

### Magnetic line groups. III. Corepresentations of the magnetic line groups isogonal to the point groups $D_n, C_{nv}, D_{nd},$ and $D_{nh}$

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(Received 28 August 1989; revised manuscript received 24 September 1990)

The irreducible corepresentations of the 42 families of the magnetic line groups isogonal to the point groups  $D_n, C_{nv}, D_{nd},$  and  $D_{nh}$  are presented. Some general properties of these representations are discussed.

#### I. INTRODUCTION

Line groups,<sup>1</sup> being the symmetry groups of the systems with translational periodicity in one direction, usually are used in investigations of polymers, quasi-one-dimensional systems, or three-dimensional crystals with high anisotropy. In the first paper of the series<sup>2</sup> (to be referred to as I), the time reversal is added to the spatial symmetries and magnetic line groups, 81 families in all, are constructed. For the first 13 families of the magnetic line groups (those isogonal to the point groups  $C_n, S_{2n},$  and  $C_{nh}$ ) the results have been published recently<sup>3</sup> (to be referred to as II). Also, the corepresentations of the 13 families of the gray magnetic line groups have been found.<sup>4</sup>

In this paper we complete this program by presenting the corepresentations for the remaining 42 families of the magnetic black-and-white line groups isogonal to the point groups  $D_n, C_{nv}, D_{nd},$  and  $D_{nh}$ .

The organization of the paper is as follows. The method and the notation are briefly explained in Sec. II (in the Appendix, there is a brief reminder of the basic facts about the line groups). The results, contained in the 42 tables, are presented in Sec. III. Section IV contains some general observations and comments about the results and possible physical applications.

#### II. NOTATION AND METHOD

The application of the \*-induction method to the magnetic line groups is analyzed in II. The most important conclusions will be reviewed here in order to introduce the notation.  $L(L')$  is a magnetic line group, whose index-2 subgroup  $L'$  is represented by linear operators in the Hilbert space. This means that  $L(L')=L'+g\Theta L'$ , where  $\Theta$  is the time reversal and  $g$  is a coset representative of  $L'$  in  $L$ . When  $g$  is the identity  $I$ , then  $L(L')$  is called the gray group. In this paper only black-and-white magnetic line groups, with  $g \neq I$ , are considered.

The construction of the corepresentations is based on the irreducible representations of  $L'$ , which are known.<sup>5</sup> For each such representation  $d(L')$ , Dimmock's character test is performed: The quantity  $X=(1/|L'|)\sum_{h \in L'} \text{Tr}[d((gh)^2)]$  is calculated. Its possi-

ble values are 1, -1, and 0, corresponding to the cases  $1^*, 2^*a,$  and  $2^*b$ . If  $X=1$ , then  $d(L')$  gives rise to the irreducible corepresentation of  $L(L')$ :

$$\bar{d}(L(L')) = \{d(h), d(g\Theta h) = Zd^*(h) | h \in L'\} . \quad (1)$$

In the other cases,  $X = -1, 0$ , \*-induction yields the irreducible corepresentation:

$$\bar{d}(h) = \begin{pmatrix} d(h) & 0 \\ 0 & d_g^*(h) \end{pmatrix}, \quad (2)$$

$$\bar{d}(g\Theta h) = \begin{pmatrix} 0 & d(g^2) \\ I & 0 \end{pmatrix} \begin{pmatrix} d^*(h) & 0 \\ 0 & d_g(h) \end{pmatrix},$$

where  $d_g(h)=d(g^{-1}hg)$ . In the case  $2^*a$ ,  $d(L')$  is equivalent to  $d_g^*(L')$ . Hence the construction begins with calculation of  $X$  and then (1) or (2) is applied. In the case  $X=1$ , matrix  $Z$  should be found, also. Again, a direct calculation of  $Z$  proved to be more efficient than the general method of construction,<sup>6</sup> because the representations of  $L'$  contained diagonal or off-diagonal matrices only.

#### III. TABLES OF COREPRESENTATIONS

The results of the aforementioned calculations are expressed in Tables I–XLII. The first column of each table contains the symbol of an irreducible corepresentation, which is basically the symbol of the irreducible representation of the index-2 subgroup  $L'$ , given in the caption of the table; it is barred once in the case  $1^*$  and twice in the cases  $2^*a$  and  $2^*b$ . The second column presents the range of the parameters  $m$  and  $k$  when they appear. The type of the representation is written in the third column of the tables (the captions of the tables contain the \*-g-

TABLE I. Irreducible corepresentations of the magnetic line groups  $L_{n_p}2'2', L_{n_p}2'$ . Here  $L'=L_{n_p}=\{(C_n^s | \text{Fr}(sp/n) + t) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}, g=(U|0)$ .  $[\text{Fr}(x)$  denotes the fractional part of  $x$ .]

Corep.		Type	$g$	$(C_n^s   \text{Fr}(sp/n) + t)$
$\bar{k}A_m$	$k \in (-\frac{n}{2}, \frac{n}{2}]$ $m \in (-\frac{n}{2}, \frac{n}{2}]$	$1^*$	1	$e^{ims\alpha} e^{ik(\text{Fr}(sp/n)+t)a}$

TABLE II. Irreducible corepresentations of the magnetic line groups  $L_n'22'$ . Here  $L' = \begin{cases} L(n/2)_{p22} & (n/2 \text{ even}) \\ L(n/2)_{p2} & (n/2 \text{ odd}) \end{cases} = \{(C_{n/2}^s | t + Fr(2sp/n)), (UC_{n/2}^s | -Fr(2sp/n) - t) | s = 0, 1, \dots, n/2 - 1; t = 0, \pm 1, \dots\}$ ,  $g = (C_n | p/n)$ .

Corep.	Type	$ g$	$(C_{n/2}^s   Fr(\frac{2sp}{n}) + t)$	$(UC_{n/2}^s   -Fr(\frac{2sp}{n}) - t)$
$p < n/2$ : $L' = L(n/2)_{p22}$ ( $n/2$ even) and $L(n/2)_{p2}$ ( $n/2$ odd). The pairs of $*-g$ -conjugated irreducible representations of $L'$ forming $2^*b$ corepresentations: $(\circ A_{n/4}^\pm, \circ A_{n/4}^\mp)$ , $(\pi/a A_{n/4-p/2}^\pm, \pi/a A_{n/4-p/2}^\mp)$ . $m' = \begin{cases} -m-p, & \text{if } -m-p \in (-\frac{n}{4}, \frac{n}{4}] \\ n/2 - m - p, & \text{if } n/2 - m - p \in (-\frac{n}{4}, \frac{n}{4}] \end{cases}$				
$\circ \bar{A}_o^\pm$	$1^*$	1	1	$\pm 1$
$\circ \bar{E}_m^{-m}$	$1^*$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
$\circ \bar{A}_{n/4}^\pm$	$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\pi/a \bar{A}_{-p/2}^\pm$	$1^*$	1	$(-1)^t (-1)^{Int(2sp/n)}$	$\pm (-1)^t (-1)^{Int(2sp/n)}$
$\pi/a \bar{E}_m^{m'}$	$1^*$	$W^*(\frac{2px}{na}, m)$	$(-1)^t M(m + p/2, s)$	$(-1)^t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m + p/2, s)$
$\pi/a \bar{A}_{n/4-p/2}^\pm$	$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} (-1)^{Int(2sp/n)} I$	$(-1)^{s+t} (-1)^{Int(2sp/n)} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\frac{-k}{k} \bar{E}_m^{-m}$	$1^*$	$W^*(\frac{2pk}{n}, m)$	$M(m, s)K(k, Fr(\frac{2sp}{n}) + t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)K(k, Fr(\frac{2sp}{n}) + t)$
$p \geq n/2$ : $L' = L(n/2)_{p-n/222}$ ( $n/2$ even) and $L(n/2)_{p-n/22}$ ( $n/2$ odd). The pairs of $*-g$ -conjugated irreducible representations of $L'$ forming $2^*b$ corepresentations: $(\circ A_{n/4}^\pm, \circ A_{n/4}^\mp)$ , $(\pi/a A_{n/4-p/2}^\pm, \pi/a A_{n/4-p/2}^\mp)$ . $m' = \begin{cases} n/2 - m - p, & \text{if } n/2 - m - p \in (-\frac{n}{4}, \frac{n}{4}] \\ n - m - p, & \text{if } n - m - p \in (-\frac{n}{4}, \frac{n}{4}] \end{cases}$				
$\circ \bar{A}_o^\pm$	$1^*$	1	1	$\pm 1$
$\circ \bar{E}_m^{-m}$	$1^*$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
$\circ \bar{A}_{n/4}^\pm$	$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\pi/a \bar{A}_{n/2-p/2}^\pm$	$1^*$	1	$(-1)^{s+t} (-1)^{Int(2sp/n)}$	$\pm (-1)^{s+t} (-1)^{Int(2sp/n)}$
$\pi/a \bar{E}_m^{m'}$	$1^*$	$W^*(\frac{2px}{na}, m)$	$(-1)^t M(m + \frac{p}{2} - \frac{n}{4}, s)$	$(-1)^t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m + \frac{p}{2} - \frac{n}{4}, s)$
$\pi/a \bar{A}_{n/4-p/2}^\pm$	$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t (-1)^{Int(2sp/n)} I$	$(-1)^t (-1)^{Int(2sp/n)} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\frac{-k}{k} \bar{E}_m^{-m}$	$1^*$	$W^*(\frac{2pk}{n}, m)$	$M(m, s)K(k, Fr(\frac{2sp}{n}) + t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)K(k, Fr(\frac{2sp}{n}) + t)$

conjugated pairs in the cases  $2^*b$ ). In the next column there is the matrix representing  $g$  in that corepresentation, while the other columns contain the corepresentation of the elements of the index-2 subgroup (the rest of the corepresentation can be easily calculated multiplying matrices corresponding to the coset representative  $g$  and

to the subgroup).

Most of the matrices are given explicitly; the upper-case (capital) and lower-case letters denote two- and four-dimensional matrices, respectively. Still, in order to make the results more comprehensive the following abbreviations were introduced ( $\alpha = 2\pi/n$ ):

TABLE III. Irreducible corepresentations of the magnetic line groups  $L_c n_p 22$ ,  $L_c n_p 2$ . Here  $L' = \begin{cases} L'_n p/222, & n \text{ even} \\ L'_n p/22, & n \text{ odd} \end{cases} = \{(C_n^s | t + Fr(sp/2n)), (UC_n^s | -Fr(sp/2n) - t) | s = 0, 1, \dots, n-1; t = 0, \pm 1, \dots\}$ ,  $g = (E | \frac{1}{2})$ . The pairs of  $*-g$ -conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentation:  $(\pi/a A_{-p/4}^\pm, \pi/a A_{-p/4}^\mp)$ ,  $(\pi/a A_{n/2-p/4}^\pm, \pi/a A_{n/2-p/4}^\mp)$ .

Corep.	Type	$ g$	$(C_n^s   Fr(\frac{sp}{2n}) + t)$	$(UC_n^s   -Fr(\frac{sp}{2n}) - t)$
$\circ \bar{A}_o^\pm$	$1^*$	1	1	$\pm 1$
$\circ \bar{E}_m^{-m}$	$1^*$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
$\circ \bar{A}_{n/2}^\pm$	$1^*$	1	$(-1)^s$	$\pm (-1)^s$
$\frac{-k}{k} \bar{E}_m^{-m}$	$1^*$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$M(m, s)K(k, Fr(\frac{sp}{2n}) + t)$	$M(m, s)K(k, Fr(\frac{sp}{2n}) + t) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\pi/a \bar{E}_m^{m'}$	$1^*$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t M(m + p/4, s)$	$(-1)^t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m + p/4, s)$
$\pi/a \bar{A}_{-p/4}^\pm$	$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t (-1)^{Int(sp/2n)} I$	$(-1)^t (-1)^{Int(sp/2n)} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\pi/a \bar{A}_{n/2-p/4}^\pm$	$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} (-1)^{Int(sp/2n)} I$	$(-1)^{s+t} (-1)^{Int(sp/2n)} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

TABLE IV. Irreducible corepresentations of the magnetic line groups  $L_s n_p 22$ ,  $L_s n_p 2$ . Here  $L' = \begin{pmatrix} L' n_{(p+n)/2, 22} \\ L' n_{(p+n)/2, 2} \end{pmatrix} = \{(C_n^s | t + \text{Fr}(sp/2n)) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (E | \frac{1}{2})$ ; The pairs of  $*$ -g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentations:  $(\pi/a A_{-(n+p)/4}^+, \pi/a A_{-(n+p)/4}^-)$ ,  $(\pi/a A_{-(n-p)/4}^+, \pi/a A_{-(n-p)/4}^-)$ .

Corep.		Type	$g$	$(C_n^s   \text{Fr}(\frac{sp}{2n}) + t)$	$(UC_n^s   -\text{Fr}(\frac{sp}{2n}) - t)$
${}^o\bar{A}_o^\pm$		$1^*$	1	1	$\pm 1$
${}^o\bar{E}_m^{-m}$	$m \in (0, \frac{n}{2})$	$1^*$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
${}^o\bar{A}_{n/2}^\pm$		$1^*$	1	$(-1)^s$	$\pm(-1)^s$
${}^k\bar{E}_m^{-m}$	$k \in (0, \frac{\pi}{a})$ $m \in (0, \frac{n}{2})$	$1^*$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$M(m, s)K(k, \text{Fr}(\frac{sp}{2n}) + t)$	$M(m, s)K(k, \text{Fr}(\frac{sp}{2n}) + t) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\pi/a \bar{E}_m^{m'}$	$m \in (-\frac{(n+p)}{4}, \frac{(n-p)}{4})$	$1^*$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t M(m + n/4 + p/4, s)$	$(-1)^t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m + n/4 + p/4, s)$
$\pi/a \bar{A}_{-(n+p)/4}$		$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t (-1)^{\text{Int}(sp/2n)} I$	$(-1)^t (-1)^{\text{Int}(sp/2n)} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\pi/a \bar{A}_{(n-p)/4}$		$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} (-1)^{\text{Int}(sp/2n)} I$	$(-1)^{s+t} (-1)^{\text{Int}(sp/2n)} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

TABLE V. Irreducible corepresentations of the magnetic line groups  $Lnm'm'$ ,  $Lnm'$ . Here  $L' = Ln = \{(C_n^s | t) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (\sigma_v | 0)$ . The pair of  $*$ -g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentation:  $({}_k A_m, -{}_k A_m)$ .

Corep.		Type	$g$	$(C_n^s   t)$
${}^o\bar{A}_m$	$m \in (-\frac{n}{2}, \frac{n}{2}]$	$1^*$	1	$e^{ims\alpha}$
$\pi/a \bar{A}_m$	$m \in (-\frac{n}{2}, \frac{n}{2}]$	$1^*$	1	$(-1)^t e^{ims\alpha}$
${}^k\bar{A}_m$	$k \in (0, \frac{\pi}{a})$ $m \in (-\frac{n}{2}, \frac{n}{2}]$	$2^*b$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$e^{ims\alpha} K(k, t)$

TABLE VI. Irreducible corepresentations of the magnetic line groups  $L(2p)'mm'$ . Here  $L' = \begin{pmatrix} L' p m m' \\ L' p m \end{pmatrix} = \{(C_p^s | t), (\sigma_v C_p^s | t) | s=0, 1, \dots, p-1; t=0, \pm 1, \dots\}$ ,  $g = (C_{2p} | 0)$ . The pairs of  $*$ -g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentations:  $({}_0 A_{p/2}, {}_0 B_{p/2})$ ,  $(\pi/a A_{p/2}, \pi/a B_{p/2})$ ,  $({}_k E_{m,-m}, -{}_k E_{m,-m})$ ,  $({}_k A_{p/2}, -{}_k B_{p/2})$ ,  $({}_k B_{p/2}, -{}_k A_{p/2})$ ,  $({}_k A_0, -{}_k A_0)$ ,  $({}_k B_0, -{}_k B_0)$ .

Corep.		Type	$g$	$(C_p^s   t)$	$(\sigma_v C_p^s   t)$
${}^o\bar{A}_o$		$1^*$	1	1	1
${}^o\bar{B}_o$		$1^*$	1	1	-1
${}^o\bar{E}_m^{-m}$	$m \in (0, \frac{p}{2})$	$1^*$	$\begin{pmatrix} 0 & e^{-im\alpha} \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
${}^{\pm}\bar{A}_{p/2}$		$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\pi/a \bar{A}_o$		$1^*$	1	$(-1)^t$	$(-1)^s$
$\pi/a \bar{B}_o$		$1^*$	1	$(-1)^t$	$-(-1)^t$
$\pi/a \bar{E}_m^{-m}$	$m \in (0, \frac{p}{2})$	$1^*$	$\begin{pmatrix} 0 & e^{-im\alpha} \\ 1 & 0 \end{pmatrix}$	$(-1)^t M(m, s)$	$(-1)^t i \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
$\pi/a \bar{A}_{p/2}$		$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} I$	$(-1)^{s+t} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
${}^k\bar{A}_o$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$K(k, t)$
${}^k\bar{B}_o$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$-K(k, t)$
${}^k\bar{E}_m^{-m}$	$k \in (0, \frac{\pi}{a})$ $m \in (0, \frac{p}{2})$	$2^*b$	$s^*(M(m, 1))$	$l(k, t)m(m, s)$	$d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} l(k, t)m'(m, s)$
${}^k\bar{A}_{p/2}$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s K(k, t)$	$(-1)^s K(k, t) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
${}^k\bar{B}_{p/2}$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s K(k, t)$	$-(-1)^s K(k, t) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\begin{aligned}
I &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad K(k,t) = \begin{bmatrix} e^{ikta} & 0 \\ 0 & e^{-ikta} \end{bmatrix}, \quad M(m,s) = \begin{bmatrix} e^{imsa} & 0 \\ 0 & e^{-imsa} \end{bmatrix}, \\
Q &= \begin{bmatrix} e^{ika} & 0 \\ 0 & e^{-ika} \end{bmatrix}, \quad W(k,m) = \begin{bmatrix} 0 & e^{ika}e^{ima} \\ 1 & 0 \end{bmatrix}, \quad b(X) = \begin{bmatrix} 0 & X \\ X & 0 \end{bmatrix}, \quad d(X) = \begin{bmatrix} X & 0 \\ 0 & X^* \end{bmatrix}, \\
g(X) &= \begin{bmatrix} I & 0 \\ 0 & X \end{bmatrix}, \quad k(k,t) = \begin{bmatrix} K(k,t) & 0 \\ 0 & K^*(k,t) \end{bmatrix}, \\
k'(k,t) &= \begin{bmatrix} K(k,t) & 0 \\ 0 & K^*(k,t+1) \end{bmatrix}, \quad l(k,t) = \begin{bmatrix} e^{ikta}I & 0 \\ 0 & e^{-ikta}I \end{bmatrix}, \\
m(m,s) &= \begin{bmatrix} M(m,s) & 0 \\ 0 & M^*(m,s) \end{bmatrix}, \quad m'(m,s) = \begin{bmatrix} M(m,s) & 0 \\ 0 & M^*(m,s+1) \end{bmatrix}, \\
n(m,s) &= \begin{bmatrix} M(m,s) & 0 \\ 0 & M(m,s) \end{bmatrix}, \quad n'(m,s) = \begin{bmatrix} M(m,s) & 0 \\ 0 & M(m,s+1) \end{bmatrix}, \\
s(X) &= \begin{bmatrix} 0 & X \\ I & 0 \end{bmatrix}, \quad z(m,k) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{ima} \\ e^{-ika} & 0 & 0 & 0 \\ 0 & e^{ima-ika} & 0 & 0 \end{bmatrix}.
\end{aligned}$$

TABLE VII. Irreducible corepresentations of the magnetic line groups  $L_c nmm$ ,  $L_c nm$ . Here  $L' = (L'_{nm}, \begin{matrix} n \text{ even} \\ n \text{ odd} \end{matrix}) = \{(C_n^s | t), (\sigma_v C_n^s | t) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (E | \frac{1}{2})$ . The pairs of \*-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentations:  $({}_k E_{m,-m}, -{}_k E_{m,-m})$ ,  $({}_k A_0, -{}_k A_0)$ ,  $({}_k B_0, -{}_k B_0)$ ,  $({}_k A_{n/2}, -{}_k A_{n/2})$ ,  $({}_k B_{n/2}, -{}_k B_{n/2})$ .

Corep.		Type	$g$	$(C_n^s   t)$	$(\sigma_v C_n^s   t)$
$\overline{{}_o A_0}$		$1^*$	1	1	1
$\overline{{}_o B_0}$		$1^*$	1	1	-1
$\overline{{}_o A_{n/2}}$		$1^*$	1	$(-1)^s$	$(-1)^s$
$\overline{{}_o B_{n/2}}$		$1^*$	1	$(-1)^s$	$-(-1)^s$
$\overline{{}_o E_m^{-m}}$	$m \in (0, \frac{n}{2})$	$1^*$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$M(m,s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m,s)$
$\overline{\pi/a A_0}$		$2^*a$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t I$	$(-1)^t I$
$\overline{\pi/a B_0}$		$2^*a$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t I$	$-(-1)^t I$
$\overline{\pi/a A_{n/2}}$		$2^*a$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} I$	$(-1)^{s+t} I$
$\overline{\pi/a B_{n/2}}$		$2^*a$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} I$	$-(-1)^{s+t} I$
$\overline{\pi/a E_m^{-m}}$	$m \in (0, \frac{n}{2})$	$2^*a$	$s(-I)$	$(-1)^t m(m,s)$	$(-1)^t d\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right) m(m,s)$
$\overline{{}_k A_0}$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$K(k,t)$	$K(k,t)$
$\overline{{}_k B_0}$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$K(k,t)$	$-K(k,t)$
$\overline{{}_k A_{n/2}}$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$(-1)^s K(k,t)$	$(-1)^s K(k,t)$
$\overline{{}_k B_{n/2}}$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$(-1)^s K(k,t)$	$-(-1)^s K(k,t)$
$\overline{{}_k E_m^{-m}}$	$k \in (0, \frac{\pi}{a})$ $m \in (0, \frac{n}{2})$	$2^*b$	$s(e^{-ika} I)$	$l(k,t) m(m,s)$	$d\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right) m(m,s) l(k,t)$

TABLE VIII. Irreducible corepresentations of the magnetic line groups  $L_m nmm$ ,  $L_m nm$ . Here  $L' = (L'_{nc}, n \text{ even}) = \{(C_n^s | t), (\sigma_v C_n^s | t + \frac{1}{2}) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (E | \frac{1}{2})$ . The pairs of \*-g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentations:  $({}_k A_{n/2}, -{}_k A_{n/2})$ ,  $({}_k B_{n/2}, -{}_k B_{n/2})$ ,  $(\pi/a A_{n/2}, \pi/a B_{n/2})$ ,  $(\pi/a A_0, \pi/a B_0)$ ,  $({}_k A_0, -{}_k A_0)$ ,  $({}_k B_0, -{}_k B_0)$ ,  $({}_k E_{m,-m}, -{}_k E_{m,-m})$ .

Corep.		Type	$g$	$(C_n^s   t)$	$(\sigma_v C_n^s   t + \frac{1}{2})$
${}_o \bar{A}_o$		$1^*$	1	1	1
${}_o \bar{B}_o$		$1^*$	1	1	-1
${}_o \bar{E}_{m,-m}$	$m \in (0, \frac{n}{2})$	$1^*$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
${}_o \bar{A}_{n/2}$		$1^*$	1	$(-1)^s$	$(-1)^s$
${}_o \bar{B}_{n/2}$		$1^*$	1	$(-1)^s$	$-(-1)^s$
$\pi/a \bar{A}_o$		$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t I$	$i(-1)^t \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\pi/a \bar{E}_{m,-m}$	$m \in (0, \frac{n}{2})$	$1^*$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t M(m, s)$	$i(-1)^t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
$\pi/a \bar{A}_{n/2}$		$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} I$	$i(-1)^{s+t} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
${}_k \bar{A}_o$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$K(k, t + \frac{1}{2})$
${}_k \bar{B}_o$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$-K(k, t + \frac{1}{2})$
${}_k \bar{A}_{n/2}$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$(-1)^s K(k, t)$	$(-1)^s K(k, t + \frac{1}{2})$
${}_k \bar{B}_{n/2}$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$(-1)^s K(k, t)$	$-(-1)^s K(k, t + \frac{1}{2})$
${}_k \bar{E}_{m,-m}$	$k \in (0, \frac{\pi}{a})$ $m \in (0, \frac{n}{2})$	$2^*b$	$s(e^{-ika} I)$	$l(k, t)m(m, s)$	$d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} l(k, t + \frac{1}{2})m(m, s)$

TABLE IX. Irreducible corepresentations of the magnetic line groups  $L_s nmm$  ( $n=2p$ ). Here  $L' = L''(2p)_p mc = \{(C_n^s | \text{Fr}(s/2) + t), (\sigma_v C_n^s | \text{Fr}(s/2) + t) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (E | \frac{1}{2})$ . The pairs of \*-g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentations:  $({}_k A_{n/2}, -{}_k A_{n/2})$ ,  $({}_k B_{n/2}, -{}_k B_{n/2})$ ,  $(\pi/a A_0, \pi/a A_{n/2})$ ,  $(\pi/a B_0, \pi/a B_{n/2})$ ,  $({}_k A_0, -{}_k A_0)$ ,  $({}_k B_0, -{}_k B_0)$ ,  $({}_k E_{m,-m}, -{}_k E_{m,-m})$ ,  $(\pi/a E_{m,-m}, \pi/a E_{p-m, m-p})$ .

Corep.		Type	$g$	$(C_n^s   \text{Fr}(\frac{s}{2}) + t)$	$(\sigma_v C_n^s   \text{Fr}(\frac{s}{2}) + t)$
${}_o \bar{A}_o$		$1^*$	1	1	1
${}_o \bar{B}_o$		$1^*$	1	1	-1
${}_o \bar{E}_{m,-m}$	$m \in (0, \frac{n}{2})$	$1^*$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
${}_o \bar{A}_{n/2}$		$1^*$	1	$(-1)^s$	$(-1)^s$
${}_o \bar{B}_{n/2}$		$1^*$	1	$(-1)^s$	$-(-1)^s$
$\pi/a \bar{A}_o$		$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t e^{i\pi \text{Fr}(s/2)} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^s$	$(-1)^t e^{i\pi \text{Fr}(s/2)} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^s$
$\pi/a \bar{B}_o$		$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t e^{i\pi \text{Fr}(s/2)} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^s$	$-(-1)^t e^{i\pi \text{Fr}(s/2)} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^s$
$\pi/a \bar{E}_{p/2, -p/2}$		$2^*a$	$s(-I)$	$(-1)^t l(\pi/a, \text{Fr}(\frac{s}{2}))m(\frac{p}{2}, s)$	$(-1)^t d(I)l(\pi/a, \text{Fr}(\frac{s}{2}))m(\frac{p}{2}, s)$
$\pi/a \bar{E}_{m,-m}$	$m \in (0, \frac{p}{2})$	$2^*b$	$s(-I)$	$(-1)^t l(\pi/a, \text{Fr}(\frac{s}{2}))m(m, s)$	$(-1)^t d(I)l(\pi/a, \text{Fr}(\frac{s}{2}))m(m, s)$
${}_k \bar{A}_o$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$K(k, t + \text{Fr}(\frac{s}{2}))$	$K(k, t + \text{Fr}(\frac{s}{2}))$
${}_k \bar{B}_o$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$K(k, t + \text{Fr}(\frac{s}{2}))$	$-K(k, t + \text{Fr}(\frac{s}{2}))$
${}_k \bar{A}_{n/2}$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$(-1)^s K(k, t + \text{Fr}(\frac{s}{2}))$	$(-1)^s K(k, t + \text{Fr}(\frac{s}{2}))$
${}_k \bar{B}_{n/2}$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$(-1)^s K(k, t + \text{Fr}(\frac{s}{2}))$	$-(-1)^s K(k, t + \text{Fr}(\frac{s}{2}))$
${}_k \bar{E}_{m,-m}$	$k \in (0, \frac{\pi}{a})$ $m \in (0, \frac{p}{2})$	$2^*b$	$s(e^{-ika} I)$	$l(k, t + \text{Fr}(\frac{s}{2}))m(m, s)$	$d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} l(k, t + \text{Fr}(\frac{s}{2}))m(m, s)$

TABLE X. Irreducible corepresentations of the magnetic line groups  $Lnc'c'$ ,  $Lnc'$ . Here  $L' = Ln = \{(C_n^s/t) | s = 0, 1, \dots, n-1; t = 0, \pm 1, \dots\}$ ,  $g = (\sigma_v | \frac{1}{2})$ . The pair of \*-g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentation:  $({}_k A_m, -{}_k A_m)$ .

Corep.		Type	$g$	$(C_n^s   t)$
${}_o \bar{A}_m$	$m \in (-\frac{n}{2}, \frac{n}{2}]$	$1^*$	1	$e^{ims\alpha}$
${}_k \bar{A}_m$	$k \in (0, \frac{\pi}{a})$ $m \in (-\frac{n}{2}, \frac{n}{2}]$	$2^*b$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$e^{ims\alpha} K(k, t)$
${}_{\pi/a} \bar{A}_m$	$m \in (-\frac{n}{2}, \frac{n}{2}]$	$2^*a$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t e^{ims\alpha} I$

TABLE XI. Irreducible corepresentations of the magnetic line groups  $L(2p)'cc$ . Here  $L' = (L_{pc}^{pcc}, \frac{p}{p} \text{ even}) = \{(C_p^s | t), (\sigma_v C_p^s | t + \frac{1}{2}) | s = 0, 1, \dots, p-1; t = 0, \pm 1, \dots\}$ ,  $g = (C_{2p} | 0)$ . The pairs of \*-g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentations:  $({}_0 A_{p/2}, {}_0 B_{p/2})$ ,  $({}_k A_{p/2}, -{}_k B_{p/2})$ ,  $({}_k B_{p/2}, -{}_k A_{p/2})$ ,  $({}_{\pi/a} A_0, {}_{\pi/a} B_0)$ ,  $({}_k A_0, -{}_k A_0)$ ,  $({}_k B_0, -{}_k B_0)$ ,  $({}_k E_{m,-m}, -{}_k E_{m,-m})$ .

Corep.		Type	$g$	$(C_p^s   t)$	$(\sigma_v C_p^s   t + \frac{1}{2})$
${}_o \bar{A}_0$		$1^*$	1	1	1
${}_o \bar{B}_0$		$1^*$	1	1	-1
${}_o \bar{E}_{m,-m}$	$m \in (0, \frac{p}{2})$	$1^*$	$\begin{pmatrix} 0 & e^{-im\alpha} \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
${}_o \bar{A}_{p/2}$		$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
${}_{\pi/a} \bar{A}_0$		$2^*b$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t I$	$i(-1)^t \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
${}_{\pi/a} \bar{A}_{p/2}$		$2^*a$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} I$	$i(-1)^{s+t} I$
${}_{\pi/a} \bar{B}_{p/2}$		$2^*a$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} I$	$-i(-1)^{s+t} I$
${}_{\pi/a} \bar{E}_{m,-m}$	$m \in (0, \frac{p}{2})$	$2^*a$	$s^*(M(m, 1))$	$(-1)^t m(m, s)$	$i(-1)^t d \left( \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right) m(m, s)$
${}_k \bar{A}_0$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$K(k, t + \frac{1}{2})$
${}_k \bar{B}_0$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$-K(k, t + \frac{1}{2})$
${}_k \bar{A}_{p/2}$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s K(k, t)$	$(-1)^s K(k, t + \frac{1}{2})$
${}_k \bar{B}_{p/2}$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s K(k, t)$	$-(-1)^s K(k, t + \frac{1}{2})$
${}_k \bar{E}_{m,-m}$	$k \in (0, \frac{\pi}{a})$ $m \in (0, \frac{p}{2})$	$2^*b$	$s^*(M(m, 1))$	$l(k, t) m(m, s)$	$d \left( \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right) m'(m, s) l(k, t + \frac{1}{2})$

TABLE XII. Irreducible corepresentations of the magnetic line groups  $L(2p)_p m'c'$ . Here  $L' = L(2p)_p = \{(C_{2p}^s | Fr(s/2) + t) | s = 0, 1, \dots, 2n-1; t = 0, \pm 1, \dots\}$ ,  $g = (\sigma_v | 0)$ . The pairs of \*-g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentation:  $({}_k A_m, -{}_k A_m)$ ,  $({}_{\pi/a} A_m, {}_{\pi/a} A_m)$ , with  $m' = \begin{cases} m+p & \text{if } m \in (-p, 0] \\ m-p & \text{if } m \in (0, p] \end{cases}$ .

Corep.		Type	$g$	$(C_n^s   Fr(s/2) + t)$
${}_o \bar{A}_m$	$m \in (-p, p]$	$1^*$	1	$e^{ims\alpha}$
${}_{\pi/a} \bar{A}_m$	$m \in (0, p)$	$2^*b$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t e^{ims\alpha} K(\pi/a, Fr(s/2))$
${}_k \bar{A}_m$	$k \in (0, \frac{\pi}{a})$ $m \in (-p, p]$	$2^*b$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$e^{ims\alpha} K(\pi/a, Fr(s/2) + t)$

TABLE XIII. Irreducible corepresentations of the magnetic line groups  $L(2p)'_p mc'$ . Here  $L' = (\frac{L'_{pm}}{L'_{pc}}, \frac{p}{p} \text{ even}) = \{(C_p^s | t), (\sigma_v C_p^s | t) | s = 0, 1, \dots, p-1; t = 0, \pm 1, \dots\}$ ,  $g = (C_{2p} | 0)$ . The pairs of \*-g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentations:  $({}_k A_{p/2}, -{}_k B_{p/2})$ ,  $({}_k B_{p/2}, -{}_k A_{p/2})$ ,  $(\pi/a A_{p/2}, \pi/a B_{p/2})$ ,  $({}_0 A_{p/2}, {}_0 B_{p/2})$ ,  $({}_k A_0, -{}_k A_0)$ ,  $({}_k B_0, -{}_k B_0)$ ,  $({}_k E_{m,-m}, -{}_k E_{m,-m})$ .

Corep.		Type	$g$	$(C_p^s   t)$	$(\sigma_v C_p^s   t)$
${}_0 \bar{A}_0$		$1^*$	1	1	1
${}_0 \bar{B}_0$		$1^*$	1	1	-1
$\bar{\bar{A}}_{p/2}$		$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
${}_0 \bar{E}_{m,-m}$	$m \in (0, \frac{\pi}{4})$	$1^*$	$\begin{pmatrix} 0 & e^{-im\alpha} \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
$\pi/a \bar{A}_0$		$1^*$	1	$(-1)^t$	$(-1)^t$
$\pi/a \bar{B}_0$		$1^*$	1	$(-1)^t$	$-(-1)^t$
$\pi/a \bar{\bar{A}}_{p/2}$		$2^*b$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} I$	$(-1)^{s+t} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\pi/a \bar{\bar{E}}_{m,-m}$	$m \in (0, \frac{\pi}{4})$	$2^*a$	$s^*(-M(m, 1))$	$(-1)^t m(m, s)$	$(-1)^t d(\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}) m'(m, s)$
${}_k \bar{A}_0$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$K(k, t)$
${}_k \bar{B}_0$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$-K(k, t)$
$\bar{\bar{A}}_{p/2}$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & -e^{-ika} \\ 1 & 0 \end{pmatrix}$	$(-1)^s K(k, t)$	$(-1)^s K(k, t) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\bar{\bar{B}}_{p/2}$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & -e^{-ika} \\ 1 & 0 \end{pmatrix}$	$(-1)^s K(k, t)$	$-(-1)^s K(k, t) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
${}_k \bar{\bar{E}}_{m,-m}$	$k \in (0, \frac{\pi}{a})$ $m \in (0, \frac{\pi}{4})$	$2^*b$	$s^*(e^{ika} M(m, 1))$	$l(k, t) m(m, s)$	$d(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}) l(k, t) m'(m, s)$

TABLE XIV. Irreducible corepresentations of the magnetic line groups  $L(2p)'_p m'c$ . Here  $L' = (\frac{L'_{pm}}{L'_{pc}}, \frac{p}{p} \text{ even}) = \{(C_p^s | t), (\sigma_v C_p^s | t + \frac{1}{2}) | s = 0, 1, \dots, p-1; t = 0, \pm 1, \dots\}$ ,  $g = (C_{2p} | \frac{1}{2})$ . The pairs of \*-g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentations:  $({}_k A_{p/2}, -{}_k B_{p/2})$ ,  $({}_k B_{p/2}, -{}_k A_{p/2})$ ,  $(\pi/a A_0, \pi/a B_0)$ ,  $({}_0 A_{p/2}, {}_0 B_{p/2})$ ,  $({}_k A_0, -{}_k A_0)$ ,  $({}_k B_0, -{}_k B_0)$ ,  $({}_k E_{m,-m}, -{}_k E_{m,-m})$ .

Corep.		Type	$g$	$(C_p^s   t)$	$(\sigma_v C_p^s   t + \frac{1}{2})$
${}_0 \bar{A}_0$		$1^*$	1	1	1
${}_0 \bar{B}_0$		$1^*$	1	1	-1
$\bar{\bar{A}}_{p/2}$		$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\pi/a \bar{\bar{A}}_0$		$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s I$	$i(-1)^t \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
${}_0 \bar{E}_{m,-m}$	$m \in (0, \frac{\pi}{2})$	$1^*$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
$\pi/a \bar{\bar{A}}_{p/2}$		$1^*$	1	$(-1)^{s+t}$	$i(-1)^{s+t}$
$\pi/a \bar{\bar{B}}_{p/2}$		$1^*$	1	$(-1)^{s+t}$	$-i(-1)^{s+t}$
${}_k \bar{A}_0$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$K(k, t + \frac{1}{2})$
${}_k \bar{B}_0$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$-K(k, t + \frac{1}{2})$
$\pi/a \bar{\bar{E}}_{m,-m}$	$m \in (0, \frac{\pi}{2})$	$1^*$	$\begin{pmatrix} 0 & -e^{-im\alpha} \\ 1 & 0 \end{pmatrix}$	$(-1)^t M(m, s)$	$i(-1)^t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
$\bar{\bar{A}}_{p/2}$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & -e^{-ika} \\ 1 & 0 \end{pmatrix}$	$(-1)^s K(k, t)$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} K(k, t + \frac{1}{2})$
$\bar{\bar{B}}_{p/2}$	$k \in (0, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & -e^{-ika} \\ 1 & 0 \end{pmatrix}$	$(-1)^s K(k, t)$	$-(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} K(k, t + \frac{1}{2})$
${}_k \bar{\bar{E}}_{m,-m}$	$k \in (0, \frac{\pi}{a})$ $m \in (0, \frac{\pi}{2})$	$2^*b$	$s^*(e^{ika} M(m, 1))$	$l(k, t) m(m, s)$	$d(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}) l(k, t + \frac{1}{2}) m'(m, s)$

TABLE XV. Irreducible corepresentations of the magnetic line groups  $L(\overline{2n})2'm'$ ,  $L\bar{n}m'$ . Here  $L' = \begin{pmatrix} L_{n2}^{2n} \\ L_{n\bar{n}} \end{pmatrix} = \{(C_n^s|t), (\sigma_h C_{2n} C_n^s|-t) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (\sigma_v|0)$ .

Corep.		Type	$g$	$(C_n^s   t)$	$(\sigma_h C_{2n} C_n^s   -t)$
${}_o\overline{A}_m^\pm$	$m \in (-\frac{n}{2}, \frac{n}{2}]$	$1^*$	1	$e^{ims\alpha}$	$\pm e^{im(s+\frac{1}{2})\alpha}$
${}_k\overline{E}_m$	$k \in (0, \frac{\pi}{a})$ $m \in (-\frac{n}{2}, \frac{n}{2}]$	$1^*$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$e^{ims\alpha} K(k, t)$	$e^{ims\alpha} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
${}_{\pi/a}\overline{A}_m^\pm$	$m \in (-\frac{n}{2}, \frac{n}{2}]$	$1^*$	1	$(-1)^t e^{ims\alpha}$	$\pm (-1)^t e^{im(s+\frac{1}{2})\alpha}$

TABLE XVI. Irreducible corepresentations of the magnetic line groups  $L(\overline{2n})'2m'$ ,  $L\bar{n}'m'$ . Here  $L' = \begin{pmatrix} L_{n2}^{2n} \\ L_{n\bar{n}} \end{pmatrix} = \{(C_n^s|t), (UC_n^s|-t) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (\sigma'_v|0)$ ; the axis of rotation of  $U$  makes an angle of  $\pi/2n$  with the plane of  $\sigma'_v$ . The pairs of  $^*g$ -conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentations:  $({}_oA_{n/2}^+, {}_oA_{n/2}^-)$ ,  $({}_{\pi/a}A_{n/2}^+, {}_{\pi/a}A_{n/2}^-)$ ,  $({}_kE_m^-, {}_{-k}E_m^-)$ .

Corep.		Type	$g$	$(C_n^s   t)$	$(UC_n^s   -t)$
${}_o\overline{A}_o^\pm$		$1^*$	1	1	$\pm 1$
${}_o\overline{E}_m^-$	$m \in (0, \frac{n}{2})$	$1^*$	$M(m, -1/2)$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
${}_o\overline{A}_{n/2}$		$2^*b$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
${}_{\pi/a}\overline{A}_o^\pm$		$1^*$	1	$(-1)^t (-1)^{In t (\frac{2\pi}{n})}$	$\pm (-1)^{s+t} (-1)^{In t (\frac{2\pi}{n})}$
${}_{\pi/a}\overline{E}_m^-$	$m \in (0, \frac{n}{2})$	$1^*$	$M(m, -1/2)$	$M(m, s) K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s) K(k, t)$
${}_{\pi/a}\overline{A}_{n/2}$		$2^*b$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} I$	$(-1)^{s+t} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
${}_k\overline{E}_m^-$	$k \in (0, \frac{\pi}{a})$ $m \in (0, \frac{n}{2})$	$2^*b$	$s(I)$	$k(k, t)n(m, s)$	$d(P)k(k, t)n(m, s)$

TABLE XVII. Irreducible corepresentations of the magnetic line groups  $L(\overline{2n})'2'm$ ,  $L\bar{n}'m$ . Here  $L' = \begin{pmatrix} L_{nm}^{nm} \\ L_{n\bar{n}} \end{pmatrix} = \{(C_n^s|t), (\sigma_v C_n^s|t) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (U'|0)$ ; the axis of rotation of  $U'$  makes an angle of  $\pi/n$  with the plane of  $\sigma_v$ . The pair of  $^*g$ -conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentation:  $({}_kA_{n/2}, {}_kB_{n/2})$ .

Corep.		Type	$g$	$(C_n^s   t)$	$(\sigma_v C_n^s   t)$
${}_k\overline{A}_o$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$	$1^*$	1	$e^{ikta}$	$e^{ikta}$
${}_kB_o$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$	$1^*$	1	$e^{ikta}$	$e^{-ikta}$
${}_k\overline{E}_m^-$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$ $m \in (0, \frac{n}{2})$	$1^*$	$M(m, -1/2)$	$e^{ikta} M(m, s)$	$e^{ikta} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
${}_k\overline{A}_{n/2}$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s e^{ikta} I$	$(-1)^s e^{ikta} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$



TABLE XVIII. Irreducible corepresentations of the magnetic line groups  $L_c(\overline{2n})2m$ ,  $L_c\overline{n}m$ . Here  $L' = (L'_{\overline{n}m}, \begin{smallmatrix} n \text{ even} \\ n \text{ odd} \end{smallmatrix}) = \{(C_n^s|t), (\sigma_v C_n^s|t), (\sigma_h C_n^s|-t), (UC_n^s|-t) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (E|\frac{1}{2})$ . The pairs of \*-g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentations:  $(\begin{smallmatrix} - \\ k \end{smallmatrix} E_{A_{n/2}}^{B_{n/2}}, \begin{smallmatrix} - \\ k \end{smallmatrix} E_{B_{n/2}}^{A_{n/2}})$ ,  $(\pi/a A_0^+, \pi/a A_0^-)$ ,  $(\pi/a B_0^+, \pi/a B_0^-)$ ,  $(\pi/a E_{m,-m}^+, \pi/a E_{m,-m}^-)$ .

Corep.	Type	$g$	$(C_n^s t)$	$(\sigma_v C_n^s t)$	$(\sigma_h C_n^s -t)$	$(UC_n^s -t)$
$\begin{smallmatrix} - \\ o \end{smallmatrix} A_o^\pm$	$1^*$	1	1	1	$\pm 1$	$\pm 1$
$\begin{smallmatrix} - \\ o \end{smallmatrix} B_o^\pm$	$1^*$	1	1	-1	$\pm 1$	$\mp 1$
$\begin{smallmatrix} - \\ o \end{smallmatrix} E_{m,-m}^\pm$	$1^*$	$m \in (0, \frac{n}{2})$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s - \frac{1}{2})$	$\pm M(m, s + \frac{1}{2})$
$\begin{smallmatrix} - \\ o \end{smallmatrix} E_{n/2}$	$1^*$	1	$(-1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
$\begin{smallmatrix} - \\ k \end{smallmatrix} E_{A_o}$	$1^*$	$k \in (0, \frac{n}{a})$	$\begin{pmatrix} 0 & e^{-ims\alpha} \\ 1 & 0 \end{pmatrix} K(k, t)$	$K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
$\begin{smallmatrix} - \\ k \end{smallmatrix} E_{B_o}$	$1^*$	$k \in (0, \frac{n}{a})$	$\begin{pmatrix} 0 & e^{-ims\alpha} \\ 1 & 0 \end{pmatrix} K(k, t)$	$-K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$-\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
$\begin{smallmatrix} - \\ k \end{smallmatrix} G_m^m$	$1^*$	$k \in (0, \frac{n}{a}), m \in (0, \frac{n}{2})$	$z^*(m, k)$	$l(k, t)m(m, s)$	$l(k, t)m(m, s)d(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix})$	$s(I)l(k, t)m(m, s)$
$\begin{smallmatrix} - \\ k \end{smallmatrix} E_{A_{n/2}}^{B_{n/2}}$	$2^*b$	$k \in (0, \frac{n}{a})$	$s^*(Q)$	$(-1)^s k(k, t)$	$(-1)^s d(\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix})k(k, t)$	$(-1)^s d(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix})k'(k, t)$
$\pi/a A_o$	$2^*b$		$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\pi/a B_o$	$2^*b$		$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s I$	$-( -1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\pi/a E_m^m$	$2^*b$	$m \in (0, \frac{n}{2})$	$s(-I)$	$(-1)^s m(m, s)$	$(-1)^s d(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix})m(m, s)$	$(-1)^s g(-I)m(m, s - \frac{1}{2})$
$\pi/a E_{n/2}$	$2^*a$		$s(-I)$	$(-1)^{s+t} \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$	$(-1)^{s+t} d(\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix})$	$(-1)^{s+t} g(-I)d(\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix})$

TABLE XIX. Irreducible corepresentations of the magnetic line groups  $L_m(\overline{2n})2m$ ,  $L_m\overline{n}m$ . Here  $L' = (L'_{\overline{n}c}, \begin{smallmatrix} n \text{ even} \\ n \text{ odd} \end{smallmatrix}) = \{(C_n^s|t), (\sigma_v C_n^s|t + \frac{1}{2}), (U' C_n^s|-t), (U' \sigma_v C_n^s|-t - \frac{1}{2}) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (E|\frac{1}{2})$ . The pairs of \*-g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentations:  $(\begin{smallmatrix} - \\ k \end{smallmatrix} E_{A_{n/2}}^{B_{n/2}}, \begin{smallmatrix} - \\ k \end{smallmatrix} E_{B_{n/2}}^{A_{n/2}})$ ,  $(\pi/a A_{n/2}^+, \pi/a B_{n/2}^-)$ .

Corep.	Type	$g$	$(C_n^s t)$	$(\sigma_v C_n^s t + \frac{1}{2})$	$(U' C_n^s -t)$	$(U' \sigma_v C_n^s -t - \frac{1}{2})$
$\begin{smallmatrix} - \\ o \end{smallmatrix} A_o^\pm$	$1^*$	1	1	1	$\pm 1$	$\pm 1$
$\begin{smallmatrix} - \\ o \end{smallmatrix} B_o^\pm$	$1^*$	1	1	-1	$\pm 1$	$\mp 1$
$\begin{smallmatrix} - \\ o \end{smallmatrix} E_{m,-m}^\pm$	$1^*$	$m \in (0, \frac{n}{2})$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s - \frac{1}{2})$	$\pm M(m, s + \frac{1}{2})$
$\begin{smallmatrix} - \\ o \end{smallmatrix} E_{n/2}$	$1^*$	1	$(-1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
$\begin{smallmatrix} - \\ k \end{smallmatrix} E_{A_o}$	$1^*$	$k \in (0, \frac{n}{a})$	$\begin{pmatrix} 0 & e^{-ims\alpha} \\ 1 & 0 \end{pmatrix} K(k, t)$	$K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t + \frac{1}{2})$
$\begin{smallmatrix} - \\ k \end{smallmatrix} E_{B_o}$	$1^*$	$k \in (0, \frac{n}{a})$	$\begin{pmatrix} 0 & e^{-ims\alpha} \\ 1 & 0 \end{pmatrix} K(k, t)$	$-K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$-\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t + \frac{1}{2})$
$\begin{smallmatrix} - \\ k \end{smallmatrix} G_m^m$	$1^*$	$k \in (0, \frac{n}{a}), m \in (0, \frac{n}{2})$	$z^*(m, k)$	$l(k, t)m(m, s)$	$d(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix})l(k, t + \frac{1}{2})m'(m, s)$	$s(I)l(k, t)m(m, s)$
$\begin{smallmatrix} - \\ k \end{smallmatrix} E_{n/2}$	$2^*b$	$k \in (0, \frac{n}{a})$	$s^*(Q)$	$(-1)^s k(k, t)$	$(-1)^s d(\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix})k(k, t + \frac{1}{2})$	$(-1)^s d(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix})k'(k, t + \frac{1}{2})$
$\pi/a E_o$	$1^*$		$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\pi/a E_{m,-m}^\pm$	$1^*$	$m \in (0, \frac{n}{2})$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s M(m, s)$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm (-1)^s \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} M(m, s - \frac{1}{2})$
$\pi/a A_{n/2}$	$2^*b$		$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} I$	$(-1)^{s+t} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\pm (-1)^{s+t} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

TABLE XX. Irreducible corepresentations of the magnetic line groups  $L(\overline{2n})2'c'$ ,  $L\overline{n}c'$ . Here  $L' = (L'_{\overline{n}}, \begin{smallmatrix} n \text{ even} \\ n \text{ odd} \end{smallmatrix}) = \{(C_n^s|t), (\sigma_h C_{2n} C_n^s|-t) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (\sigma_v|\frac{1}{2})$ . The pair of \*-g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentation:  $(\pi/a A_m^+, \pi/a A_m^-)$ .

Corep.	Type	$g$	$(C_n^s t)$	$(\sigma_h C_{2n} C_n^s -t)$
$\begin{smallmatrix} - \\ o \end{smallmatrix} A_m^\pm$	$1^*$	$m \in (-\frac{n}{2}, \frac{n}{2})$	1	$e^{ims\alpha}$
$\begin{smallmatrix} - \\ k \end{smallmatrix} E_m$	$1^*$	$k \in (0, \frac{n}{a}), m \in (0, \frac{n}{2})$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, \frac{1}{2})$	$e^{ims\alpha} K(k, t)$
$\pi/a A_m$	$2^*b$	$m \in (-\frac{n}{2}, \frac{n}{2})$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t e^{ims\alpha} I$

TABLE XXI. Irreducible corepresentations of the magnetic line groups  $L(\overline{2n})'2c'$ ,  $L\overline{n}'c'$ . Here  $L' = (L_{n2}^{n2}, n \text{ even} / L_{n2}, n \text{ odd}) = \{(C_n^s | t), (UC_n^s | -t) | s = 0, 1, \dots, n-1; t = 0, \pm 1, \dots\}$ ,  $g = (\sigma_v | \frac{1}{2})$ . The pairs of \*-g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentations:  $({}_0A_{n/2,0}^+, {}_0A_{n/2,0}^-)$ ,  $({}_{\pi/a}A_0^+, {}_{\pi/a}A_0^-)$ ,  $({}^kE_m^-, {}^kE_m^-)$ .

Corep.		Type	$g$	$(C_n^s   t)$	$(UC_n^s   -t)$
${}_0\overline{A}_0^\pm$		$1^*$	1	1	$\pm 1$
${}_0\overline{E}_m^-$	$m \in (0, \frac{n}{2})$	$1^*$	$M(m, \frac{1}{2})$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
${}_0\overline{A}_{n/2}$		$2^*b$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
${}_{\pi/a}\overline{A}_0$		$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
${}_{\pi/a}\overline{A}_{n/2}$		$2^*a$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} I$	$\pm (-1)^{s+t} I$
${}_{\pi/a}\overline{E}_m^-$	$m \in (0, \frac{n}{2})$	$2^*a$	$s(-I)$	$(-1)^t n(m, s)$	$(-1)^t g(-I) d(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}) n(m, s)$
${}^k\overline{E}_m^-$	$k \in (0, \frac{\pi}{a})$ $m \in (0, \frac{n}{2})$	$2^*b$	$s(Q)$	$k(k, t) n(m, s)$	$d(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}) k'(k, t) n'(m, s)$

TABLE XXII. Irreducible corepresentations of the magnetic line groups  $L(\overline{2n})'2'c'$ ,  $L\overline{n}'c'$ . Here  $L' = (L_{nc}^{nc}, n \text{ even} / L_{nc}, n \text{ odd}) = \{(C_n^s | t), (\sigma_v C_n^s | t + \frac{1}{2}) | s = 0, 1, \dots, n-1; t = 0, \pm 1, \dots\}$ ,  $g = (U' | 0)$ ; the axis of rotation of  $U'$  makes an angle of  $\pi/n$  with the plane of  $\sigma_v$ . The pair of \*-g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentation:  $({}_kA_{n/2}, {}_kB_{n/2})$ .

Corep.		Type	$g$	$(C_n^s   t)$	$(\sigma_v C_n^s   t + \frac{1}{2})$
${}_k\overline{A}_0$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$	$1^*$	1	$e^{ikta}$	$e^{ik(t+\frac{1}{2})a}$
${}_k\overline{B}_0$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$	$1^*$	1	$e^{ikta}$	$-e^{ik(t+\frac{1}{2})a}$
${}_k\overline{E}_m^-$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$ $m \in (0, \frac{n}{2})$	$1^*$	$M(m, -\frac{1}{2})$	$e^{ikta} M(m, s)$	$e^{ik(t+\frac{1}{2})a} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
${}_k\overline{A}_{n/2}$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$	$2^*b$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s e^{ikta} I$	$(-1)^s e^{ik(t+\frac{1}{2})a} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

TABLE XXIII. Irreducible corepresentations of the magnetic line groups  $L_n/mmm'm'$ ,  $L(\overline{2n})2'm'$ . Here  $L' = (L_{(2n)}^{n/m}, n \text{ even} / L_{(2n)}, n \text{ odd}) = \{(C_n^s | t), (\sigma_h C_n^s | -t) | s = 0, 1, \dots, n-1; t = 0, \pm 1, \dots\}$ ,  $g = (\sigma_v | 0)$ .

Corep.		Type	$g$	$(C_n^s   t)$	$(\sigma_h C_n^s   -t)$
${}_0\overline{A}_m^\pm$	$m \in (-\frac{n}{2}, \frac{n}{2}]$	$1^*$	1	$e^{ims\alpha}$	$\pm e^{ims\alpha}$
${}^k\overline{E}_m$	$k \in (0, \frac{\pi}{a})$ $m \in (-\frac{n}{2}, \frac{n}{2}]$	$1^*$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$e^{ims\alpha} K(k, t)$	$e^{ims\alpha} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
${}_{\pi/a}\overline{A}_m^\pm$	$m \in (-\frac{n}{2}, \frac{n}{2}]$	$1^*$	1	$(-1)^t e^{ims\alpha}$	$\pm (-1)^t e^{ims\alpha}$

TABLE XXIV. Irreducible corepresentations of the magnetic line groups  $L_n/m'm'm'$ ,  $L(2n)'2m'$ . Here  $L' = \begin{pmatrix} L_n^{22}, & n \text{ even} \\ L_n^2, & n \text{ odd} \end{pmatrix} = \{(C_n^s | t), (UC_n^s | -t) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (\sigma_v | 0)$ . The pair of  $^*g$ -conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentation:  $(\begin{smallmatrix} -k \\ k \end{smallmatrix} E_m^{-m}, \begin{smallmatrix} k \\ -k \end{smallmatrix} E_m^{-m})$ .

Corep.		Type	$g$	$(C_n^s   t)$	$(UC_n^s   -t)$
$\overline{A}_o^\pm$		$1^*$	1	1	$\pm 1$
$\overline{E}_m^{-m}$	$m \in (0, \frac{n}{2})$	$1^*$	$I$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
$\overline{A}_{n/2}^\pm$		$1^*$	1	$(-1)^s$	$\pm(-1)^s$
$\pi/a \overline{A}_o^\pm$		$1^*$	1	$(-1)^t$	$\pm(-1)^t$
$\pi/a \overline{A}_{n/2}^\pm$		$1^*$	1	$(-1)^{s+t}$	$\pm(-1)^{s+t}$
$\pi/a \overline{E}_m^{-m}$	$m \in (0, \frac{n}{2})$	$1^*$	$I$	$(-1)^t M(m, s)$	$(-1)^t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
$\begin{smallmatrix} -k \\ k \end{smallmatrix} \overline{E}_m^{-m}$	$k \in (0, \frac{n}{2})$ $m \in (0, \frac{n}{2})$	$2^*b$	$s(I)$	$k(k, t)n(m, s)$	$d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} k(k, t)n(m, s)$

TABLE XXV. Irreducible corepresentations of the magnetic line groups  $L_n/m'mm$ ,  $L(2n)'2'm$ . Here  $L' = \begin{pmatrix} L_n^{mm}, & n \text{ even} \\ L_n m, & n \text{ odd} \end{pmatrix} = \{(C_n^s | t), (\sigma_v C_n^s | t) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (\sigma_h | 0)$ .

Corep.		Type	$g$	$(C_n^s   t)$	$(\sigma_v C_n^s   t)$
$k \overline{A}_o$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$	$1^*$	1	$e^{ikta}$	$e^{ikta}$
$k \overline{B}_o$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$	$1^*$	1	$e^{ikta}$	$-e^{ikta}$
$k \overline{E}_m^{-m}$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$ $m \in (0, \frac{n}{2})$	$1^*$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$e^{ikta} M(m, s)$	$e^{ikta} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
$\overline{A}_{n/2}$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$	$1^*$	1	$(-1)^s e^{ikta}$	$(-1)^s e^{ikta}$
$\overline{B}_{n/2}$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$	$1^*$	1	$(-1)^s e^{ikta}$	$-(-1)^s e^{ikta}$

TABLE XXVI. Irreducible corepresentations of the magnetic line groups  $L_n'/m'mm'$  ( $n=2p$ ). Here  $L' = \begin{pmatrix} L(2p)2m, p \text{ even} \\ L p m, p \text{ odd} \end{pmatrix} = \{(C_p^s | t), (\sigma_v C_p^s | t), (U' C_p^s | -t), (U' \sigma_v C_p^s | -t) | s=0, 1, \dots, p-1; t=0, \pm 1, \dots\}$ ,  $g = (\sigma_h | 0)$ .

Corep.		Type	$g$	$(C_p^s   t)$	$(\sigma_v C_p^s   t)$	$(U' C_p^s   -t)$	$(U' \sigma_v C_p^s   -t)$
$\overline{A}_o^\pm$		$1^*$	1	1	1	$\pm 1$	$\pm 1$
$\overline{B}_o^\pm$		$1^*$	1	1	-1	$\pm 1$	$\mp 1$
$\overline{E}_{m,-m}^\pm$	$m \in (0, \frac{n}{4})$	$1^*$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s - \frac{1}{2})$	$\pm M(m, s + \frac{1}{2})$
$\overline{E}_{p/2}$		$1^*$	$I$	$(-1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
$\begin{smallmatrix} -k \\ k \end{smallmatrix} \overline{E}_{A_s}$	$k \in (0, \frac{\pi}{a})$	$1^*$	$I$	$K(k, t)$	$K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
$\begin{smallmatrix} -k \\ k \end{smallmatrix} \overline{E}_{B_s}$	$k \in (0, \frac{\pi}{a})$	$1^*$	$I$	$K(k, t)$	$-K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$-\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
$\begin{smallmatrix} -k \\ k \end{smallmatrix} \overline{G}_{m,-m}$	$k \in (0, \frac{\pi}{4})$ $m \in (0, \frac{\pi}{4})$	$1^*$	$d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$l(k, t)m(m, s)$	$d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} l(k, t)m'(m, s)$	$s(I)l(k, t)m(m, s)$	$b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} l(k, t)m'(m, s)$
$\begin{smallmatrix} -k \\ k \end{smallmatrix} \overline{E}_{A_{p/2}}^{B_{p/2}}$	$k \in (0, \frac{\pi}{a})$	$1^*$	$I$	$(-1)^s K(k, t)$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} K(k, t)$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$(-1)^s \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} K(k, t)$
$\begin{smallmatrix} -k \\ k \end{smallmatrix} \overline{E}_{B_{p/2}}^{A_{p/2}}$	$k \in (0, \frac{\pi}{a})$	$1^*$	$I$	$(-1)^s K(k, t)$	$-(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} K(k, t)$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$-(-1)^s \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} K(k, t)$
$\pi/a \overline{A}_o^\pm$		$1^*$	1	$(-1)^t$	$(-1)^t$	$\pm(-1)^t$	$\pm(-1)^t$
$\pi/a \overline{B}_o^\pm$		$1^*$	1	$(-1)^t$	$-(-1)^t$	$\pm(-1)^t$	$\mp(-1)^t$
$\pi/a \overline{E}_{m,-m}^\pm$	$m \in (0, \frac{n}{4})$	$1^*$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t M(m, s)$	$(-1)^t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm(-1)^t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s - \frac{1}{2})$	$\pm(-1)^t M(m, s + \frac{1}{2})$
$\pi/a \overline{E}_{p/2}$		$1^*$	$I$	$(-1)^{s+t} I$	$(-1)^{s+t} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^{s+t} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

TABLE XXVII. Irreducible corepresentations of the magnetic line groups  $L_n'/mmm'$  ( $n=2p$ ). Here  $L' = (L_p/mmm', p \text{ even}) = \{(C_p^s | t), (\sigma_v C_p^s | t), (\sigma_h C_p^s | -t), (UC_p^s | -t) | s=0, 1, \dots, p-1; t=0, \pm 1, \dots\}$ ,  $g = (C_n | 0)$ . The pairs of \*-g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentations:  $({}_0A_{p/2}^{\pm}, {}_0B_{p/2}^{\pm}), ({}^kE_{A_{p/2}}^{\pm}, {}^kE_{B_{p/2}}^{\pm}), (\pi/a A_{p/2}^{\pm}, \pi/a B_{p/2}^{\pm})$ .

Corep.	Type	$g$	$(C_p^s   t)$	$(\sigma_v C_p^s   t)$	$(\sigma_h C_p^s   -t)$	$(UC_p^s   -t)$
${}_0\bar{A}_0^{\pm}$	$1^*$	1	1	1	$\pm 1$	$\pm 1$
${}_0\bar{B}_0^{\pm}$	$1^*$	1	1	-1	$\pm 1$	$\mp 1$
${}_0\bar{E}_{m,-m}^{\pm}$	$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\pm (-1)^s I$	$\pm (-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
${}_0\bar{E}_{m,-m}^{\pm}$	$1^*$	$\begin{pmatrix} 0 & e^{im\alpha} \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm M(m, s)$	$\pm \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
${}^k\bar{E}_{A_0}^{\pm}$	$1^*$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
${}^k\bar{E}_{B_0}^{\pm}$	$1^*$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$-K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$-\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
${}^k\bar{E}_{m,-m}^{\pm}$	$1^*$	$b \begin{pmatrix} 0 & e^{im\alpha} \\ 1 & 0 \end{pmatrix}$	$l(k, t)n(m, s)$	$d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} l(k, t)n(m, s)$	$s(I)l(k, t)n(m, s)$	$b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} l(k, t)n(m, s)$
${}^k\bar{E}_{p/2}^{\pm}$	$2^*b$	$s(-I)$	$(-1)^s k(k, t)$	$(-1)^s g(-I)k(k, t)$	$(-1)^s d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} k(k, t)$	$(-1)^s d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} g(-I)k(k, t)$
$\pi/a \bar{A}_0^{\pm}$	$1^*$	1	$(-1)^t$	$(-1)^t$	$\pm (-1)^t$	$\pm (-1)^t$
$\pi/a \bar{B}_0^{\pm}$	$1^*$	1	$(-1)^t$	$-(-1)^t$	$\pm (-1)^t$	$\mp (-1)^t$
$\pi/a \bar{E}_{m,-m}^{\pm}$	$1^*$	$\begin{pmatrix} 0 & e^{im\alpha} \\ 1 & 0 \end{pmatrix}$	$(-1)^t M(m, s)$	$(-1)^t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm (-1)^t M(m, s)$	$\pm (-1)^t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
$\pi/a \bar{E}_{p/2}^{\pm}$	$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} I$	$(-1)^{s+t} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\pm (-1)^{s+t} I$	$\pm (-1)^{s+t} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

A. Corepresentations of the groups isogonal to  $D_n$

$D_n$  are non-Abelian point groups, which can be expressed as the semidirect products  $D_n = C_n \wedge \{e, U\}$  (in the notation of I) of two cyclic groups. Hence the representations of the isogonal line groups are obtained by a single induction, which reflects in their dimension: the

maximal dimension of a representation of a line group  $L'$  is 2; performing \*-induction one obtains one-, two-, and four-dimensional corepresentations.

The line groups isogonal to  $D_n$  ( $n=0, 1, \dots$ ) are  $L_{n_p}2$  for  $n$  odd and  $L_{n_p}22$  for  $n$  even ( $p=0, 1, \dots, n-1$ ). It is shown in I that the black-and-white magnetic line groups obtained from  $L_{n_p}2$  ( $L_{n_p}22$ ), respectively) are

TABLE XXVIII. Irreducible corepresentations of the magnetic line groups  $L_c n/mmm$ ,  $L_c(\bar{2}n)2m$ . Here  $L' = (L_c n/mmm, n \text{ even}) = \{(C_n^s | t), (\sigma_v C_n^s | t), (\sigma_h C_n^s | -t), (UC_n^s | -t) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (E | \frac{1}{2})$ . The pairs of \*-g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentations:  $(\pi/a A_0^+, \pi/a A_0^-), (\pi/a B_0^+, \pi/a B_0^-), (\pi/a A_{n/2}^+, \pi/a A_{n/2}^-), (\pi/a B_{n/2}^+, \pi/a B_{n/2}^-), (\pi/a E_{m,-m}^+, \pi/a E_{m,-m}^-)$ .

Corep.	Type	$g$	$(C_n^s   t)$	$(\sigma_v C_n^s   t)$	$(\sigma_h C_n^s   -t)$	$(UC_n^s   -t)$
${}_0\bar{A}_0^{\pm}$	$1^*$	1	1	1	$\pm 1$	$\pm 1$
${}_0\bar{B}_0^{\pm}$	$1^*$	1	1	-1	$\pm 1$	$\mp 1$
${}_0\bar{E}_{m,-m}^{\pm}$	$1^*$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm M(m, s)$	$\pm \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
${}_0\bar{A}_{n/2}^{\pm}$	$1^*$	1	$(-1)^s$	$(-1)^s$	$\pm (-1)^s$	$\pm (-1)^s$
${}_0\bar{B}_{n/2}^{\pm}$	$1^*$	1	$(-1)^s$	$-(-1)^s$	$\pm (-1)^s$	$\mp (-1)^s$
${}^k\bar{E}_{A_0}^{\pm}$	$1^*$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
${}^k\bar{E}_{B_0}^{\pm}$	$1^*$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$-K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$-\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
${}^k\bar{E}_{m,-m}^{\pm}$	$1^*$	$d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} g \begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$l(k, t)n(m, s)$	$d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} l(k, t)n(m, s)$	$s(I)l(k, t)n(m, s)$	$b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} l(k, t)n(m, s)$
${}^k\bar{E}_{A_{n/2}}^{\pm}$	$1^*$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$(-1)^s K(k, t)$	$(-1)^s K(k, t)$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
${}^k\bar{E}_{B_{n/2}}^{\pm}$	$1^*$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$(-1)^s K(k, t)$	$(-1)^s K(k, t)$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
$\pi/a \bar{A}_0^{\pm}$	$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t I$	$(-1)^t I$	$(-1)^t \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^t \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\pi/a \bar{B}_0^{\pm}$	$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t I$	$(-1)^t I$	$(-1)^t \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^t \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\pi/a \bar{E}_{m,-m}^{\pm}$	$2^*b$	$s(-I)$	$(-1)^t m(m, s)$	$(-1)^t d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} m(m, s)$	$(-1)^t g(-I)m(m, s)$	$(-1)^t d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} g(-I)m(m, s)$
$\pi/a \bar{A}_{n/2}^{\pm}$	$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} I$	$(-1)^{s+t} I$	$(-1)^{s+t} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^{s+t} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\pi/a \bar{B}_{n/2}^{\pm}$	$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} I$	$(-1)^{s+t} I$	$(-1)^{s+t} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^{s+t} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

TABLE XXIX. Irreducible corepresentations of the magnetic line groups  $L_s n/mmm$  ( $n=2p$ ). Here  $L' = L''(2p)_p/mcm = \{(C_n^s | Fr(s/2) + t), (\sigma_v C_n^s | Fr(s/2) + t), (\sigma_h C_n^s | -Fr(s/2) - t), (UC_n^s | -Fr(s/2) - t) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (E | \frac{1}{2})$ ;  $f=2 Fr(s/2) = \begin{cases} 0, & s \text{ even} \\ 1, & s \text{ odd} \end{cases}$ . The pair of \*-g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentation:  $(\pi/2 E_{p/2}^+, \pi/2 E_{p/2}^-)$ .

Corep.	Type   g	$(C_n^s   Fr(\frac{1}{2}) + t)$	$(\sigma_v C_n^s   Fr(\frac{1}{2}) + t)$	$(\sigma_h C_n^s   -Fr(\frac{1}{2}) - t)$	$(UC_n^s   -Fr(\frac{1}{2}) - t)$
$\overset{\pm}{\circ} \overset{\pm}{A}_n$	1*	1	1	$\pm 1$	$\pm 1$
$\overset{\pm}{\circ} \overset{\pm}{B}_n$	1*	1	-1	$\pm 1$	$\mp 1$
$\overset{\pm}{\circ} \overset{\pm}{E}_{m,-m}$	$m \in (0, n/2)$ 1*	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm M(m, s)$
$\overset{\pm}{\circ} \overset{\pm}{A}_{n/2}$	1*	1	$(-1)^s$	$\pm(-1)^s$	$\pm(-1)^s$
$\overset{\pm}{\circ} \overset{\pm}{B}_{n/2}$	1*	1	$(-1)^s$	$\pm(-1)^s$	$\mp(-1)^s$
$\overset{\pm}{k} \overset{\pm}{E}_{A_s}$	$k \in (0, \pi/a)$ 1*	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$K(k, t + Fr(\frac{1}{2}))$	$K(k, t + Fr(\frac{1}{2}))$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t + Fr(\frac{1}{2}))$
$\overset{\pm}{k} \overset{\pm}{E}_{B_s}$	$k \in (0, \pi/a)$ 1*	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$K(k, t + Fr(\frac{1}{2}))$	$-K(k, t + Fr(\frac{1}{2}))$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t + Fr(\frac{1}{2}))$
$\overset{\pm}{k} \overset{\pm}{G}_{m,-m}$	$\begin{matrix} k \in (0, \pi/a) \\ m \in (0, n/2) \end{matrix}$ 1*	$s(Q)$	$l(k, t + Fr(\frac{1}{2}))n(m, s)$	$d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} l(k, t + Fr(\frac{1}{2}))n(m, s)$	$s(I)l(k, t + Fr(\frac{1}{2}))n(m, s)$
$\overset{\pm}{k} \overset{\pm}{E}_{A_s}$	$k \in (0, \pi/a)$ 1*	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$(-1)^s K(k, t + Fr(\frac{1}{2}))$	$(-1)^s K(k, t + Fr(\frac{1}{2}))$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t + Fr(\frac{1}{2}))$
$\overset{\pm}{k} \overset{\pm}{E}_{B_s}$	$k \in (0, \pi/a)$ 1*	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$(-1)^s K(k, t + Fr(\frac{1}{2}))$	$(-1)^s K(k, t + Fr(\frac{1}{2}))$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t + Fr(\frac{1}{2}))$
$\pi/a \overset{\pm}{E}_A$	1*	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s (i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix})^s$	$(-1)^s (i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix})^s$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix})^s$
$\pi/a \overset{\pm}{E}_B$	1*	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s (i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix})^s$	$(-1)^s (i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix})^s$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix})^s$
$\pi/a \overset{\pm}{G}_{m,-m}$	$m \in (0, n/4)$ 1*	$s(I)$	$(-1)^s (ig(-I))^s n(m, s)$	$(-1)^s (ig(-I))^s d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} n(m, s)$	$(-1)^s (-ig(-I))^s b(I)n(m, s)$
$\pi/a \overset{\pm}{E}_{p/2}$	2*b	$s(-I)$	$(-1)^s (-1)^{n(s/2)} d \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^s$	$(-1)^s (-1)^{n(s/2)} d \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^s$	$(-1)^s g(-I) d \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}^s (-1)^{n(s/2)}$

TABLE XXX. Irreducible corepresentations of the magnetic line groups  $L_m n/mmm$ ,  $L_m(\overline{2n})2m$ . Here  $L' = (L'_n/mcc, \begin{matrix} n \text{ even} \\ n \text{ odd} \end{matrix}) = \{(C_n^s | t), (\sigma_v C_n^s | t), (\sigma_h C_n^s | -t), (UC_n^s | -t) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (E | \frac{1}{2})$ .

Corep.	Type   g	$(C_n^s   t)$	$(\sigma_v C_n^s   t)$	$(\sigma_h C_n^s   -t)$	$(UC_n^s   -t)$
$\overset{\pm}{\circ} \overset{\pm}{A}_n$	1*	1	1	$\pm 1$	$\pm 1$
$\overset{\pm}{\circ} \overset{\pm}{B}_n$	1*	1	-1	$\pm 1$	$\mp 1$
$\overset{\pm}{\circ} \overset{\pm}{E}_{m,-m}$	$m \in (0, \frac{n}{2})$ 1*	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm M(m, s)$
$\overset{\pm}{\circ} \overset{\pm}{A}_{n/2}$	1*	1	$(-1)^s$	$\pm(-1)^s$	$\pm(-1)^s$
$\overset{\pm}{\circ} \overset{\pm}{B}_{n/2}$	1*	1	$(-1)^s$	$\pm(-1)^s$	$\mp(-1)^s$
$\overset{\pm}{k} \overset{\pm}{E}_{A_s}$	$k \in (0, \frac{\pi}{a})$ 1*	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
$\overset{\pm}{k} \overset{\pm}{E}_{B_s}$	$k \in (0, \frac{\pi}{a})$ 1*	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$-K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
$\overset{\pm}{k} \overset{\pm}{G}_{m,-m}$	$\begin{matrix} k \in (0, \frac{\pi}{a}) \\ m \in (0, \frac{n}{2}) \end{matrix}$ 1*	$d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} g \begin{pmatrix} e^{ika} & 0 \\ 0 & e^{ika} \end{pmatrix}$	$l(k, t)n(m, s)$	$d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} l(k, t)n(m, s)$	$s(I)l(k, t)n(m, s)$
$\pi/a \overset{\pm}{E}_o$	1*	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s I$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\pm(-1)^s I$
$\pi/a \overset{\pm}{E}_{m,-m}$	$m \in (0, \frac{n}{2})$ 1*	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s M(m, s)$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm(-1)^s M(m, s)$
$\overset{\pm}{k} \overset{\pm}{E}_{A_{n/2}}$	$k \in (0, \frac{\pi}{a})$ 1*	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$(-1)^s K(k, t)$	$(-1)^s K(k, t)$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
$\overset{\pm}{k} \overset{\pm}{E}_{B_{n/2}}$	$k \in (0, \frac{\pi}{a})$ 1*	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$(-1)^s K(k, t)$	$(-1)^s K(k, t)$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
$\pi/a \overset{\pm}{E}_{n/2}$	1*	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s +t I$	$(-1)^s +t i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^s +t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

TABLE XXXI. Irreducible corepresentations of the magnetic line groups  $L_n/mc'c'$ ,  $L(\overline{2n})2'c'$ . Here  $L' = (L'_n/m, \begin{matrix} n \text{ even} \\ n \text{ odd} \end{matrix}) = \{(C_n^s | t), (\sigma_h C_n^s | -t) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (\sigma_v | \frac{1}{2})$ . The pair of \*-g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentation:  $(\pi/a A_m^+, \pi/a A_m^-)$ .

Corep.	Type   g	$(C_n^s   t)$	$(\sigma_h C_n^s   -t)$
$\overset{\pm}{\circ} \overset{\pm}{A}_m$	$m \in (-\frac{n}{2}, \frac{n}{2}]$ 1*	1	$e^{ims\alpha}$
$\overset{\pm}{k} \overset{\pm}{E}_m$	$\begin{matrix} k \in (0, \frac{\pi}{a}) \\ m \in (-\frac{n}{2}, \frac{n}{2}] \end{matrix}$ 1*	$K(k, -\frac{1}{2})$	$e^{ims\alpha} K(k, t)$
$\pi/a \overset{\pm}{A}_m$	$m \in (-\frac{n}{2}, \frac{n}{2}]$ 2*b	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s e^{ims\alpha} I$

TABLE XXXII. Irreducible corepresentations of the magnetic line groups  $L_n/m'c'c'$ ,  $L(2n)'2c'$ . Here  $L' = (L_{n2}^{n2}, n \text{ even}) = \{(C_n^s | t), (UC_n^s | -t) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (\sigma_v | \frac{1}{2})$ . The pairs of \*-g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentations:  $(\pi/a A_{n/2}^+, \pi/a A_{n/2}^-)$ ,  $(\pi/a A_0^+, \pi/a A_0^-)$ ,  $(k^k E_m^{-m}, -k E_m^{-m})$ .

Corep.		Type	$g$	$(C_n^s   t)$	$(UC_n^s   -t)$
${}_o\bar{A}_o^\pm$		1*	1	1	$\pm 1$
${}_o\bar{B}_o^\pm$		1*	1	1	$\mp 1$
${}_o\bar{E}_m^{-m}$	$m \in (0, \frac{n}{2})$	1*	$I$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
$\pi/a \bar{A}_o$		$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t I$	$(-1)^t \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\pi/a \bar{E}_m^{-m}$	$m \in (0, \frac{n}{2})$	$2^*a$	$s(-I)$	$(-1)^t n(m, s)$	$(-1)^t d \left( \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right) g(-I) n(m, s)$
$\pi/a \bar{A}_{n/2}$		$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} I$	$(-1)^{s+t} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\frac{-k}{k} \bar{E}_m^{-m}$	$k \in (0, \frac{\pi}{a})$ $m \in (0, \frac{n}{2})$	$2^*b$	$s(Q)$	$k(k, t) n(m, s)$	$d \left( \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right) k'(k, t) n(m, s)$

TABLE XXXIII. Irreducible corepresentations of the magnetic line groups  $L_n/m'cc$ ,  $L(2n)'2c$ . Here  $L' = (L_{nc}^{ncc}, n \text{ even}) = \{(C_n^s | t), (\sigma_v C_n^s | t + \frac{1}{2}) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (\sigma_h | 0)$ .

Corep.		Type	$g$	$(C_n^s   t)$	$(\sigma_v C_n^s   t + \frac{1}{2})$
${}_k\bar{A}_o$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$	1*	1	$e^{ikta}$	$e^{ik(t+\frac{1}{2})a}$
${}_k\bar{B}_o$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$	1*	1	$e^{ikta}$	$-e^{ik(t+\frac{1}{2})a}$
${}_k\bar{E}_m^{-m}$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$ $m \in (0, \frac{n}{2})$	1*	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$e^{ikta} M(m, s)$	$e^{ik(t+\frac{1}{2})a} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
${}_o\bar{A}_{n/2}$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$	1*	1	$(-1)^s e^{ikta}$	$(-1)^s e^{ik(t+\frac{1}{2})a}$
${}_o\bar{B}_{n/2}$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$	1*	1	$(-1)^s e^{ikta}$	$-(-1)^s e^{ik(t+\frac{1}{2})a}$

TABLE XXXIV. Irreducible corepresentations of the magnetic line groups  $L_n/m'cc'$  ( $n=2p$ ). Here  $L' = (L_{pc}^{(2p)2c}, p \text{ even}) = \{(C_p^s | t), (\sigma_v C_p^s | t + \frac{1}{2}), (U' C_p^s | -t), (U' \sigma_v C_p^s | -t - \frac{1}{2}) | s=0, 1, \dots, p-1; t=0, \pm 1, \dots\}$ ,  $g = (\sigma_h | 0)$ .

Corep.		Type	$g$	$(C_p^s   t)$	$(\sigma_v C_p^s   t + \frac{1}{2})$	$(U' C_p^s   -t)$	$(U' \sigma_v C_p^s   -t - \frac{1}{2})$
${}_o\bar{A}_o^\pm$		1*	1	1	1	$\pm 1$	$\pm 1$
${}_o\bar{B}_o^\pm$		1*	1	1	-1	$\pm 1$	$\mp 1$
${}_o\bar{E}_{m,-m}^\pm$	$m \in (0, \frac{p}{4})$	1*	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s - \frac{1}{2})$	$\pm M(m, s + \frac{1}{2})$
${}_o\bar{E}_{p/2}$		1*	$I$	$(-1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
$\frac{-k}{k} \bar{E}_{A_o}$	$k \in (0, \frac{\pi}{a})$	1*	$I$	$K(k, t)$	$K(k, t + \frac{1}{2})$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t + \frac{1}{2})$
$\frac{-k}{k} \bar{E}_{B_o}$	$k \in (0, \frac{\pi}{a})$	1*	$I$	$K(k, t)$	$-K(k, t + \frac{1}{2})$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$-\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t + \frac{1}{2})$
$\frac{-k}{k} \bar{G}_m^{-m}$	$k \in (0, \frac{\pi}{a})$ $m \in (0, \frac{p}{4})$	1*	$d \left( \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right)$	$l(k, t) m(m, s)$	$d \left( \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right) l(k, t + \frac{1}{2}) m'(m, s)$	$s(I) l(k, t) m(m, s)$	$b \left( \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right) l(k, t + \frac{1}{2}) m'(m, s)$
$\frac{-k}{k} \bar{E}_{A_p/2}$	$k \in (0, \frac{\pi}{a})$	1*	$I$	$(-1)^s K(k, t)$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} K(k, t + \frac{1}{2})$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$(-1)^s \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} K(k, t + \frac{1}{2})$
$\frac{-k}{k} \bar{E}_{B_p/2}$	$k \in (0, \frac{\pi}{a})$	1*	$I$	$(-1)^s K(k, t)$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} K(k, t + \frac{1}{2})$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$(-1)^s \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} K(k, t + \frac{1}{2})$
$\pi/a \bar{E}_o$		1*	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s I$	$i(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s i \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
$\pi/a \bar{E}_{m,-m}^\pm$	$m \in (0, \frac{p}{4})$	1*	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t M(m, s)$	$i(-1)^t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm (-1)^t \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} M(m, s - \frac{1}{2})$	$\mp (-1)^t \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} M(m, s + \frac{1}{2})$
$\pi/a \bar{A}_p^\pm$		1*	1	$(-1)^{s+t}$	$i(-1)^{s+t}$	$\pm (-1)^{s+t}$	$\pm i(-1)^{s+t}$
$\pi/a \bar{B}_p^\pm$		1*	1	$(-1)^{s+t}$	$-i(-1)^{s+t}$	$\pm (-1)^{s+t}$	$\mp i(-1)^{s+t}$

TABLE XXXV. Irreducible corepresentations of the magnetic line groups  $Ln'/mcc'$  ( $n=2p$ ). Here  $L' = (L(2p)_{mcc'}^p)^{p \text{ even}} = \{(C_p^s | t), (\sigma_v C_p^s | t + \frac{1}{2}), (\sigma_h C_p^s | -t), (UC_p^s | -t - \frac{1}{2}) | s=0, 1, \dots, p-1; t=0, \pm 1, \dots\}$ ,  $g = (C_n | 0)$ . The pairs of \*g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentations:  $({}_0A_{p/2,0}^\pm, {}_0B_{p/2}^\pm)$ ,  $({}_k^k E_{A_{p/2},k}^\pm, {}_k^k E_{B_{p/2},k}^\pm)$ ,  $(\pi/a E_{m,-m}^+, \pi/a E_{m,-m}^-)$ .

Corep.	Type   g	$(C_p^s   t)$	$(\sigma_v C_p^s   t + \frac{1}{2})$	$(\sigma_h C_p^s   -t)$	$(UC_p^s   -t - \frac{1}{2})$
${}_0\bar{A}_0^\pm$	$1^*$   1	1	1	$\pm 1$	$\pm 1$
${}_0\bar{B}_0^\pm$	$1^*$   1	1	-1	$\mp 1$	$\mp 1$
${}_0\bar{E}_{m,-m}^\pm$	$1^*$   $\begin{pmatrix} 0 & e^{im\alpha} \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm M(m, s)$	$\pm \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
${}_0\bar{A}_{p/2}^\pm$	$2^*b$   $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\pm (-1)^s I$	$\pm (-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
${}_k^k \bar{E}_{A_s}$	$1^*$   $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$K(k, t + \frac{1}{2})$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t + \frac{1}{2})$
${}_k^k \bar{E}_{B_s}$	$1^*$   $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$-K(k, t + \frac{1}{2})$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$-\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t + \frac{1}{2})$
${}_k^k \bar{G}_{m,-m}$	$1^*$   $b \begin{pmatrix} 0 & e^{im\alpha} \\ 1 & 0 \end{pmatrix}$	$l(k, t)n(m, s)$	$d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} l(k, t + \frac{1}{2})n(m, s)$	$s(I)l(k, t)n(m, s)$	$b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} l(k, t + \frac{1}{2})n(m, s)$
$\pi/a \bar{E}_{p/2}$	$2^*b$   $s(-I)$	$(-1)^{s+t} \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$	$i(-1)^{s+t} d \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^{s+t} d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$i(-1)^{s+t} d \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
$\pi/a \bar{E}_0$	$1^*$   $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t I$	$i(-1)^t \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$i(-1)^t \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
$\pi/a \bar{E}_m^m$	$2^*b$   $s(M(m, 1))$	$(-1)^t m(m, s)$	$i(-1)^t d \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} g(-I)m'(m, s)$	$(-1)^t d \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} m(m, s)$	$-i(-1)^t d \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} m'(m, s)$
${}_k^k \bar{E}_{p/2}$	$2^*a$   $s(-I)$	$(-1)^s k(k, t)$	$(-1)^s g(-I)k(k, t + \frac{1}{2})$	$(-1)^s d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} k(k, t)$	$(-1)^s d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} g(-I)k(k, t + \frac{1}{2})$

TABLE XXXVI. Irreducible corepresentations of the magnetic line groups  $L(2p)_p/mc'm'$  ( $n=2p$ ). Here  $L' = L(2p)_p/m = \{(C_n^s | Fr(s/2) + t), (\sigma_h C_n^s | -Fr(s/2) - t) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (\sigma_v | 0)$ .

Corep.	Type   g	$(C_n^s   Fr(\frac{s}{2}) + t)$	$(\sigma_h C_n^s   -Fr(\frac{s}{2}) - t)$
${}_0\bar{A}_m^\pm$	$1^*$   1	$e^{ims\alpha}$	$\pm e^{ims\alpha}$
${}_k^k \bar{E}_m$	$1^*$   $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$e^{ims\alpha} K(k, t + Fr(\frac{s}{2}))$	$e^{ims\alpha} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t + Fr(\frac{s}{2}))$
$\pi/a \bar{E}_m^{m-p}$	$1^*$   $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t e^{ims\alpha} K(\frac{\pi}{a}, Fr(\frac{s}{2}))$	$(-1)^t e^{ims\alpha} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(\frac{\pi}{a}, Fr(\frac{s}{2}))$

TABLE XXXVII. Irreducible corepresentations of the magnetic line groups  $L(2p)_p/m'c'm'$  ( $n=2p$ ). Here  $L' = L(2p)_p/22 = \{(C_n^s | Fr(s/2) + t), (UC_n^s | -Fr(s/2) - t) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (\sigma_v | 0)$ , and  $m' = \begin{cases} -m-p & \text{if } -p-n/2 \leq m \leq -p+n/2 \\ n-m-p & \text{if } -p+n/2 \leq m \leq -p+3n/2 \end{cases}$ . The pairs of \*g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentations:  $(\pi/a E_m^m, \pi/a E_{-m}^{-m'})$ ,  $(\pi/a A_{p/2}^\pm, \pi/a A_{n/2-p/2}^\pm)$ ,  $({}_k^k E_m^m, {}_{-k}^{-k} E_m^{-m'})$ .

Corep.	Type   g	$(C_n^s   Fr(\frac{s}{2}) + t)$	$(UC_n^s   -Fr(\frac{s}{2}) - t)$
${}_0\bar{A}_0^\pm$	$1^*$   1	1	$\pm 1$
${}_0\bar{E}_m^m$	$1^*$   I	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
${}_0\bar{A}_p^\pm$	$1^*$   1	$(-1)^s$	$\pm (-1)^s$
$\pi/a \bar{E}_0^p$	$1^*$   $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t M(\frac{p}{2}, s)$	$(-1)^t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(\frac{p}{2}, s)$
$\pi/a \bar{A}_{-p/2}^\pm$	$2^*b$   $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t (-1)^{\text{Int}(\frac{s}{2})} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^s$	$\pm (-1)^t (-1)^{\text{Int}(\frac{s}{2})} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^s$
$\pi/a \bar{E}_m^m$	$2^*b$   $s(I)$	$(-1)^t n(m + \frac{p}{2}, s)$	$(-1)^t d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} n(m + \frac{p}{2}, s)$
${}_k^k \bar{E}_m^m$	$2^*b$   $s(I)$	$k(k, t + Fr(\frac{s}{2}))n(m, s)$	$d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} k(k, t + Fr(\frac{s}{2}))n(m, s)$

TABLE XXXVIII. Irreducible corepresentations of the magnetic line groups  $L(2p)_p/m'cm$  ( $n=2p$ ). Here  $L'=L(2p)_p mc = \{(C_n^s | Fr(s/2) + t), (\sigma_v C_n^s | Fr(s/2) + t) | s=0, 1, \dots, n-1; t=0, \pm 1, \dots\}$ ,  $g = (\sigma_h | 0)$ .

Corep.		Type	$g$	$(C_n^s   Fr(\frac{s}{2}) + t)$	$(\sigma_v C_n^s   Fr(\frac{s}{2}) + t)$
$\overline{k}A_o$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$	1*	1	$e^{ik(Fr(\frac{s}{2})+t)a}$	$e^{ik(Fr(\frac{s}{2})+t)a}$
$\overline{k}B_o$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$	1*	1	$e^{ik(Fr(\frac{s}{2})+t)a}$	$-e^{ik(Fr(\frac{s}{2})+t)a}$
$\overline{k}E_m^{-m}$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$ $m \in (0, \frac{p}{2})$	1*	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$e^{ik(Fr(\frac{s}{2})+t)a} M(m, s)$	$e^{ik(Fr(\frac{s}{2})+t)a} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
$\overline{o}A_{n/2}$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$	1*	1	$(-1)^s e^{ik(Fr(\frac{s}{2})+t)a}$	$(-1)^s e^{ik(Fr(\frac{s}{2})+t)a}$
$\overline{o}B_{n/2}$	$k \in (-\frac{\pi}{a}, \frac{\pi}{a})$	1*	1	$(-1)^s e^{ik(Fr(\frac{s}{2})+t)a}$	$-(-1)^s e^{ik(Fr(\frac{s}{2})+t)a}$

$Ln_p 2', L_c n_p 2, L_s n_p 2 (Ln_p 2' 2', L_c n_p 22, L_s n_p 22, Ln_p' 22')$ .

For each of the aforementioned magnetic groups their corepresentations have been calculated and displayed in Tables I-IV.

**B. Corepresentations of the groups isogonal to  $C_{nv}$**

$C_{nv}$  and  $D_n$  are mutually isomorphic. This reflects both in the structure,  $C_{nv} = C_n \wedge \{e, \sigma_v\}$ , and in the dimensions of the representations of the isogonal magnetic line groups: 1, 2, or 4.

$Lnm, Lnc$  for  $n$  odd and  $Lnmm, Lncc, L(2p)_p mc$  for  $n$  even ( $n=2p$ ) are the line groups isogonal to  $C_{nv}$ . From  $Lnmm$  ( $Lnmm$ ) the magnetic line groups  $Lnmm', L_c nm, L_m nm$  ( $Lnmm', L_c nmm, L_m nmm, Ln'mm'$ ,  $L_s nmm$  only for  $n$  even) are derived. The family  $Lnc$  ( $Lncc$ ) yields magnetic groups  $Lnc'$  ( $Lnc'c', Ln'cc$ ). Finally, one finds the groups  $L(2p)_p m'c', L(2p)'_p mc',$  and  $L(2p)'_p m'c'$  in the family of  $L(2p)_p mc$ .

For each of the mentioned magnetic groups their corepresentations have been calculated and exhibited in Tables V-XIV.

**C. Corepresentations of the groups isogonal to  $D_{nd}$**

The groups  $D_{nd}$  are double semidirect products:  $D_{nd} = C_{nv} \wedge \{e, U'\}$  (the axis of rotation of  $U'$  makes the angle of  $\pi/n$  with the phase of  $\sigma_v$  from  $C_{nv}$ ). The consequence of this is that the representations of the line groups isogonal to  $D_{nd}$  are induced in two steps, and some of them are four dimensional. Still, this does not result in eight-dimensional corepresentations of the corresponding magnetic groups, because in all such cases the four-dimensional representation of  $L'$  appears to be of the type 1\*.

The line groups isogonal to  $D_{nd}$  ( $n=1, 2, \dots$ ) are  $L\bar{n}m, L\bar{n}c$  for  $n$  odd, and  $L(2n)2m, L(2n)2c$  for  $n$  even. As can be seen in I, the black-and-white magnetic line groups derived from  $L(2n)2m$ , ( $L\bar{n}m$ , respectively) are  $L(2n)2'm', L(2n)'2m', L(2n)'2'm, L_c(2n)2m, L_m(2n)2m$  ( $L\bar{n}m', L\bar{n}'m', L\bar{n}'m, L_c\bar{n}m, L_m\bar{n}m$ ). Similarly, from  $L(2n)2c$  ( $L\bar{n}c$ ), the magnetic line groups  $L(2n)2'c', L(2n)'2c', L(2n)'2'c$  ( $L\bar{n}c', L\bar{n}'c', L\bar{n}'c$ ) are obtained.

TABLE XXXIX. Irreducible corepresentations of the magnetic line groups  $L(2p)'_p/m'c'm$  ( $n=2p$ ). Here  $L' = (L(2p)'_p m, p \text{ even}) = \{(C_p^s | t), (\sigma_v C_p^s | t), (U' C_p^s | -t), (U' \sigma_v C_p^s | -t) | s=0, 1, \dots, p-1; t=0, \pm 1, \dots\}$ ,  $g = (\sigma_h | 0)$ .

Corep.		Type	$g$	$(C_p^s   t)$	$(\sigma_v C_p^s   t)$	$(U' C_p^s   -t)$	$(U' \sigma_v C_p^s   -t)$
$\overline{o}A_o^{\pm}$		1*	1	1	1	$\pm 1$	$\pm 1$
$\overline{o}B_o^{\pm}$		1*	1	1	-1	$\pm 1$	$\mp 1$
$\overline{o}E_{m,-m}^{\pm}$	$m \in (0, \frac{p}{2})$	1*	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s - \frac{1}{2})$	$\pm M(m, s + \frac{1}{2})$
$\overline{o}E_{p/2}$		1*	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
$\overline{k}E_{A_s}$	$k \in (0, \frac{\pi}{a})$	1*	$I$	$K(k, t)$	$K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
$\overline{k}E_{B_s}$	$k \in (0, \frac{\pi}{a})$	1*	$I$	$K(k, t)$	$-K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$-\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
$\overline{k}E_{m,-m}^{\pm}$	$k \in (0, \frac{\pi}{a})$ $m \in (0, \frac{p}{2})$	1*	$d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$l(k, t)m(m, s)$	$d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} l(k, t)m'(m, s)$	$s(I)l(k, t)m(m, s)$	$b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} l(k, t)m'(m, s)$
$\overline{k}E_{A_p/2}^{\pm}$	$k \in (0, \frac{\pi}{a})$	1*	$I$	$(-1)^s K(k, t)$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} K(k, t)$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$(-1)^s \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} K(k, t)$
$\overline{k}E_{B_p/2}^{\pm}$	$k \in (0, \frac{\pi}{a})$	1*	$I$	$(-1)^s K(k, t)$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} K(k, t)$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$(-1)^s \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} K(k, t)$
$\overline{\pi/a}A_o^{\pm}$		1*	1	$(-1)^t$	$(-1)^t$	$\pm(-1)^t$	$\pm(-1)^t$
$\overline{\pi/a}B_o^{\pm}$		1*	1	$(-1)^t$	$(-1)^t$	$\pm(-1)^t$	$\mp(-1)^t$
$\overline{\pi/a}E_{m,-m}^{\pm}$	$m \in (0, \frac{p}{2})$	1*	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t M(m, s)$	$(-1)^t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm(-1)^t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s - \frac{1}{2})$	$\pm(-1)^t M(m, s + \frac{1}{2})$
$\overline{\pi/a}E_{p/2}$		1*	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} I$	$(-1)^{s+t} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^{s+t} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$



TABLE XL. Irreducible corepresentations of the magnetic line groups  $L(2p)'_p/m'cm'$  ( $n=2p$ ). Here  $L' = (L(2p)'_{2c, p \text{ even}}) = \{(C_p^s|t), (\sigma_v C_p^s|t + \frac{1}{2}), (U' C_p^s|-t), (U' \sigma_v C_p^s|t - \frac{1}{2}) | s=0, 1, \dots, p-1; t=0, \pm 1, \dots\}$ ,  $g = (\sigma_h|0)$ .

Corep.	Type	$ g$	$(C_p^s t)$	$(\sigma_v C_p^s t + \frac{1}{2})$	$(U' C_p^s -t)$	$(U' \sigma_v C_p^s t - \frac{1}{2})$
${}^{\pm}A_0$	$1^*$	1	1	1	$\pm 1$	$\pm 1$
${}^{\pm}B_0$	$1^*$	1	1	-1	$\pm 1$	$\mp 1$
${}^{\pm}E_{m,-m}$	$1^*$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm M(m, s + \frac{1}{2})$
${}^{\pm}E_{p/2}$	$1^*$	$I$	$(-1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
${}^{\pm}E_{A_s}$	$1^*$	$I$	$K(k, t)$	$K(k, t + \frac{1}{2})$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t + \frac{1}{2})$
${}^{\pm}E_{B_s}$	$1^*$	$I$	$K(k, t)$	$-K(k, t + \frac{1}{2})$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$-\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t + \frac{1}{2})$
${}^{\pm}G_{m,-m}$	$1^*$	$d\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right)$	$l(k, t)m(m, s)$	$d\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right)l(k, t + \frac{1}{2})m'(m, s)$	$s(I)l(k, t)m(m, s)$	$b\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right)l(k, t + \frac{1}{2})m'(m, s)$
${}^{\pm}E_{A_{p/2}}$	$1^*$	$I$	$(-1)^s K(k, t)$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} K(k, t + \frac{1}{2})$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$(-1)^s \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} K(k, t + \frac{1}{2})$
${}^{\pm}E_{B_{p/2}}$	$1^*$	$I$	$(-1)^s K(k, t)$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} K(k, t + \frac{1}{2})$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$(-1)^s \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} K(k, t + \frac{1}{2})$
$\pi/a \bar{E}_0$	$1^*$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t I$	$i(-1)^t \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$i(-1)^t \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
$\pi/a \bar{E}_{m,-m}$	$1^*$	$I$	$(-1)^t M(m, s)$	$i(-1)^t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm(-1)^t \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} M(m, s - \frac{1}{2})$	$\mp(-1)^t \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} M(m, s + \frac{1}{2})$
$\pi/a \bar{A}_{p/2}$	$1^*$	1	$(-1)^{s+t}$	$i(-1)^{s+t}$	$\pm(-1)^{s+t}$	$i \pm (-1)^{s+t}$
$\pi/a \bar{B}_{p/2}$	$1^*$	1	$(-1)^{s+t}$	$-i(-1)^{s+t}$	$\pm(-1)^{s+t}$	$i \mp (-1)^{s+t}$

The corepresentations of all these groups are presented in Tables XV–XXII.

D. Corepresentations of the groups isogonal to  $D_{nh}$

The groups  $D_{nh}$  are double products:  $D_{nh} = C_{nv} \otimes \{e, \sigma_h\}$ . Analogously to the previous case, this is the reason the representations of the isogonal line groups are induced in two steps, implying that some of them are four dimensional. Again, there are no eight-dimensional representations of the corresponding mag-

netic groups, since all four-dimensional representation of  $L'$  appears to be of the type  $1^*$ .

The line groups isogonal to  $D_{nh}$  ( $n=1, 2, \dots$ ) are  $L_n/mmm$ ,  $L_n/mcc$ ,  $L(2p)_p/mcm$  for  $n (=2p)$  even, and  $L(2n)2m$ ,  $L(2n)2c$  for  $n$  odd. As can be seen in I, the black-and-white magnetic line groups derived from  $L_n/mmm$  [ $L(2n)2m$ , respectively] are  $L_n/m'm'm'$ ,  $L_n/m'm'm'$ ,  $L_n/m'm'm$ ,  $L_n'/m'm'm'$ ,  $L_n'/m'm'm'$ ,  $L_n/m'mmm$ ,  $L_n/m'mmm$ , and  $L_n/m'mmm$  [ $L(2n)2'm'$ ,  $L(2n)'2m'$ ,  $L(2n)'2'm$ ,  $L_c(2n)2m$ , and  $L_m(2n)2m$ ]. From  $L_n/mcc$  [ $L(2n)2c$ , respectively] one obtains

TABLE XLI. Irreducible corepresentations of the magnetic line groups  $L(2p)'_p/mc'm$  ( $n=2p$ ). Here  $L' = (L(2p)'_{2c, p \text{ even}}) = \{(C_p^s|t), (\sigma_v C_p^s|t), (\sigma_h C_p^s|-t), (UC_p^s|-t) | s=0, 1, \dots, p-1; t=0, \pm 1, \dots\}$ ,  $g = (C_{2p}|\frac{1}{2})$ . The pairs of \*-g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentations:  $({}_0A_{p/2}^{\pm}, {}_0B_{p/2}^{\pm})$ ,  $(\pi/a A_0^{\pm}, \pi/a B_0^{\mp})$ ,  $({}^{\pm}E_{A_{p/2}}, {}^{\pm}E_{B_{p/2}})$ ,  $(\pi/a A_{p/2}^{\pm}, \pi/a B_{p/2}^{\pm})$ ,  $(\pi/a E_{m,-m}^{\pm}, \pi/a E_{m,-m}^{\mp})$ .

Corep.	Type	$ g$	$(C_p^s t)$	$(\sigma_v C_p^s t)$	$(\sigma_h C_p^s -t)$	$(UC_p^s -t)$
${}^{\pm}A_0$	$1^*$	1	1	1	$\pm 1$	$\pm 1$
${}^{\pm}B_0$	$1^*$	1	1	-1	$\pm 1$	$\mp 1$
${}^{\pm}E_{m,-m}$	$1^*$	$\begin{pmatrix} 0 & e^{im\alpha} \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm M(m, s)$	$\pm \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$
${}^{\pm}A_{p/2}$	$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\pm(-1)^s I$	$\pm(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
${}^{\pm}E_{A_s}$	$1^*$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
${}^{\pm}E_{B_s}$	$1^*$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$-K(k, t)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$	$-\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
${}^{\pm}G_{m,-m}$	$1^*$	$z^*(m, 0)d\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right)$	$l(k, t)n(m, s)$	$d\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right)l(k, t)n(m, s)$	$s(I)l(k, t)n(m, s)$	$b\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right)l(k, t)n(m, s)$
${}^{\pm}E_{p/2}$	$2^*b$	$s(-Q)$	$(-1)^s k(k, t)$	$(-1)^s g(-I)k(k, t)$	$(-1)^s d\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right)k'(k, t)$	$(-1)^s d\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right)g(-I)k'(k, t)$
$\pi/a \bar{A}_0$	$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t I$	$(-1)^t I$	$(-1)^t \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^t \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\pi/a \bar{B}_0$	$2^*b$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^t I$	$(-1)^t I$	$(-1)^t \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^t \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\pi/a \bar{E}_{m,-m}$	$2^*b$	$s(-M(m, 1))$	$(-1)^t m(m, s)$	$(-1)^t d\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right)m'(m, s)$	$(-1)^t g(-I)m(m, s)$	$(-1)^t d\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right)m'(m, s)g(-I)$
$\pi/a \bar{A}_{p/2}$	$2^*b$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} I$	$(-1)^{s+t} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\pm(-1)^{s+t} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\pm(-1)^{s+t} I$

TABLE XLII. Irreducible corepresentations of the magnetic line groups  $L(2p)'_p/mcm'$  ( $n=2p$ ). Here  $L' = (L(2p)'_{mcc,p\text{even}}) = \{(C_p^s|t), (\sigma_v C_p^s|t + \frac{1}{2}), (\sigma_h C_p^s|-t), (UC_p^s|-t - \frac{1}{2})\} | s=0, 1, \dots, p-1; t=0, \pm 1, \dots\}$ ,  $g = (C_{2p}|\frac{1}{2})$ . The pairs of \*-g-conjugated irreducible representations of  $L'$  forming  $2^*b$  corepresentations:  $({}_0A_{p/2}^\pm, {}_0B_{p/2}^\pm), ({}_kE_{A_{p/2}}^\pm, {}_kE_{B_{p/2}}^\pm)$ .

Corep.	Type	$g$	$(C_p^s t)$	$(\sigma_v C_p^s t + \frac{1}{2})$	$(\sigma_h C_p^s -t)$	$(UC_p^s -t - \frac{1}{2})$
$\overline{{}_0A_0^\pm}$	$1^*$	1	1	1	$\pm 1$	$\pm 1$
$\overline{{}_0B_0^\pm}$	$1^*$	1	1	-1	$\pm 1$	$\mp 1$
$\overline{{}_0E_{m,-m}^\pm}$	$1^*$	$m \in (0, \frac{n}{4})$	$\begin{pmatrix} 0 & e^{im\alpha} \\ 1 & 0 \end{pmatrix}$	$M(m, s)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm M(m, s)$
$\overline{{}_0A_{p/2}}$	$2^*b$		$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s I$	$(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\pm(-1)^s I$
$\overline{{}_kE_{A_s}}$	$1^*$	$k \in (0, \frac{n}{4})$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$K(k, t + \frac{1}{2})$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
$\overline{{}_kE_{B_s}}$	$1^*$	$k \in (0, \frac{n}{4})$	$\begin{pmatrix} 0 & e^{-ika} \\ 1 & 0 \end{pmatrix}$	$K(k, t)$	$-K(k, t + \frac{1}{2})$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K(k, t)$
$\overline{{}_kG_{m,-m}}$	$1^*$	$\begin{matrix} k \in (0, \frac{n}{4}) \\ m \in (0, \frac{n}{4}) \end{matrix}$	$z^*(m, k)d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$l(k, t)n(m, s)$	$d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} l(k, t + \frac{1}{2})n(m, s)$	$s(I)l(k, t)n(m, s)$
$\overline{{}_kE_{p/2}}$	$2^*b$	$k \in (0, \frac{n}{4})$	$s(-Q)$	$(-1)^s k(k, t)$	$(-1)^s g(-I)k(k, t + \frac{1}{2})$	$(-1)^s d \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} k'(k, t)$
$\pi/a \overline{E_0}$	$1^*$		$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s I$	$i(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\pi/a \overline{E_{m,-m}^\pm}$	$1^*$	$m \in (0, \frac{n}{4})$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^s M(m, s)$	$i(-1)^s \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M(m, s)$	$\pm(-1)^s \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} M(m, s)$
$\pi/a \overline{E_{p/2}}$	$1^*$		$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(-1)^{s+t} I$	$i(-1)^{s+t} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$(-1)^{s+t} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

black-and-white groups  $Ln/mc'c'$ ,  $Ln/m'c'c'$ ,  $Ln/m'cc'$ ,  $Ln'/m'cc'$ , and  $Ln'/mcc'$  [ $L(2n)2'c'$ ,  $L(2n)2c'$ , and  $L(2n)'2'c'$ ]. Finally, black-and-white groups corresponding to  $L(2p)_p/mcm'$  are  $L(2p)_p/mc'm'$ ,  $L(2p)_p/m'c'm'$ ,  $L(2p)_p/m'cm'$ ,  $L(2p)'_p/m'c'm'$ ,  $L(2p)'_p/m'cm'$ ,  $L(2p)'_p/mc'm'$ , and  $L(2p)'_p/mcm'$ .

The corepresentations of all these groups are presented in Tables XXIII–XLII.

IV. DISCUSSION

One of the most standard applications of time-reversal symmetry to predict possible doubling of the degeneracies of the energy levels. Namely, let us consider the black-and-white group  $L(L')$ . If an energy level belongs to the  $d$ -dimensional representation  $D$  of the group  $L'$  (ordinary line group), and this representation is of the type  $1^*$ , then the degeneracy of this level is  $d$ , while in the other cases ( $2^*a$  and  $2^*b$ ) the degeneracy is  $2d$  (the accidental degeneracy is not considered). For this reason it is important to note the black-and-white groups in which the time reversal yields no additional degeneracy, i.e., when all the representations of the group  $L'$  are of the type  $1^*$ . These groups are  $Ln_p 2'2'$ ,  $Ln_p 2'$ ,  $L(2n)2'm'$ ,  $L\bar{n}m'$ ,  $Ln/mm'm'$ ,  $L(2n)2'm'$ ,  $Ln/m'mm'$ ,  $L(2n)'2'm'$ ,  $Ln'/m'mm'$ ,  $L_{m,n}/mmm'$ ,  $L_m(2n)2m$ ,  $Ln/m'cc'$ ,  $L(2n)'2'c'$ ,  $Ln'/m'cc'$ ,  $L(2p)_p/mc'm'$ ,  $L(2p)_p/m'cm'$ ,  $L(2p)'_p/m'c'm'$ ,  $L(2p)'_p/m'cm'$ .

This is connected to conservation laws. In fact, the irreducible representations of the ordinary line groups are quite generally characterized by two quantum numbers  $m$  and  $k$ , which are related to the  $z$  component of the angular momentum and the quasimomentum along the  $z$  axis, respectively. It is easy to obtain the following rules for the transformation of the quantum numbers during the action of the time reversal and the typical transformations of the line groups:

$$\Theta: \begin{cases} m \rightarrow -m \\ k \rightarrow -k \end{cases},$$

$$(\sigma_v|t): \begin{cases} m \rightarrow -m \\ k \rightarrow k \end{cases},$$

$$(\sigma_h|0): \begin{cases} m \rightarrow m \\ k \rightarrow -k \end{cases},$$

$$(U|0): \begin{cases} m \rightarrow -m \\ k \rightarrow -k \end{cases}.$$

The construction of the irreducible representations of the line groups is the induction procedure, based on the representations  ${}_k A_m$  of the simplest line groups  $Ln_p$  (see the Appendix); therefore the general behavior is that in the groups isogonal to  $C_{nv}$  the two-dimensional irreducible representations come from the connecting quantum numbers  $m$  and  $-m$ , in the groups isogonal to  $C_{nh}$  and  $S_{2n}$ ,  $k$  and  $-k$  become coupled, while in the groups isogonal to  $D_n$ , the pairs  $(k, m)$  and  $(-k, -m)$  are attached. In the next step, for the groups isogonal to  $D_{nd}$  and  $D_{nd}$ , all the quantum numbers  $k, -k, m, -m$  give rise to the same four-dimensional irreducible representations (for special values of  $k$  or  $m$ —e.g., at the end of the Brillouin zone—some of the rules hold, although, due to equivalences of different values of  $k$  and  $m$  in this point, they give rise to the representations of lower dimension).

After introducing the time reversal, one can see that, depending on the factor  $g$ , additional coupling of the quantum numbers should occur. But if in the previous steps of the induction the same coupling has been done already, then the corresponding representation is of type  $1^*$  or  $2^*a$ . Moreover, if the representation is “real,” in the sense that  $d_g^*(L')$  is not only equivalent, but equal to

$d(\mathbf{L}')$ , then the representation is of type  $1^*$ , and no doubling of degeneracy occurs.

As an example, we note that ferromagnetic structure  $\text{Ni}_3\text{Er}$  possesses<sup>7</sup> the symmetry  $P6/m\bar{m}'m'$ , with the magnetic moment along the  $z$  axis, enabling us to use only a subgroup  $\mathbf{L}6/m\bar{m}'m'$ , while considering the spin subsystem of this anisotropic crystal. Much more important is the example of the ferromagnetic structure  $\text{CsNiF}_3$ , with the symmetry<sup>7</sup>  $P6_3/m\bar{m}'c'$ . Due to its quasi-one-dimensionality,<sup>8</sup> the magnetic line group  $\mathbf{L}6_3/m\bar{m}'c'$  is the only relevant symmetry of this structure.

Application of the corepresentation of the magnetic line groups is possible in the standard way, whenever the anisotropic or quasi-one-dimensional magnetic crystal is studied. This includes phase transition considerations in the systems with the resulting magnetic moment along the  $z$  axis in both phases (note the second-order phase transitions have been extensively considered for ordinary line groups,<sup>9</sup> and only the influence of time reversal should be incorporated<sup>10</sup>), spin arrangements,<sup>11</sup> and selection rules.<sup>12</sup>

#### APPENDIX

The line groups are the groups of symmetry of the systems translationally periodical along the  $z$  axis. Their

general element can be written as the Koster's symbol  $(R|\tau)$ , where  $\tau$  is a translation along the  $z$  axis and  $R$  is an element of  $O(3)$ , leaving the  $z$  axis invariant:  $C_n^s$  (the rotation around the  $z$  axis for the angle  $2\pi s/n$ ),  $\sigma_v$  and  $\sigma_h$  (reflections in the vertical and horizontal plane),  $U$  (rotation for  $\pi$  around a horizontal axis), and their compositions. There are 13 families of the line groups.<sup>1</sup>

Pure translations in each line group  $\mathbf{L}$  form the invariant subgroup  $\mathbf{T}$ , and the factor group  $\mathbf{L}/\mathbf{T}$  is the isogonal point group (one of  $C_n$ ,  $S_{2n}$ ,  $D_n$ ,  $C_{nv}$ ,  $C_{nh}$ ,  $D_{nd}$ , and  $D_{nh}$ ).

For the construction of the representations of the line group, the method of the induction has been quite efficient.<sup>5</sup> In fact, the line groups  $\mathbf{L}n_p$  are Abelian, and their representations  ${}_k A_m(C_n^s | \text{Fr}(sp/n)+t)$  are easily found. Further,  $\mathbf{L}n_p$  are index-2 subgroups in the groups isogonal to  $S_{2n}$ ,  $D_n$ ,  $C_{nv}$ , and  $C_{nh}$ , which allows a straightforward application of induction. Finally, each line group isogonal to  $D_{nd}$  and  $D_{nh}$  contains a line group isogonal to  $C_{nv}$  as a subgroup of index 2, and the procedure can be applied once again.

The magnetic line groups include beside the spatial symmetry, the time reversal  $\Theta$ , also. Each such group  $\mathbf{L}(\mathbf{L}')$  contains an index-2 subgroup  $\mathbf{L}'$  which is an ordinary line group, while the coset representative is of the type  $g\Theta$ , with  $g$  being again one of the transformations  $(R|\tau)$ , leaving the  $z$  axis invariant.

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