a different dissipative mechanism, which yields a much larger phase-slip rate, is dominant at temperatures more than ≈ 0.2 K below T_c (for samples composed of materials such as In and Pb, which have $T_c \sim 3-7$ K). A number of workers[26,21] have considered the possibility of quantum phase slip, according to which the fictitious particle tunnels through the potential barrier. This is often referred to as macroscopic quantum tunneling[12-14] since it involves the coherent "motion" of the electron condensate in a volume of order ($\xi\sigma$), which typically contains $\sim 10^8$ or more electrons. In recent work,[8,11] we proposed a qualitative form for the quantum tunneling rate, based largely on analogy with the work of LA and MH, and with previous work on MQT in Josephson junctions. We were led to an expression of the form

$$R_{\rm MQT}=\frac{\Phi_0^2\beta_1\tau_{\rm GL}}{\hbar}\frac{L}{\xi}\left[\frac{\Delta F_0}{\hbar\tau_{\rm GL}}\right]^{1/2}\exp\left[-\beta_2\frac{\Delta F_0\,\tau_{\rm GL}}{\hbar}\right], \tag{5}$$

where β_1 and β_2 are constants of order unity. In general, both thermal activation and quantum tunneling will be present, and to a first approximation the total resistance should be the sum of (1) and (5).

In our previous work, we have analyzed the results using the thermal activation and quantum tunneling predictions given in (1) and (5), and for comparison we will do so again in this paper. However, we will also compare the measurements with recent theoretical work which has treated the quantum-phase-slip process quantitatively. Saito and Murayama[21,22] have discussed quantum phase slip using time-dependent Ginzburg-Landau theory. They have discussed the behavior for small currents and for currents just below the critical current, at both low and high temperatures. They have drawn a careful distinction between the cases of small and large dissipation. Physically, "large" dissipation means that the coupling of the system to outside degrees of freedom is dominant, leading to diffusive time evolution of the order-parameter, and it turns out that (5) is implicitly in this regime. When the dissipation is weak, the order parameter motion is "oscillatory." Saito and Murayama predicted that these two limits are determined by the relative sizes of $\tau_{\rm GL}$ and τ_0, where

$$\tau_0=\sqrt{3}\xi/v_F\ . \tag{6}$$

In the high dissipation limit one has $\tau_{\rm GL} \gg \tau_0$, while the ballistic case corresponds to the opposite inequality. Since $\tau_{\rm GL}\sim(T_c-T)^{-1}$, while $\tau_0\sim(T_c-T)^{-1/2}$, there will be a crossover from low to high dissipation as $T\rightarrow T_c$. This is predicted to occur at $\approx 0.3T_c$, which is in the experimentally accessible regime.

Saito and Murayama predict a resistance due to quantum tunneling which is of the form

$$R\sim\exp(-B)\ , \tag{7}$$

where

$$B\approx\frac{H_c^2\xi\sigma\tau_{\rm GL}}{4\pi\hbar}\ , \tag{8}$$

for the case of large dissipation, while in the opposite limit

$$B\approx\frac{H_c^2\xi\sigma\tau_0}{4\pi\hbar}\ . \tag{9}$$

The result (8) is very similar to (5), while (9) differs in the temperature dependence of the factor B.

The theory of thermal activation developed by LA and MH, and the work on quantum tunneling by Saito and Murayama are based on the usual time-independent and time-dependent Ginzburg-Landau (GL) theories. It has recently been pointed out[27] that these conventional forms of GL theory may not be appropriate for describing a phase-slip process. The usual derivation of GL theory assumes[23-25] that the external degrees of freedom (the quasiparticles, etc.) are always in thermal equilibrium with the system. However, a phase-slip process will occur on a time scale[27] which is presumably of order $\hbar/\Delta$, where Δ is the energy gap. This time scale will approach infinity as $T\rightarrow T_c$, and hence sufficiently close to T_c the time required for the quasiparticles to come into equilibrium will be shorter than $\hbar/\Delta$, and the usual GL theory can be used. However, it is estimated[27] that this is only the case within about $\sim 10^{-3}$ K of T_c. At temperatures farther from T_c (which will be of interest to us here) the quasiparticles are unable to respond during a phase-slip event. It is thus more appropriate to assume[27] that the quasiparticles are "frozen" during the phase-slip process, and this leads to a GL theory which has a somewhat different form than the conventional one. The time evolution of the order parameter is in this case oscillatory, since the quasiparticles are not able to exchange energy with the system during the phase-slip process; the characteristic time scale for order-parameter evolution is τ_0. In addition, the energy barrier has a somewhat different form than that found by LA, (2). As a result, *both* the thermal-activation *and* quantum-phase-slip rates are affected. The contribution of thermal activation (TA) to the resistance has the form

$$R_{\mathrm{TA}} \approx \frac{1}{\tau_0}\frac{L}{\xi}\exp(-\Delta F_{\mathrm{FQ}}/k_B T) , \tag{10}$$

which is similar to (1), except that the energy barrier [in frozen-quasiparticle (FQ) theory] is now[27]

$$\Delta F_{\mathrm{FQ}} \approx 1.1\left[1-\frac{T}{T_c}\right]^{-1/4}\Delta F_0 , \tag{11}$$

where ΔF_0 is the barrier calculated by LA, (2), using the conventional GL theory. The quantum-phase-slip rate is given by

$$R_{\mathrm{MQT}} \approx \frac{1}{\tau_0}\frac{L}{\xi}\exp(-\gamma\Delta F_{\mathrm{FQ}}\tau_0/\hbar) , \tag{12}$$

where $\gamma = 3\sqrt{3}\pi/2$ is a constant factor. We also note that the prefactors in (10) and (12) are only qualitative estimates;[27] however, they should be sufficient for our purposes, since the exponential factors dominate the temperature dependence in both cases.

In the next section, we will compare these different predictions with our results. The frozen-quasiparticle approach appears to be the appropriate one, and it will therefore play a primary role in our discussions in the remainder of this paper. However, since in our previous work we used the older predictions (1) and (5), we will also compare our results with those expressions, in order to determine if the experiments can distinguish between them and the predictions of the frozen-quasiparticle model. We will find that the experiments can be described reasonably well by either theory.

In addition to our measurements of the resistance at low currents, we have also studied the voltage-current characteristics at currents below the "mean-field" critical current I_c. According to Ginzburg-Landau theory, I_c is related to the energy barrier by

$$I_c = \pi\left[\frac{2}{3}\right]^{1/2}\frac{\Delta F_0}{\Phi_0/c} . \tag{13}$$

The physical meaning of I_c can be seen from Fig. 1, which shows schematically the free energy of the system as a function of current. This result follows from the usual GL free energy;[2] the minima correspond to stable current-carrying states, for which the order parameter has the form $\psi = f\exp(i\phi)$, with the phase $\phi = \kappa z$, where z is position along the system and κ is a function of the current. Note that for convenience we have followed LA and MH, and assumed periodic boundary conditions for ψ. The wave vectors κ of these uniform solutions for ψ are then quantized, and the different superconducting states correspond to different integral numbers of turns in helix[2] which describes the spatial variation of ψ. The free energies of the zero-voltage states (the minima in Fig. 1) are a quadratic function of the current. The oscillations of the potential (i.e., free energy) correspond to the cost required for the system to move from one superconducting state to another (remove a loop of the helix), i.e., a phase slip.

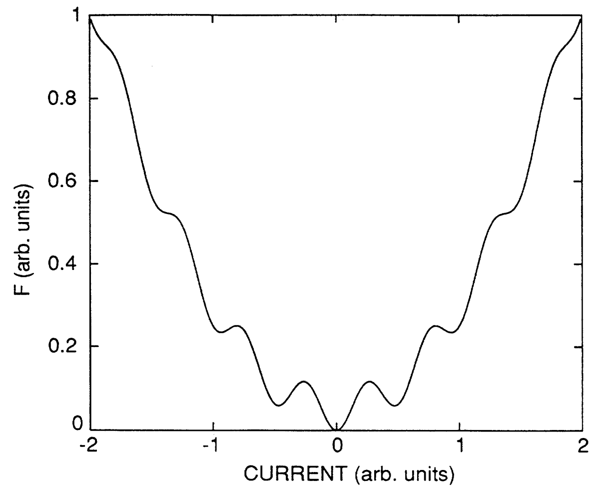

FIG. 1. Schematic of the Ginzburg-Landau free energy as a function of current for a one dimensional superconductor, after Ref. 2.

For slow variations of ψ, the amplitude of the oscillations in the potential in Fig. 1 are just ΔF_0, while for rapid variations (as in a phase slip), this amplitude is ΔF_{FQ}. At a given temperature this amplitude is fixed. As one moves to higher currents in Fig. 1 one will eventually reach a point at which the minima disappear, and this corresponds to the critical current. While I_c is well defined in terms of the behavior of the potential, Fig. 1, it is not an easy quantity to measure directly. If the system is initially located at potential minimum, one could imagine simply increasing the current gradually until a finite voltage is observed. At this point the minimum will presumably have just disappeared, and the system would be "moving" down the potential curve. However, one must also allow for fluctuations, and these will cause the system to escape from a minimum before it becomes absolutely unstable. That is, the system will switch into the finite-voltage state before I reaches I_c. This effect is well documented in work on MQT.[15–20]

When the system switches into the finite-voltage state, the effects of dissipation can be important. In particular, if there is no dissipation, which corresponds to no friction in the particle analogy, the system will accelerate down the potential curve, and never be "retrapped" in any of the minima it encounters. However, if the dissipation is sufficiently large, the system will lose energy fast enough to be captured by another minimum. These two possibilities can be distinguished experimentally by the value of the voltage. If the voltage has a value appropriate for the normal state, this implies that retrapping does not occur, while if the voltage is lower than this, retrapping must presumably be significant.

To this point, we have treated the particle quantum mechanically only with regard to its interwell motion (i.e., tunneling). If the "mass" of the particle is sufficiently small, the quantized nature of the energy levels within a potential well may also be important. While our system is more complicated, and has more degrees of freedom than a simple particle, one would presumably still expect the energy levels to be quantized. This could be manifest in several ways. First, the quantum-phase-slip process involves tunneling of the system through the barriers in Fig. 1. If the initial and final states have the same energies, one will have a sort of resonant tunneling,[28–30,10] leading to a large tunneling rate, and a large voltage. On the other hand, if the levels are not coincident, the tunneling rate will be reduced. One would therefore expect to see some sort of oscillations or other structure in the V-I characteristics, as variations of the current will shift the levels in one well relative to those in adjacent wells. One might also expect to see effects due to the change in the number of bound levels in a given well as the depth of the well is changed. Such a change can occur through changes in the current, or temperature, and could be reflected in the resistance as a function of temperature, or the escape rate as a function of current. Finally, irradiation with photons of the appropriate frequency should induce transitions between the levels within a well[31,32].

The possibility of resonant tunneling was discussed by Hatakenaka and Kurihara,[28–30] who considered explicitly the case of Josephson junctions. So as we know, resonant tunneling has not yet been observed in that system, or, prior to our work,[10] in any other system which involves a macroscopic degree of freedom. However, in some very elegant experiments employing Josephson junctions, Clarke and co-workers[31,32] have demonstrated the existence of discrete levels of the "particle," through the observation of transitions between levels induced by microwave radiation. Another interesting discussion relating to the effects of discrete levels has been given recently by Silvestrini, Ovchinnikov, and Cristiano.[33] They have shown that the rate of escape from a well as a function of well depth can exhibit structure due to level quantization. Their arguments are very similar to those we will employ below to account for some of our results.

As will be discussed in the next section, our results seem to indicate the existence of discrete levels of the system within a well. To compare our results with the theory, it is necessary to calculate the energy level spacing, etc. Such calculations require quantitative knowledge of the shape and depth of the potential, etc., which is not currently available. However, it turns out that the relative position of adjacent wells is, according to the theory, independent of the details of the potential; the prediction is[2,34]

$$\Delta F_I = \pm \Phi_0 \sigma J / c \ . \tag{14}$$

This relation will be used in our analysis below.

To this point we have discussed only the thermal and quantum-phase-slip models and their consequences. However, other explanations could well explain our results, and we now discuss two that appear most plausible.

We first consider what are known as "charging" effects.[35] Our samples have very small diameters, and since the PbIn films from which the one-dimensional samples were made were polycrystalline, the samples could be viewed as connected grains. If the intergrain connections are "strong," it is appropriate to view the samples as continuous metallic strips, as we have assumed to this point. However, if the connections are weak, the intergrain capacitance and tunneling between grains could be very important. Since the minimum charge which can be transferred is one electron, the associated charging energy is e^2/C, where C is the intergrain capacitance. To obtain a rough estimate for C, we assume that the capacitor has a spacing of 20 Å, a diameter of 300 Å, and a dielectric constant of unity. This leads to $C \sim 4\times 10^{-18}$ F, which corresponds to a charging voltage of $V_C \sim 40$ mV. This effect would be manifest in the V-I relation through the behavior at low V. If a small voltage ($< V_C$) is applied, the current will be "blocked" since the transfer of an electron would result in an opposing voltage V_C, which would make the total voltage across the junction negative. Hence there will be a depressed current until $V \gtrsim V_C$. In addition, one might also observe the so-called Coulomb staircase at higher voltages. This is an oscillation in the V-I relation which arises due to the discrete transfer of charge. Again, V_C will be the characteristic scale, as these oscillations should be uniformly spaced in V. Since this effect is a geometrical one, the voltages at which these features occur in the V-I characteristic should be temperature independent.[36]

Our samples were typically $\lesssim 300$ Å in diameter and 40 μm long, and hence could be considered to be "mesoscopic." One might therefore expect to find mesoscopiclike fluctuations,[37] and recent theoretical work[38] has suggested that there will be such fluctuations in the critical current. Fluctuations in I_c will presumably be reflected in the V-I characteristics. The predicted fluctuations in the supercurrent are[38]

$$\frac{\langle (\delta I_s)^2 \rangle}{\langle I_s \rangle^2} \approx \frac{\xi(0)}{L_e (k_F L_e)^2} \ , \tag{15}$$

where L_e is the elastic mean free path, and $\xi(0)$ is the zero-temperature coherence length. Note that (15) assumes that the length of the system is less than the electron phase coherence length (we will return to this point below). In addition, these fluctuations should be sample specific.[37]

III. EXPERIMENTAL TECHNIQUE

The samples were fabricated from Pb-In films produced by coevaporation, with an In composition of approximately 10% by weight.[39] This alloy was chosen for several reasons. First, thin films of PbIn appear to be more homogeneous and have a smaller grain size than the In films used in our previous work. This has permitted us to study samples with significantly smaller cross-sectional areas. Second, films of PbIn are fairly resistant to oxidation; the samples were relatively stable and could be studied for long periods of time without significant degradation. The values of T_c were in the range 6.9–7.1 K, depending on the precise composition (which varied slightly from batch to batch). The samples considered here were made from films with thicknesses in the range 110–200 Å. The films were evaporated onto glass substrates which had been precoated with a thin $\sim$50 Å layer of thermally evaporated Ge. The Ge and PbIn evaporations were performed in succession without breaking vacuum. The substrates were held at 77 K during the evaporations to reduce the grain size, and a small amount of O_2 ($\sim$200 Torr) was admitted to the evaporator while the films warmed to room temperature, as this was found to reduce agglomeration.[40] These films had normal-state resistivities of $\sim 20\ \mu\Omega$ cm. This value was essentially independent of film thickness for films as thin as $\sim$60 Å, which was much thinner than those used to make the one-dimensional samples. This implies (but certainly does not prove) that granularity should not be a problem in our one-dimensional samples.

A step-edge technique[41,42] was used to pattern the films into narrow strips of PbIn. Electron micrographs of similarly prepared samples have been presented elsewhere.[41,42] The one-dimensional samples had resistivity ratios (i.e., ratio of the room-temperature resistance to that in the normal state at low temperatures) which were the same ($\approx$2.0) as those of the films, which implies that they had the same resistivity. The step-edge method produces samples with approximately right triangular cross sections. The cross-sectional area σ was estimated from the measured length and resistance, and the resistivity, which, for the reason just noted, was assumed to be the same as that of the films. The values of σ obtained in this way were consistent with the known step heights and starting film thickness, and are believed accurate to better than 20%. However, none of our conclusions depend on precise knowledge of σ. In what follows we will refer to $\sqrt{\sigma}$ as the sample diameter.

The step-edge technique was used to directly produce samples as small as about 150 Å. Several of the smallest samples were exposed to air at room temperature for extended periods (typically a few days). They exhibited a gradual increase in resistance, implying that the effective diameters had decreased, presumably due to oxidation or agglomeration. All samples with diameters under 150 Å were produced in this manner. In addition, samples with diameters in the range 150–200 Å were produced either directly with the step-edge method or with the step-edge technique followed by exposure to air, and the behavior in the two cases was very similar.

The measurements were performed using the instrumentation described previously.[11] The only major difference was that in the present work the sample was located in a microwave cavity, whose resonant frequency was near 8 GHz, and which was connected to a room-temperature microwave source via semirigid coaxial cable. The source was tunable, and it was possible to vary the microwave frequency over the range 7–11 GHz. Unfortunately the direction of the microwave field was not known, due to distortion from the voltage and current leads connected to the sample. The present apparatus also allowed the application of a magnetic field.

IV. RESULTS AND ANALYSIS

A. Resistance in low currents

Figure 2 shows typical results for the resistance as a function of temperature, for samples of different sizes. These measurements were performed with currents of order 10^{-8} A or less. The results were found to be independent of the current for currents below about 10^{-7} A. That is, the V-I relation was linear for currents in this range (for measurements at much higher currents, however, this was not the case; see below). In addition, the results for R as a function of T shown here and below were independent of the direction in which the temperature was swept. It can be seen that the behavior of the resistance varies substantially as the diameter is reduced. In the largest samples, with diameters d above about 280 Å, R dropped to zero relatively rapidly below T_c, while for $\sim 200 < d < 280$ Å the resistance approached zero much more slowly as T was reduced.

Figure 3 shows the results for the two largest samples in Fig. 2 on logarithmic scales, along with the theoretical predictions of the frozen-quasiparticle theory. For the 310 Å sample the results are well described[43] by thermal activation, (10), over the entire range, with no evidence for quantum phase slippage. We also note that the theory of LA-MH, (1), fits the results just as well.[43] Thus,

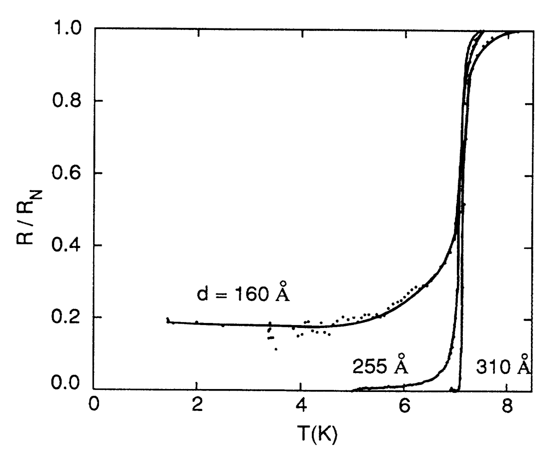

FIG. 2. Resistance as a function of temperature for several samples. The sample diameters are indicated in the figure. The measuring current was typically $\lesssim 10^{-8}$ A, which was found to be low enough that the behavior was independent of the current. The solid curves are guides to the eye.

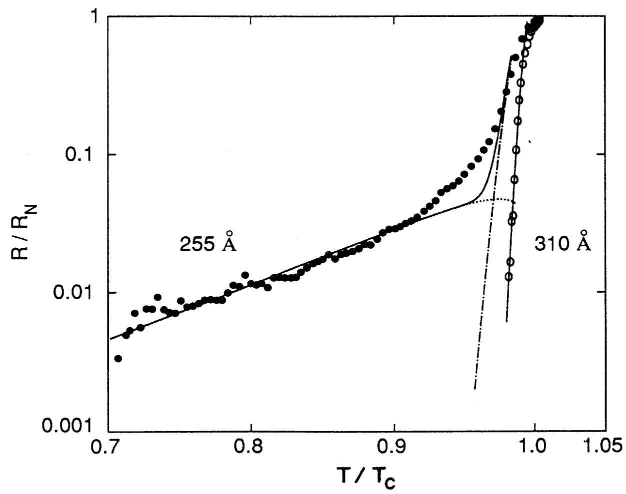

FIG. 3. The results for the two largest samples considered in Fig. 2, with a logarithmic scale for the resistance. The solid lines are the predictions of the frozen-quasiparticle theory allowing for both thermal activation, (10), and quantum tunneling, (12). The dot-dashed line is the thermal-activation prediction for the 255 Å sample, while the dotted line is the quantum tunneling prediction for the same sample. The fitted parameters are discussed in Ref. 43.

the predictions of these two theories cannot be distinguished on the basis of our experiments, although as noted above, the frozen-quasiparticle model certainly seems preferable on physical grounds. In addition, this means that all of our previous results in the thermal-activation regime[8,11] are consistent with the predictions of the frozen-quasiparticle model.

As can be seen from Fig. 3, the behavior of the 255 Å sample near T_c can also be described by the thermal-activation prediction, (10). However, at temperatures more than about ~ 0.2 K below T_c the resistance (and hence the phase-slip rate) is much too large to be accounted for in terms of thermal activation. This is illustrated in Fig. 3, which shows the prediction of thermal-activation theory alone, (10), along with the quantum tunneling result, (12); the solid line shows the sum of the two contributions.[43] The prediction for quantum tunneling is seen to be quite consistent with the results well below T_c. The agreement is not as good at intermediate temperatures, where the behavior crosses over from quantum tunneling to thermal activation. This may be due to an interplay between the two mechanisms, which has been ignored in deriving (10) and (12).[45] The predictions, (1) and (5), can also be made consistent with these results for reasonable values of the parameters. Hence, we again find that our results cannot be used to distinguish between predictions based on the frozen-quasiparticle model, (10) and (12), and the functional forms used in our previous analyses, (1) and (5).

Returning to Fig. 2, we see that for the 160 Å sample the resistance approaches a nonzero constant below T_c, and does not go to zero even as $T \rightarrow 0$. This can be easily understood in terms of the quantum-phase-slip model. According to this picture, the resistance is given by an expression of the form $R_{\text{MQT}} \sim \exp(-\Delta F_{\text{FQ}}\tau_0/\hbar)$, (12). Near T_c, ΔF_{FQ} is proportional to a power of $(T_c - T)$, but well away from T_c it will be temperature independent. Since $\Delta F_{\text{FQ}} \sim \sigma$, there will, if the sample is sufficiently small, be a significant nonzero resistance even at $T = 0$. Note also that this behavior cannot possibly be accounted for by any kind of thermal mechanism, since they must always yield a vanishing resistance as $T \rightarrow 0$.

Behavior similar to that seen in the 160 Å sample in Fig. 2 has also been observed in granular superconducting films.[46,47] Our explanation of this behavior is quite different from that generally used to explain the results for granular films. One could, of course, argue that our smallest samples contain "weak" spots which never become superconducting, at any temperature, and it is hard to completely rule out such an explanation. However, such a weak spot, if it were just a normal metal region, would have to be ~ 10 μm long, which seems quite unlikely. We also note that, as will be discussed in Sec. IV B, larger wires (like the other two samples considered in Fig. 2) also exhibit a relatively large, nonzero resistance as $T \rightarrow 0$, when exposed to a microwave field. We will return to this point in Sec. IV B.

B. Resistance in the presence of microwaves

The analogy with a particle moving in the potential sketched in Fig. 1 suggests that our system should have quantized energy levels. As noted above, the existence of

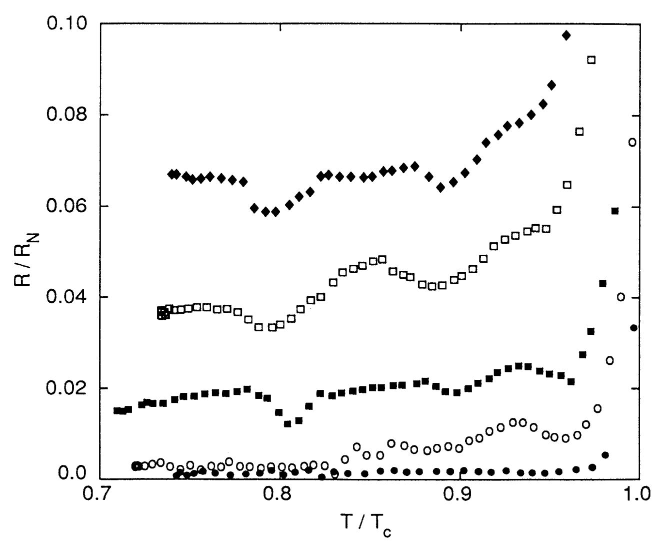

FIG. 4. Resistance of a 265 Å sample as a function of temperature in the presence of a 7 GHz microwave field. The relative microwave intensities were, going from the lowest curve to the highest, 1, 4, 10, 50, and 90.

such levels in Josephson junctions has been demonstrated very nicely by experiments in which the junctions were exposed to microwave radiation.[31,32] In those experiments, the usual resonance condition $\hbar\omega=\Delta E$ was satisfied, where ω is the microwave frequency and ΔE is the level spacing. For our system the value of ΔE was not known prior to the experiments described below. We had hoped that ΔE would be of order $\hbar\tau_{\mathrm{GL}}^{-1}$ or $\hbar\tau_0^{-1}$, since τ_{GL} and τ_0 are the characteristic time scales in the problem. Both of these times diverge as $T\rightarrow T_c$, which would imply that $\Delta E\rightarrow 0$ as $T\rightarrow T_c$. Hence, for a given value of ω the resonance condition would always be satisfied at some temperature. While in retrospect it does not appear that this was the case for attainable distances from T_c, we have nevertheless found that microwaves have a dramatic effect on the behavior.

Figure 4 shows some results for R as a function of T at low currents in the presence of 7 GHz radiation of different intensities. The behavior at the lowest microwave intensity is quantitatively similar to that seen for the 310 Å sample in Fig. 2, as R rapidly approaches a very small value below T_c. However, as the microwave intensity is increased, the behavior becomes quite different. First, there are distinct resonantlike peaks in R as a function of T. Second, R does *not* go to zero far below T_c; measurements to lower temperatures (as low as $T/T_c\sim 0.15$) indicate that R approaches a nonzero constant (comparable to the values seen at the lowest temperatures in Fig. 4) as $T\rightarrow 0$. This behavior is very similar to that seen for the 160 Å sample in Fig. 2. In that case we could not rule out the presence of nonsuperconducting regions. However, such regions are certainly not responsible for the results in Fig. 4, since without microwaves the resistance of this sample is negligible more than about 0.2 K below T_c. This suggests that the two results have a similar explanation.

The effect of varying the microwave frequency is shown in Fig. 5. Since the sample was located in a resonant cavity, it was difficult to maintain a constant microwave intensity as the frequency was changed. Here we have adjusted the microwave intensity to obtain roughly comparable values of R well below T_c at the different frequencies. Measurements at different microwave intensities yield similar results (i.e., Fig. 4). We see from Fig. 5 that changing the frequency does change the behavior somewhat, but from these and many other measurements, it does not appear that the peaks in R are systematic functions of frequency. For example, in Fig. 5 well-defined peaks are seen at $T/T_c\sim 0.93$ at both 7 and 10 GHz. In contrast, the peaks at this temperature with $\omega=8$ and 9 GHz are not as well defined, although peaks and "shoulders" are observed at other temperatures.

Figure 6 shows the effect of increasing the measuring current, again in the presence of microwaves. It is seen that for $I\lesssim 2$ μA the peak positions are essentially fixed, but that at higher currents the various features, especially the one at $T/T_c\sim 0.8$, shift somewhat to lower T.

Figures 4–6 clearly show that microwaves have a large effect on the behavior, and the resonantlike peaks in R as a function of T are clearly suggestive of quantized energy levels. It is natural to suppose that the peaks in R occur when $\hbar\omega$ matches the energy-level spacing in a particular well. If the population of an excited level were increased

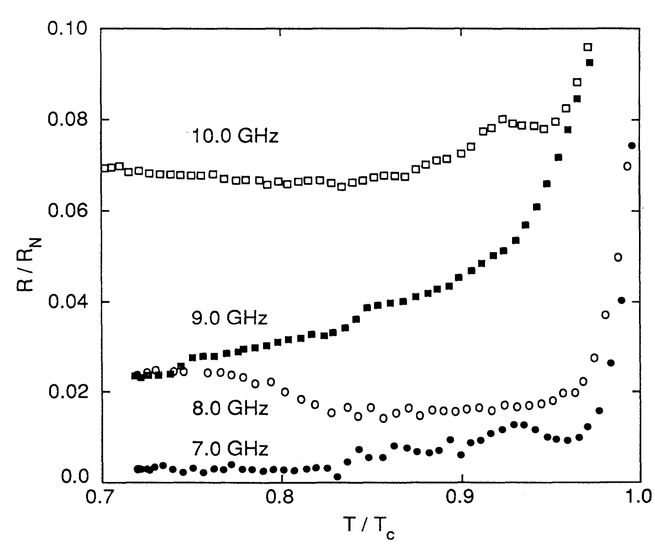

FIG. 5. Resistance as a function of temperature for microwave fields of different frequencies, as indicated in the figure. This is the same sample as considered in Fig. 4.

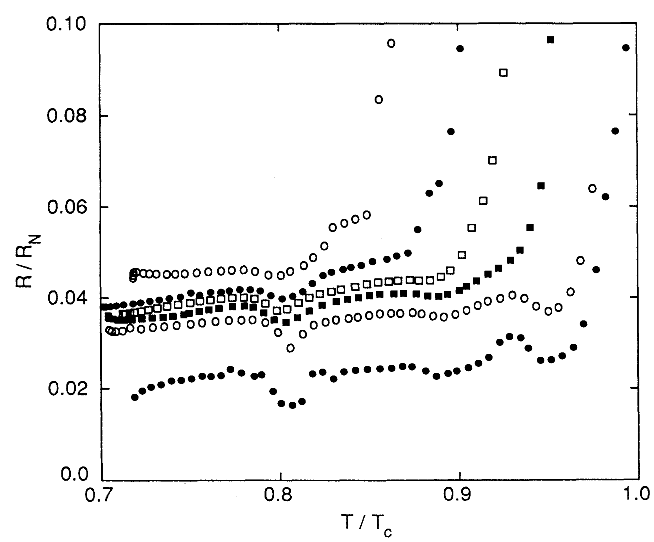

FIG. 6. Resistance as a function of temperature with different measuring currents for the 265 Å sample considered in Figs. 4 and 5. A microwave field with frequency 7 GHz, and intensity 4 on the scale of Fig. 4 was applied. Nonlinearities of the V-I relation have been ignored here; we have simply plotted V/I. The different curves have been offset vertically for clarity. The values of the current were, going from the lowest curve to the highest, 10^{-7}, 2×10^{-6}, 3×10^{-6}, 4×10^{-6}, 5×10^{-6}, and 6×10^{-6} A.

through resonant absorption, the effective resistance would also be increased, since the tunneling rate out of an excited level would be higher than that from the ground state. The level spacing is temperature dependent, so this would lead to resonantlike behavior as a function of T, like that seen in Figs. 4–6. However, the energy scales involved are not consistent with such an explanation. The microwave energy is $\hbar\omega \sim 5\times 10^{-17}$ erg. We argued above that the energy-level spacing should be of order $\hbar\tau_0^{-1}$, which is of order 10^{-14} erg at typical temperatures in Figs. 4–6, and thus *much* larger than $\hbar\omega$ (note that $\hbar\tau_{GL}^{-1}$ is also much larger than $\hbar\omega$). Furthermore, we will see below that the V-I measurements independently imply a level spacing comparable to $\hbar/\tau_0$. Hence the microwave energy is an *extremely* poor match to the level spacing at these temperatures, and it is hard to see why the microwaves should have any effect at all. Nevertheless, microwaves clearly do have a large effect, and we now consider other mechanisms which could be responsible.

The arguments in the previous paragraph tend to rule out the importance of direct intrawell transitions. We now consider several other explanations. Figure 7(a) shows schematically the quantized levels in two adjacent wells. As the temperature is varied, the well depth and also the level spacing will change, and both will become larger as T is decreased. If the system is in the ground state of the higher lying well, then as T is lowered, the excited levels in the adjacent (lower) well will be swept past

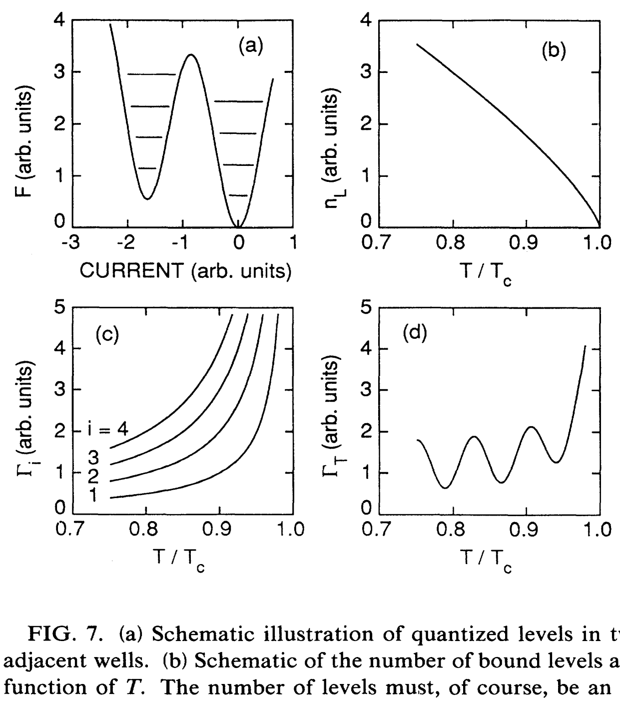

FIG. 7. (a) Schematic illustration of quantized levels in two adjacent wells. (b) Schematic of the number of bound levels as a function of T. The number of levels must, of course, be an integer. Here we show a curve proportional to $\Delta F_{FQ}\tau_0$, (17). (c) Schematic variation of the escape rate from level i as a function of T. Γ_i diverges when level i becomes unbound. (d) Schematic of the total escape rate as a function of T.

it. One will thus pass in and out of resonance for tunneling from the ground level as a function of T, leading to peaks in R as a function of T. However, a problem with this explanation is that the microwaves play no role, in contrast to the experiments which clearly show that the resonantlike behavior is only found when microwaves are present. The fact that the resistance well below T_c is extremely small without microwaves implies that the tunneling probability out of the ground level is very small under these conditions. One might then suppose that the main effect of the microwaves is to excite the system out of the ground level, i.e. induce a significant population in the excited levels. The nonzero resistance seen in Figs. 4–6 would then be due to tunneling from these excited levels, which would have smaller energy barriers, and hence larger tunneling rates, than the ground level. In this picture, the resonant tunneling would occur from the excited levels in the upper well. A problem with this proposal, though, is that the *relative* positions of the excited levels in adjacent wells should not change with temperature.

These arguments suggest that an *inter*well process, such as resonant tunneling, cannot give rise to the behavior seen in Fig. 4. In addition, this conclusion is reinforced by the results in Fig. 6, which show that the peaks in R versus T change very little as the measuring current is varied in the range $I \lesssim 2\ \mu$A. Since varying the measuring current shifts the position of a well relative to adjacent ones, this would change the position of any interwell resonances, but such a shift is not seen [from (14) we estimate that the shifts for these currents should be larger than observed in Fig. 6]. Let us therefore consider if an *intra*well process might be responsible. One possibility involves the temperature dependence of the level spectrum within a well. As T decreases, the well depth ΔF_{FQ} and the spacing of levels within a well $\hbar\tau_0^{-1}$ both increase.[48] Since ΔF_{FQ} increases faster than $\hbar\tau_0^{-1}$, the number of bound levels in a well will increase as T is lowered, as indicated schematically in Fig. 7(b). The effect of microwaves will be to increase the population of the excited levels above what would be found in thermal equilibrium. Since the microwave frequency is far off resonance, we will assume that this population is characterized by an effective temperature T^*, so that the probability of being in level i is

$$P_i \sim \exp(-E_i/k_B T^*) \ . \tag{16}$$

The system can leave level i by tunneling out of the well, by relaxing to a lower level in the same well, or by absorbing microwaves and thereby being excited to a higher level. The voltage will be determined by the tunneling rate, and since the amplitude of the barrier seen by a level will vary with T, the tunneling rate out of level i will vary schematically as shown in Fig. 7(c). The total tunneling rate Γ_T will be a combination of these individual tunneling rates and the level occupancies, (16), with a result qualitatively like that sketched in Fig. 7(d). The basic idea is that Γ_T exhibits oscillations when a level leaves (or enters) the well, i.e., when the number of bound levels changes by one. Since R will be proportional to Γ_T, the resistance will also display peaks like those in Fig. 7(d). Of course, this argument is only qualitative; a quantitative calculation would certainly be of great interest. After we had arrived at this explanation of our results, we learned of the work of Silvestrini, Ovchinnikov, and Cristiano,[33] which discusses, in a much more quantitative manner, a very similar mechanism in the context of Josephson junctions. Our experiments seem to be the first to observe the sort of behavior predicted in Ref. 33.

This picture appears to be at least qualitatively consistent with the experimental results. One could go a step further in the analysis, and use a model potential to calculate the rates Γ_i and Γ_T, but there is so much uncertainty in the parameters characterizing the potential that such a calculation does not seem worthwhile at this time. However, this mechanism does imply that each peak in R versus T (Fig. 4) corresponds to the addition of a bound level as T decreases. The number of bound levels is, to a first approximation, just[49]

$$n_L = \Delta F_{\rm FQ} / \hbar \tau_0^{-1} \ . \qquad (17)$$

Near T_c we have $\Delta F_{\rm FQ} \sim (T_c - T)^{5/4}$, while $\tau_0^{-1} \sim (T_c - T)^{1/2}$, so the number of bound levels will tend to zero as $T \rightarrow T_c$, as assumed in Fig. 7. Using (6) and (11) to evaluate (17), we find $n_L \sim 40$ at $T/T_c = 0.8$. The experiments, Fig. 4, imply that the number of bound levels should be ~ 3, so the comparison is not terribly good. However, the arguments which yield the level spacing are only qualitative, so this discrepancy may be acceptable. Moreover, in the analysis of the resistance at low currents, Fig. 3, we found[43] that (11) [in conjunction with (2)] overestimated the free energy barrier by a factor of about 4. In addition, we will find below that our V-I results imply a level spacing a factor of 3 larger than obtained from the value of τ_0 in (6). With these factors included, our estimate for the number of levels in the well becomes ~ 3, which is much more consistent with the behavior seen in Fig. 4.

While the tunneling model is thus capable of explaining the behavior of R-T with microwaves, the mesoscopic fluctuations predicted by Spivak and Zyuzin[38] could conceivably lead to similar behavior. We will discuss that model in more detail in the next section. Here we only note one piece of evidence against such an explanation; results quantitatively similar to those in Figs. 4–6 were obtained with a number of other samples. In contrast, mesoscopic fluctuations should be sample specific.

C. V-I characteristics

Figure 8(a) shows some typical results for the V-I characteristics for a wide range of I. The behavior is like that expected for phase-slip centers.[50] The fact that the switch to the normal state occurs over a range of $\sim \pm 25\%$ in I suggests a similar variation of the cross-sectional area along the length of the sample. This is one of the few experimental checks we have on the homogeneity of the samples, and the result seems quite acceptable. The value of the effective critical current obtained from Fig. 8(a) is also consistent with that calculated from (13) using the energy barrier obtained from the behavior of R near T_c.[51] Let us now consider the behavior at low currents and voltages, Fig. 8(b). Here we observe pronounced oscillations in V-I, both with and without mi-

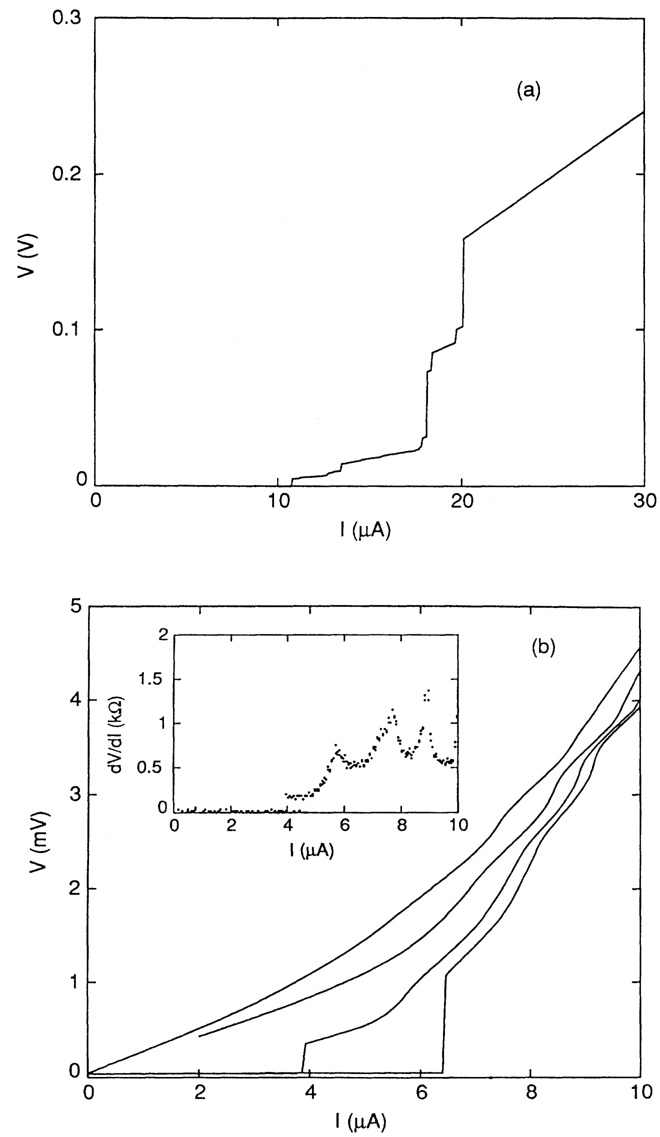

FIG. 8. (a) V-I relation for a 275 Å sample at high currents at $T=4.2$ K. Here the current was swept up starting from $I=0$. (b) Same, but now at low currents. For the lowest curve the microwave intensity was zero, and the current was swept down from high values where $V>0$ [for sweeps without microwaves in which the current was increased from zero, V did not become different from zero until somewhat larger values of I than those shown here; see (a)]. The relative intensities for the other curves were (top to bottom) 10, 3, and 1. For the measurements with microwaves present, identical results were found for increasing and decreasing currents. The frequency was 7 GHz. The inset in (b) shows dV/dI for the V-I curve with a microwave intensity of $\times 1$.

crowaves.[52] It can be seen that the behavior changes smoothly with increasing microwave intensity. In particular, the locations of the oscillations move to slightly lower currents (and voltages) with increasing intensity (see Fig. 9). This may be due to some Joule heating from the microwave field. The fact that the oscillations are present without microwaves implies that they are not caused by the microwaves. Similar results are observed at other microwave frequencies (in the range accessible in these experiments, 7–11 GHz), and it does not appear that the microwaves themselves are involved in any sort of resonant process. The inset in Fig. 8(b) shows the dV/dI for one of the V-I curves, and it is seen that the spacing of the peaks in dV/dI decreases as I increases. The peak locations vary continuously with temperature, and the results for one sample are given in Fig. 9(a), which shows the currents at which the first three peaks occur, in the limit of low microwave intensity. Figure 9(b) shows corresponding results for the voltages at which the peaks occur. These quantities are all seen to approach zero as $T \rightarrow T_c$.

Let us now consider how the V-I results can be understood, beginning with the tunneling model. As already

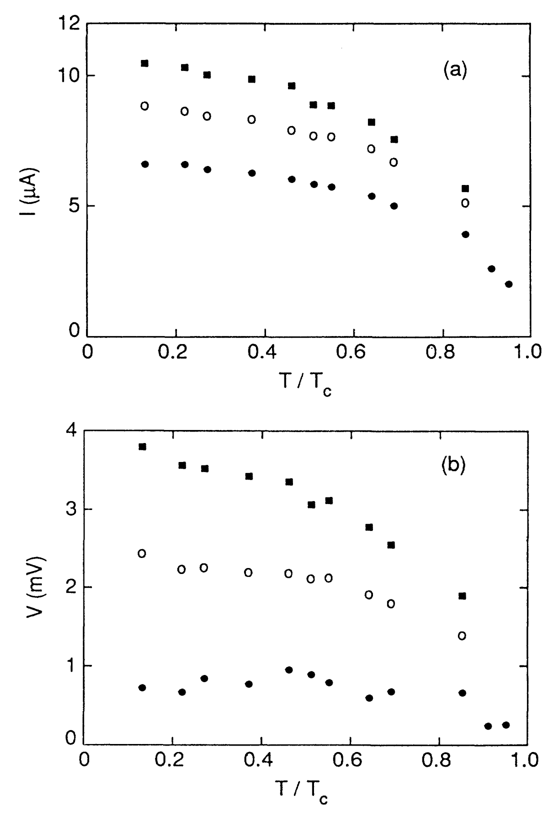

FIG. 9. (a) Current at which peaks in dV/dI occur, as a function of T, for the 275 Å sample, considered in Fig. 8. The symbols correspond to the first peak ●, second peak ○, and third peak ■. (b) Same as (a), but here the corresponding voltages are plotted.

noted, the results indicate that the microwaves do *not* cause the oscillations. We assume again that the main role of the microwaves is to increase the populations of the higher levels, and hence make tunneling or thermal activation more likely. We first consider intrawell processes. To explain the R-T results, we invoked an intrawell effect involving changes in the number of bound levels in a well as a function of T. If T is fixed and I is increased (as in the V-I measurements), the number of bound levels will decrease as the well is tipped more and more (Fig. 1). From the arguments associated with Fig. 7 we would therefore expect features in the V-I relation whenever a level is pushed out of the well. However, this explanation seems to have a problem explaining the results for the peak positions in Fig. 9. According to this model, the first few peaks in dV/dI (i.e., at the lowest currents) correspond to levels which were initially at the top of the well. For a harmonic well, the levels will all be evenly spaced with current, but in our case the potential will be slightly anharmonic, which should cause the level spacing to be smaller at the top of the well. The levels which are pushed out of the well first should therefore be more closely spaced than the levels which are pushed out at higher currents. Since the tipping of the well is proportional to I [see (14)], this implies that the spacing of the oscillations as a function of I should be smaller at low currents. However, Fig. 9 shows just the opposite; the spacing of the peaks is larger at small I. We therefore do not believe that this intrawell effect is responsible for the behavior of V-I, but a quantitative calculation of the level spectrum as a function of I is needed before this explanation can be completely ruled out.

We next consider an interwell effect, which is essentially just resonant tunneling. As discussed above, increasing I will sweep the levels in one well past those in an adjacent well, thus sweeping through resonant and nonresonant conditions. According to this model, the first peak in dV/dI occurs when the second level in one well is coincident in energy with the lowest level in the neighboring well, etc., for the second, third, etc., peaks. Hence, by the above arguments, the spacing of the peaks should be largest for the peaks at lowest I, since they probe the locations of the lowest-lying levels. The shift of the potential with current is given by (14), so we can compare the experimental peak locations with the estimated energy-level spacing $\hbar\tau_0^{-1}$. At the temperature in Fig. 8(b), the first peak occurs at $I \approx 5$ μA, which from (14) corresponds to 1×10^{-13} erg. This compares fairly well with the value of $\hbar\tau_0^{-1}$ [obtained from (6)], which is 3×10^{-14} erg at this temperature. Hence this model seems at least roughly consistent with the experiments. Of course, it would be of great interest to verify theoretically that the level spacing should really be of order $\hbar\tau_0^{-1}$, as our results seem to imply; this calculation has not yet been performed. Moreover, if the shape of the potential is known accurately enough, it should be possible to calculate the effect of anharmonicity on the level spacing, and compare in more detail with the results in Fig. 9.

Let us now consider other possible explanations of the V-I results. The mesoscopic effects considered by Spivak and Zyuzin[38] lead to fluctuations in the critical current which we estimate to be of order a few percent for our samples.[53] Given the uncertainties, this is at least consistent with the size of the oscillations seen in V-I. However, the very regular spacing seen in Figs. 8(b) and 9, and also the fact that other samples exhibit quantitatively similar results, would seem to rule out sample specific fluctuations like those expected from mesoscopic effects. Nevertheless, such mesoscopic fluctuations should be present at some level, and it would be interesting to try to observe them.

Another possible explanation involves charging effects arising from the assumed formation of tunnel junctions along the length of our samples. A difficulty in explaining our results in this manner is that, so far as we can tell, charging models predict that the features in V-I should occur at energies, i.e., voltages, which are temperature independent, since they would then be just a geometrical effect arising from the junction capacitance. From Fig. 9(b) we see that the V-I features are strongly temperature dependent. Moreover, at a given temperature, we would expect charging effects to lead to a "staircaselike" structure which is evenly spaced in V. Figure 9(b) shows that the peak spacing as a function V is far from constant. Finally, the rough estimate of the capacitance given in Sec. II yields a much larger characteristic voltage for charging effects (by more than an order of magnitude) than that seen experimentally. We therefore conclude that the charging model is not consistent with our results for the relatively large samples we have considered to this point. However, we will argue below that charging may be important in our smallest samples. It is also interesting to note that the phase-slip model does, in a sense, involve a kind of charging effect, since during a phase-slip event the current must be carried by the quasiparticles, with a resultant spatial buildup of charge.

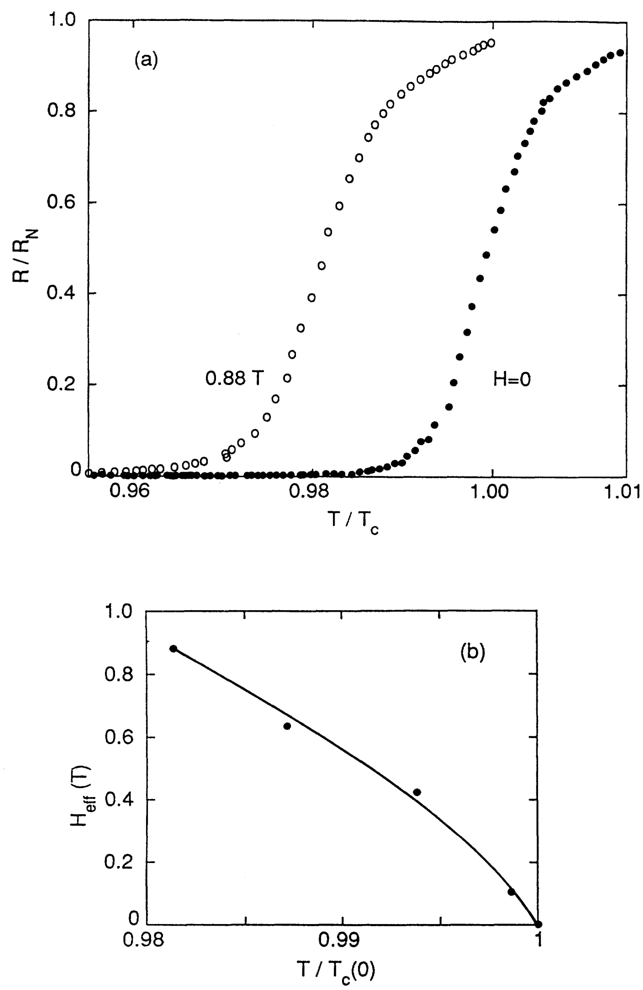

FIG. 10. (a) Resistance as a function of T with $H=0$ and $H=0.88$ T, for the 275 Å sample considered in Figs. 8 and 9. The magnetic field was applied parallel to the sample. No microwave field was applied. (b) H_{eff} as a function of T. The solid curve is a guide to the eye.

D. Effect of a magnetic field

Figure 10(a) shows results for the resistive transition in the presence of a magnetic field. It can be seen that a field simply shifts the transition to lower temperatures, with no appreciable change in the form of R as a function of T.[54] Figure 10(b) shows the effective critical field H_{eff} [not to be confused with the bulk critical field H_c introduced in connection with (2)] as a function of T. Here we have taken T_c to be the value of T at which R is half the normal-state value; since the R-T curves all have the same shape, any other criteria would yield similar results

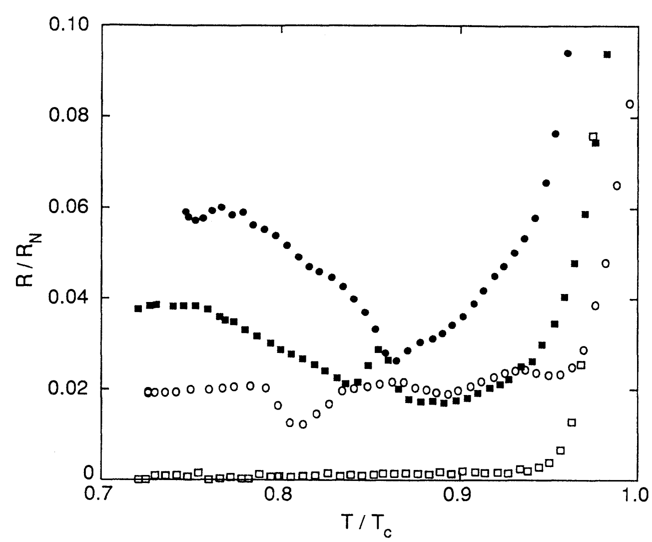

FIG. 11. Resistance as a function of T for the 275 Å sample considered in Figs. 8–10, with both a magnetic field and a microwave field present. (●) $H=0.8$ T, with microwaves; (○) $H=0$, with microwaves; (■) $H=0.6$ T, with microwaves; (□) $H=0.8$ T, no microwaves. The microwave frequency was 7 GHz, and the intensity was 4 on the scale of Fig. 4. The solid curves are guides to the eye.

for the variation of H_{eff} with temperature. There is a large enhancement of H_{eff} above the thermodynamic critical field (which is 0.08 T at $T=0$), as expected from the small size of the system. Estimates from Ginzburg-Landau theory[44] yield critical fields which are within a factor of 2 of the values we observe. This prediction depends on the value of the elastic mean free path, and on the detailed shape of the sample, so this level of agreement seems reasonable.[55]

Figure 11 shows results for R as a function of T in the presence of both microwaves and a magnetic field. The latter clearly has a large effect on the resonant peaks in R-T. It is not clear at present if this can be accounted for using the model discussed in connection with Fig. 4.

E. Behavior of extremely small samples

In a previous report,[10] we presented results for samples somewhat smaller than those considered above. A surprising aspect of those results was that at low temperatures the resistance at low currents was *larger* than the normal-state value. This is illustrated in Fig. 12 which shows the V-I characteristics for a 155 Å sample at several temperatures. It is seen that dV/dI becomes very large as $I \rightarrow 0$, and the effect is enhanced as the temperature is reduced. In addition, there are oscillations in the V-I relation which are quite similar to those seen above in Fig. 8(b). It is not clear why only one or two oscillations are seen in Fig. 12, but it may have been due to insufficient sensitivity in these measurements.[56]

In Fig. 13 we show results for V-I for an 80 Å sample, our smallest sample to date. Many oscillations are evident here, and the large differential resistance near $I=0$,

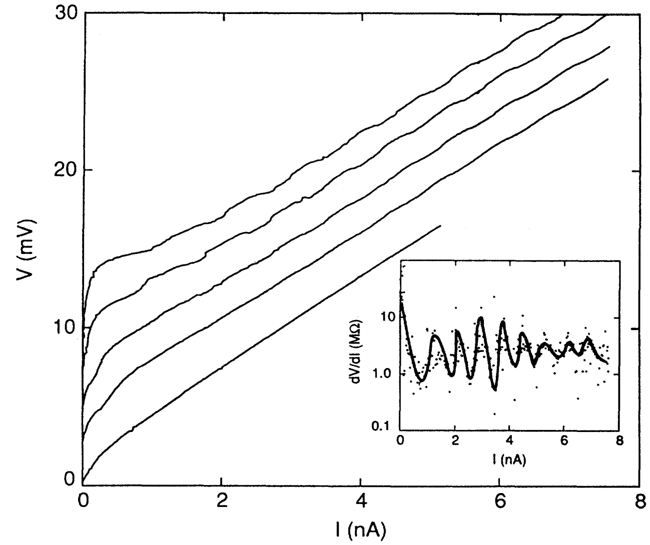

FIG. 13. Same as Fig. 12, but for an 80 Å sample at $T=6.02$, 4.52, 3.82, 2.68, and 1.36 K (bottom to top). For clarity, the curves have been offset vertically. The inset shows dV/dI at 1.36 K. Note that the vertical scale of the inset is logarithmic. The solid curve is a guide to the eye.

especially at the lowest temperature, is even more apparent. The oscillations seen in Figs. 12 and 13 are certainly similar to those seen in the larger samples, Fig. 8(b), and may therefore have a similar origin. However, the large value of dV/dI at low currents is also quite striking. We previously proposed[10] that this could be be due to some sort of coherent (i.e., Bloch-like) state composed of the ground levels of several wells. However, a difficulty with this explanation is that the interlevel coupling, i.e., overlap, must be large for such states to form.[57] The fact that many oscillations are seen in Fig. 13 imply that there are many levels in each well, and hence that the overlap of the ground levels must be quite small.[58] Another explanation of this behavior has been suggested by Zwerger,[59] who has shown[60] that a sort of competition between dissipation and the quantum nature of the effective "particle" can lead to a localization transition at $T=0$. It also gives rise to the possibility of a nonmonotonic variation of the resistance as a function of both voltage and temperature. In our discussions to this point we have ignored such effects. A completely different explanation in terms of charging also seems plausible. Unlike with the larger samples [i.e., Fig. 8(b)], the levels in Fig. 13 are not far from being evenly spaced in V. Moreover, the large dV/dI near $I=0$ would be expected from a Coulomb blockade effect. Problems with granularity will be more severe with the extremely small samples considered in Figs. 12 and 13, and such an explanation seems much more likely than in the case of the larger samples considered in previous sections. However, the estimated charging energy is still somewhat larger than observed, so this explanation is not entirely satisfactory.

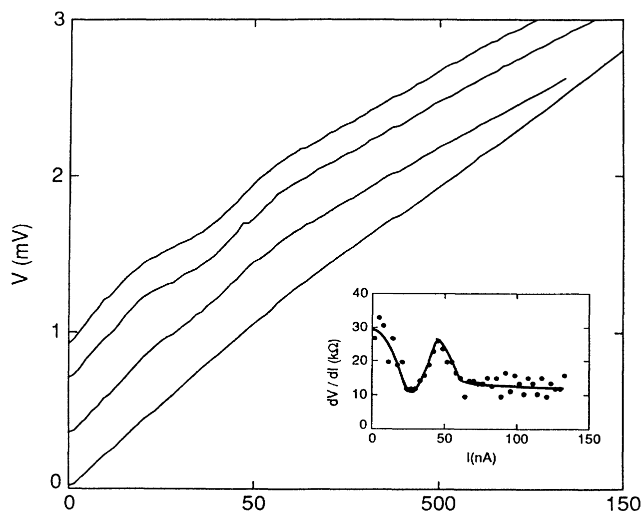

FIG. 12. V-I characteristics for a 155 Å sample at $T=6.00$, 3.99, 2.92, and 1.38 K (bottom to top). For clarity, the curves have been offset vertically. The inset shows dV/dI at 1.38 K. The smooth curve is a guide to the eye.

V. DISCUSSION

In this paper we have presented a number of new results concerning dissipation in one-dimensional superconductors. The majority of our previous results in this area[8–11] could be explained in terms of the LA-MH theory, and simple extensions to include quantum phase slip (i.e., MQT). We have shown here that the same results are consistent with the frozen-quasiparticle model, and that the experimental results to date are not able to distinguish between the different predictions. A number of the results presented here seem also to require the existence of quantized levels within the Ginzburg-Landau potential well. While it appears possible to account for many aspects of our results in terms of quantized levels, most of our arguments are extremely qualitative. A more complete theoretical treatment is needed, and the recent development of the frozen-quasiparticle model is certainly a major step in that direction.

In our analysis we have also tried to give a careful comparison with other possible explanations. We do not believe that explanations in terms of charging effects can account for the bulk of our results, although they may be important in our smallest samples (Figs. 12 and 13). The predicted size of the mesoscopic fluctuations is not far from our sensitivity, and we expect that they should be observable under the right conditions.

As we have tried to emphasize throughout this paper, the evidence in favor of the quantum-phase-slip model is by no means overwhelming, especially since a quantitative theory of this process is only now emerging. However, at present this model seems to provide the most plausible explanation of the main body of our results. In particular, it is difficult to see how any thermal activation-type model could account for the behavior of the low-current resistance of our smaller samples (Figs. 2 and 3). Our interpretation of the effects of microwaves on the resistance (Figs. 4–6), and of the the V-I results, in terms of quantized levels is also somewhat speculative. However, if one accepts the quantum tunneling model, it is hard to see how our results could be explained without assuming the existence of quantized levels. In any event, this problem certainly seems deserving of further study.

ACKNOWLEDGMENTS

I thank P. Ao, R. P. Barber, Jr., A. J. Leggett, K. Mullen, P. Muzikar, L.-Y. Shieh, and B. Z. Spivak for stimulating and critical discussions, and H. Hatakenaka, S. Saito, P. Silvestrini, and W. Zwerger for helpful correspondence. I also thank the authors of Refs. 27 and 33 for communicating their results prior to publication. This work was supported by the National Science Foundation through Grants No. DMR-86-14862 and No. DMR-89-15574.

---

[1]W. A. Little, Phys. Rev. **156**, 396 (1967).

[2]J. S. Langer and V. Ambegaokar, Phys. Rev. **164**, 498 (1967).

[3]D. E. McCumber and B. I. Halperin, Phys. Rev. B **1**, 1054 (1970).

[4]J. E. Lukens, R. J. Warburton, and W. W. Webb, Phys. Rev. Lett. **25**, 1180 (1970).

[5]R. S. Newbower, M. R. Beasley, and M. Tinkham, Phys. Rev. B **5**, 864 (1972).

[6]J. R. Miller and J. M. Pierce, Phys. Rev. B **8**, 4164 (1973); in *Proceedings of the 13th International Conference on Low Temperature Physics, Boulder,* edited by W. S. O'Sullivan (Plenum, New York, 1974), Vol. III, p. 659.

[7]For a review, see W. J. Skocpol and M. Tinkham, Rep. Prog. Phys. **38**, 1049 (1975).

[8]N. Giordano, Phys. Rev. Lett. **61**, 2137 (1988).

[9]N. Giordano, in *Proceedings of the International Symposium on Nanostructure Physics and Fabrication,* edited by W. P. Kirk and M. A. Reed (Academic, San Diego, 1989), p. 473.

[10]N. Giordano and E. R. Schuler, Phys. Rev. Lett. **63**, 2417 (1989).

[11]N. Giordano, Phys. Rev. B **41**, 6350 (1990).

[12]J. Kurkijärvi, in *SQUID-80: Superconducting Quantum Interference Devices and Their Applications,* edited by H. D. Hahlbohm and H. Lubbig (de Gruyter, Berlin, 1980), p. 247.

[13]A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. **46**, 211 (1981); Ann. Phys. (N.Y.) **149**, 374 (1983).

[14]P. Hänggi, J. Stat. Phys. **42**, 105 (1986).

[15]R. F. Voss and R. A. Webb, Phys. Rev. Lett. **47**, 265 (1981).

[16]L. D. Jackel, J. P. Gordon, E. L. Hu, R. E. Howard, L. A. Fetter, D. M. Tennant, R. W. Epworth, and J. Kurkijärvi, Phys. Rev. Lett. **47**, 697 (1981).

[17]S. Washburn, R. A. Webb, R. F. Voss, and S. M. Faris, Phys. Rev. Lett. **54**, 2712 (1985).

[18]D. B. Schwartz, B. Sen, C. N. Archie, and J. E. Lukens, Phys. Rev. Lett. **55**, 1547 (1985).

[19]M. H. Devoret, J. M. Martinis, and J. Clarke, Phys. Rev. Lett. **55**, 1908 (1985).

[20]J. M. Martinis, M. H. Devoret, and J. Clarke, Phys. Rev. B **35**, 4682 (1987).

[21]S. Saito and Y. Murayama, Phys. Lett. A **135**, 55 (1989).

[22]S. Saito and Y. Murayama, Phys. Lett. A **139**, 85 (1989).

[23]E. Abrahams and T. Tsuneto, Phys. Rev. **152**, 416 (1966).

[24]N. R. Werthamer, in *Superconductivity,* edited by R. D. Parks (Dekker, New York, 1969), Chap. 6.

[25]M. Cyrot, Rep. Prog. Phys. **36**, 103 (1973).

[26]A. J. Van Run, J. Romijn, and J. E. Mooij, Jpn. J. Appl. Phys. Pt. 1, **26**, 1765 (1987).

[27]L.-Y. Shieh, P. Ao, J.-M. Duan, and A. J. Leggett, Bull. Am. Phys. Soc. **35**, 250 (1990); P. Ao, J.-M. Duan, L.-Y. Shieh, and A. J. Leggett, Bull. Am. Phys. Soc. **35**, 677 (1990); and (unpublished).

[28]N. Hatakenaka and S. Kurihara, Jpn. J. Appl. Phys. **26**, 1413 (1987).

[29]N. Hatakenaka and S. Kurihara, Solid State Commun. **64**, 943 (1987).

[30]N. Hatakenaka and S. Kurihara, Solid State Commun. **68**, 131 (1988).

[31]M. H. Devoret, J. M. Martinis, D. Esteve, and J. Clarke, Phys. Rev. Lett. **53**, 1260 (1984).

[32]M. H. Devoret, D. Esteve, J. M. Martinis, A. Cleland, and J.

Clarke, Phys. Rev. B **36**, 58 (1987).
[33]P. Silvestrini, Yu. N. Ovchinnikov, and Cristiano, Phys. Rev. B **41**, 7341 (1990).
[34]There may actually be a problem with this expression, since its derivation (Ref. 2) seems to assume that ψ has a chance to relax over the entire system on a time scale comparable to that of a phase slip. However, we will ignore this complication, and assume that this expression is appropriate here.
[35]See, for example, K. K. Likharev, IBM J. Res. Dev. **32**, 144 (1988).
[36]While the charging energy should be temperature independent, the features in the *V-I* characteristics associated with charging may vary with *T*. In addition, the coupling of the system to its environment could act to modify the charging effects [see, for example, A. N. Cleland, J. M. Schmidt, and J. Clarke, Phys. Rev. Lett. **64**, 1565 (1990)] although in our case such effects should presumably be small since the sample acts effectively as its own high-impedance "leads."
[37]See, for example, *Mesoscopic Phenomena in Solids,* edited by B. L. Al'tshuler, P. A. Lee, and R. A. Webb (Elsevier, Amsterdam, in press).
[38]B. Z. Spivak and A. Yu. Zyuzin, Pis'ma Zh. Eksp. Teor. Fiz. **47**, 221 (1988) [JETP Lett. **47**, 267 (1988)]. See also the chapter by B. Z. Spivak in Ref. 37.
[39]For a discussion of the properties of similar films, see J. H. Greiner, C. J. Kirchner, S. P. Klepner, S. K. Lahiri, A. J. Warnecke, S. Basavaiah, E. T. Yen, J. M. Baker, P. R. Brosious, H.-C. W. Huang, M. Murakami, and I. Ames, IBM J. Res. Dev. **24**, 195 (1980).
[40]See, for example, W. J. Skocpol, Ph.D. thesis, Harvard University, 1974 (unpublished).
[41]D. E. Prober, M. D. Feuer, and N. Giordano, Appl. Phys. Lett. **37**, 94 (1980).
[42]N. Giordano, Phys. Rev. B **22**, 5635 (1980).
[43]The fits shown in Fig. 3 employ the frozen-quasiparticle (FQ) theory (10) and (12). In performing these fits, we have adjusted the prefactor and the term in the exponent in both expressions to obtain reasonable agreement, and there are thus four parameters involved in these comparisons. For the 310 Å sample, the thermal-activation expression (1) fits the data well with an assumed energy barrier smaller than (10) by a factor of 5±2. For the 255 Å sample, it was necessary to assume a barrier smaller than (10) by a factor of 6±2, in order to obtain reasonable agreement in the thermal activation region. In both cases the attempt frequency was about 1×10^{-3} of that given in (10); since the theory estimates this prefactor only qualitatively, it is not clear if this is reasonable agreement or not. In the quantum tunneling regime, no adjustment of the exponential factor was necessary; that is, given the reduced barrier obtained in the thermal-activation regime, the exponential factor in (12) fits the data reasonably well with no further adjustment. This confirms the value of the parameter γ in (12) to within a factor ≈ 2 (similar results for the energy barrier and for γ were found for a number of other samples). The value of γ is a key result of the frozen-quasiparticle theory, and the fact that the experimental results agree well with the predicted value of γ lends strong support to the theory. We also obtained acceptable fits to (1) and (5). We again had to assume barrier heights reduced from (2) by a factor of $\sim$2–3, and the parameter β_2 was $\sim 2\times10^{-2}$. These values are comparable to those found in our work on In wires (Ref. 11). In all of these fits we assumed a thermodynamic critical field $H_c(T=0)=803$ Oe and the coherence length was estimated from the standard relation (see Ref. 44) $\xi(T=0)=0.86\sqrt{\lambda\xi_0}$, where for Pb $\xi_0=8.3\times10^{-6}$ cm and λ is the mean free path. λ was estimated from the resistivity to be $\sim$75 Å.
[44]See, for example, M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1975).
[45]For example, the system might be thermally activated part of the way out of the well, and then tunnel the rest of the way out. Such effects are not included in (10) and (12), but have been studied with regard to tunnel junctions. See, for example, Ref. 14.
[46]H. M. Jaeger, D. B. Haviland, B. G. Orr, and A. M. Goldman, Phys. Rev. B **40**, 182 (1989).
[47]R. P. Barber, Jr. and R. E. Glover III, Phys. Rev. B **42**, 6754 (1990).
[48]The slope of the quadratic background in Fig. 1 will also change, but that should have little effect on the relative position of levels within a given well.
[49]This assumes that the well is harmonic so that the levels are equally spaced, which should be a reasonable first approximation.
[50]See, for example, W. J. Skocpol, M. R. Beasley, and M. Tinkham, J. Low Temp. Phys. **16**, 145 (1974).
[51]Thermal activation and quantum tunneling should cause the system to switch to the finite-voltage state at a current below the "mean-field" critical current; see, for example, T. A. Fulton and L. N. Dunkleberger, Phys. Rev. B **9**, 4760 (1974). The mean-field critical current is related to ΔF_0, see (13), and we expect the "true" and "effective" critical currents to be at least comparable. From our fits to the frozen-quasiparticle model, we have obtained energy barriers smaller than those obtained from (11), and hence (2), by a factor of $\sim$5. Using this barrier height to calculate the critical current for the sample in Fig. 8(a), we find $I_c \approx 18$ μA, which compares quite favorably with the measurements.
[52]These oscillations are similar to those we have reported previously (Ref. 10) in somewhat smaller samples. The behavior of those samples will be discussed below.
[53]This estimate is obtained from (15) with the additional reduction by a factor of $(L_\phi/L)^{1/2}$, where L_ϕ is the electron phase coherence length. This factor is necessary since our samples are longer than L_ϕ, and is similar to factors which arise from the theory of universal conductance fluctuations; see, for example, P. A. Lee, A. D. Stone, and H. Fukuyama, Phys. Rev. B **35**, 1039 (1987).
[54]This was verified by a detailed comparison of the different curves in Fig. 10.
[55]The effects of disorder on superconductivity in one dimension [see, for example, H. Ebisawa, H. Fukuyama, and S. Maekawa, J. Phys. Soc. Jpn. **55**, 4408 (1986)] would also be expected to reduce the critical field, but the size of this effect has not yet been calculated.
[56]The results shown in Fig. 12 were our first observations of the oscillations in *V-I*, and we have subsequently improved our resolution somewhat.
[57]We thank A. J. Leggett (private communication) for emphasizing the importance of this point.
[58]An alternative explanation of the oscillations seen here, and perhaps also in our larger samples, involves "transverse" degrees of freedom of the order parameter. The potential diagrams shown in Figs. 1 and 7 are contours along one direction in order-parameter space. There could conceivably be motion of the system in transverse directions, and this motion might take the form of oscillations, which would be quantized. An examination of the shape of the potential in these

transverse directions both near (Ref. 2) and far from [N. Giordano (unpublished)] the extrema indicates that the curvature of the potential in these directions is always larger than in the primary direction shown in Fig. 7. Hence, transverse oscillations would presumably give rise to levels with a larger spacing than those along the contours in Figs. 1 and 7, which does not explain the more closely spaced levels indicated by the results in Fig. 13.

[59] W. Zwerger (private communication).
[60] W. Zwerger, Phys. Rev. B **35**, 4737 (1987).