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Period and amplitude halving in mesoscopic rings with spin

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We consider the flux dependence of persistent currents in mesoscopic rings threaded by magnetic flux, and extend well-known arguments to include particles of spin $\frac{1}{2}$. We find several interesting consequences of spin, such as period and amplitude halving in a single ring, without including electron-electron interactions, transverse channels, or disorder. These consequences depend sensitively on the fixed number (*modulo* 4) of particles on a given ring, and lead to strong fluctuations between samples containing a small number of rings.

The purpose of this paper is to address the consequences of the electron spin, treated simply as a nondynamical (i.e., flavor) index,[1] for persistent currents flowing in normal-metal rings threaded by magnetic flux.[2] To do this we adopt the lines of reasoning used recently by several groups[3] in addressing the experimental results of Lévy *et al.*[4] on the total magnetization of a collection of approximately 10^7 mesoscopic metal rings. We implement these arguments in the simplest possible context, that of a single, strictly one-dimensional ring, containing a fixed number of free, noninteracting spin-$\frac{1}{2}$ fermions, such as electrons. We find striking results which depend sensitively on the number of particles (*modulo* 4) and which lead to interesting mesoscopic effects, such as period halving and amplitude halving due to the spin degree of freedom. We emphasize that these effects would show up in experiments examining a *single* isolated mesoscopic ring and have, to the best of our knowledge, not yet been predicted by other approaches. If addressed experimentally, such effects might be useful in eliminating one or more proposals[3,5] for the primary explanation of period halving in a collection of mesoscopic rings.

To gain some confidence that such simple ideas may indeed accurately reflect some of the physics of mesoscopic rings, we first restate arguments contained in Ref. 3 to explain the four particularly striking issues which have emerged from the experiments of Lévy *et al.* on the total magnetization of a collection of approximately 10^7 mesoscopic normal-metal rings. We then go on to extend these simple ideas, taking into account the electron spin.

The observations of Lévy *et al.* which we wish to address are (i) the oscillatory response of the magnetization, with a period half the flux quantum (i.e., $\phi_0/2$, where $\phi_0=h/e$), to the magnetic flux through each ring; (ii) the consistently diamagnetic nature of the magnetization; (iii) the scaling of the magnetization with the number of rings N; and (iv) the magnitude of the total magnetization of the collection of rings.

We consider M spinless, noninteracting particles of mass m on a strictly one-dimensional ring of radius a, threaded by a magnetic flux ϕ. The Hamiltonian is given by

$$H=\Delta\sum_{\beta=1}^{M}\left(-i\frac{\partial}{\partial\theta_\beta}-\frac{\phi}{\phi_0}\right)^2, \tag{1}$$

where $\Delta=\hbar^2/2ma^2$ characterizes the energy-level spacing and θ_β is the angular coordinate of the βth particle. We impose periodic boundary conditions in order to obtain single-valued many-body wave functions. The single-particle energy levels are given by $\varepsilon_n(\phi)=\Delta(n-\phi/\phi_0)^2$, with n integral, from which we obtain the ground-state energy $E=\sum_n\varepsilon_n$, where the summation includes the M lowest-lying single-particle energy levels, consistent with the Pauli principle. The equilibrium current for a single ring $j=(2\Delta/\phi_0)\sum_{\beta=1}^{M}\langle -i\partial/\partial\theta_\beta-\phi/\phi_0\rangle$ can be obtained through $j=-\langle\partial H/\partial\phi\rangle$, where $\langle\cdots\rangle$ denotes an expectation value in the canonical ensemble, i.e., with *fixed* particle number M. We shall concentrate here only on ground-state (i.e., $T=0$) properties; in this case, the current is given by $j=-\partial E/\partial\phi$. Note that the current (and all other equilibrium expectation values) are periodic functions of ϕ, with period (at most) ϕ_0. This follows from the invariance of the spectrum of eigenvalues and eigenfunctions[6] under $\phi\rightarrow\phi+\phi_0$. Thus, we may restrict our attention to the first Brillouin flux zone $(-\phi_0/2,\phi_0/2)$.

At zero temperature and for a single ring, one finds

$$j(M)=\tfrac{1}{2}[1+(-1)^M]j_{\rm even}(M)+\tfrac{1}{2}[1-(-1)^M]j_{\rm odd}(M),$$

where

$$j_{\rm even}(M)=\frac{M\Delta}{\phi_0}\left(1-2\frac{|\phi|}{\phi_0}\right)\mathrm{sgn}\phi,\ \text{ for } M \text{ even}, \tag{2}$$

$$j_{\rm odd}(M)=\frac{2\Delta}{\phi_0}\left[\frac{M-1}{2}\left(1-2\frac{\phi}{\phi_0}\right)+\left(-\frac{M-1}{2}-\frac{\phi}{\phi_0}\right)\right]=-\frac{2M\Delta}{\phi_0}\frac{\phi}{\phi_0},\ \text{ for } M \text{ odd}.$$

These formulas hold for $0<|\phi|<\phi_0/2$ and must be periodically extended elsewhere, the current vanishing at the $2\phi/\phi_0$ integral. In the even-M case, the current results from the $M/2$ successive pairs of particles in the Fermi sea, the difference between the slope of $\varepsilon_n(\phi)$ for $n=p$ and $n=1-p$, yielding $(2\Delta/\phi_0)(1-2\phi/\phi_0)$ *from each pair.* In the odd-M case, the first contribution to the current results from the $(M-1)/2$ successive pairs of particles beneath the Fermi surface; the second contribution is from the remaining particle at the Fermi surface. The contributions from the Fermi sea and the Fermi surface are each of order M, but are different in nature, the latter determining the overall sign of j_{odd}. We emphasize that for odd M the current is diamagnetic (i.e., $j<0$ for small $\phi>0$), while for even M it is paramagnetic (i.e., $j>0$ for small $\phi>0$). In both cases the current is periodic only over the first Brillouin zone. Note that the expressions for the current given in Eqs. (2) have been derived for ballistic motion on a ring.

In order to estimate the amplitude of the current, $j_{\max}=M\Delta/\phi_0$, we apply the result given in Eqs. (2) in a phenomenological way to a metal. First, we eliminate M in favor of the three-dimensional spin-$\frac{1}{2}$ Fermi velocity v_F using $v_F \approx v_F^{(1d)} \equiv 2\pi\hbar M/2mL$, where $v_F^{(1d)}$ is the one-dimensional Fermi velocity, and $L=2\pi a$ is the circumference of the ring. Thus, we can express the amplitude in terms of known bulk-metal parameters, i.e., $j_{\max}\approx ev_F/L$. Second, in the diffusive regime, where the elastic mean-free path l (i.e., due to static impurity scattering of electrons), the circumference L, and the localization length ξ satisfy $l \ll L \ll \xi$, it is reasonable to replace the ballistic length L by the diffusive length[7] $L_D=L^2/l$. One then expects to observe persistent currents with period ϕ_0, provided that L_D and the dephasing length l_φ (typically due to inelastic scattering from phonons and electrons) satisfy $L_D<l_\varphi$. With these modifications, the maximum values of both j_{even} and j_{odd} become $j_{\max}^D=ev_F/L_D$, where D indicates the diffusive regime. For Cu, using $v_F\approx 1.57\times 10^6$ m s^{-1} (Ref. 8), $L\approx 2.2$ μm (Ref. 4) and $l\approx 30$ nm (Ref. 9), one obtains $j_{\max}^D\approx 1.6$ nA.

We now describe the properties of a collection of N disconnected rings, each carrying a fixed number M_k of particles ($k=1,2,\dots,N$), in order to address the experimental results of Lévy *et al.*[4] For convenience, we assume that the number of particles on each ring is governed by identical, independent Poisson distributions with mean $\overline{M}$. The current j_k in each ring produces a magnetic moment Aj_k, where A is the area of each ring. It is the total moment of the collection of rings which is observed experimentally; this total moment is given by $AJ=\sum_{k=1}^{N}Aj_k$. Performing a quenched average over the number of particles on each ring, denoted by square brackets $[\cdots]_{\text{av}}$, it yields an average (effective) current

$$[J]_{\text{av}}=\left[\sum_{k=1}^{N} j_k\right]_{\text{av}} = \tfrac{1}{2}Nj_{\max}(\overline{M})\left(1-\frac{2|\phi|}{\phi_0/2}-\tfrac{1}{2}e^{-2\overline{M}}\right)\operatorname{sgn}\phi\,. \qquad (3)$$

Note that the amplitude of the average current per ring is half the amplitude of the current in a single ring. When the mean number of particles per ring $\overline{M}$ is large, such as approximately 10^8 as it is in the Cu rings studied in Ref. 4, then the last term in Eq. (3) is utterly negligible. Furthermore, a simple calculation shows that the relative fluctuations of the total magnetization are also exponentially suppressed, $[(J-[J]_{\text{av}})^2]_{\text{av}}/[J]_{\text{av}}^2 \sim \exp(-2\overline{M})$. As above, we determine $\overline{M}$ by the experimental value of the three-dimensional Fermi velocity v_F. In the metallic regime, the total current is given by $[J^D]_{\text{av}}$, which is obtained from $[J]_{\text{av}}$ simply through the replacement of $j_{\max}$ by $j_{\max}^D$.

As can be seen from Eq. (3), and also from Fig. 1, (i) one consequence of considering a collection of N rings with *some* width to the distribution of the number of particles fixed on each ring[10] is that the period of the flux dependence of the total magnetization is *halved, with period* $\phi_0/2=h/2e$. Moreover, (ii) the nature of the total magnetization is consistently *paramagnetic*, in contrast to the tentative diamagnetic assignment of Ref. 4. Equation (3) also shows that (iii) the total magnetization scales with the number of rings N and the mean number of particles per ring $\overline{M}$; and (iv) the magnitude of the total magnetization can be pictured as coming from N identical effective current loops, each carrying an oscillatory current with maximum value $j_{\max}^D=ev_F/L_D\approx 1.6$ nA. For 10^7 rings each with area $A\approx 0.3$ $(\mu\text{m})^2$ (Ref. 4), this predicts a maximum total magnetic moment $\mu_{\max}\approx 2.3\times 10^{-15}$ A m^2. The experimental value[4] $\mu_{\max}^{\text{expt}}$, *to within a factor of* 2, is $\mu_{\max}^{\text{expt}}\approx 1.2\times 10^{-15}$ A m^2, in good agreement with the theoretical prediction. We suspect that if this is the primary explanation of the experiments of Lévy *et al.*,[4] then there should be some deeper, underlying reason why these results are sufficiently robust to survive the inclusion of transverse channels, static impurities, phonons, electron-electron interactions, etc.[11]

Having gained confidence in the validity of such a simple model we now develop a straightforward extension of the discussion given above in order to establish the consequences of the spin of the particle. For obvious reasons, we concentrate on the case of spin $\frac{1}{2}$. In this case, the current is given by $j^\sigma=-\partial E/\partial\phi$, where σ simply denotes

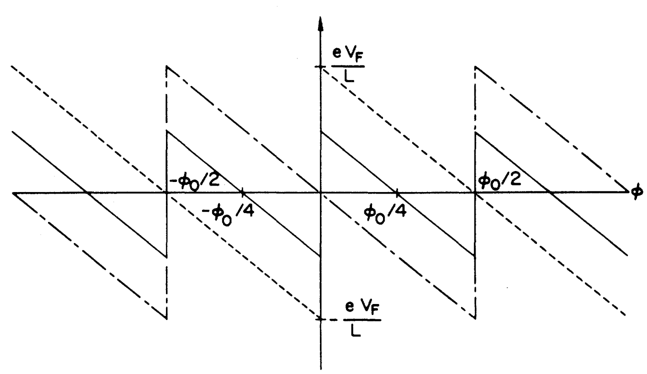

FIG. 1. The dependence of the current on the flux in a single ring containing M particles for the three cases (described in the text) arising through the presence of the spin-$\frac{1}{2}$ degree of freedom: case (0), $M/2$ even (dotted line); case (2), $M/2$ odd (dashed-dotted line); cases (1) and (3), M odd (solid line). Note the period and amplitude halving for M odd. This figure also applies to a collection of N rings containing *spinless* particles, showing j_{even} (dotted line), j_{odd} (dashed-dotted line), and $[J]_{\text{av}}/N$ (solid line). We stress that period and amplitude halving occur in the spin-$\frac{1}{2}$ case in a single ring, without the necessity of averaging over a collection of rings.

that we are now incorporating spin. In contrast with the spinless case discussed above, the ground-state energy E is given by $E=\sum_{n,\alpha}\varepsilon_n$, where the summation includes the M lowest-lying single-particle energy levels, which can now be occupied by (at most) two particles with opposite spins. Here, α refers to the spin eigenvalues. Note that we are treating spin merely as a label; we take the Hamiltonian to be spin independent.[1]

Repeating the argument given above, but now accounting for the spin, we find that there are essentially three distinct ways,[12] depicted in the figure, in which a single ring can respond to the external magnetic flux, determined by the total number M of spin-$\frac{1}{2}$ particles.

Case (0). An even number of pairs alone, i.e., $M=0$ (mod 4)—it should be clear that the currents resulting from up spins and down spins are identical and each equal to the current in the case of $M/2$ spinless particles, given in Eqs. (2). Thus, the current is paramagnetic, with period ϕ_0, and given by

$$j^{\sigma;0}=2j_{\text{even}}(M/2)=\frac{M\Delta}{\phi_0}\left[1-2\frac{|\phi|}{\phi_0}\right]\text{sgn}\phi\,. \tag{4}$$

Case (1). An even number of pairs plus one extra particle, i.e., $M=1$ (mod 4)—there is now one additional particle, whose spin is unpaired compared with case (0), and the current is composed from j_{even} and j_{odd}. Thus, the current is paramagnetic, with period $\phi_0/2$, and given by

$$\begin{aligned} j^{\sigma;1}&=j_{\text{even}}((M-1)/2)+j_{\text{odd}}((M+1)/2)\\ &=\frac{M\Delta}{2\phi_0}\left[1-\frac{2|\phi|}{\phi_0/2}-\frac{1}{M}\right]\text{sgn}\phi\,. \end{aligned} \tag{5}$$

Case (2). An odd number of pairs alone, i.e., $M=2$ (mod 4)—the currents resulting from up spins and down spins are identical and each is equal to the current in the case of $M/2$ spinless particles, given in Eqs. (2). Thus, the current is diamagnetic, with period ϕ_0, and given by

$$j^{\sigma;2}=2j_{\text{odd}}(M/2)=-(2M\Delta/\phi_0)\phi/\phi_0\,. \tag{6}$$

Case (3). An odd number of pairs plus one extra particle, i.e., $M=3$ (mod 4)—there is now one missing particle compared with case (0), and the current is composed from j_{even} and j_{odd}. Thus, the current is paramagnetic, with period $\phi_0/2$, and given by

$$\begin{aligned} j^{\sigma;3}&=j_{\text{even}}((M+1)/2)+j_{\text{odd}}((M-1)/2)\\ &=\frac{M\Delta}{2\phi_0}\left[1-\frac{2|\phi|}{\phi_0/2}+\frac{1}{M}\right]\text{sgn}\phi\,. \end{aligned} \tag{7}$$

All four currents are to be periodically extended, as depicted in the figure. Note that the currents in cases (1) and (3) (for which M is odd) are equal, to relative order $1/M$. Omitting such $1/M$ corrections, the four cases can be compactly summarized as

$$\begin{aligned} j^{\sigma}\approx\ &\tfrac{1}{4}(j^{\sigma;0}+j^{\sigma;1}+j^{\sigma;2}+j^{\sigma;3})\\ &+\tfrac{1}{2}(j^{\sigma;0}-j^{\sigma;2})\cos(M\pi/2)\,. \end{aligned} \tag{8}$$

Furthermore, in cases (1) and (3) the amplitude of the current is given by $j^{\sigma;1,3}=M\Delta/(2\phi_0)\approx ev_F/2L$, i.e., half the value for cases (0) and (2), i.e., $j^{\sigma;0,2}=M\Delta/\phi_0\approx ev_F/L$. Thus, we see that both the period and the amplitude of the persistent current are halved in a single ring containing an *odd* number M of spin-$\frac{1}{2}$ particles, compared with a single ring containing an even number of spin-$\frac{1}{2}$ particles. Therefore experiments on samples containing only a few mesoscopic rings should exhibit strong sample-to-sample fluctuations in the periodicity (between ϕ_0 and $\phi_0/2$), amplitude (between ev_F/L and $ev_F/2L$), and direction (between diamagnetic and paramagnetic) of the current, depending on the precise realization of fixed particle numbers on the rings. The former two fluctuations are novel mesoscopic phenomena, being consequences of spin.

For experiments on semiconductors, the low density of carriers reduces the amplitude of the current compared with *pure* metals. However, the possible absence of static disorder in semiconductors, compared with *real* metals, may compensate this reduction. Other factors being equal, it may be preferable to perform experiments on semiconductors in the ballistic regime $l>L$, since one would be testing a simpler situation, for which theoretical predictions would not need to account for disorder. One may also consider distinct valleys of a Fermi surface in the same way that we have been considering spin. For metal rings of the size and purity of those used in current experiments $l\ll L$, one may introduce a phenomenological description, as discussed above, replacing the ballistic ring circumference L by the diffusive one, i.e., $L\to L_D=L^2/l$, leading to the modified amplitudes ev_F/L_D (and $ev_F/2L_D$).

We now turn to an important question: Should such spin-determined effects show up in experiments on collections of a large number of rings? The answer to this question depends crucially on the width of the distribution of the number of particles on each ring. Using Poisson statistics, as above, we find that (to within terms which are exponentially small in the mean number of particles $\overline{M}$) the total quenched-average current $[J^\sigma]_{\text{av}}$ is *exactly* as described above for the spinless case, Eq. (3), i.e.,

$$[J^\sigma]_{\text{av}}=\sum_{k=1}^{N}[j_k^\sigma]_{\text{av}} \tag{9a}$$

$$\approx\frac{1}{2}N\frac{\overline{M}\Delta}{\phi_0}\left[1-\frac{2|\phi|}{\phi_0/2}-\frac{1}{2}e^{-\overline{M}}\sin\overline{M}\right]\text{sgn}\phi,\quad \overline{M}\gg 1 \tag{9b}$$

$$\approx[J]_{\text{av}},\quad \overline{M}\gg 1\,. \tag{9c}$$

Thus, we see that spin effects do not show up in experiments which average over a large number of rings. As with the spinless case, the result for the metallic regime is simply given by $[J^D]_{\text{av}}$.

In the simple model considered here, it is clear that Fermi statistics plays an essential role. It is tempting to suspect that if the phenomena discussed here are to survive the incorporation of transverse channels, static impurities, phonons, electron-electron interactions, etc., then they must have originated in some robust topological property that Fermi statistics confers upon the ground-state many-body wave function.[11] We conclude by emphasizing the fact that this simple picture would be directly tested by experiments on a single ring. In addition, such experiments would help to clarify the true origin of period halving of persistent currents in normal-metal rings.

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[9]L. P. Lévy (unpublished).

[10]Of course, in practice it would be difficult to avoid any width to the distribution of the number of particles per ring.

[11]For a discussion of these issues in the spin-independent case, see, e.g., A. J. Leggett, in *Proceedings of the NATO Advanced Study Institute on Granular Nanoelectronics, Il Ciocco, Italy, 1990* (Plenum, New York, in press).

[12]For M particles the relevant case is M (*modulo* 4).