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Two-stream instabilities in solid-state plasmas caused by conventional and unconventional mechanisms

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We employ the linear-response theory of collisionless plasmas and the linear-response theory of carriers in a static, homogeneous electric field, with collisions approximated by the relaxation-time approximation [Phys. Rev. B **39**, 8464 (1989)] to study instabilities with respect to charge-density perturbations of counterstreaming charged particles. We treat both bulk (three-dimensional) systems and systems where the carriers drift along adjacent two-dimensional conducting planes (as in a semiconductor heterostructure). Instabilities occur in both the three- and two-dimensional systems, for both the collisionless plasma case (as in conventional plasma theory) and for the case of carriers driven by an electric field (which we call "electric-field-induced instability"). The physical mechanism that causes the electric-field-induced two-stream instability is linked to the presence of the driving electric field and scattering and is different from that of the conventional collisionless plasma instability. In a pair of adjacent quantum wells with $Al_{1-x}Ga_xAs/GaAs$-type parameters, we obtain an instability at very large drift velocities that may not be experimentally attainable. We speculate that by drifting carriers in a superlattice of alternating electron and hole layers, an instability could be obtained experimentally, and that such an instability could be used to produce a terahertz oscillator.

I. INTRODUCTION

Plasma instabilities that are caused by the counterstreaming of particles in a plasma, which generally go by the cognomen "two-stream instabilities," are a well-known phenomena.[1,2] The two-stream instability is an instability with respect to density perturbations in the plasma; under certain conditions, density perturbations in the plasma with counterstreaming particles will initially grow exponentially. The presence of these instabilities in plasmas has been confirmed experimentally,[3] and their appearance in some areas of plasma physics, such as the loss-cone instability in magnetic confinement,[4] has been considered an outright nuisance.

For reasons described in Sec. II, it is interesting to investigate the possibility of seeing an analogous instability in solid-state plasmas. A simple scheme would be to drift electrons and holes in a solid in opposite directions through the application of a static electric field. Spurred by this possibility, many theoretical studies of the two-stream instability in solid-state plasmas have been attempted.[5–14] In general, these theories were simply a rehashing of the classical two-stream instability theory that was well-known in plasma physics, with the physical parameters adjusted to describe a solid-state system. In all the above treatments, in place of explicitly treating the effect of the driving electric field on the carriers, an *ad hoc* drift of the carriers relative to the lattice was assumed. Also, in many cases the scattering of the carriers due to the lattice was ignored or glossed over. Clearly, these theories do not give an adequate description of the physics of carriers drifted by an electric field and scattered by the lattice. Fortunately, technology and fabrication techniques have progressed to the point where the simplifying assumptions of these old theories can be attained in some devices (see Sec. II), and for this reason we direct some of our effort in reproducing these theories in this paper.

Nevertheless, after all these years, the initial concept of achieving an instability by drifting the oppositely charged carriers with an electric field remained theoretically unexplored. Recently, a nonequilibrium linear-response formalism has been developed[15–18] that allows us to calculate, among other things, the collective modes of carriers in a solid that are being drifted by a large static homogeneous electric field and being scattered by the lattice. In this paper, we apply this formalism to the case where we have two species of carriers which are counterstreaming, as in the two-stream instability. We show both that this system admits a two-stream instability and that the mechanism for this instability is *different* from that for a collisionless plasma.

We devote a significant amount of our efforts to studying the two-stream instability of two-dimensional charged planes placed close to one another. The reason is as follows: with the existing technology of experimental semiconductor-device physics, in particular that of molecular beam epitaxy, experimentalists can make high mobility quasi-two-dimensional conducting layers in artificial semiconductor structures. By putting these conducting layers close together, and drifting the carriers in the layers against each other, it should be possible to obtain an instability in these quasi-two-dimensional structures.

The outline of this paper is as follows. In Sec. II we discuss the possible experimental realizations and the uses of the two-stream instability in solid-state devices. In Sec. III we review the theory of collective modes in bulk (three-dimensional) systems and the method for obtaining linear response from transport equations. In Sec. IV we study the two-stream instability in collisionless and electric-field-driven semiconductor plasmas in bulk (three-dimensional) systems. We also discuss the differences between the collisionless plasma and the electric-field-induced instabilities. In Sec. V we introduce the formalism for collective modes in coupled two-dimensional planes, and in Sec. VI we calculate the instabilities in collisionless and electric-field-driven semiconductor plasmas in coupled two-dimensional planes. Section VII contains a discussion of the results and a summary of the paper.

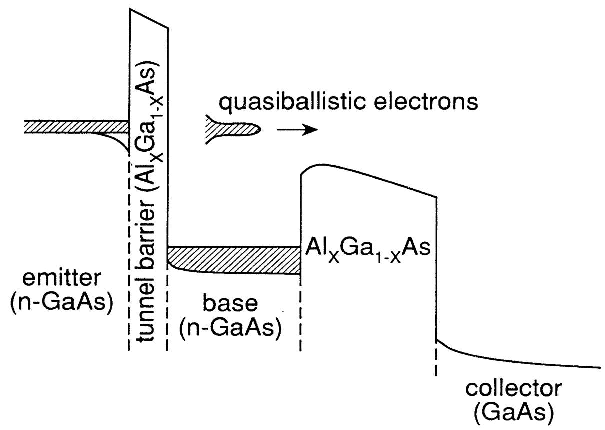

FIG. 1. Schematic diagram of the Heiblum THETA device. The electrons are injected from the emitter through a tunneling barrier into a base that is n-doped at 10^{18} cm^{-3}. The injected electrons are quasiballistic in the base region. The injected electrons and "cold" electrons in the base form a two-stream plasma which should be unstable to density perturbations.

II. POSSIBLE REALIZATIONS AND USES OF THE TWO-STREAM INSTABILITY IN SOLID-STATE DEVICES

In this section we briefly describe two possible experimental solid-state device realizations of plasmas with components that drift relative to one another, which could show the two-stream instability. We then discuss why such a device might be technologically useful.

A two-stream instability might be seen in a device in which electrons are injected into a relatively short doped base region from a tunnel barrier. Such a device, called the tunneling hot-electron transfer amplifier (THETA) device, has been fabricated by Heiblum,[19] and in a modified form, by Levi.[20] This device is shown schematically in Fig. 1. When the device is biased, electrons from the emitter tunnel through the barrier into the base region. These injected electrons are essentially monoenergetic, and they stream quasiballistically relative to the "cold" electrons that are present in the base. This situation could lead to a two-stream instability, as was pointed out by Gruzinskis *et al.*,[21] who used the two-stream instability to explain the anomalously short mean free paths of the injected electrons in these THETA devices. The physics of the two-stream instability in such a device is essentially identical to that of classical collisionless plasma physics.[2] However, in practice, there are a few difficulties associated with obtaining an instability with this scheme. First, the injected electrons have relatively short mean free paths of a few hundred angstroms. Therefore, the base region must be thin, which makes the instability difficult to observe. Furthermore, as we show in Sec. IV, the system needs to be a minimum size for instabilities to occur, and, for the parameters of the device fabricated by Heiblum,[19] this minimum size is larger than the mean free path of the electrons. Since the theory that is used to predict this instability assumes that the carriers are collisionless, the fact that the carriers undergo several collisions as they stream through the base might invalidate the prediction of the instability.

A second scheme for obtaining a two-stream instability in a solid-state plasma would be to drift electrons and holes in a semiconductor (or perhaps a semimetal) in opposite directions by application of a static electric field. One could photoexcite electrons and holes, and drift them in opposite directions, but the electrons and holes would recombine on the time scale of the order of hundreds of nanoseconds[22] and continuous "pumping" of electrons to the conduction band would be necessary. An alternate idea would be to put electrons and holes in adjacent conducting planes (see Fig. 2). The electrons and holes cannot recombine, but since there is still a Coulomb coupling between the electrons and holes in the separate layers (which is one of the key ingredients in the two-

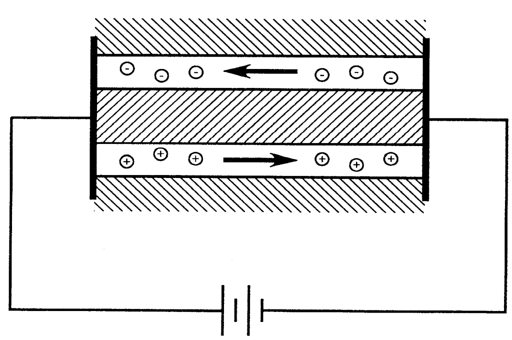

FIG. 2. Schematic diagram of the two-stream instability device in semiconductors. The electrons and holes are confined to adjacent two-dimensional planes and are drifted relative to one another by application of a static electric field parallel to the planes.

stream instability), it is possible that one will observe an instability in this structure. There are important differences between this electric-field-induced two-stream instability and the THETA device. Namely, in the region in which the carriers are counterstreaming, the carriers in the electric-field-induced two-stream instability device are (1) accelerated by the electric field, and (2) scattered by phonons, lattice imperfections and other carriers, whereas in the THETA device, acceleration and scattering in the base region (where the carriers are counterstreaming) are minimal. Since there is an electric field and strong scattering, the formalism that we have developed[15–18] is suitable for describing the electric-field-induced two-stream instability.

In this paper we investigate the possibility of obtaining instabilities in both these device configurations described above. The theory of instabilities in the THETA device is given by the conventional collisionless plasma theory, and this case has been explored previously.[5–14] The origin of the electric-field-induced instability, where the carriers are driven by an electric field, is somewhat different from the collisionless plasma instability because of the presence of the driving electric field and the scattering. In Sec. IV we compare the characteristics of the instabilities for the collisionless plasma and the field-induced cases.

Why should we attempt to make these unstable devices at all? Ultimately, the devices described above might be good infrared radiation sources. There seems to be a lack of a good robust coherent source of radiation in the infrared-frequency regime, since the highest frequency of radiation that can be produced from the venerable Gunn oscillator is approximately 100 GHz.[23] Various other devices have been proposed, from resonant-tunneling diodes to Josephson junction arrays to inversion population of Landau levels in a magnetic field, but none of these devices to date has performed as envisaged. The motivation for studying two-stream instabilities in two-dimensional structures is this: the plasmons in two-dimensional electron gases in GaAs MOSFET's (metal-oxide-semiconductor field-effect transistors) with wavelengths on the order of thousands of angstroms have frequencies in the terahertz regime.[24] Therefore, if these plasmons could be made to grow exponentially in amplitude, they could radiate at terahertz frequencies, giving us an infrared radiation source.

III. COLLECTIVE MODES AND LINEAR RESPONSE

In this section we review the conditions for the existence of collective modes in a charged system. A collective mode of a system is a self-sustaining coherent oscillation in the system, i.e., the system oscillates without having an external driving "force." We show that a collective mode exists when the dielectric function vanishes. We also review the method for calculating linear response from transport equations, since the linear response of a charged system determines its dielectric function.

A. Collective modes in bulk (three-dimensional) systems

In a charged system the relationship between the potential of an external perturbing source $V_{\rm ext}$ and the total potential (sum of external and induced potential in the system) $V_{\rm tot}$ for a longitudinal electric field is[25]

$$V_{\rm tot}(\mathbf{q},\omega)=\frac{V_{\rm ext}(\mathbf{q},\omega)}{\epsilon(\mathbf{q},\omega)} \; . \tag{1}$$

The expression for the dielectric function for a bulk plasma, $\epsilon(\mathbf{q},\omega)$, in terms of the linear density response of the system's carriers to the total potential (or "susceptibility"), $\chi(\mathbf{q},\omega)=n(\mathbf{q},\omega)/V_{\rm tot}(\mathbf{q},\omega)$, is[26]

$$\epsilon(\mathbf{q},\omega)=1-V_c(q)\chi(\mathbf{q},\omega)=1-\frac{4\pi e^2}{q^2}\chi(\mathbf{q},\omega) \; . \tag{2}$$

In the case of a solid-state plasma, in which the carriers move about in a solid-state lattice, the expression for $\epsilon(\mathbf{q},\omega)$ differs slightly from the form given by Eq. (2) because of the polarizability of the lattice. The lattice polarizability reduces both the external potential and the Coulomb interaction $V_c(q)$ by the lattice dielectric constant ϵ_0. Therefore, in Eq. (2), the overall dielectric function is increased by a factor of ϵ_0, while the $V_c(q)=4\pi e^2/q^2$ is decreased by a factor of ϵ_0. Hence, the dielectric function for carriers in a lattice is given by

$$\epsilon(\mathbf{q},\omega)=\epsilon_0\left[1-\frac{4\pi e^2}{\epsilon_0 q^2}\chi(\mathbf{q},\omega)\right] \; . \tag{3}$$

Since $V_{\rm tot}=V_{\rm ext}+V_{\rm ind}$ (where $V_{\rm ind}$ is the potential induced by the charge-density oscillations of the carriers in the system), Eq. (1) yields

$$\epsilon(\mathbf{q},\omega)V_{\rm ind}(\mathbf{q},\omega)=[1-\epsilon(\mathbf{q},\omega)]V_{\rm ext}(\mathbf{q},\omega) \; . \tag{4}$$

Equation (4) implies that if $\epsilon=0$, a nonzero $V_{\rm ind}$, caused by the oscillation of the charge density of the carriers in the system, can exist when $V_{\rm ext}=0$. This oscillation of the charge density of the carriers in the absence of an external potential is called a collective mode of the system. Hence, the condition of the occurrence of a collective mode with wave vector $\mathbf{q}$ which oscillates at the angular frequency $\omega(\mathbf{q})$ is

$$\epsilon(\mathbf{q},\omega(\mathbf{q}))=\epsilon_0\left[1-\frac{4\pi e^2}{\epsilon_0 q^2}\chi(\mathbf{q},\omega(\mathbf{q}))\right]=0 \; . \tag{5}$$

The time evolution of the mode with wave vector $\mathbf{q}$ goes as[27] $e^{-i\omega(\mathbf{q})t}$. The frequency $\omega(\mathbf{q})$ for a given collective mode is in general complex,

$$\omega(\mathbf{q})=\omega_r(\mathbf{q})+i\omega_i(\mathbf{q}) \; . \tag{6}$$

Hence, the time evolution of the induced potential of the collective mode is proportional to

$$V_{\rm ind}(t)\propto {\rm Re}(e^{-i\omega_r t})e^{\omega_i t} \; .$$

In general, for a system in stable equilibrium, the collective mode that is excited in the system will be damped exponentially, i.e., $\omega_i<0$. However, there are certain situations where one or more collective modes exist with

$\omega_i > 0$. Then, the collective mode is *unstable* and *grows exponentially*. (Clearly, the exponential growth of a collective mode cannot persist indefinitely. Since the linear-response theory only deals with small perturbations, the theory fails when the amplitude of the mode becomes too large. All the linear-response analysis tells you is if a particular mode will *initially* start to be damped or to grow.)

This paper deals with instabilities that may occur when two streams of charged particles are drifted relative to one another. We find the unstable collective modes in these systems by searching for the roots of $\epsilon(\omega)=0$ on the complex ω plane. The presence of one (or more) of the roots ω in the upper half complex plane indicates the existence of an unstable mode (or modes). As indicated by Eq. (3), to obtain the expression for the dielectric function $\epsilon(\mathbf{q},\omega)$ of the system, one needs to know the system's susceptibility, $\chi(\mathbf{q},\omega)$. In Sec. III B we describe the method for calculating the susceptibility from the Boltzmann transport equation.

B. Linear response from the Boltzmann equation

The Boltzmann equation is a very versatile tool for describing systems with carriers, especially in nonequilibrium situations. In particular, carrier linear response in both equilibrium and nonequilibrium situations have been successfully described by the Boltzmann equation and its extensions.[28–30,15–18]

The summary of the method for calculating $\chi(\mathbf{q},\omega)$ from the Boltzmann equation is as follows: (1) Solve the Boltzmann equation for the situation being investigated. (2) Perturb the Boltzmann equation with a small sinusoidal force term, $i\mathbf{q}V_1 e^{i(\mathbf{q}\cdot\mathbf{x}-\omega t)}$, to produce a response in the distribution function $f_1(\mathbf{v})e^{i(\mathbf{q}\cdot\mathbf{x}-\omega t)}$ that is linear in the V_1, and solve for $f_1(\mathbf{v})$. (3) Integrate $f_1(\mathbf{v})$ with respect to $\mathbf{v}$ to obtain n_1, the linear density response to V_1. The susceptibility χ is given by the ratio n_1/V_1. This method can be used to calculate the χ for both collisionless plasmas and for carriers drifting in an electric field. Below, we quote the results obtained for both these cases.

(i) $\chi(\mathbf{q},\omega)$ *for a collisionless plasma.* For a collisionless plasma with a distribution function[31] of $f_0(\mathbf{v})$, the susceptibility is given by[29]

$$\chi(\mathbf{q},\omega)=\frac{1}{m}\int_{\mathcal{C}} du \frac{\partial F_0(u)/\partial u}{u-\omega/|q|} . \quad (7)$$

Here, m is the mass of the carrier,

$$F_0(u)=\int d\mathbf{v}\, f_0(\mathbf{v})\delta(u-\mathbf{v}\cdot\hat{\mathbf{q}}) \quad (8)$$

is the projection of the distribution on the $\mathbf{q}$ axis, and $\mathcal{C}$ denotes an integration contour that goes below the pole at $u=\omega/|q|$.

(ii) $\chi(\mathbf{q},\omega)$ *for carriers in an electric field.* Recently, we calculated the susceptibility of nondegenerate carriers drifting in a static, homogeneous electric field exerting a force $\mathbf{F}_0$ on each carrier, with the collisions described by a relaxation-time approximation.[15] The Boltzmann equation for this situation is

$$\mathbf{F}_0\cdot\frac{\partial f_0}{\partial \mathbf{p}}=-\frac{f_0(\mathbf{p})-f_{\text{eq}}(\mathbf{p})}{\tau} , \quad (9)$$

where $f_{\text{eq}}(\mathbf{p})$ is a Maxwell-Boltzmann distribution. By applying the method summarized above to calculate the susceptibility, we obtained

$$\chi(\mathbf{q},\omega)=-\left[\frac{n_0}{k_B T}\right]\frac{s^2}{2}\int_0^\infty dx \frac{x\exp[-i\mathbf{s}\cdot\mathbf{w}_d x^2/2-x(1-i\Omega)-s^2x^2/4]}{1+ix\mathbf{s}\cdot\mathbf{w}_d}\left[1-\frac{1}{1-i\Omega}W\left[\frac{1-i\Omega}{\sqrt{s^2+2i\mathbf{s}\cdot\mathbf{w}_d}}\right]\right]^{-1} , \quad (10)$$

where $\mathbf{w}_d=\mathbf{v}_d/v_{\text{th}}=\mathbf{F}_0\tau/mv_{\text{th}}$ is the normalized drift velocity, $\mathbf{s}=\mathbf{q}v_{\text{th}}\tau$ is the normalized wave vector, $\Omega=\omega\tau$ is the normalized frequency, and $W(\zeta)=\sqrt{\pi}\zeta\exp(\zeta^2)\text{erfc}(\zeta)$. Here, $\text{erfc}(\zeta)$ is the complimentary error function[32] and the radical in Eq. (10) denotes the principal part of the square root. In the following sections, the expressions (7) and (10) are used to obtain the dispersion relation for the collective modes of systems with counterstreaming charged particles.

IV. TWO-STREAM INSTABILITIES IN BULK SYSTEMS

In this section we discuss the instabilities that result from streaming two sets of charged particles relative to one another in bulk (three-dimensional) solid-state systems. The two-stream instability is an instability with respect to density perturbations, and under certain conditions (as described below), perturbations in the density of the plasma will (initially) *grow exponentially.* We find the unstable modes for counterstreaming charged particles in the following situations.

(1) *Collisionless plasma (no driving electric field),* where the distribution function of the particles are (a) δ functions in velocity (which we call the "pure beam instability") and (b) thermally broadened (which we call the "thermalized beam instability"). The pure beam instability is, of course, the limit of the thermalized beam instability where the thermal spread of the distribution goes to zero. Because the mathematics of the pure beam instability is considerably simpler than the thermalized beam instability, it is discussed first. This theory describes a plasma instability that might occur in a THETA device, where carriers are injected into a doped base region, and where the injected carriers travel quasiballistically over a

distance of several hundred angstroms.

(2) *Carriers in a driving electric field,* with lattice scattering (which we call the "electric-field-induced instability"). This theory describes a plasma instability that might occur in a continuously photoexcited bulk intrinsic semiconductor, or a semimetal.

Unfortunately, good physical explanations for the occurrence of two-stream instabilities in the literature is lacking. At the end of this section, we review a few explanations that have been advanced for the physical mechanisms responsible for the two-stream instability.

A. The pure beam two-stream instability in bulk systems

The simplest case of a two-stream instability is the pure beam instability, where there are two monoenergetic beams of charged particles drifting relative to one another in the absence of both collisions and a driving static electric field. Here, we review the theory of this instability.[2]

Let one beam of particles be called component a and the other component b. The distribution functions for this system are

$$f_a(\mathbf{v})=n_{0,a}\delta(\mathbf{v}-\mathbf{v}_{d,a}) ,$$
$$f_b(\mathbf{v})=n_{0,b}\delta(\mathbf{v}-\mathbf{v}_{d,b}) . \qquad (11)$$

We show that this system is unstable to perturbations in the density.

Equation (2) in the previous section gave the relationship between the dielectric function for a bulk plasma and the susceptibility of a one-component bulk plasma. The generalization of this relationship for a many-component bulk plasma is simply

$$\frac{\epsilon(\mathbf{q},\omega)}{\epsilon_0}=1-V_c(q)\sum_\alpha \chi_\alpha(\mathbf{q},\omega) , \qquad (12)$$

where $V_c(q)=4\pi e^2/\epsilon_0 q^2$ is the three-dimensional Fourier transform of the Coulomb potential, and χ_α is the susceptibility of the αth component of the plasma.

Substituting the distribution functions of the components for the pure-beam case, Eq. (11), into Eq. (8) gives the projected distribution function

$$F_{0,\alpha}(u)=n_{0,\alpha}\delta(u-\hat{\mathbf{q}}\cdot\mathbf{v}_{d,\alpha}) . \qquad (13)$$

Substituting this into Eq. (7), the general expression for the susceptibility for a collisionless plasma, gives the susceptibility for each component of the pure beam of

$$\chi_\alpha(\mathbf{q},\omega)=\frac{n_{0,\alpha}q^2}{m_\alpha}\frac{1}{(\omega-\mathbf{q}\cdot\mathbf{v}_{d,\alpha})^2} . \qquad (14)$$

From Eq. (14) and Eq. (12), the relationship between the susceptibility and the dielectric function, we obtain

$$\epsilon(\mathbf{q},\omega)=1-\frac{\omega_{p,a}^2}{(\omega-\mathbf{q}\cdot\mathbf{v}_{d,a})^2}-\frac{\omega_{p,b}^2}{(\omega-\mathbf{q}\cdot\mathbf{v}_{d,b})^2} , \qquad (15)$$

where $\omega_{p,\alpha}^2=4\pi n_{0,\alpha}e^2/\epsilon_0 m_\alpha$ denotes the plasma frequency of component α.

The collective modes are given by the zeros of $\epsilon(\mathbf{q},\omega)$. To get an idea of how one obtains an instability from a plasma with two counterstreaming beams, we first discuss the simplest case, where the plasma frequencies of the two components are equal. Let component a be drifting at velocity $\mathbf{v}_d$ and let component b be at rest (we can do this without loss of generality because we can always make a Galilean transformation into the moving frame of reference of one of the components). With these parameters,

$$\omega_{p,a}=\omega_{p,b}=\omega_p ,$$
$$\mathbf{v}_{d,a}=\mathbf{v}_d , \qquad (16)$$
$$\mathbf{v}_{d,b}=0 ,$$

Eq. (15) implies that collective modes are given by the equation

$$\epsilon(\mathbf{q},\omega)=1-\frac{\omega_p^2}{(\omega-\mathbf{q}\cdot\mathbf{v}_d)^2}-\frac{\omega_p^2}{\omega^2}$$
$$=1-\frac{1}{2(y-Q)^2}-\frac{1}{2(y+Q)^2}$$
$$=0 , \qquad (17)$$

where

$$y=\frac{\omega-\mathbf{q}\cdot\mathbf{v}_d/2}{\sqrt{2}\omega_p} ,$$
$$Q=\frac{\mathbf{q}\cdot\mathbf{v}_d}{2\sqrt{2}\omega_p} . \qquad (18)$$

From Eq. (17), we obtain

$$(y^2)^2-y^2(2Q^2+1)+(Q^4-Q^2)=0 , \qquad (19)$$

which can be solved to yield

$$y^2=\tfrac{1}{2}[1+2Q^2\pm(1+8Q^2)^{1/2}] . \qquad (20)$$

Equation (20) gives the four roots for the collective mode frequencies

$$\omega=\frac{\mathbf{q}\cdot\mathbf{v}_d}{2}\pm\omega_p\left\{1+\left[\frac{\mathbf{q}\cdot\mathbf{v}_d}{2\omega_p}\right]^2\pm\left[1+\left[\frac{\mathbf{q}\cdot\mathbf{v}_d}{\omega_p}\right]^2\right]^{1/2}\right\}^{1/2} . \qquad (21)$$

These four roots are plotted in Fig. 3. If $\mathbf{q}\cdot\mathbf{v}_d<2\sqrt{2}\omega_p$, then two of the roots have imaginary components,

$$\omega=\frac{\mathbf{q}\cdot\mathbf{v}_d}{2}\pm i\omega_p\left\{\left[1+\left[\frac{\mathbf{q}\cdot\mathbf{v}_d}{\omega_p}\right]^2\right]^{1/2}-1-\left[\frac{\mathbf{q}\cdot\mathbf{v}_d}{2\omega_p}\right]^2\right\}^{1/2} . \qquad (22)$$

Since one of these imaginary roots is in the upper-half complex plane, the mode it describes grows exponentially and is therefore unstable.

Note that the range of wave vectors that are unstable scale linearly with the *inverse* of the drift velocity. The maximum wave vector at which the system is unstable to

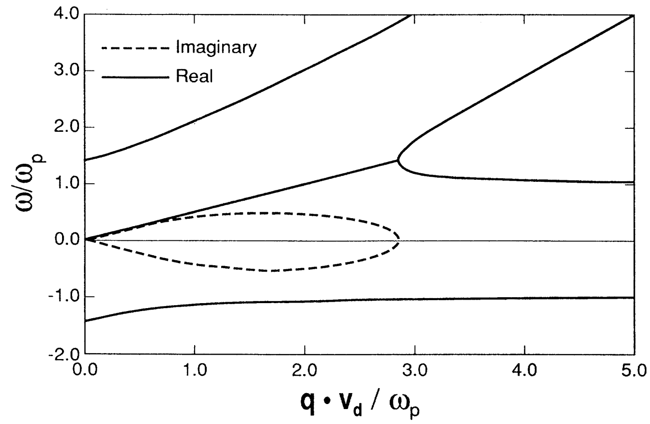

FIG. 3. The dispersion relation for a three-dimensional collisionless plasma comprising two pure beams of particles, each having the same plasma frequency, ω_p. All the particles in the first beam are moving with velocity $\mathbf{v}_d$ while all the particles in the second beam are at rest. There are a total of four collective modes in this system. The two modes starting at $\pm\sqrt{2}\omega_p$ at $q=0$ are the plasmon modes, where the charge oscillations of the two beams are in phase. The other two modes have imaginary parts (which are shown by the dashed lines) and the same real part for $\mathbf{q}\cdot\mathbf{v}_d<2\sqrt{2}\omega_p$. These two modes correspond to the charge oscillations of the two beams being almost π out of phase for small $\mathbf{q}\cdot\mathbf{v}_d$, resulting in a nearly total cancellation of the Coulomb potential, and hence a mode that is acoustic in character. The mode with the positive imaginary part is the unstable mode.

density perturbations, $q_{\max}$, is given by

$$q_{\max}=\frac{2\sqrt{2}\omega_p}{v_d} . \tag{23}$$

Therefore, the larger the drift velocity, the smaller the $q_{\max}$. This can be explained heuristically as follows. The continuity equation is

$$\nabla\cdot\mathbf{j}+\frac{\partial n}{\partial t}=0 , \tag{24}$$

where $\mathbf{j}$ is the current. Consider the static case for simplicity. For a small perturbation in the average velocity (caused by a small modulation in the potential, say) $\mathbf{v}(\mathbf{x})=\mathbf{v}_0+\delta\mathbf{v}e^{i\mathbf{q}\cdot\mathbf{x}}$ and in the density, $n(\mathbf{x})=n_0+\delta n e^{i\mathbf{q}\cdot\mathbf{x}}$, the continuity equation gives

$$\frac{\delta n}{n_0}=\frac{\hat{\mathbf{q}}\cdot\delta\mathbf{v}}{\hat{\mathbf{q}}\cdot\mathbf{v}_d} . \tag{25}$$

This equation states that the larger the drift velocity, the smaller the effect of a velocity modulation on the density modulation. We expect that a smaller modulation of the density hinders the occurrence of the instability (since instabilities are density modulations that increase in magnitude), and therefore Eq. (25) implies that large drift velocities tend to *suppress* instabilities. Therefore, we expect that an increase of the drift velocities decreases the range of wave vectors over which the instability occurs, which is consistent with Eq. (23).

The analysis given above seems to be unphysical, since it seems to imply that even the smallest drift v_d of one component relative to another makes the plasma unstable to the growth of density perturbations. Indeed, Eq. (23) seems to indicate that the smaller v_d is, the larger the range over which q is unstable. The reason for this strange result lies in the derivation of the dielectric function[29]—based on the linearized Boltzmann equation—which assumed that the perturbation was weak. The linearized Boltzmann-Vlasov equation does *not* conserve energy—the total kinetic energy of the particles, $\int d\mathbf{v}(mv^2/2)\int d\mathbf{x}\,f(\mathbf{v},\mathbf{x},t)$, does not change, but the electric-field energy $\int d\mathbf{x}\,\mathbf{E}^2(\mathbf{x},t)/4\pi$ changes with time. Because energy is not conserved in the linearized theory, the theory predicts that a minuscule drift in one component relative to another will cause the density perturbation (and hence the electric-field energy) to grow exponentially forever. However, if higher-order terms in the perturbation are added, as in the quasilinear Boltzmann-Vlasov theory,[33] one regains conservation of energy, and hence the size of the electric-field energy grows at the expense of the total kinetic energy of the particles. Therefore, if there is a minuscule drift of one component relative to another, the kinetic energy that is free for "conversion" to the plasma-wave electric-field energy is small, and so the exponential nature of the growth shall be limited to very short times before energy conservation stops the increase of the growth of the plasma wave. Another important feature that is neglected in the pure beam case is the fact that, in general, the particles in a given component of the plasma are not all moving with the same velocity. The effect of a spread in the velocities of the particles is treated in Sec. IV B.

In the unstable mode, the charge oscillations of the two components are *out of phase* and tend to *cancel*. We illustrate this with the symmetric pure beam instability (where the plasma frequencies of the two beams are equal). By expanding the inner square root, Eq. (22), in powers of $\mathbf{q}$, we find that the frequency of the unstable mode in the limit of small $\mathbf{q}\cdot\mathbf{v}_d/\omega_p$ is

$$\omega\approx\frac{\mathbf{q}\cdot\mathbf{v}_d}{2}(1+i) \quad (q\to 0) . \tag{26}$$

Substituting Eq. (26) into the expressions for the linear response of components a and b,

$$\chi_a(\mathbf{q},\omega)=\frac{\delta n_a}{\delta V}=\frac{n_{0,a}q^2}{m_a(\omega-\mathbf{q}\cdot\mathbf{v}_d)^2} ,$$
$$\chi_b(\mathbf{q},\omega)=\frac{\delta n_b}{\delta V}=\frac{n_{0,b}q^2}{m_b\omega^2} \tag{27}$$

gives the expression for the density response in the low-q limit,

$$\frac{\delta n_a}{\delta V}=\frac{2in_{0,a}}{m_a(\hat{\mathbf{q}}\cdot\mathbf{v}_d)^2} ,$$
$$\frac{\delta n_b}{\delta V}=-\frac{2in_{0,b}}{m_b(\hat{\mathbf{q}},\mathbf{v}_d)^2} . \tag{28}$$

Since $\delta n_a/\delta V = -\delta n_b/\delta V$, the charge-density oscillations of components a and b in the unstable mode are π out of phase and tend to cancel each other, which weakens the Coulomb interaction and results in a mode with a dispersion relation of the real part of ω that is linear in $\mathbf{q}$ (i.e., an acoustic mode). In fact, in all the unstable modes described in this paper, the charge oscillations of the two components cancel and the dispersion relation of the real part of ω is linear in $\mathbf{q}$.

Of course, the instability is not limited to the case where the plasma frequencies of the constituent components are equal. For arbitrary plasma frequencies, with component a drifting at velocity $\mathbf{v}_d$ and component b at rest, we define

$$y = \frac{\omega - \mathbf{q}\cdot\mathbf{v}_d/2}{[(\omega_{p,a}^2 + \omega_{p,b}^2)]^{1/2}} ,$$

$$Q = \frac{\mathbf{q}\cdot\mathbf{v}_d}{2[\omega_{p,a}^2 + \omega_{p,b}^2)]^{1/2}} , \tag{29}$$

$$r = \frac{\omega_{p,a}^2 - \omega_{p,b}^2}{\omega_{p,a}^2 + \omega_{p,b}^2} .$$

Setting ϵ equal to zero in Eq. (15), we obtain the quartic equation for the frequencies of the collective modes

$$y^4 - y^2(2Q^2+1) - y(2rQ) + (Q^4 - Q^2) = 0 . \tag{30}$$

It can be shown[34] that the instability criterion for two streams is

$$|\mathbf{q}\cdot\mathbf{v}_d| < \tfrac{1}{2}(\omega_{p,e}^{2/3} + \omega_{p,h}^{2/3})^{3/2} . \tag{31}$$

We know of no physical explanation for this somewhat perplexing criterion.

We solved Eq. (30) using a numerical polynomial root-finding program.[35] Figure 4 shows a three-dimensional plot of the domain of instability as a function of the normalized wave vector Q and the difference in the plasma frequencies r.

We apply this theory to the experimental condition in the THETA device of Heiblum *et al.*[19] Let a be the electrons being injected into the base region, and b be the electrons that are sitting in the base. We use the parameters of the experiment,

$$v_{d,a} \approx 10^8 \text{ cm/s} ,$$

$$n_a \approx 10^{16} \text{ cm}^{-3} , \tag{32}$$

$$n_b \approx 10^{18} \text{ cm}^{-3} .$$

The density n_a of the injected current had to be inferred by assuming the cross section of the device was on the order of 10^{-6} cm^2 (the same value of n_a was used in Ref. 21). The lattice dielectric constant ϵ_0 and the effective electron mass m_e in GaAs is

$$\epsilon_0 \approx 10, \quad m_e \approx 0.067 m_{\text{bare}} , \tag{33}$$

where $m_{\text{bare}} = 9.1\times10^{-28}$ g is the bare electron mass. With these parameters, the plasma frequencies are

$$\omega_{p,a} = \left[\frac{4\pi n_a e^2}{m_e\epsilon_0}\right]^{1/2} \approx 5\times10^{12}\ \text{s}^{-1} ,$$

$$\omega_{p,b} = \left[\frac{4\pi n_b e^2}{m_e\epsilon_0}\right]^{1/2} \approx 5\times10^{13}\ \text{s}^{-1} , \tag{34}$$

$$\omega_{p,t} \equiv (\omega_{p,a}^2 + \omega_{p,b}^2)^{1/2} \approx 5\times10^{13}\ \text{s}^{-1} .$$

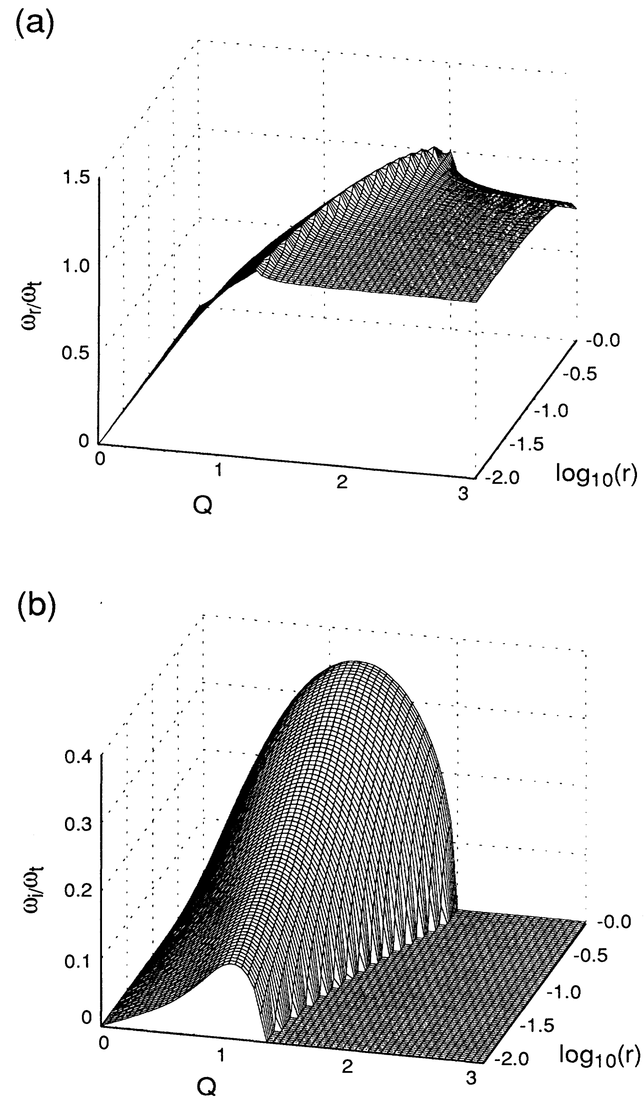

FIG. 4. The unstable mode in a collisionless "pure beam" two-stream plasma, as a function of wave vector and difference in plasma frequencies of the two beams. The real part (oscillation angular frequency) is shown in (a) and the imaginary part (growth rate) is shown in (b). This mode corresponds to the changes in the two beams oscillating almost π out of phase, resulting in a linear dispersion relation for the real part. Note that the growth rate is still substantial even when one of the components has a plasma frequency that is two orders of magnitude lower than the other.

Using these numbers, we calculate the conditions for collective modes, shown in Fig. 5, in the Heiblum *et al.* THETA device. The x axis is in units of $\omega_{p,t}/v_d \approx 5\times10^5$ cm. Therefore, since the maximum wave vector $q_{\max}$ for the instability is on the order of $\omega_{p,t}/v_d$, the minimum wavelength $\lambda_{\min}$ of an unstable mode is on the order of $\lambda_{\min}=2\pi/q_{\max}\approx 1000$ Å. The oscillation frequency and growth rate at this wave vector are on the order of $\omega \simeq \omega_{p,t} \simeq 5\times10^{13}$ s^{-1} and $\gamma \simeq 0.1\times\omega_{p,t} \simeq 5\times10^{12}$ s^{-1}, respectively.

Since the base of the Heiblum THETA device is only approximately 300 Å long, it is not long enough for an instability to occur. By making a base with a higher doping level (to decrease $\lambda_{\min}$) and/or engineering a longer base with an increased mean free path, an instability might be observed.

B. The thermalized beam two-stream instability in bulk systems

In Sec. IV A the distribution functions of the two components were assumed to be δ functions in velocity. In any real plasma, the distribution functions will have some width in momentum space, since not all the particles will be traveling at exactly the same velocity. Since one way to drift a collisionless plasma is to uniformly boost an equilibrium plasma with some drift velocity, the most natural distribution function that one can put in is a drifted Maxwellian

$$f_{0,\alpha}=\frac{n_{0,\alpha}}{\sqrt{\pi}v_{\mathrm{th},\alpha}}\exp\left[-\frac{(\mathbf{v}-\mathbf{v}_{d,\alpha})^2}{v_{\mathrm{th},\alpha}^2}\right], \quad (35)$$

where $v_{\mathrm{th},\alpha}$ is the width of the distribution of component α. (In the limit, $v_{\mathrm{th},\alpha}\to 0$, we regain the δ-function distribution of Sec. IV A.) The instability that occurs when

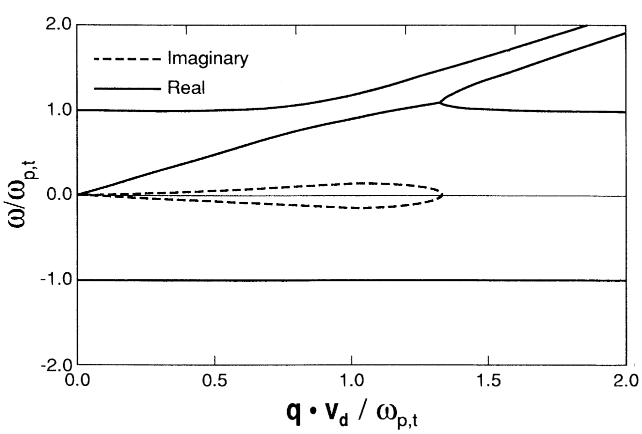

FIG. 5. The unstable modes in a collisionless pure beam two-stream plasma, for the parameters corresponding to the Heiblum THETA device (Ref. 19). As in Fig. 2, the dashed lines correspond to the imaginary parts of the acoustic modes. In the Heiblum device, $\omega_{p,t}\equiv(\omega_{p,a}^2+\omega_{p,b}^2)^{1/2}\approx 5\times10^{13}$ s^{-1} and $v_d/\omega_{p,t}\approx 2\times10^{-6}$ cm$=200$ Å. This implies that the minimum wavelength that is unstable is on the order of $\lambda_{\min}>(1/1.3)(2\pi v_d/\omega_{p,t})\approx 1000$ Å.

collisionless Maxwellian plasmas are counterstreamed is called the thermalized beam two-stream instability. In this section, we review the results of the instability in the thermalized beam two-stream instability in bulk systems.[2,5]

The question is: does the thermal broadening have any effect on the range of drift momenta and wave vectors over which the system is unstable? The answer is, for $v_{d,a}+v_{d,b}\lesssim v_{\mathrm{th},a}+v_{\mathrm{th},b}$ there is a profound effect—at drift velocities that are small or comparable to the thermal velocities, there is *no* instability, as we expect. This immediately leads us to another question: does the thermal and degeneracy broadening of the distribution function in the Heiblum THETA device result in the disappearance of the two-stream instability? Using the results in this section, we argue that the answer is "no."

To describe the linear response of a thermalized beam, we use the plasma dispersion function[36]

$$Z(\zeta)=\frac{1}{\sqrt{\pi}}\int_{\mathcal{C}}\frac{\exp(-z^2)}{z-\zeta}, \quad (36)$$

where the contour of integration $\mathcal{C}$ in Eq. (36) goes under the pole at ζ, as in Eq. (7), the expression for the susceptibility of a collisionless plasma. Substituting the drifted Maxwellian distribution, Eq. (35), into Eq. (7), and integrating by parts, we obtain

$$\chi_\alpha(\mathbf{q},\omega)=\frac{n_{0,\alpha}}{m_\alpha}\frac{1}{\sqrt{\pi}}\times\int_{\mathcal{C}}\frac{du}{v_{\mathrm{th},\alpha}}\frac{\frac{\partial}{\partial u}\exp[-(u-\hat{\mathbf{q}}\cdot\mathbf{v}_{d,\alpha}/v_{\mathrm{th},\alpha})^2]}{u-\omega/|q|}$$
$$=\frac{n_{0,\alpha}}{mv_{\mathrm{th},\alpha}^2}Z'(\tilde{v}_{\phi,\alpha}-\tilde{v}_{d,\alpha}), \quad (37)$$

where $\tilde{v}_{\phi,\alpha}=\omega/(|q|v_{\mathrm{th},\alpha})$ and $\tilde{v}_{d,\alpha}=\mathbf{v}_{d,\alpha}\cdot\hat{\mathbf{q}}/v_{\mathrm{th},\alpha}$ are the normalized phase and drift velocities, respectively, and $Z'(\zeta)$ is the derivative of the plasma dispersion function. The dielectric function, from Eq. (12), is

$$\epsilon(\mathbf{q},\omega)=1-\frac{q_{\mathrm{sc},a}^2}{2q^2}Z'(\tilde{v}_{\phi,a}-\tilde{v}_{d,a})-\frac{q_{\mathrm{sc},b}^2}{2q^2}Z'(\tilde{v}_{\phi,b}-\tilde{v}_{d,b}), \quad (38)$$

where

$$q_{\mathrm{sc},\alpha}^2=4\pi n_{0,\alpha}e^2/(\epsilon_0 m v_{\mathrm{th},\alpha}^2/2)=4\pi n_{0,\alpha}e^2/\epsilon_0 k_B T_\alpha$$

is the screening wave vector of a classical plasma with temperature T_α. The function $Z'(\zeta)$ for complex ζ can be numerically evaluated without much difficulty[36] [$Z'(x)$ for real x is plotted in Fig. 6].

To find the collective modes of this plasma, we numerically searched for the roots $\omega(\mathbf{q})$ of $\epsilon(\mathbf{q},\omega(\mathbf{q}))=0$ by utilizing the downhill simplex method[37] to search for the minima of the function $1+|\epsilon|^2$ on the complex ω plane. Figure 7 shows the region of instability in phase space for

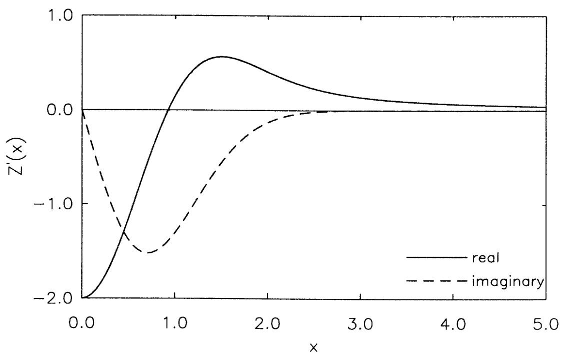

FIG. 6. The real (solid line) and imaginary (dashed line) parts of the derivative of the plasma dispersion function, $Z'(x)$, as a function of real x. The function $Z(x)$ is defined in Eq. (36). $Z'(\zeta)$ is used to describe the linear response of classical plasmas with equilibrium (Maxwellian) distribution functions.

a *symmetric* thermally broadened two-stream plasma (i.e., the plasma thermal broadening $v_{\text{th},\alpha}$ and the plasma frequencies $\omega_{p,\alpha}=q_{\text{sc},\alpha}v_{\text{th},\alpha}/\sqrt{2}$ of the two beams are equal). Let $\mathbf{v}_d$ be the relative drift between the center of the two beams. For $v_d/v_{\text{th}} \gg 1$, the thermal broadening becomes irrelevant, and the nature of the instability essentially is given by that of the instability described with the δ-function distributions. However, if $v_d/v_{\text{th}} \lesssim 1.9$, there is no instability, as expected. The heuristic argument for the disappearance of the instability at small drift velocities is as follows: when one has a large spread in velocity of the distribution function, the effect of the particles of very different velocities will tend to add up incoherently in the density perturbations, which tends to diminish the possibility of an instability. The actual explanation of course is much more complicated, given that these collective modes come from a "self-consistent" set of equations (the density modulation produces a potential modulation, which changes the distribution function, which in turn adds to the density modulation, as so on).

In the Heiblum THETA device, the thermal and degeneracy broadening should *not* be a factor in destroying the instability. According to Ref. 19, the energy spread of the injected beam is on the order of 50 meV, and the density of the base approximately $n_b=10^{18}$ cm^{-3}. This implies that the thermal broadening of the injected beam and degeneracy broadening of the base electrons is

$$\delta v_a = 10^7 \text{ cm/s}, \quad \delta v_b = 3\times 10^7 \text{ cm/s} . \tag{39}$$

Since the drift velocity is on the order of 10^8 cm/s, the thermal broadening should play no role in reducing the instability.

Finally, we address a question that is often asked about these unstable modes: "Why doesn't Landau damping eliminate these unstable modes?" An instability in the case of a two-stream collisionless plasma is simply a reversed Landau damped mode. This is shown quite dramatically by Fig. 8, where we plot the frequency of a collective mode at fixed wave vector against the relative drift velocity of the two components. We see that a mode that is heavily Landau damped when the relative velocities are zero (equilibrium) evolves into an unstable mode when the components are drifted against each other. This behavior implies that the mechanism that is causing the modes in a thermalized beam to be unstable is "Landau undamping"; i.e., Landau damping is somehow reversed in these situations so that the unstable modes *gain* (rather than *lose*) energy from the single particles.

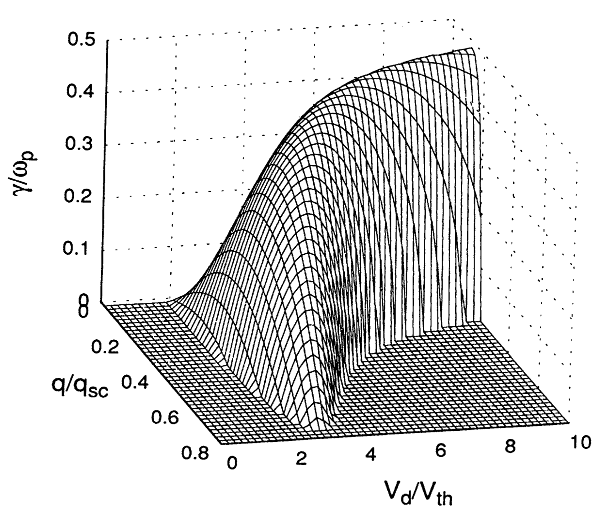

FIG. 7. The linear-response growth rate γ of a density perturbation in a bulk (three-dimensional) collisionless plasma with counterstreaming electrons and holes, whose plasma frequencies and thermal velocities are identical. This figure shows that at drift velocities that are comparable or smaller than the thermal velocity, the instability disappears altogether.

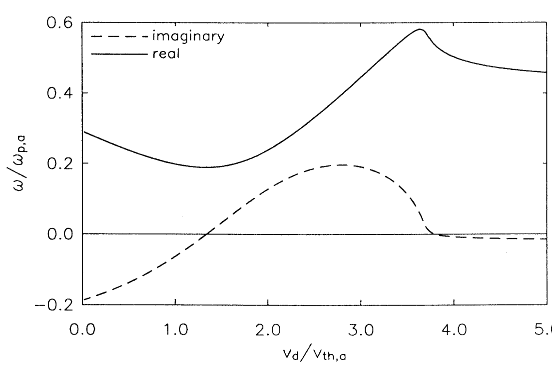

FIG. 8. The real (solid line) and imaginary (dotted line) parts of the collective mode frequency of the acoustic mode of a two-component plasma, at *fixed wave vector*, $q=0.4q_{\text{sc},a}$, as a function of the *relative drift* velocity $\mathbf{v}_d$. Both components have Maxwellian distributions with equal thermal velocities. Component a is at rest, while component b drifts with velocity $\mathbf{v}_d$. The mass of component a is nine times that of component b. When the relative drift of the two components is zero (as in equilibrium), the mode is heavily Landau damped [Im(ω)<0]. As the relative drift velocity increases, the mode evolves into an unstable mode [Re(ω)>0]. This implies that when the two components are drifted relative to one another, the mechanism responsible for Landau damping is reversed and causes the electrostatic wave to grow instead of being damped.

C. Electric-field-induced instability in bulk semiconductors

We have described two-stream instabilities for particles that are collisionless and not driven by an external applied electric field. This accurately describes the situation of a plasma of ions and electrons, and it describes the physics of the THETA device, where electrons that are injected into the base region stream quasiballistically across the electrons at rest in the base. However, the situation changes when one tries to drive the instability by the application of a large static electric field on carriers in a solid-state device. The carriers are accelerated by the field and scattered by phonons, impurities and lattice defects, and therefore, the formalism must take into account both the effect of the applied external electric field and the scattering. The linear response of nondegenerate semiconductors with carriers driven by a static, homogeneous electric field and with collisions described by a single-rate relaxation-time approximation was calculated in Ref. 15. In this section, we use the results from Ref. 15 (which we quoted in Sec. III B) to investigate instabilities in solid-state plasmas driven by an electric field.

In this section we treat bulk (three-dimensional) semiconductors, where we assume that there are electrons and holes occupying the same volume of space. In practice, it is impossible for the electrons and holes in a semiconductor to survive indefinitely in the same volume of space, since recombination occurs on the time scale of hundreds of nanoseconds.[22] One could envisage photoexciting an intrinsic semiconductor, and performing the experiment before recombination occurs, or continuously photoexciting the system so that the generation is matched by the recombination. One could also imagine using a semimetal like bismuth, which has both a small pocket of electrons and holes. By applying an electric field to system, the electrons and holes are drifted in opposite directions, creating two streams of charged particles flowing relative to one another. Is this system unstable to density perturbations, as in the case of the collisionless plasmas? The answer is yes, but we argue below that the mechanisms for the instability for the field-induced case and the collisionless plasma case are *not* the same.

The collective modes are found from the zeros of the dielectric function. We obtained the dielectric function from Eq. (12), using the $\chi_\alpha(\mathbf{q},\omega)$ given in Eq. (10), which was derived from the Boltzmann equation for a nondegenerate semiconductor in a static homogeneous electric field, within the relaxation-time approximation. The collective modes are those $\omega(\mathbf{q})$ that satisfy $\epsilon(\mathbf{q},\omega(\mathbf{q}))=0$. We find these numerically, using the simplex method (as in Sec. III B) for finding the roots in the complex ω plane. Figure 9 shows the plot of the real and imaginary frequencies of the unstable collective mode, as a function of the drift velocity and wave vector, in a bulk system with electrons and holes. The densities and relaxation times are assumed to be equal, while the mass ratio of the holes to the electrons is taken to be 7 (as in GaAs). The contour line on the Im[ω] plot is at Im[ω]=0, and therefore it separates the regions where the mode is damped (Im[ω]<0) and where it is unstable (Im[ω]>0). For these parameters, the collective modes become unstable at approximately $\mathbf{v}_{d,e}\approx 0,8v_{\mathrm{th},e}$. For typical values of densities and relaxation times

$$
\begin{aligned}
&n_{0,e}=n_{0,h}=3\times10^{17}\ \mathrm{cm}^{-3}\ ,\\
&\tau_e=\tau_h=10^{-13}\ \mathrm{s}\ ,\\
&(\omega_{p,e}\tau_e)^2=10\ ,
\end{aligned}
\tag{40}
$$

and at 100 K, the thermal velocity of electrons in GaAs is approximately 10^7 cm/s. Given a relaxation time of approximately 10^{-13} s, it would take an electric field on the

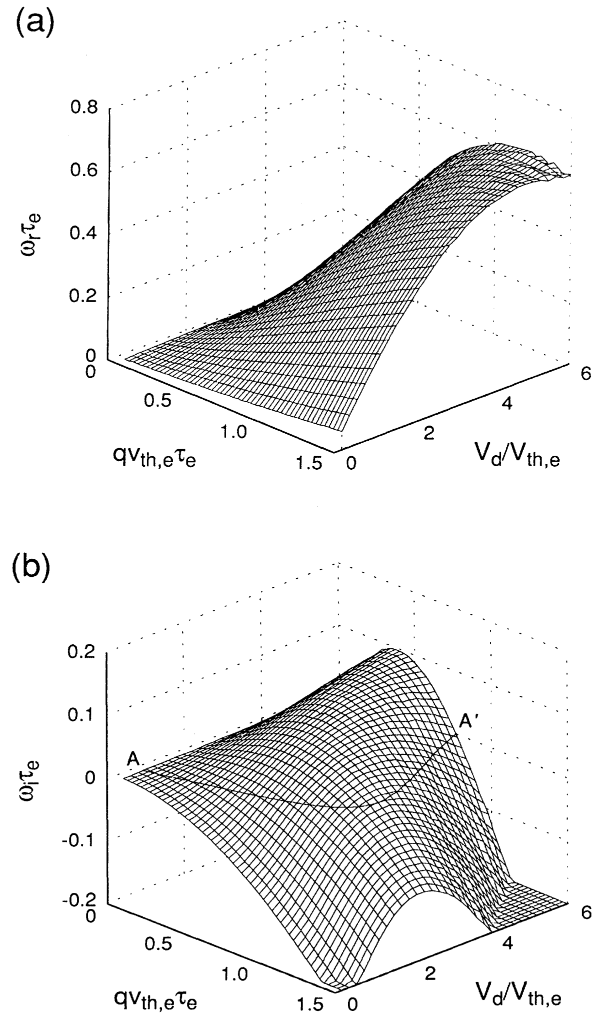

FIG. 9. The (a) real and (b) imaginary parts of the unstable collective mode frequency ω as a function of drift velocity and wave vector, in a bulk (three-dimensional) nondegenerate semiconductor with equal density of electrons and holes drifted by a static electric field. The linear response $\chi(\mathbf{q},\omega)$ was calculated from the Boltzmann equation within the relaxation-time approximation. The ratio of the masses, $m_h/m_e=7$, and the ratio of the relaxation times $\tau_h/\tau_e=1$, and $(\omega_{p,e}\tau)^2=10$. The contour line AA' in (b) separates the $\omega_i<0$ (stable) and $\omega_i>0$ (unstable) regions.

order of $E = 5\times10^3$ V/cm to obtain the instability.

We claim that the mechanism for the instability in the electric-field-induced two-stream instability is different from the mechanism for the instability in the collisionless plasma two-stream instability. We present three arguments to support this statement.

(1) Figure 9 shows that the instability occurs in the $q \lesssim (v_{\rm th}\tau)^{-1}$ and $\omega \lesssim \tau^{-1}$ regimes. In Ref. 15, we showed that the susceptibility $\chi(\mathbf{q},\omega)$ in this wave vector and frequency range is *not* given by the collisionless plasma form [the $\chi(\mathbf{q},\omega)$ reduces to the collisionless plasma form only in the large q and/or large ω regimes]. Therefore, in the phase-space region of Fig. 9 where the collective modes are unstable, the dielectric function (which gives the collective modes) is determined by $\chi(\mathbf{q},\omega)$ which is *not* of the form for collisionless plasmas. Since the instability in the electric-field-induced case occurs in regime where the dielectric function is not described by the collisionless plasma form, we conclude that the mechanism of the instability in the field-induced case is different from the collisionless plasma case.

(2) In the case where the electrons and holes are identical except for charge (i.e., same masses, thermal velocities, scattering rates, etc., but opposite charges), if we use the $\chi_\alpha(\mathbf{q},\omega)$ from Eq. (10) (the calculation that takes the effects of the electric field and scattering into account), we can show that there is a two-stream instability for $v_d > v_{\rm th}/2$. However, if exactly the same electron and hole distribution functions that were used in the calculation to obtain Eq. (10) were substituted into Eq. (7) [the expression for $\chi(\mathbf{q},\omega)$ for collisionless plasmas], we get *no unstable* modes at *any* drift velocity (see the Appendix). Therefore, the fact that $\chi(\mathbf{q},\omega)$ given by Eq. (10) (the expression for χ in which the effects of the electric field and scattering are included) predicts the occurrence of an instability, while the $\chi(\mathbf{q},\omega)$ given by Eq. (7) (the susceptibility for a collisionless plasma in the absence of any driving electric field) does not, implies that the electric field and scattering are somehow responsible for the occurrence of the instability. Hence, the cause of the electric-field-induced instability described in this section must be different from the cause of the collisionless plasma two-stream instability.

(3) The dependence of the growth rate γ of the instabilities as a function of q for the collisionless plasma and the field-induced cases are different. Table I shows the small-q behavior of γ for these cases, as well as for the Gunn effect[38,39] for comparison.

D. Attempts at a physical explanation of the two-stream instability

What is the origin of the two-stream instability in collisionless plasmas? This question, to the best of our knowledge, does not have a satisfactory answer. The mathematics of the pure beam instability is simple enough, as was shown in Sec. IV A, but the physical explanation of the instability is not a trivial matter. The problem with understanding this phenomena is that this is an inherently *nonequilibrium* phenomenon, and in general, there is a dearth of real understanding of nonequilibrium situations. For example, Bohm and Gross[40] describe the instability as a bunching of space charge in the direction of motion of each beam, which modulates the beam and feeds the disturbance back to the source in an amplified form. While their description is probably technically correct, it gives no real physical insight into why the instability occurs, and no quantitative prediction of where the instability onsets [e.g., no explanation for Eq. (31)].

Another way to explain the instability is from the point of view of Landau damping. The same mechanisms that are responsible for Landau damping (i.e., the exponential decay of electrostatic waves) in equilibrium collisionless plasmas are probably responsible for the unstable growing waves in the two-stream plasmas. In Landau damping, a plasma wave *loses* energy to the individual particles, and hence it eventually damps away. The unstable modes in a two-stream plasma, on the other hand, *gain* energy from the particles. Therefore, in some sense, one can think of the unstable modes as "reversed" Landau damped modes, and so perhaps an explanation for the instability might be found in the physical description of Landau damping.

The usual explanation for Landau damping in equilibrium is the "surf-riding" explanation,[41] which states that damping occurs because particles "surf-ride" a plasma wave. The particles moving slightly faster than the plasma wave give energy to the wave, while the particles moving slightly slower than the wave remove energy from it. In equilibrium, since the distribution is monotonically decreasing, there are more particles moving slightly slower than faster than the wave, and therefore the wave loses energy and damps away. In some of the two-stream thermalized beam instabilities, the mode occurs with a phase velocity such that there are more particles traveling slightly faster than particles traveling

TABLE I. Characteristics of instabilities in bulk samples.

Type of instability	Behavior of γ at small q	Instability mechanism
Gunn effect	γ independent of q	Negative dv_d/dF due to scattering to upper valleys
Collisionless two-stream	$\gamma \propto q$	Newton's laws
Field-induced two-stream	$\gamma \propto q^2$	$A(dF/dx)$ term in the drift-diffusion equation due to ballistic carriers

slightly slower, and so the reverse situation occurs and the plasma wave grows. However, this explanation does not suffice for the pure beam instability, where there are usually *no* particles at the phase velocity of the plasma wave. Neither does it explain the instability in the totally symmetric thermalized beam case, as discussed Sec. IV B, since the phase velocity of the unstable wave is at the same velocity as the minimum of the distribution, and so there are just as many particles moving slightly faster than slower than the wave. Thus, the "surf-riding" description is not a universal explanation for the unstable modes in two-stream collisionless plasmas. Furthermore, in any case where it might be relevant, it gives no quantitative prediction of where the instability onsets.

One thing that *can* be said generically about instabilities in charged systems is that in order for an instability to exist, it is necessary (but not sufficient) that the ϵ be *pure real* and *negative* at some real ω (this is a direct corollary of the Nyquist criterion—see the Appendix). Therefore, since $\epsilon(\mathbf{q},\omega)=\epsilon_0[1-(4\pi e^2/\epsilon_0 q^2)\chi(\mathbf{q},\omega)]$, the linear response $\chi(\mathbf{q},\omega)$ must be pure real and positive at some real ω for an instability to occur. This condition makes sense physically, since a χ with a positive real part implies that the charged particles tend towards the crests of the potentials where these particles further enhance the potential and draw in more particles, leading to an exponential growth in the density.

The linear response χ for an unstable two-stream plasma certainly meets the condition stated above. To illustrate this, let us look at the symmetric thermalized beam case, as discussed in Sec. IV B. A Maxwellian distribution with a drift velocity $\mathbf{v}_d$ has a linear response,

$$\chi(\mathbf{q},\omega)=\frac{n_0}{mv_{\text{th}}^2}Z'\left[\frac{\omega/|q|-\hat{\mathbf{q}}\cdot\mathbf{v}_d}{v_{\text{th}}}\right]. \qquad (41)$$

From Fig. 6, one can see that the real part of this χ is greater than zero for $|\omega/|q|-\hat{\mathbf{q}}\cdot\mathbf{v}_d|\gtrsim 0.95v_{\text{th}}$. The zero-frequency response for this drifting Maxwellian distribution is $\chi_a(\mathbf{q},\omega=0)=(n_0/mv_{\text{th}}^2)Z'(-\hat{\mathbf{q}}\cdot\mathbf{v}_d/v_{\text{th}})$. Similarly, a Maxwellian distribution of the same particles with the same density and thermal width, but with a drift velocity $-\mathbf{v}_d$ (i.e., in the opposite direction) has a static linear response $\chi_b(\mathbf{q},\omega=0)=(n_0/mv_{\text{th}}^2)Z'(\hat{\mathbf{q}}\cdot\mathbf{v}_d/v_{\text{th}})$. Since the imaginary part of $Z'(x)$ is an odd function of x, the imaginary parts of χ_a and χ_b cancel, and therefore the total static linear response, $\chi(\mathbf{q},\omega=0)=\chi_a(\mathbf{q},\omega=0)+\chi_b(\mathbf{q},\omega=0)$, is pure real. Furthermore, the real part of $Z'(x)$ is an even function of x, so the real parts of χ_a and χ_b add up. Since $\text{Re}[Z(x)]>0$ for $|x|\gtrsim 0.95$, when $|\hat{\mathbf{q}}\cdot\mathbf{v}_d|\gtrsim 0.95v_{\text{th}}$ (so that the relative drift velocity between the two is greater than $1.9v_{\text{th}}$), $\chi(\mathbf{q},\omega=0)$ is pure real and *positive.* As one can see from Fig. 6 instabilities occur when the relative drift velocity between the two components is greater than approximately $1.9v_{\text{th}}$.

From the above example, we see that the key ingredients that make an instability in a charged system possible are as follows: (1) one component must have a χ with a positive real part over some frequency range, and (2) in this frequency range, the second component must have a χ which cancels the imaginary part of the first component, to make the total χ pure real. The real part of the χ of the second component must not be so large and negative that it cancels the positive real χ of the first component. As we have shown above, the linear response for a collisionless plasma with a Maxwellian distribution satisfies ingredient (1), and by drifting another component relative to the first, we can achieve ingredient (2).

These two ingredients are also present in the field-induced two-stream instability. In Ref. 17, we showed that the real part of the static χ for a nondegenerate semiconductor drifting in an applied static electric field, within the relaxation-time approximation, is positive around $q\sim 0$ for $v_d>v_{\text{th}}/2$. Furthermore, from Eq. (10), one sees that $\chi(\mathbf{q},\omega=0,\mathbf{v}_d)$ and $\chi(\mathbf{q},\omega=0,-\mathbf{v}_d)$ are complex conjugates (as is required by reflection symmetry), and hence two streams of oppositely charged, but otherwise identical, particles drifting in opposite directions under application of a static electric field have imaginary parts of $\chi_\alpha(\mathbf{q},\omega=0)$ that cancel each other. Therefore, within the relaxation-time approximation, by counterstreaming two oppositely charged, but otherwise identical (i.e., equal masses, relaxation times, etc.) components by the application of a large static electric field, one obtains a total χ that is pure real and positive at $\omega=0$, implying that an instability is possible. Of course, there is a possibility of an instability even when the parameters of the two components are different, so long as $\text{Re}[\chi(\mathbf{q},\omega)]>0$ for some real ω. Figure 9 shows that an instability occurs for the components with mass ratio of 7.

While both the collisionless plasma and the field-induced two-stream instabilities share the same property that at least one component has a $\text{Re}[\chi]>0$ over some real frequency range, the mechanisms that cause $\text{Re}[\chi]>0$ in both these cases are very different. In the collisionless plasma case, it is simply the fact that the particles move faster over the troughs of the potential (because they have more kinetic energy) than over the crests. By the continuity equation, this means that the particles spend more time on the crests than on the troughs, and hence $\delta n/\delta V>0$. (Incidentally, this is not true at equilibrium because some particles get "trapped" in the troughs of the potential, which increases the density at the troughs, resulting in $\delta n/\delta V<0$.) In the field driven case, $\text{Re}[\chi]>0$ because of the presence in the Thornber-Price drift-diffusion equation of the field-gradient term,[17] $A(dF/dx)$. This field-gradient term is present because of the interplay between the streaming motion of the carriers in the electric field and the scattering of the lattice. Therefore, in a sense, the instability in the field-driven case is indirectly caused by the scattering, which is somewhat surprising since scattering normally tends to damp out collective modes. Some years ago, Tosima and Hirota[6] also came to the conclusion that scattering would actually *enhance* rather than diminish the two-stream instability in a semiconductor. While their conclusion was based on a somewhat unrealistic model, it serves to show that scattering can affect instabilities in unpredictable ways. Another example of an instability which has its roots in scattering is the Gunn instability,[38,39] in which $\mathrm{Re}(\chi)>0$ because of the negative differential mobility caused by scattering of the electrons from the Γ to the upper valleys.

As shown in Table I, the characteristics of the growth rates γ for the Gunn instability, the collisionless plasma two-stream instability, and the electric-field-induced two-stream instability are all different. This is expected since the mechanisms for the three instabilities, as listed in Table I, are all different.

V. COULOMB COUPLING, DIELECTRIC FUNCTION, AND COLLECTIVE MODES IN TWO-DIMENSIONAL STRUCTURES

To drift the carriers in opposite directions by application of an electric field, it is necessary to have oppositely charged carriers. In a solid-state system both electrons and holes must be present. As mentioned previously, the problem with putting the electrons and holes in a semiconductor in the same space in a bulk sample is that they will ultimately recombine. To avoid recombination, one would have to physically separate the electrons from the holes. On the other hand, to obtain an instability, the electrons and holes have to be as close as possible to each other so that there is significant Coulomb coupling between them (since it is the Coulomb coupling that is primarily responsible for the instability). So, we want the electrons and holes to be close together, yet physically separated. Is there a solution to this dilemma?

The solution lies in recently developed fabrication techniques, such as molecular-beam epitaxy (MBE). With MBE, precisely controlled layers of different solids can be grown on top of one another.[42] By controlling the doping profiles, one could envisage juxtaposing two modulation-doped GaAs quantum wells, one filled with electrons and the other with holes. Figure 10 shows a possible experimental realization of this device. n- and p-doped $Al_{1-x}Ga_xAs$ layers are placed adjacent to the two juxtaposed intrinsic GaAs quantum wells. The carriers migrate from the $Al_{1-x}Ga_xAs$ layers into the GaAs quantum wells, which serve as high-mobility quasi-two-dimensional planes. Application of an electric field along the plane drifts the carriers in opposite direction. We show in the next section that, within the relaxation-time approximation, such a configuration is unstable to density perturbations.

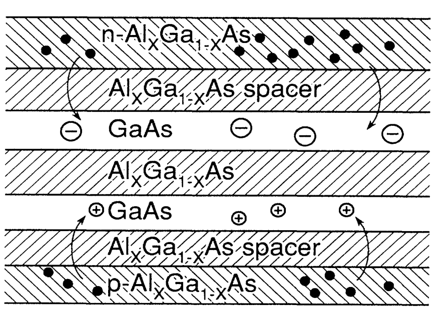

FIG. 10. A possible experimentally feasible realization of the field-induced two-stream instability. The electrons and holes from the $Al_{1-x}Ga_xAs$ layers migrate into the intrinsic GaAs quantum wells, producing adjacent high-mobility wells of electrons and holes. An electric field placed parallel to the planes causes the electrons and holes to drift in opposite directions. This device configuration was suggested by F. Capasso and A. Vengurlekar.

Since we are studying two-dimensional structures, we must first familiarize ourselves with the results of the Coulomb interaction and screening in two-dimensions. In this section, we review these results.[43,44] In particular, we concentrate on the formalism for screening and collective modes for two adjacent conducting planes of charges.

A. Coulomb coupling in two-dimensional structures

The two-dimensional Fourier transform of the Coulomb potential source located a distance d away from the Fourier transform plane is

$$\begin{aligned} V_C(q,d) &= \int d^2r\, e^{i\mathbf{q}\cdot\mathbf{r}} \frac{e^2}{(d^2+r^2)^{1/2}} \\ &= 2\pi e^2 \int_0^\infty \frac{r\,dr}{(r^2+d^2)^{1/2}} J_0(qr) \\ &= \frac{2\pi e^2}{q} e^{-qd} . \end{aligned} \tag{42}$$

For the case of adjacent planes of electrons and holes separated by a distance d, the intraplane and interplane Coulomb interactions are, respectively,

$$\begin{aligned} V_{C,\mathrm{intra}}(q) &= \frac{2\pi e^2}{\epsilon_0 q} , \\ V_{C,\mathrm{inter}}(q) &= \frac{2\pi e^2}{\epsilon_0 q} e^{-qd} \\ &= V_{C,\mathrm{intra}}(q) e^{-qd} . \end{aligned} \tag{43}$$

Equation (43) shows that the two-dimensional Coulomb interaction at large distances, given by $q\rightarrow 0$ limit, is weaker than the corresponding three-dimensional Coulomb interaction. As we show in Sec. VI, this fact makes it much harder to obtain an instability in adjacent two-dimensional charged planes than in the three-dimensional bulk samples.

B. Dielectric function for a pair of adjacent two-dimensional conducting planes

Denote the linear density response to the total perturbing potential for the electrons and holes by $\chi_e(\mathbf{q},\omega)$ and $\chi_h(\mathbf{q},\omega)$, respectively. Hence, the change in the electron and hole densities due to the total potentials $V_{\mathrm{tot},e}$ and $V_{\mathrm{tot},h}$ on the electron and hole planes are

$$\delta n_e = \chi_e V_{\mathrm{tot},e} ,$$
$$\delta n_h = \chi_h V_{\mathrm{tot},h} . \tag{44}$$

The total potential in the electron plane, $V_{\mathrm{tot},e}(\mathbf{q},\omega)e^{i(\mathbf{q}\cdot\mathbf{x}-\omega t)}$, in the presence of an external potential $V_{\mathrm{ext},e}(\mathbf{q},\omega)e^{i(\mathbf{q}\cdot\mathbf{x}-\omega t)}$ in the electron plane is the sum of the external potential (screened by the lattice polarizability) and the induced potentials caused by the density response in both the electron and hole planes, i.e.,

$$\begin{aligned} V_{\mathrm{tot},e}(\mathbf{q},\omega) &= \frac{V_{\mathrm{ext},e}(\mathbf{q},\omega)}{\epsilon_0} + V_{C,\mathrm{intra}}(q)\delta n_e(\mathbf{q},\omega) + V_{C,\mathrm{inter}}(q)\delta n_h(\mathbf{q},\omega) \\ &= \frac{V_{\mathrm{ext},e}(\mathbf{q},\omega)}{\epsilon_0} + V_{C,\mathrm{intra}}(q)\chi_e(\mathbf{q},\omega)V_{\mathrm{tot},e}(\mathbf{q},\omega) + V_{C,\mathrm{inter}}(q)\chi_h(\mathbf{q},\omega)V_{\mathrm{tot},h}(\mathbf{q},\omega) , \end{aligned} \tag{45}$$

which implies

$$[1 - V_{C,\mathrm{intra}}(q)\chi_e(\mathbf{q},\omega)]V_{\mathrm{tot},e}(\mathbf{q},\omega) - V_{C,\mathrm{inter}}(q)\chi_h(\mathbf{q},\omega)V_{\mathrm{tot},h}(\mathbf{q},\omega) = \frac{V_{\mathrm{ext},h}(\mathbf{q},\omega)}{\epsilon_0} . \tag{46}$$

Similarly, the equation for the total potential in the hole plane is

$$[1 - V_{C,\mathrm{intra}}(q)\chi_h(\mathbf{q},\omega)]V_{\mathrm{tot},h}(\mathbf{q},\omega) - V_{C,\mathrm{inter}}(q)\chi_e(\mathbf{q},\omega)V_{\mathrm{tot},e}(\mathbf{q},\omega) = \frac{V_{\mathrm{ext},e}(\mathbf{q},\omega)}{\epsilon_0} . \tag{47}$$

Equations (45) and (46) can be written as a matrix equation

$$\epsilon \mathbf{V}_{\mathrm{tot}} = \mathbf{V}_{\mathrm{ext}} , \tag{48}$$

where $\mathbf{V}$ is a two-component vector whose elements are the potentials on the electron and hole layers,

$$\mathbf{V} = \begin{bmatrix} V_e \\ V_h \end{bmatrix} , \tag{49}$$

and the $\epsilon(\mathbf{q},\omega)$ is the dielectric matrix given by

$$\epsilon(\mathbf{q},\omega) = \epsilon_0 \begin{bmatrix} 1 - V_{C,\mathrm{intra}}\chi_e(\mathbf{q},\omega) & -V_{C,\mathrm{inter}}\chi_h(\mathbf{q},\omega) \\ -V_{C,\mathrm{inter}}\chi_e(\mathbf{q},\omega) & 1 - V_{C,\mathrm{intra}}\chi_h(\mathbf{q},\omega) \end{bmatrix} . \tag{50}$$

While we have called the carriers on the adjacent layers electrons and holes, we should note that Eq. (50) is valid also when carriers in the adjacent planes are of the same charge.

The formalism described in this section assumes that the charges are completely confined to planes with zero thickness. Actually, the quantum wells which confine the carriers have finite widths and therefore there are generally several subbands of carrier states, each with a different transverse wave function. However, for wells that are sufficiently thin so that only the lowest subband is filled, the assumption of zero-thickness planes is satisfactory. For simplicity, this assumption will be used throughout this paper.

C. Collective modes in a pair of adjacent conducting planes

A collective mode is marked by a nonzero total potential in the *absence* an external potential. In a three-dimensional translationally invariant system, where

$$\epsilon(\mathbf{q},\omega)V_{\mathrm{tot}}(\mathbf{q},\omega) = V_{\mathrm{ext}}(\mathbf{q},\omega) ,$$

the condition $\epsilon(\mathbf{q},\omega)=0$ implies that V_{tot} can be nonzero when V_{ext} is zero, and hence implies the existence of a collective mode. In the case of two adjacent two-dimensional planes, a nonzero $\mathbf{V}_{\mathrm{tot}}$ in the absence of $\mathbf{V}_{\mathrm{ext}}$ can occur if the *determinant* of the matrix $\epsilon(\mathbf{q},\omega)$ in Eq. (48) is zero. Therefore, the condition for occurrence of a collective mode in this two-dimensional system is

$$\det|\epsilon(\mathbf{q},\omega)| = 0 . \tag{51}$$

From the form of the dielectric matrix, Eq. (50), the condition for the collective mode is

$$\det\left|\frac{\epsilon(\mathbf{q},\omega)}{\epsilon_0}\right| = 1 - \frac{2\pi e^2}{\epsilon_0 q}[\chi_e(\mathbf{q},\omega)+\chi_h(\mathbf{q},\omega)] + \left[\frac{2\pi e^2}{\epsilon_0 q}\right]^2 (1-e^{-2qd})\chi_e(\mathbf{q},\omega)\chi_h(\mathbf{q},\omega) = 0 . \tag{52}$$

In the next section, we use Eq. (52) to identify the collective modes of two-stream semiconductor plasmas in adjacent two-dimensional planes.

VI. TWO-STREAM INSTABILITIES IN ADJACENT TWO-DIMENSIONAL CONDUCTING PLANES

In this section we repeat the sequence of calculations described in Secs. IV A–IV C, this time for counterstreaming plasmas in adjacent two-dimensional planes. We first treat the collisionless plasma case, where we discuss the difference between the pure beam and thermalized beam two-stream instabilities, and we follow that with a discussion of the electric-field-induced instability.

A. Pure beam instability two-dimensional structures

We repeat the derivation of the instability in Sec. IV A, now for the case of a pair of adjacent two-dimensional planes. As in Sec. IV A, we assume that the distribution functions are of the form $f_{0,\alpha}=n_{0,\alpha}\delta(\mathbf{v}-\mathbf{v}_{d,\alpha})$, i.e., all the particles in a plane are moving with the same velocity. The two-dimensional planes are separated by a distance d. This system has been studied by Krasheninnikov and Chaplik[9] and by Crowne,[11] and we review their results here.

To what physical system does this theory correspond? This theory might correspond to electrons being injected through a tunneling barrier into a quasi-two-dimensional well which is in close proximity to a doped base, i.e., the two-dimensional version of the THETA device. Such devices have been fabricated,[45] and one advantage of going to two dimensions is that the mean free paths of the electrons in such two-dimensional systems seem to be considerably longer than the mean free paths of their three-dimensional counterparts (the two-dimensional mean free path is estimated to be approximately 500 nm, while the three-dimensional mean free path is about an order of magnitude shorter).

Assume that $n_{0,e}/m_e=n_{0,h}/m_h=n_0/m$, and the distribution functions are

$$f_{0,e}(\mathbf{v})=n_{0,e}\delta(\mathbf{v}-\mathbf{v}_d)\ ,\qquad f_{0,h}(\mathbf{v})=n_{0,h}\delta(\mathbf{v})\ . \tag{53}$$

The linear response for collisionless δ-function distributions is given by Eq. (14). When this expression for χ is substituted into Eq. (52), we obtain

$$0=1-\omega_{p,2\mathrm{D}}^2(q)\left[\frac{1}{(\mathbf{v}_d\cdot\mathbf{q}-\omega)^2}+\frac{1}{\omega^2}\right]+\omega_{p,2\mathrm{D}}^4(q)(1-e^{-2qd})\left[\frac{1}{(\mathbf{v}_d\cdot\mathbf{q}-\omega)^2\omega^2}\right]\ , \tag{54}$$

where $\omega_{p,2\mathrm{D}}(q)$ is the two-dimensional plasmon dispersion function,[43] given by

$$\omega_{p,2\mathrm{D}}^2(q)=\frac{2\pi e^2 n_0}{\epsilon_0 m}q\ . \tag{55}$$

This leads to an equation quadratic in $(\omega-\mathbf{v}_d\cdot\mathbf{q}/2)^2$,

$$\left[\omega-\frac{\mathbf{v}_d\cdot\mathbf{q}}{2}\right]^4-2\left[\omega-\frac{\mathbf{v}_d\cdot\mathbf{q}}{2}\right]^2\left[\left[\frac{\mathbf{v}_d\cdot\mathbf{q}}{2}\right]^2+\omega_{p,2\mathrm{D}}(q)^2\right]+\left[\frac{\mathbf{v}_d\cdot\mathbf{q}}{2}\right]^4-2\left[\frac{\mathbf{v}_d\cdot\mathbf{q}}{2}\right]^2\omega_{p,2\mathrm{D}}^2(q)+\omega_{p,2\mathrm{D}}^4(q)(1-e^{-2qd})=0\ . \tag{56}$$

This can be solved to yield the collective-mode dispersion relation

$$\left[\omega(\mathbf{q})-\frac{\mathbf{q}\cdot\mathbf{v}_d}{2}\right]^2=\left[\frac{\mathbf{v}_d\cdot\mathbf{q}}{2}\right]^2+\omega_{p,2\mathrm{D}}^2(q)\pm[(\mathbf{v}_d\cdot\mathbf{q})^2\omega_{p,2\mathrm{D}}^2(q)+\omega_{p,2\mathrm{D}}^4(q)e^{-2qd}]^{1/2}\ . \tag{57}$$

An instability occurs when the right-hand side is negative (implying an imaginary root for ω), i.e., when

$$(1-e^{-qd})^{1/2}\omega_{p,2\mathrm{D}}(q)<\mathbf{v}_d\cdot\mathbf{q}<(1+e^{-qd})^{1/2}\omega_{p,2\mathrm{D}}(q)\ . \tag{58}$$

Shown in Fig. 11 is a contour plot of $\mathrm{Im}[\omega(\mathbf{q})]=\gamma$ (the growth rate) of the unstable mode as a function of q and v_d, for the pure beam two-stream instability in adjacent two-dimensional planes. The real part of ω for the unstable mode is, from Eq. (57), $\mathrm{Re}[\omega(\mathbf{q})]=\mathbf{q}\cdot\mathbf{v}_d/2$. One can see that the region in q and v_d space in which the instability occurs is rather unintuitive. In the two-dimensional case, for $d\neq0$, the instability does not go down to zero wave vector for small v_d, because in two-dimensional systems, the Coulomb coupling is reduced from $\sim q^{-2}$ to $\sim q^{-1}$, and since the Coulomb coupling is an important factor in producing the instability, a lowered Coulomb coupling will tend to make the plasma less unstable. Only for $v_d>(2\pi ne^2d/\epsilon_0 m)^{1/2}$ is the plasma unstable for

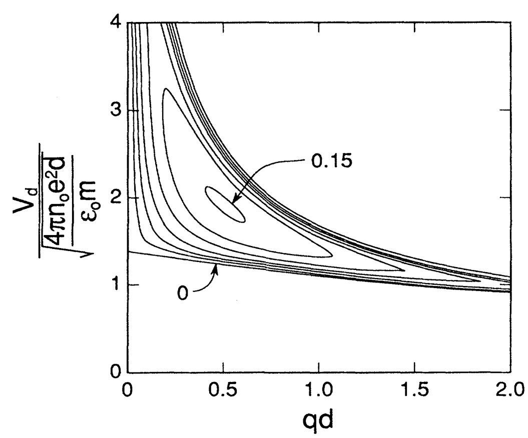

FIG. 11. Contour plot of the linear-response growth rates γ of a collisionless pure beam two-stream plasma in which the counterstreaming components are flowing in adjacent two-dimensional planes separated by a distance d, as a function of wave vector q and relative drift velocity v_d. The ratio n_0/m of the two components are equal. The contours are spaced in steps of $0.025(4\pi e^2 n_0/\epsilon_0 m_d)^{1/2}$. For electrons in GaAs, with $d=100$ Å, and $n_0=10^{12}$ cm^{-2}, this spacing is 10^{12} s^{-1}.

wave vectors down to zero. In the case of electrons in GaAs, with $d=100$ Å and $n_0=10^{12}$ cm^{-2}, the maximum growth rate, $\gamma_{\rm max}\approx 5\times 10^{12}$ s^{-1} occurs when $q\approx 0.5\times 10^6$ cm^{-1} and $v_d\approx 10^8$ cm/s.

B. Thermalized beam instability in two-dimensional structures

As in Sec. IV, we generalize the study of the collisionless plasma instabilities by inserting a thermal broadening in the distribution functions and studying how this broadening affects the parameter-space regions of instability. As in the three-dimensional case, we expect that the thermal effect we serve to reduce the regions of instability in q- and v_d-parameter space around $v_d/v_{\rm th}\lesssim 1$.

Assume that the plasmas in the adjacent planes are identical, except that one is at rest and the other is drifting at some drift velocity $\mathbf{v}_d$. Each distribution has a thermal width $v_{\rm th}$, so that their distribution functions are

$$f_e(\mathbf{v})=\frac{n_0}{\pi v_{\rm th}^2}\exp\left[-\frac{(\mathbf{v}-\mathbf{v}_d)^2}{v_{\rm th}^2}\right],\qquad f_h(\mathbf{v})=\frac{n_0}{\pi v_{\rm th}^2}\exp\left[-\frac{\mathbf{v}^2}{v_{\rm th}^2}\right]. \tag{59}$$

The expression for $\chi_\alpha(\mathbf{q},\omega)$ for these distributions is given by Eq. (37). Substituting these expressions for χ into Eq. (52) gives

$$0=1-\frac{q_{\rm sc}}{q}[Z'(\tilde{v}_\phi)+Z'(\tilde{v}_\phi-\tilde{v}_d)]+\left[\frac{q_{\rm sc}}{q}\right]^2(1-e^{-2qd})Z'(\tilde{v}_\phi)Z'(\tilde{v}_\phi-\tilde{v}_d), \tag{60}$$

where $q_{\rm sc}=4\pi e^2 n_0/\epsilon_0 m v_{\rm th}^2$ is the two-dimensional screening wave vector,[43] and $\tilde{v}_\phi=\omega/(|q|v_{\rm th})$ and $\tilde{v}_d=\mathbf{v}_d\cdot\hat{\mathbf{q}}/v_{\rm th}$ are the normalized phase and drift velocities.

We numerically searched for the roots of Eq. (60). In this case, where n_0/m and $v_{\rm th}$ of both components are equal, the real part of the phase velocity of the unstable mode, by symmetry, is the average of the drift velocities of the two Maxwellians. The numerical search is therefore limited to one dimension, and is much simpler than a two-dimensional search.[46] Figure 12 is a contour plot of the imaginary part of ω (the growth rate) as a function of q and v_d, for the thermalized beam two-stream instability in adjacent two-dimensional planes. The $v_{\rm th}$ for this plot is given by $v_{\rm th}/(4\pi e^2 n_0 d/\epsilon_0 m)^{1/2}=3$, which, for electrons in GaAs with $n_0=10^{12}$ cm^{-2} and $d=100$ Å, corresponds to an effective temperature of $T=mv_{\rm th}^2/(2k_B)\approx 100$ K. Note that, as in the bulk case, at small drift velocities (i.e., $v_d\lesssim v_{\rm th}$) the instability disappears.

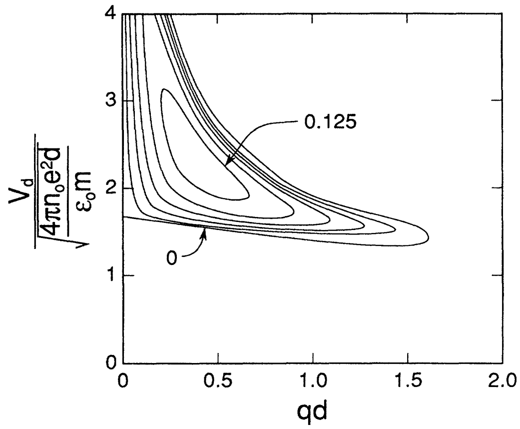

FIG. 12. Contour plot of the linear-response growth rates γ of a collisionless thermalized beam two-stream plasma in which the counterstreaming components are flowing in adjacent planes separated by a distance d, with $v_{\rm th}(4\pi e^2 n_0 d/\epsilon_0 m)^{1/2}=3$. The distributions are assumed to be Maxwellians, and the ratio n_0/m and thermal velocities of the two components are equal. For large v_d, when the drift velocity is much greater than the thermal velocity, the plot is almost identical to the pure beam case (Fig. 11), but for $v_d\lesssim v_{\rm th}$, the instability disappears. As in Fig. 11, the contours are spaced in steps of $0.025(4\pi e^2 n_0/\epsilon_0 md)^{1/2}$.

C. Electric-field-induced instability in two-dimensional structures

As mentioned in Sec. V, using MBE, it may be possible to grow a device with a pair of adjacent quasi-two-dimensional quantum wells, one containing mobile electrons, the other containing mobile holes. By application of a static electric field parallel to the planes, the electrons and holes drift in opposite directions. We show below that, within the relaxation-time approximation, this device is unstable when a large enough electric field is applied.

We used the expression in Eq. (10) for χ (which was derived for carriers in a static, homogeneous electric field within the relaxation-time approximation) in Eq. (52), the equation giving the collective modes of the system. We used the simplex method[37] to search for the roots of Eq. (52) to obtain the dispersion relation of the collective modes.

Figure 13 shows the real and imaginary parts of the collective-mode frequency of the mode that is unstable at certain drift velocities. The parameters used for this calculation are as follows:

$$\frac{m_e}{m_h}=\frac{1}{7},\quad \frac{n_{0,e}}{n_{0,h}}=1,\quad \frac{\tau_e}{\tau_h}=1,\qquad \left[\frac{2\pi n_{0,e}e^2}{\epsilon_0 k_B T}\right]^{1/2} v_{{\rm th},e}\tau_e=200,\quad \frac{d}{v_{{\rm th},e}\tau_e}=1. \tag{61}$$

These parameters correspond roughly to the following physical parameters in a pair of GaAs wells:

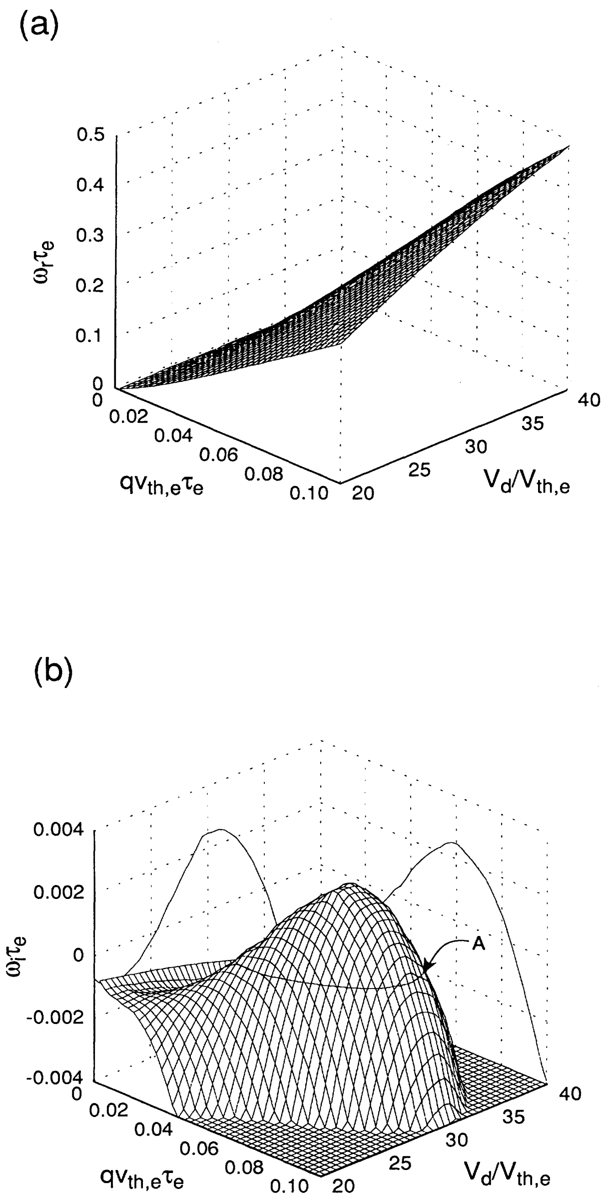

FIG. 13. The (a) real part (angular oscillation frequency) and (b) imaginary part (growth rate) of the unstable collective-mode frequency for adjacent zero-thickness planes separated by a distance d, doped with electrons and holes that are drifted by an electric field. The scattering is treated within the relaxation-time approximation. The contour line A in (b) separates the regions where the modes are damped from where they are unstable. The parameters used are as follows: $n_{0,e}=n_{0,h}$, $\tau_e=\tau_h$, $m_h=7m_e$. $q_{\mathrm{sc},e}v_{\mathrm{th},e}\tau_e=200$ and $d=v_{\mathrm{th},e}\tau_e$. Such parameters could correspond to the case of a heterostructure that has layers of GaAs a distance $d=200$ Å apart, with densities on the order of $n_{0,e}=n_{0,h}=5\times10^{12}$ cm^{-2}. Relaxation times on the order of $\tau_e=\tau_h=10^{-13}$ s, and a temperature on the order of $T=100$ K would imply that the drift velocities are on the order of 5×10^{8} cm/s, which is unfortunately much higher than the drift velocity at which negative differential mobility occurs. The oscillation frequency in the unstable region is $\mathrm{Re}[\omega(\mathbf{q})]\approx2\times10^{12}$ s^{-1}.

$$n_{0,e}=n_{0,h}=5\times10^{12}\ \mathrm{cm}^{-2},\quad d=200\ \text{Å}\ ,$$
$$m_e=\tfrac{1}{7}m_h=0.067m_{\mathrm{bare}},\quad T=100\ \mathrm{K}\ , \tag{62}$$
$$\tau_e=\tau_h=10^{-13}\ \mathrm{s},\quad \epsilon_0=10\ .$$

The contour line A in Fig. 13(b) separates the regions where the modes are damped $[\mathrm{Im}(\omega)<0]$ and where they are unstable $[\mathrm{Im}(\omega)>0]$. As one can see, this region is an "island" in $\mathbf{q}$ and $\mathbf{v}_d$ space, i.e., the unstable region does not go down to $q=0$, unlike the three-dimensional case (see Fig. 9). The cause of this strange feature seems to be the weakening of the two-dimensional Coulomb interaction at $q\sim0$. For the parameters that we used, the instability occurs at $v_{d,e}=25-35v_{\mathrm{th}}$, where $v_{\mathrm{th}}=(2k_BT/m_e)^{1/2}\approx2\times10^{7}$ cm/s for $T=100$ K. The oscillation frequency in the unstable region is on the order of $\mathrm{Re}[\omega(\mathbf{q})]\approx2\times10^{12}$ s^{-1}. Since the negative differential mobility regime in GaAs occurs at $v_d\approx4\times10^{7}$ cm/s,[47] the drift velocity needed to produce an instability in this model seems experimentally out of reach.

This negative result would seem to be rather discouraging. However, there are positive aspects to this result. First, it shows the theoretical possibility of obtaining an instability in an idealized situation. Furthermore, the relaxation-time approximation does not accurately describe collisions in GaAs, and therefore a prediction based on a theory using this approximation can only be trusted on a qualitative level. It may be the case that, in GaAs, we need not go to such high drift velocities to see such an instability. Second, it should be possible to increase the parameter space over which the instabilities occur by having a superlattice of adjacent electron and hole conducting planes, instead of just a pair of them. It has been shown that, in a superlattice, when the wave vector of the collective mode that is perpendicular to the superlattice is small compared to the inverse of the interlayer spacing, the collective-mode dispersion goes to the bulk (three-dimensional) limit.[48,49] Therefore, in this small perpendicular wave-vector limit for a superlattice, the parameter-space region of instability should be given by Fig. 9 in Sec. IV C, and hence the drift velocities needed to obtain the instability should be experimentally attainable.

VII. DISCUSSION AND SUMMARY

In this section we discuss some future directions for the continuation of this project. We also speculate how this work might lead to a viable terahertz oscillator. We then give a brief summary of the paper.

A. Future directions

In the field-induced instability, we made extensive use of the relaxation-time approximation. It would be interesting to see if the results we obtained with this approximation can be reproduced when a more realistic scattering term is used, e.g., one that takes into account the discreteness of the loss of energy from optic-phonon emissions. Furthermore, the question of electron-hole scattering has not be addressed. These interactions are responsible for the negative mobility of minority carriers in photoexcited quantum wells,[50] as the majority carriers "dragged" the minority carriers along in their wake. The presence of these scatterings would unfortunately tend to decrease the possibility of the two-stream instability since they would reduce the mobility of both species. Future investigations into the two-stream instability using Monte Carlo or other such numerical techniques could include the more realistic scattering terms and carrier-carrier scattering.

As mentioned at the end of Sec. VI, it is interesting to see how the instability behaves when there is a superlattice of alternating planes of counterstreaming electrons and holes. A calculation for unstable modes in superlattice, within the pure beam collisionless plasma approximation, has been done.[12] Performing the calculation for thermalized beams and for the field-induced instability in a superlattice should be a worthwhile project.

B. A possible terahertz oscillator?

A motivating factor behind the effort to investigate instabilities in solid-state plasmas is the possibility that these instabilities could be utilized to provide an inexpensive source of infrared radiation. The Gunn oscillator[38,51] is an example of an instability that has given the world a reliable source of radiation up to the 100-Ghz regime. For the spectrum close to the visible range, there are lasers. However, in between these two regimes, there is a dearth of good sources.

Assuming that the device described in Sec. VI C is indeed unstable to density perturbations, the charge-density oscillations in the two-dimensional conducting sheets must be coupled in some manner to the radiation field if a successful oscillator is to be fabricated. This coupling has been experimentally achieved by placing an antennalike structure in close proximity to the two-dimensional conducting planes.[24] This antenna, comprised of a metal grating of period a, is placed close to the electron layer so that it couples the radiation field to density oscillations of wavelength na in the electron layer. Some work has been done on the theory of the coupling of the radiation field to a two-dimensional charged conducting plane situated near a metallic grating,[24] and to similar types of structures.[52-54] However, a *complete* theory of the coupling of the electromagnetic field to a two-dimensional charged conducting sheet in close proximity to a metallic grating sheet does not exist yet, and therefore this problem should be addressed in the future.

C. Summary

We have used the theories of linear response for both collisionless plasmas and carriers in an electric field within the relaxation-time approximation to study instabilities with respect to charge-density perturbations in the presence of counterstreaming charged particles—the two-stream instability. We find that both these theories predict that instabilities occur in both three-dimensional structures and two-dimensional heterostructures when the charges are counterstreamed; however, the mechanisms that cause the instabilities in the collisionless plasma case and the electric-field-induced case are different and are not fully understood. For parameters chosen to correspond roughly to an $Al_{1-x}Ga_xAs$-GaAs heterostructure of a pair of adjacent two-dimensional conducting planes, an instability is predicted at very high drift velocities, which might not be experimentally attainable. We speculate that a superlattice provides a better possibility for experimentally achieving the instability, and that this instability might be used as a terahertz radiation source. To conclude, we have shown that interesting possibilities for creating plasma instabilities in quantum-well structures exist, and this matter should be pursued further.

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APPENDIX: THE NYQUIST CRITERION FOR INSTABILITY

In this appendix we sketch the argument behind the Nyquist criterion for the occurrence of an unstable mode in a plasma. Roughly, the Nyquist criterion states that if the path that the dielectric function $\epsilon(\omega)$ traces as ω goes from $-\infty$ to ∞ winds around the origin of the complex plane, then there exists an unstable collective mode. We also furnish an example for the usefulness of this criterion.

The system is unstable to small perturbations if there are roots to the equation

$$\epsilon(\mathbf{q},\omega)=0 \tag{A1}$$

in the upper-half complex plane, i.e., if there are roots $\omega(\mathbf{q})$ such that

$$\mathrm{Im}[\omega(\mathbf{q})]>0 \; . \tag{A2}$$

Define the quantity

$$G(\omega)=\frac{1}{\epsilon(\omega)}\frac{\partial\epsilon(\omega)}{\partial\omega}=\frac{\partial\ln\epsilon(\omega)}{\partial\omega} \; . \tag{A3}$$

The zeros of $\epsilon(\omega)$ are the poles of $G(\omega)$. Assume that $G(\omega)$ falls faster than $|\omega|^{-1}$ for $|\omega|\to\infty$ in the upper-half complex plane.[55] Then, since an integral of $G(\omega)$ over a large semicircle on the upper-half plane contributes nothing, we have

$$\frac{1}{2\pi i}\int_{-\infty}^{\infty}G(\omega)d\omega=\frac{1}{2\pi i}\int_{\mathcal{C}}G(\omega)d\omega \; , \tag{A4}$$

where $\mathcal{C}$ is a contour enclosing the upper-half plane. By the residue theorem,[56] and Eq. (A4), the integral $(2\pi i)^{-1}\int_{-\infty}^{\infty}G(\omega)d\omega$ is equal to the sum of the residues of the poles of $G(\omega)$ in the upper-half plane. The reader can convince him or herself that the residues at the poles of $G(\omega)$ are equal to the order of the zeros of $\epsilon(\omega)$. Therefore, we have the relationship

$$\frac{1}{2\pi i}\int_{-\infty}^{\infty} G(\omega)d\omega = N_0 \ , \tag{A5}$$

where N_0 is the number of zeros of ϵ in the upper-half plane.

Furthermore, $(2\pi i)^{-1}\int_{-\infty}^{\infty} G(\omega)d\omega$ is equal to the number of times that the path that $\epsilon(\omega)$ traces on the complex plane wraps around the origin, as ω is varied continuously along the real axis from $-\infty$ to ∞.[57] This is because

$$\begin{aligned}\int_{-\infty}^{\infty} G(\omega)d\omega &= [\ln\epsilon(\omega)]_{-\infty}^{\infty} \\ &= \{\ln[|\epsilon(\omega)|e^{i\theta(\omega)}]\}_{-\infty}^{\infty} \\ &= [\ln|\epsilon(\omega)| + i\theta(\omega)]_{-\infty}^{\infty} \ .\end{aligned} \tag{A6}$$

Since $\epsilon(\pm\infty)=1$ (there is no screening at very large frequencies), the $\ln|\epsilon(\pm\infty)|$ terms in Eq. (A6) vanish. Also, the phase $\theta(\pm\infty)$ must be equal to $2\pi N$ where N is an integer. Since $\theta(\omega)$ must vary continuously as ω varies from $-\infty$ to ∞, the difference between $\theta(\infty)$ and $\theta(-\infty)$ is given by 2π times the number of times the path has wrapped around the origin. Therefore, we have

$$\frac{1}{2\pi i}\int_{-\infty}^{\infty} G(\omega)d\omega = N_{\mathrm{loc}} \ , \tag{A7}$$

where N_{loc} is the number of times the locus of ϵ winds around the origin.

Putting Eq. (A5) and Eq. (A7) together, we conclude that the number of times the locus that $\epsilon(\omega)$ traces on the complex plane as ω varies along the entire real axis is equal to the number of zeros that ϵ has in the upper-half complex plane. Since a zero of ϵ in the upper-half plane indicates that there is an instability, a corollary to the above statement is as follows: in order for a mode to be unstable, the locus that $\epsilon(\omega)$ traces on the complex plane as ω varies from $-\infty$ to ∞ must wrap around the origin at least once.

Example of a use of the Nyqust criterion

In Sec. IV C we stated that the electric-field-induced two-stream instability was caused by a different mechanism from the collisionless plasma two-stream instability. Here, using the Nyquist criterion, we sketch an argument to justify this statement.

We study a system of nondegenerate electrons and holes in a bulk sample with distributions given by the Boltzmann equation, Eq. (9) (i.e., the relaxation-time approximation in a static electric field). The masses and relaxation-time approximations of the electrons and holes are assumed to be equal, so that for all values of the electric field, the distribution function of the electrons is a reflection of the distribution function of the holes about the origin in velocity space, i.e.,

$$f_e(\mathbf{v}) = f_h(-\mathbf{v}) \ . \tag{A8}$$

Using these distribution functions, and *assuming collisionless plasma linear-response theory,* we evaluate the dielectric function ϵ for this system. We show, using the Nyquist criterion, that the ϵ obtained by using the collisionless plasma linear-response theory in conjunction with the relaxation-time approximation distribution functions gives *no* unstable collective modes.

The Nyquist criterion states that, in order for a system to be unstable, the dielectric function $\epsilon(\mathbf{q},\omega) = 1-4\pi e^2\chi(\mathbf{q},\omega)/q^2$ has to encircle the origin as ω varies from $-\infty$ to ∞. To do this, $\epsilon(\mathbf{q},\omega)$ has to be pure real and negative for some finite ω, and hence $\chi(\mathbf{q},\omega)$ has to be *pure real and positive* for some finite ω. We show that, in this system, $\chi(\mathbf{q},\omega)$ is *never both pure real and positive,* and hence a system with this ϵ has no unstable modes.

Define $F_0(u)$ as the projection of both distributions onto the $\mathbf{q}$ axis

$$F_0(u) = \int d\mathbf{v}[f_e(\mathbf{v}) + f_h(\mathbf{v})]\delta(u - \hat{\mathbf{q}}\cdot\mathbf{v}) \ . \tag{A9}$$

The collisionless plasma theory gives that the linear response for real ω as

$$\begin{aligned}\chi(\mathbf{q},\omega) &= \int_{-\infty}^{\infty} \frac{\partial F_0/\partial u}{u-\omega/|q|-i0^+}du \\ &= \mathrm{P}\int_{-\infty}^{\infty}\frac{\partial F_0/\partial u}{u-\omega/|q|}du + i\pi\frac{\partial F_0}{\partial u}(\omega/|q|) \\ &= \int_{-\infty}^{\infty}\frac{F_0(u)-F_0(\omega/|q|)}{(u-\omega/|q|)^2}du + i\pi\frac{\partial F_0}{\partial u}(\omega/|q|) \ .\end{aligned} \tag{A10}$$

The last equality in Eq. (A10) came from an integration by parts [where the $F_0(\omega/|q|)$ term is necessary to prevent a spurious divergence of the integral at $u=\omega/|q|$].

Recall that we want to show that $\mathrm{Re}[\chi(\mathbf{q},\omega)]$ is *never* positive when $\mathrm{Im}[\chi(\mathbf{q},\omega)]$ is zero. The imaginary part is zero at frequencies ω such that $\partial F_0(\omega/|q|)/\partial u=0$, i.e., at the maxima and minima of $F_0(u)$. The projected distribution function $F_0(u)$, being the superposition of two singly peaked distribution functions, is twin peaked with a minimum in the middle. Therefore, there are three values of $\omega/|q|$ (not counting $\pm\infty$) at which $F_0(\omega/|q|)$ is at an extremum. For the two $\omega/|q|$ such that $F_0(\omega/|q|)$ is at its maximum, it is obvious from Eq. (A10) that $\chi(\mathbf{q},\omega)$ is negative, since the integrand $[F_0(u) - F_0(\omega/|q|)]/(u-\omega/|q|)^2$ is always negative. The question is, for $\omega/|q|$ such that $F_0(\omega/|q|)$ is at its minimum, what is the sign of $\chi(\mathbf{q},\omega)$?

Since we assumed the masses and relaxation times of the holes and electrons are equal, by symmetry, the minimum of $F_0(u)$ occurs at $u=0$. Therefore, we must evaluate the sign of $\chi(\mathbf{q},\omega)$ at $\omega/|q|=0$. The $\chi(\mathbf{q},\omega=0)$ is given by

$$\chi(\mathbf{q},\omega=0)=\frac{1}{m}\int_{-\infty}^{\infty}\frac{\partial F_0/\partial u}{u-i0^+}du$$
$$=\frac{i}{m}\int_{-\infty}^{0}dy\int_{-\infty}^{\infty}du\, e^{iuy}\frac{\partial F_0(u)}{\partial u}$$
$$=\frac{1}{m}\int_{-\infty}^{0}dy\, y\int_{-\infty}^{\infty}du\, e^{iuy}F_0(u)$$
$$=\frac{1}{m}\int_{-\infty}^{0}dy\, y\widetilde{F}_0(y)\ , \qquad \text{(A11)}$$

where

$$\widetilde{F}_0(y)=\int_{-\infty}^{\infty}e^{iuy}F_0(u) \qquad \text{(A12)}$$

is the Fourier transform of the projected distribution function of the electrons and holes. From Eq. (9), the Fourier transform of a nondegenerate relaxation-time semiconductor drifting in an electric field with drift velocity $\mathbf{v}_d$ is

$$\widetilde{f}(y)=\frac{\widetilde{f}_{\rm eq}(y)}{1-i\hat{\mathbf{q}}\cdot\mathbf{v}_d y}\ , \qquad \text{(A13)}$$

where

$$\widetilde{f}_{\rm eq}(y)=n_0\exp\left[-\frac{y^2}{4v_{\rm th}^2}\right] \qquad \text{(A14)}$$

is the Fourier transform of the equilibrium distribution for nondegenerate carriers. Hence, the Fourier transform of the sum of the electron and holes distributions with drift velocities $\mathbf{v}_d$ and $-\mathbf{v}_d$, respectively, is found to be

$$\widetilde{F}_0(y)=\frac{\widetilde{f}_{\rm eq}(y)}{1-i\hat{\mathbf{q}}\cdot\mathbf{v}_d y}+\frac{\widetilde{f}_{\rm eq}(y)}{1+i\hat{\mathbf{q}}\cdot\mathbf{v}_d y}$$
$$=\frac{2\widetilde{f}_{\rm eq}(y)}{1+(\hat{\mathbf{q}}\cdot\mathbf{v}_d y)^2}\ . \qquad \text{(A15)}$$

From Eqs. (A11), (A15), and (A14), we see that

$$\chi(\mathbf{q},\omega=0)=-\frac{2n_0}{m}\int_0^{\infty}dy\frac{y\exp(-y^2/4v_{\rm th}^2)}{1+(\hat{\mathbf{q}}\cdot\mathbf{v}_d y)^2}<0\ . \qquad \text{(A16)}$$

We have shown that for $\chi(\mathbf{q},\omega)$ calculated using the collisionless plasma formalism, with the electron and hole distribution functions from the relaxation-time approximation Boltzmann equation, at all real (and finite) ω for which $\mathrm{Im}[\chi(\mathbf{q},\omega)]=0$, the real part of χ is *negative* at all drift velocities. This implies that the locus of ϵ does *not* encircle the origin as ω goes from $-\infty$ to ∞. This means that the ϵ calculated using the collisionless plasma formalism, with the electron and hole distribution functions from the relaxation-time approximation Boltzmann equation, does *not* support an unstable mode.

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