Dephasing by a dynamic asymmetric environment

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We investigate the manner in which quantum interference is suppressed when a particle interacts with a spatially localized, dynamical environment. To do so we examine a model with two classical paths along which an electron can travel and allow it to interact with a bath of harmonic oscillators on one path and travel freely on the other. In particular, we show that the quantum fluctuations of the path of the particle can couple to the environment and thus lead to dephasing and calculate the dephasing time in the high-temperature limit. We compare this result to other views of how propagating electrons lose phase coherence.

I. INTRODUCTION

The crossover from coherent quantum-mechanical behavior in microscopic systems to classical dynamics in macroscopic ones has long generated interest. With the advent of microfabrication techniques and cryogenic technology, such discussions are no longer simply speculative, but have bearing on practical measurements. For example, experiments have shown that as the temperature is lowered, nonlocal effects can be seen in transport measurements in some systems due to the wave nature of the electron.[1,2] It has been shown theoretically that such interference effects, the hallmark of quantum systems, can be suppressed when a quantum-mechanical system interacts with an environment.[3–7] However, it must be stressed that not all interactions destroy such interference. It is perfectly possible (and has indeed been experimentally demonstrated[8]) for a quantum system to exchange energy with its environment and still display interference. Thus, it is incorrect to simply equate an inelastic scattering time with the characteristic time over which the system loses phase coherence, or "dephasing time," τ_ϕ.

A discussion of some of these issues was recently presented by Stern, Aharanov, and Imry[9] (SAI) in the context of an electron traveling across a one-dimensional ring (Fig. 1). After introducing a local environment on only one side of the ring, they show that it is possible for the electron to interact *elastically* with its environment in such a way that interference is destroyed. On the other hand, they show theoretically that in certain cases a particle can absorb energy from its environment and still demonstrate interference. Furthermore, they derive a relation between the trace the electron leaves on the environment and the expectation value of the phase shift of the electron.

However, in this approach, the effects due to the quantum fluctuations of the path of the particle about its classical trajectory are not taken into account. In essence, this does not allow the environment to alter the path the electron travels. Although this approximation is useful in certain cases, it is not, in general, consistent. When all paths have the *same* environment, the quantum-fluctuation contributions to the dephasing can cancel out. However, when we are considering the case of paths encountering disjoint, distinct environments, they, in general, do not. This raises the intriguing question of when an environment should be treated as a single entity, or when it is best to describe it as two different heat baths.

In Sec. II of this paper, we first briefly discuss some common *misunderstandings* of the origin of dephasing. While much of this discussion is implicit in other works, we have not seen them stated simply in print. We will then briefly summarize the SAI discussion and some of their results. In Sec. III, we introduce our own model with a dynamic environment in an approach similar to that of Caldeira and Leggett. This model cannot be solved exactly, and we are forced to make certain physically motivated assumptions about the propagator. We can then solve the model exactly in the high-temperature limit and find that dephasing (in addition to that found

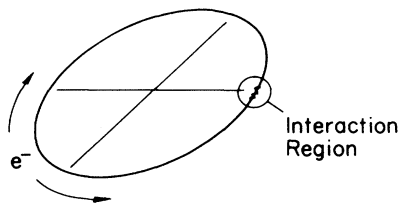

FIG. 1. An example of an interference experiment in which the system interacts with an environment on one trajectory but not on the other.

by SAI) results from the quantum fluctuations of the path of the electron about its classical trajectory. In particular, we find in the strong damping limit a dephasing time $\tau_\phi^{-1} \propto \sqrt{T}$, where T is the temperature; this is in distinction from the result for the single, uniform environment,[6] in which $\tau_\phi^{-1} \propto T$. In the zero-temperature limit, we show that the model predicts rapid destruction of the interference and discuss how this can be interpreted.

In Sec. IV, we discuss the implications of our results and possible experiments in which they could be tested. We find that the definition of what we mean by an environment and when one environment can be considered to be two disjoint environments can be problematic. Such issues not only have a direct bearing on many basic philosophical issues in quantum mechanics, but also upon specific theoretical predictions of conductivity.[10,11] We discuss the limitations of our approach and possible future directions.

II. DEPHASING AND INELASTIC COLLISIONS

It is widely accepted that dephasing and inelastic collisions are synonymous. Two plausible (though incorrect) lines of argument are often given to support this view. To be concrete, consider, for example, an electron passing through a ring device similar to Fig. 1, so that the probability amplitude to travel on the left- or right-hand side of the ring are comparable. Assume further that, on one path, the electron has an inelastic collision generating a phonon energy ΔE. The following are often offered as explanations of the loss of coherence:

(1) *The change in the electron energy makes all interference terms oscillatory and they average to zero.* If we introduce the notation $\psi_r(x,t)$ and $\psi_l(x,t)$ for the right- and left-going partial electron wave functions, then it can be argued that at the final position x_f the probability to observe the electron at time t is given by

$$\begin{aligned} P(x_f,t) &= |\psi_r(x_f,t)+\psi_l(x_f,t)|^2 \\ &= |\psi_r(x_f,t)|^2+|\psi_l(x_f,t)|^2 \\ &\quad +2\,\mathrm{Re}[\psi_r(x_f,t)^*\psi_l(x_f,t)] \,. \end{aligned} \tag{2.1}$$

If the right partial wave creates a phonon in the surrounding lattice, then ψ_r will pick up a time-dependent phase factor $\exp(i\Delta E t/\hbar)$ relative to ψ_l, so that we obtain

$$\begin{aligned} P(x_f,t) &= |\psi_r(x_f,t)e^{i\Delta E t/\hbar}+\psi_l(x_f,t)|^2 \\ &= |\psi_r(x_f,t)|^2+|\psi_l(x_f,t)|^2 \\ &\quad +2\,\mathrm{Re}[\psi_r(x_f,t)^*\psi_l(x_f,t)e^{-i\Delta E t/\hbar}] \,. \end{aligned} \tag{2.2}$$

It is then argued that the time-dependent interference term will average to zero.

This argument is unsatisfactory because it neglects the environment. To see this, let us introduce χ_r and χ_l, the wave function of the environment coupled to the right- and left-partial waves, respectively. We note that the emitted phonon is part of this environment, so that χ_r will also pick up a time-dependent phase, $\exp(-i\Delta E t/\hbar)$, relative to χ_l, a phase that will exactly cancel that of ψ_r. Thus, any appeal to such time-dependent oscillations of the wave functions must fail if the energy of the total system is conserved. In addition, the fact that inelastic scattering does not change the time development of the phase of the total wave function casts suspicion on such concepts as "phase diffusion" and "quasielastic scattering."[10,11]

(2) *The emission of a phonon makes the environment orthogonal with respect to an environment without the phonon.* This argument appeals to the fact that the phonon part of the environment can be expanded in a basis $|\{n_i\}\rangle$, where the $\{n_i\}$ denote the state with n_i phonons in mode i. Since

$$\langle\{n_i\}|\{n_i'\}\rangle = \prod_i \delta_{n_i,n_i'} \,, \tag{2.3}$$

any initial environmental eigenstate would be orthogonal to one with one more phonon, and the interference term in Eq. (2.2) would be zero. If this argument holds, then the emission of any phonon, no matter how small in energy, would destroy the interference pattern. This does not agree with our intuition.

The flaw in this approach is that there is no reason *a priori* to assume that the initial state of the system is a number eigenstate; we could equally assume a superposition of such states,

$$\sum_{\{n_i\}} c_{\{n_i\}}|\{n_i\}\rangle \,. \tag{2.4}$$

In this case, shifting the occupation number of one state need not make the final environmental wave function orthogonal to the initial one. Having such a superposition of states means that there is an uncertainty in the phonon occupation number. The issue is not whether the electron alters the environment, but rather whether or not it alters it in a way that is larger than the uncertainty in the initial state. Such a condition is sometimes referred to as "leaving a trace on the environment."[9]

Of course, this is not a new formulation of the problem; such requirements have been discussed elsewhere in the literature.[12] However, both of the above two approaches are commonly presented as the origin of dephasing. Furthermore, it is not often realized that dephasing is as much a function of the initial conditions of the system as of the interaction itself.

In order to put our work in context, we first consider the case of an electron moving on a ring that has a single spin impurity located on one side (Fig. 1) in the fashion of SAI. The two interact with a potential:

$$V(x) = \begin{cases} V_0\sigma_z & \text{for } |x-x_0| \ll 1 \,, \\ 0 & \text{otherwise} \,, \end{cases} \tag{2.5}$$

where σ_z is the spin of the impurity (the dynamics of the spin of the electron is neglected), and x_0 is its location. The interaction is assumed to be perfectly elastic short ranged so that the impurity only interacts with the electron on the right branch and extremely weak so that reflections are negligible. If the impurity is initially in an eigenstate of

$$|\sigma_x=+1\rangle=(|\sigma_z=+1\rangle+|\sigma_z=-1\rangle)/\sqrt{2}\ , \tag{2.6}$$

then by Eq. (2.5), we see that the electron experiences an uncertain potential. This uncertain potential can destroy the interference, and indeed, interference is completely suppressed when the interaction leaves the impurity spin in a state $|\sigma_x=-1\rangle$, leaving a trace on the environment marking which path the electron has taken. We note that this is *elastic dephasing* and that no net energy need be exchanged between the electron and the scatterer. For the case of a nondynamic environment, SAI arrive at the result:

$$\langle e^{i\phi}\rangle=\int d\eta\,\chi_l^*(\eta)\chi_r(\eta)\ , \tag{2.7}$$

where in their notation η denotes the degrees of freedom of the environment, and the phase angle ϕ has the simple form:

$$\phi=-\int dt\ V[x(t)] \tag{2.8}$$

where $x(t)$ is the classical path of the electron, which is assumed to be unaltered by the potential.

In the case where the environment has its own dynamics, SAI write down the equation corresponding to Eq. (2.7),

$$\langle e^{i\phi}\rangle=\int d\eta\,\chi_0^*(\eta)\hat{T}\exp\left[-i\int dt\ V_I[x(t),t]\right]\chi_0(\eta)\ , \tag{2.9}$$

where $\hat{T}$ is the time ordering operator, χ_0 is the initial environment wave function, and V_I is the potential in the interaction picture.[13]

However, implicit in Eq. (2.9) is the concept of the classical path $x(t)$. Its presence alone should make us suspicious since we are interested in quantum-mechanical effects. This treatment is inconsistent since it treats the quantum-mechanical nature of the bath, but not of the trajectory of the particle. In particular, this approach neglects the loss of interference due to the coupling of the bath to the quantum fluctuations of the trajectory of the particle about its classical path. SAI demonstrate that a particle moving along a certain trajectory in an uncertain potential leads to dephasing; it should also be possible then for a particle moving along an *uncertain* trajectory in a *certain* potential to lead to dephasing. These effects should be of comparable order, and one should not neglect the latter. In Sec. III below, we propose an approach in which we calculate the dephasing or loss of interference in a system, including those contributions due to quantum fluctuations of the particle about its classical path.

III. A MODEL WITH A DYNAMICALLY INTERACTING ENVIRONMENT

We wish to study a case where an electron can follow two classical paths, only one of which will interact with an "environment" given by a bath of harmonic oscillators. We, therefore, consider an electron moving in a two-dimensional system, with a harmonic potential in the x direction, but no potential in the y direction, as shown in Fig. 2. Our wave packets start together at the point A in Fig. 2, move apart until they reach turning points at B and C, and rejoin at the point D: The projected motion along the x axis is that of a one-dimensional oscillator. We are interested if there is interference at the point D after half a cycle, $t=2\pi/(2\omega)$, so that we do not have multiple crossings of the classical path. We include a region on the right-hand side over which the electron interacts with the bath of harmonic oscillators. Given an electron initially in a superposition of a left- and right-going wave packet, $|l\rangle+|r\rangle$, at point A, we wish to determine the magnitude of the interference term when the electron reaches point D.

Since the bath variables are not of interest, we solve for the reduced density matrix of the electron, and more simply just those contributions that stem from the interference terms. We assume that the initial density matrix of the system factorizes into a product of system and bath matrices:

$$\hat{\rho}(0)=\hat{\rho}_s\hat{\rho}_b\ ; \tag{3.1}$$

the bath starts out in thermal equilibrium at temperature T (with $\beta\equiv 1/kT$),

$$\hat{\rho}_b=\frac{1}{Z_b}e^{-\beta H_b}\ , \tag{3.2}$$

where $Z_b\equiv \mathrm{Tr}_b e^{-\beta H}$ is the canonical distribution for the bath, and Tr_b denotes the trace over the bath coordinates. The electron is initially in a superposition of left- and right-going states:

$$\begin{aligned}\hat{\rho}_s&=(|l\rangle+|r\rangle)(\langle l|+\langle r|)\\&=\hat{\rho}_{rr}+\hat{\rho}_{ll}+\hat{\rho}_{rl}+\hat{\rho}_{lr}\ .\end{aligned} \tag{3.3}$$

We are specifically interested in the interference terms resulting from $\hat{\rho}_{lr}$ and $\hat{\rho}_{rl}$. The time development of the full density matrix is then given by

$$\hat{\rho}(t)=e^{-iHt/\hbar}\hat{\rho}_s\hat{\rho}_b e^{iHt/\hbar}\ , \tag{3.4}$$

where the Hamiltonian H has three contributions, from the system (the electron), the bath, and the interaction between the two:

$$H=H_s+H_b+V\ . \tag{3.5}$$

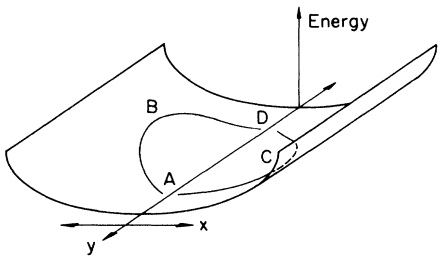

FIG. 2. A schematic drawing of the potential for the Hamiltonian given in Eq. (3.6). The parabolic potential serves to force all classical paths leaving A to intersect later at point D. We choose our electron to be in an initial state that is a superposition of states with equal and opposite momenta in the x direction, so that there are two "classical paths," one passing through B and one through C.

The system and bath Hamiltonians are harmonic oscillators:

$$H_s=\frac{\hat{p}^2}{2m}+\frac{1}{2}m\omega^2\hat{x}^2\ , \tag{3.6}$$

$$H_b=\sum_{k=1}^{N}\frac{\hat{P}_k^2}{2m_k}+\frac{1}{2}m_k\omega_k^2\hat{R}_k^2\ , \tag{3.7}$$

and they interact with a potential

$$V=\hat{x}\sum_{k=1}^{N}c_k(\hat{x})\hat{R}_k+\hat{x}^2\sum_{k=1}^{N}\frac{c_k(\hat{x})^2}{2m_k\omega_k^2}\ , \tag{3.8}$$

where the spatial dependence of the $c_k(\hat{x})$ is chosen so that an interaction occurs only along the trajectories on the right-hand side of the potential. If the $c_k(\hat{x})$ were constants in space, this formulation reduces to the Caldeira and Leggett model[4] (CL), with a counterterm, or what is sometimes referred to as the "independent oscillator model."[15] We are interested in just the interference contributions to the reduced density matrix of the electron,

$$I(x,t)=2\,\mathrm{Re}\langle x|\mathrm{Tr}_b e^{-iHt/\hbar}\hat{\rho}_{lr}\hat{\rho}_b e^{iHt/\hbar}|x\rangle\ . \tag{3.9}$$

The problem as defined above is analytically intractable due to the highly nonlinear spatial variation of the potential. We make the mathematically simplifying assumption that the left-going path has no interaction with the heat bath, so that $V|l\rangle=0$, while the right-going path always interacts with the bath. This implies

$$\begin{aligned}e^{-iHt/\hbar}\hat{\rho}_{lr}\hat{\rho}_b e^{iHt/\hbar}&=e^{-iHt/\hbar}|l\rangle\langle r|\hat{\rho}_b e^{iHt/\hbar}\\&\approx e^{-i(H_s+H_b)t/\hbar}\hat{\rho}_{lr}\hat{\rho}_b e^{iHt/\hbar}\ ,\end{aligned} \tag{3.10}$$

and the $c_k(\hat{x})$ are replaced by a set of constants, c_k in Eq. (3.8). Approximations very similar to this have been advanced elsewhere.[9,16] This assumption is crucial to the subsequent analysis and, therefore, must be understood in detail.

Although we start with a two-dimensional problem, we have already effectively reduced it to a one-dimensional problem by considering only the motion in the x direction. If we were to solve the full path integral problem, we would consider all paths that leave A, crossing into either the left or right region and end at D a time π/ω later. In Eq. (3.10), we have effectively "split" our problem into two "parallel" sheets, one on which the initial state $|r\rangle$, propagates over the entire horizontal extent of the surface, interacting with the environment, and a second one on which $|l\rangle$ propagates and does not interact with the environment (Fig. 3). To justify this approximation, we note that the classical paths of $|r\rangle$ and $|l\rangle$ do remain on the right- (interacting) and left-hand (noninteracting) sides of the system for $t<2\pi/2\omega$. The issue is whether or not fluctuations in the path in the full problem, (for example, those fluctuations in which $|l\rangle$ strays into the interaction region) contribute strongly to the final interference term. We argue that these fluctuations lead to large variations in the action, and so their contributions to the path integral will average to zero. Furthermore, our analysis is consistent so long as we find exponential destruction of the interference on a time scale smaller than $2\pi/(2\omega)$. We will return to this issue in Sec. IV.

Our model is similar to the ring problem in SAI with a harmonic oscillator bath coupling on one side of the ring, but with the assumptions that (i) higher order winding number paths are suppressed, and (ii) paths that cross from one region to the other do not contribute strongly to the interference. These assumptions seem reasonable in the limit that the damping is sufficiently strong to suppress paths that deviate strongly from the classical path.

In making the approximation of Eq. (3.10), we have rendered the time-development operator nonunitary. We can simply repair this by including an overall time-dependent normalization function $C(t)$. This will be discussed in more detail below.

With this assumption granted, we can proceed to expand Eq. (3.9):

$$\begin{aligned}I(x,t)&=2\,\mathrm{Re}C(t)\langle x|\mathrm{Tr}_b e^{-iH_0t/\hbar}\hat{\rho}_{lr}\hat{\rho}_b e^{iHt/\hbar}|x\rangle\\&=2\,\mathrm{Re}C(t)\int dR\,dR'\,dQ'\,dx'\,dy'\langle x,R|e^{-iH_0t/\hbar}|x',R'\rangle\langle x',R'|\hat{\rho}_{lr}\hat{\rho}_b|y',Q'\rangle\langle y',Q'|e^{iHt/\hbar}|x,R\rangle\ ,\end{aligned} \tag{3.11}$$

where $H_0\equiv H_s+H_b$, and $c_k(\hat{x})=c_k$ along the right-hand path. We then express the transition amplitudes as path integrals:[17,18]

$$\langle x,R|e^{-iH_0t/\hbar}|x',R'\rangle=\int_{x'}^{x}\int_{R'}^{R}\mathcal{D}x\,\mathcal{D}R\,e^{iS_0[x,R]/\hbar} \tag{3.12a}$$

$$\langle y',Q'|e^{iHt/\hbar}|x,R\rangle=\int_{y'}^{x}\int_{Q'}^{R}\mathcal{D}^*y\mathcal{D}^*Q\,e^{-iS[y,Q]/\hbar}\ , \tag{3.12b}$$

where $\mathcal{D}^*x$ represents the complex conjugate of the measure $\mathcal{D}x$, and the actions S_0 and S are derived from the Hamiltonians H_0 and H. We then follow the usual procedure of integrating out the contribution from the heat bath,[17]

$$I(x,t)=2\,\mathrm{Re}\int dx'\,dy'J(x,x,t;x',y',0)\rho_{lr}(x',y')\ , \tag{3.13}$$

where we define

$$\begin{aligned}&J(x,x,t;x',y',0)\\&\quad\equiv C(t)\int_{x'}^{x}\int_{y'}^{x}\mathcal{D}x\mathcal{D}^*y\,e^{i(S_s[x]-S_x[y])/\hbar}F[y]\ ,\end{aligned} \tag{3.14}$$

and $F[y]$ is the "influence functional"

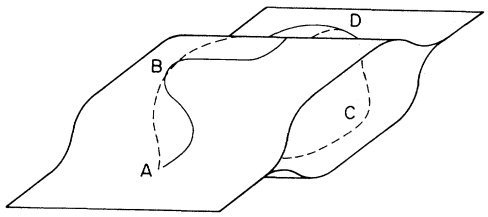

FIG. 3. A schematic interpretation of the approximation involved in Eq. (3.10). One packet (passing on the upper sheet) is coupled to the environment as it travels, while the other (on the lower) is not. Each packet has fluctuations (e.g., the solid line) about its classical path (dashed lines); it is assumed that paths with large fluctuations that should take a packet from one region to the other (say, noninteracting to interacting) do not dominate the path integral. Note that this picture is not literally true, since it implies that the two wave packets do not overlap before the point D. The vertical separation of the sheets does not imply an energy difference between the two.

$$F[y]=\int dR\,dR'\,dQ'\,\rho_b(R',Q') \times\int_{R'}^{R}\mathcal{D}R\int_{Q'}^{R}\mathcal{D}^*Q\exp\frac{i}{\hbar}(S_b[R]-S_b[Q] -S_V[y,Q])\ . \quad (3.15)$$

We note that there is no term $S_V[x,R]$ in the exponent of Eq. (3.15) because of our assumption in Eq. (3.10). This means that $F[y]$ is not a "true" influence functional in that integrals over it are not unitary. Following the Feynman-Vernon approach and integrating over the bath coordinates yields:[4,19]

$$F[y]=\exp\left[-\frac{1}{\hbar}\int_0^t d\tau\left[\int_0^\tau ds\,y(\tau)\alpha(\tau-s)y(s) -\frac{i\mu}{2}y(\tau)^2\right]\right], \quad (3.16)$$

where we have defined the coefficient of the counterterm:

$$\mu\equiv\sum_{k=1}^{N}\frac{c_k^2}{m_k\omega_k^2}\ , \quad (3.17)$$

and we have introduced the memory kernel,

$$\alpha(\tau-s)=\sum_k\frac{c_k^2}{2m_k\omega_k}\left[e^{i\omega_k(\tau-s)}+\frac{e^{-i\omega_k(\tau-s)}}{e^{\beta\hbar\omega_k}-1} +\frac{e^{i\omega_k(\tau-s)}}{e^{\beta\hbar\omega_k}-1}\right]. \quad (3.18)$$

We split $\alpha^*(\tau-s)$ into its real and imaginary components,

$$\alpha_R(\tau-s)\equiv\sum_k\frac{c_k^2}{2m_k\omega_k}\coth\frac{\beta\hbar\omega_k}{2}\cos\omega_k(\tau-s)\ , \quad (3.19)$$

$$\alpha_I(\tau-s)\equiv\sum_k\frac{c_k^2}{2m_k\omega_k}\sin\omega_k(\tau-s)$$

Inserting this form of $F[y]$ into Eq. (3.14) and factoring, we obtain

$$J=C(t)\int_{x'}^{x}\mathcal{D}x\exp\frac{i}{\hbar}\left[\int_0^t d\tau\left[\frac{m}{2}\dot{x}(\tau)^2-\frac{m\omega^2}{2}x(\tau)^2\right]\right] \times\int_{y'}^{x}\mathcal{D}^*y e^{-(S_R[y]+iS_I[y])/\hbar}\ , \quad (3.20)$$

where

$$S_I=\int_0^t d\tau\left[\frac{m}{2}\dot{y}(\tau)^2-\frac{m\omega^2}{2}y(\tau)^2-\frac{\mu}{2}y(\tau)^2 +\int_0^\tau ds\,y(\tau)y(s)\alpha_I(\tau-s)\right], \quad (3.21a)$$

$$S_R=\int_0^t d\tau\int_0^\tau ds\,y(\tau)y(s)\alpha_R(\tau-s)\ . \quad (3.21b)$$

In this form, the path integral over $x(t)$ is separated out and is just that of a simple harmonic oscillator, since these paths do not interact with the heat bath. Performing this integral, we can write[17]

$$J=C(t)A(x,x',t)\int_{y'}^{x}\mathcal{D}^*y\exp-\frac{1}{\hbar}(iS_I+S_R)\ , \quad (3.22)$$

where

$$A(x,x',t)=\left[\frac{m\omega}{2\pi i\hbar\sin\omega t}\right]^{1/2} \times\exp\left[\frac{im\omega}{2\hbar\sin\omega t}[(x^2+x'^2)\cos\omega t-2xx']\right]. \quad (3.23)$$

We now examine the behavior of $\alpha_I(\tau-s)$. For this, we replace the sum over oscillators with weights c_k by an integral over ω with a spectral distribution $G(\omega)$ defined as

$$G(\omega)=\pi\sum_{k=1}^{N}\frac{c_k^2}{2m_k\omega_k}\delta(\omega-\omega_k)\ \ (\omega\geq 0)\ . \quad (3.24)$$

We introduce a function $\gamma(t)$ defined so that

$$\frac{m}{2}\frac{d}{dt}\gamma(t)=\alpha_I(t)\ . \quad (3.25)$$

Note that

$$m\gamma(t)=-\frac{2}{\pi}\int_0^\infty d\omega\frac{G(\omega)}{\omega}\cos\omega t\ , \quad (3.26)$$

and so $m\gamma(0)=-\mu$. The distribution $G(\omega)$ is yet unspecified. We choose the response of the bath at low frequencies to be independent of ω, so that the bath is *ohmic*.[4] This fixes $G(\omega)=m\tilde{\gamma}\omega$, where $\tilde{\gamma}$ is an effective dissipation constant with dimensions of inverse time. This, of course, holds only for low frequencies; the spectral distribution function must be finite as $\omega\to\infty$ or else the system has infinite energy. We can assume that the distribution is ohmic up to some high-frequency cutoff Ω_c and investigate the resulting behavior. In this case, we note that

$$\lim_{\Omega_c\to\infty}\gamma(t)=-\frac{2\tilde{\gamma}}{\pi}\lim_{\Omega_c\to\infty}\int_0^{\Omega_c}d\omega\cos\omega t$$
$$=-2\tilde{\gamma}\delta(t)\ . \tag{3.27}$$

Taking such a limit means that physically we are interested in times $t\gg\Omega_c^{-1}$.

Some care must be used in manipulating time and frequency integrations; it is well known that the thermodynamic limit must be taken before the long time limit $t\to\infty$. By choosing a continuous spectral distribution function, we have effectively taken the thermodynamic limit for the heat bath. If we do not perform such a limit, then our heat bath would never be dissipative; there would always be Poincaré cycles of finite period.

If we examine the integral over s in Eq. (3.21a) and integrate by parts, we have:

$$\int_0^\tau\alpha_I(\tau-s)y(s)=-\frac{m}{2}[y(\tau)\gamma(0)-y(0)\gamma(\tau)]$$
$$+\frac{m}{2}\int_0^\tau ds\gamma(\tau-s)\dot{y}(s)\ . \tag{3.28}$$

Inserting this back into Eq. (3.21a), canceling the counterterm, and integrating over the δ functions, we obtain from S_I an effective action $\tilde{S}_I$:

$$\tilde{S}_I=-\frac{m\tilde{\gamma}}{4}[y(t)^2+y(0)^2]+\int_0^t d\tau\frac{m}{2}[\dot{y}(\tau)^2-\omega^2y(\tau)^2]\ . \tag{3.29}$$

We already see one striking result of this model. In the normal CL approach, the imaginary terms in the effective action yield the equations of motion of damped oscillators; here the imaginary terms display no such damping. Clearly, if there is to be any damping it must result from $\tilde{S}_R$. Yet, in the CL approach, these terms do not change the classical equation of motion for the path.[4,7,14]

The real part of the kernel, $\alpha_R(\tau)$, can now be written as

$$\alpha_R(\tau)=\frac{1}{\pi}\int_0^{\Omega_c}d\omega\,G(\omega)\coth\frac{\hbar\omega\beta}{2}\cos\omega t$$
$$=\frac{m\tilde{\gamma}}{\pi}\int_0^{\Omega_c}d\omega\,\omega\coth\frac{\hbar\omega\beta}{2}\cos\omega t\ . \tag{3.30}$$

We first treat the high-temperature limit, assuming first $\beta\hbar\omega\ll1$ and later taking the limit $\Omega_c\to\infty$. We can approximate $\alpha_R(\tau)$ as

$$\alpha_R(\tau)\approx\frac{2m\tilde{\gamma}}{\pi\hbar\beta}\int_0^{\Omega_c}d\omega\cos\omega t\ , \tag{3.31}$$

which in the limit $\Omega_c\to\infty$, yields

$$\alpha_R(\tau)\approx\frac{2m\tilde{\gamma}}{\hbar\beta}\delta(t)\ . \tag{3.32}$$

Integrating over this δ function in S_R, we finally obtain the following effective action in the high-temperature limit with ohmic dissipation

$$\tilde{S}[y]=\tilde{S}_I-i\tilde{S}_R=-\frac{m\tilde{\gamma}}{4}(y'^2+x^2)$$
$$+\int_0^t d\tau\frac{m}{2}[\dot{y}(t)^2-\omega'^2y(t)^2]\ , \tag{3.33}$$

where ω' is a complex frequency, given by

$$\omega'^2\equiv\omega^2+i\frac{2\tilde{\gamma}}{\hbar\beta}\ . \tag{3.34}$$

We have reduced the problem to that of a simple harmonic oscillator, albeit with a complex frequency. We can then use Eq. (3.23) to write down the final expression for $J(x,x,t;x',y',0)$

$$J=C(t)\frac{m}{2\pi\hbar}\left[\frac{\omega\omega'}{\sin\omega t\sin\omega' t}\right]^{1/2}\exp\left[\frac{im\omega}{2\hbar\sin\omega t}[(x^2+x'^2)\cos\omega t-2xx']\right]$$
$$\times\exp\left[\frac{im\omega'}{2\hbar\sin\omega' t}[(x^2+y'^2)\cos\omega' t-2y'x]+\frac{im\tilde{\gamma}}{4\hbar}(y'^2+x^2)\right]\ . \tag{3.35}$$

In order to obtain an explicit example for the interference term, we must assume a form for the initial wave packets. For calculational simplicity, we assume that our initial state consists of wave packets that are *already* displaced from the origin (i.e., at the points B and C):

$$\psi(x)=\psi_l(x)+\psi_r(x)=N\left[\exp\frac{-(x+x_l)^2}{4\sigma^2}+\exp\frac{-(x+x_r)^2}{4\sigma^2}\right]\ , \tag{3.36}$$

where x_l and x_r are the initial displacements of the wave packets, σ is their initial (identical) width, and N is the normalization constant. With these initial conditions the interference occurs at a time $t=2\pi/4\omega$. Performing the Gaussian integrals over x' and y' in Eq. (3.13), we arrive at

$$I(x,t)=C(t)\frac{N^2m}{\hbar}\mathrm{Re}a_1(t)^{-1/2}a_2(t)^{-1/2}\left[\frac{\omega\omega'}{\sin\omega t\sin\omega' t}\right]^{1/2}\exp\left[-a_0(t)x^2-\frac{x_l^2}{4\sigma^2}-\frac{x_r^2}{4\sigma^2}\right]$$
$$\times\exp-\left[\frac{1}{4a_1(t)}\left[\frac{-im\omega}{2\hbar\sin\omega t}-\frac{2x_l}{2\sigma^2}\right]^2+\frac{1}{4a_2(t)}\left[\frac{-i\omega'}{2\hbar\sin\omega' t}-\frac{2x_r}{2\sigma^2}\right]^2\right]\ , \tag{3.37}$$

where

$$a_0 \equiv \frac{i}{4\hbar}(2m\omega' \cot\omega' t - 2m\omega \cot\omega t - m\tilde{\gamma}) , \quad (3.38a)$$

$$a_1 \equiv \frac{-im\omega \cot\omega t}{2\hbar} + \frac{1}{4\sigma^2} , \quad (3.38b)$$

$$a_2 \equiv \frac{im\omega \cot\omega' t}{2\hbar} - \frac{im\tilde{\gamma}}{4\hbar} + \frac{1}{4\sigma^2} . \quad (3.38c)$$

If we define $I'(x,t)$ to be the above result, without the normalization function $C(t)$, [that is, $I'(x,t) = I(x,t)/C(t)$], then we can determine the overall normalization function $C(t)$ simply by integrating over $I'(x,t)$:

$$C(t) = \left[1 + \int_{-\infty}^{\infty} dx\, I'(x,t)\right]^{-1} . \quad (3.39)$$

Typical plots of our results are presented in Fig. 4. Strictly speaking, we should only plot times $t < 2\pi/4\omega$; however, we plot up to $t = 2\pi/\omega$ to help show the effect of the overall damping. Since ω' is complex, the term $\sin\omega' t$ in the denominator of the square-root grows exponentially. This behavior is not compensated by terms in the exponent, so the whole expression is damped as $e^{-\Gamma t/2}$; we can calculate this damping constant Γ; if we set $\omega' = \Omega + i\Gamma$, then

$$\Omega = \frac{1}{\sqrt{2}} \left\{ \left[\omega^4 + \left[\frac{2\tilde{\gamma}}{\beta\hbar} \right]^2 \right]^{1/2} + \omega^2 \right\}^{1/2} , \quad (3.40a)$$

$$\Gamma = \frac{1}{\sqrt{2}} \left\{ \left[\omega^4 + \left[\frac{2\tilde{\gamma}}{\beta\hbar} \right]^2 \right]^{1/2} - \omega^2 \right\}^{1/2} . \quad (3.40b)$$

In the strongly damped limit, $\tilde{\gamma}/\beta \gg \hbar\omega^2$, this reduces to

$$\Omega \approx \Gamma \approx \left[\frac{\tilde{\gamma}}{\hbar\beta} \right]^{1/2} = \left[\frac{D}{2m\hbar} \right]^{1/2} , \quad (3.41)$$

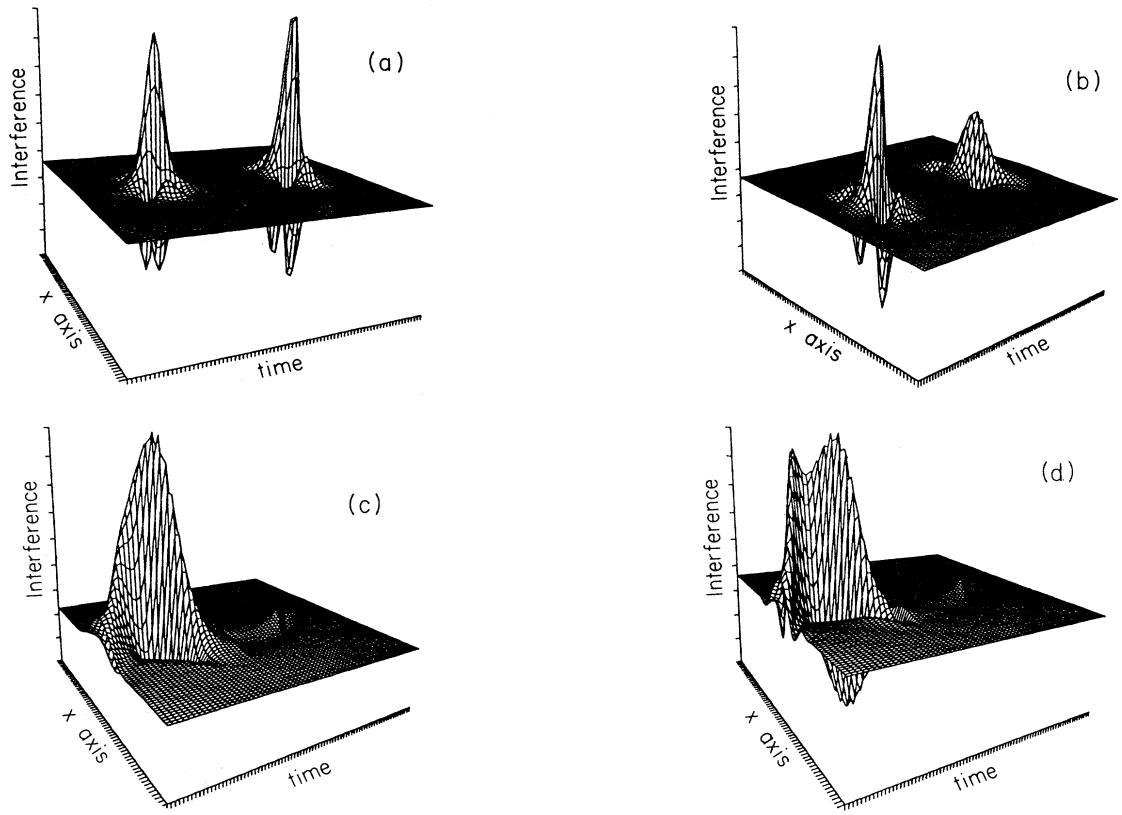

FIG. 4. Typical plots of the interference term [Eq. (3.37)] as a function of time and position in the high-temperature limit. In these plots, the two packets are assumed to be separated initially (i.e., at points B and C in Fig. 1) and intersect after a time $t = \pi/2\omega$. The plots are carried out further in time to the second such crossing, which is beyond the point where the approximation in Eq. (3.10) is valid; however, the second peak gives an impression of the strength of the damping. In all plots the x axis covers the range $-4\sqrt{2\hbar/m\omega} < x < 4\sqrt{2\hbar/m\omega}$, and the t axis covers the range $0 < t < 2\pi/\omega$. The interference (vertical axis) is scaled differently for each plot so that results are visible. (a) Plot of $I(x,t)$ for $\tilde{\gamma} = 0$; there is no damping so the two peaks are equal in height. The vertical axis covers the range $-2.14 < I(x,t) < 3.21$. Note that the initial overlap of the wave packets is unnoticeable on this scale. (b) Plot of $I(x,t)$ for $\tilde{\gamma}/\omega = 0.1$, and $kT/\hbar\omega = 4.0$. The vertical axis covers the range $-0.539 < I(x,t) < 0.779$. (c) Plot of $I(x,t)$ for $\tilde{\gamma}/\omega = 1.0$, and $kT/\hbar\omega = 4.0$. The vertical axis covers the range $-0.42 < I(x,t) < 0.127$. Note that the initial overlap, invisible in (a), is now visible on this magnified scale. (d) Plot of $I(x,t)$ for $\tilde{\gamma}/\omega = 4.0$ and $kT/\hbar\omega = 4.0$. The vertical axis covers the range $-0.012 < I(x,t) < 0.034$. The initial overlap figures prominently and is comparable with the maximum interference that develops.

where we have related these quantities to the diffusion constant D by the Einstein relation[4] $D=2m\tilde{\gamma}/\beta$. In the weak damping limit $\tilde{\gamma}/\beta \ll \hbar\omega^2$, we find $\Gamma \approx \tilde{\gamma} kT/\hbar\omega$.

Thus, we see that there is a damping factor $e^{-\Gamma t/2}$ that arises in the prefactor. This prefactor is generated by the path integral over the quantum fluctuations about the classical path of the particle. These fluctuations couple to the heat bath and, thus, lead to damping. Without these effects, there would be no damping in the high-temperature limit. We note that since $I'(x,t)$ is damped exponentially, the normalization function $C(t)$ rapidly approaches unity. It has no systematic effect on the interference term.

We now consider the zero-temperature limit. Since the kernel α_I is temperature independent, we may proceed as before. Since integrating over it leads only to finite contributions at the boundaries, as in Eq. (3.29), we concentrate on α_R. Using the definition of α_R in Eq. (3.18) and noting that $\lim_{T\to 0}\coth\beta\hbar\omega/2=1$, we then have in the ohmic limit

$$\alpha_R(\tau)=\frac{m\tilde{\gamma}}{\pi}\int_0^\infty d\omega\,\omega\cos\omega t = \frac{m\tilde{\gamma}}{\pi}\frac{d}{dt}\kappa(t)\ , \tag{3.42}$$

where $\kappa(t)$ is the distribution defined by,

$$\int dt\, f(t)\kappa(t)=\mathrm{P}\int dt\frac{f(t)}{t}\ , \tag{3.43}$$

where P denotes the principal value of the integral. Since $\kappa(t)$ is singular, it is possible that the action will be infinite. To see that this is indeed the case we first introduce a high-frequency cutoff Ω_c into the spectral distribution in the Drude fashion so that $G(\omega)=m\tilde{\gamma}\omega e^{-\omega/\Omega_c}$, thereby defining a new function $k(t)$,

$$k(t)\equiv\int_0^\infty dt\, e^{-\omega/\Omega}\sin\omega t = \frac{t}{t^2+\Omega_c^{-2}} \tag{3.44}$$

so that $\lim_{\Omega_c\to\infty} k(t)=\kappa(t)$. Substituting this back into our expression for the action, we have

$$S=\frac{m}{2}\int_0^t d\tau\left[\dot{y}(\tau)^2-\omega^2 y(\tau)^2+\frac{2i\tilde{\gamma}}{\pi}\times\int_0^t ds\, y(\tau)y(s)\frac{d}{dt}k(\tau-s)\right], = \bar{S}_I+i\bar{S}_R\ , \tag{3.45}$$

where we have dropped the constant boundary terms and exploited the symmetry of the kernel to extend the upper limit to t. The propagator is then

$$J=C(t)A(x,x',t)\int_{y'}^{x}\mathcal{D}^*y\,\exp-\frac{1}{\hbar}(i\bar{S}_I+\bar{S}_R)\ . \tag{3.46}$$

We will endeavor to show that all paths in the integral produce divergent terms in $\bar{S}_R$ so that the interference is totally suppressed as $\Omega_c\to\infty$. Integration over the imaginary part of the action can only yield oscillatory factors that cannot compensate for this damping.

We focus on $\bar{S}_R$, writing it as

$$\bar{S}_R=\bar{S}_1-\bar{S}_2 = \frac{m\tilde{\gamma}}{\pi}\int_0^t d\tau\int_0^t ds\,[y(\tau)^2+y(s)^2]\frac{d}{d\tau}k(\tau-s) - \frac{m\tilde{\gamma}}{\pi}\int_0^t d\tau\int_0^t ds\,[y(\tau)-y(s)]^2\frac{d}{d\tau}k(\tau-s)\ . \tag{3.47}$$

Clearly, the second term is less singular than the first, since the numerator will have a zero at the pole of the kernel, and, therefore, it cannot alter the leading order behavior of the integral. Performing one integration in $\bar{S}_1$, we obtain

$$\bar{S}_1=\frac{m\tilde{\gamma}}{\pi}\int_0^t dt\, k(t)[y(\tau)^2+y(t-\tau)^2]\ . \tag{3.48}$$

The dominant contributions come from the terms at the boundaries

$$\bar{S}_1\approx\frac{m\tilde{\gamma}}{\pi}[y(0)^2+y(t)^2]\int_0^t d\tau\, k(\tau) = \frac{m\tilde{\gamma}}{2\pi}(x^2+y'^2)[\ln(t^2+\Omega_c^{-2})-\ln(\Omega_c^{-2})]\ . \tag{3.49}$$

If we now take the limit $\Omega_c\to\infty$, we find that $\bar{S}_1$ diverges logarithmically, and so, the interference is suppressed. All other contributions to the path integral will be less singular, so this is the dominant behavior. After performing some Gaussian integrals, one finds that at $x=0$ the interference term is proportional to $1/\sqrt{m\tilde{\gamma}\ln\Omega_c}$. We stress that this result holds only for zero temperature and $t>\Omega_c^{-1}$.

It is tempting to try to extract a cutoff dependent dephasing time from $\bar{S}_1$. However, for times $t>\Omega_c^{-1}$, this does not yield a *rate* but rather an overall, time-independent factor that suppresses interference, a factor that tends to zero as $\Omega_c\to\infty$. This can be interpreted as stemming from our choice of factored initial conditions. At $T=0$ and $t=0$, the oscillator bath is in its ground state, decoupled from the particle [Eq. (3.2)]. For $0<t<\Omega_c^{-1}$, the oscillators must adjust to the position of the particle, with the high-frequency oscillators adjusting most rapidly, and the amount of the adjustment depending upon the displacement of the particle. The new equilibrium point of each oscillator is shifted with respect to its original equilibrium point, resulting in a reduced overlap between the initial and final states of the bath. This transient response ($t<\Omega_c$) will be dependent upon the exact form of the cutoff and may not be observable in real physical systems, although it has been seen in numerical calculations.[20] It is, therefore, inappropriate to call this dominant behavior found in Eq. (3.49) "dephasing," even though it does lead to the suppression of interference.

IV. DISCUSSION

The dephasing time τ_ϕ is an important parameter for understanding the behavior of mesoscopic systems. And yet, it has heretofore often been treated as a parameter, rather than calculated from the microscopic dynamics of the system. While it is common to equate this with the inelastic scattering time, as we have shown in Sec. II, this is by no means obvious. SAI have already shown that dephasing can occur in elastic interactions when the potential is uncertain. Our work complements this by showing how the quantum fluctuations in the path can lead to dephasing even in a certain potential. We find a dephasing time

$$\tau_\phi^{-1} \approx \sqrt{\tilde{\gamma}/\hbar\beta} = \sqrt{D/2m\hbar}$$

in the strong damping limit ($\tilde{\gamma}/\beta \gg \hbar\omega^2$), and

$$\tau_\phi^{-1} \approx \Gamma \approx \tilde{\gamma} kT/\hbar\omega$$

in the weak damping limit ($\tilde{\gamma}/\beta \ll \hbar\omega^2$). While in our particular model, dephasing may also be accompanied by a loss of energy of the electron, this is not in principle necessary. A system (consisting of a particle interacting with its environment) can lose phase coherence by its own internal dynamics, and need not transfer energy.[9]

We find that in the zero-temperature limit, interference terms are suppressed by a factor proportional to $1/\sqrt{m\tilde{\gamma}\ln\Omega_c}$ for times $t > \Omega_c^{-1}$. One cannot extract a cutoff dependent dephasing time in this limit. The transient response ($t < \Omega_c$) will be dependent upon the exact form of the cutoff and is not observable in real systems. Ideally, one should attempt to factor off this behavior and examine the subsequent time dependence.

SAI are able to write a general expression for dephasing in the limited cases where the dynamics of the environment are classical, i.e., when

$$[V_I(x,t), V_I(x',t')] = 0 \ .$$

The case of a truly dynamic environment traversed by a particle with an uncertain path does not lend itself to such simplification. In general, it is not possible to write down an expression similar to Eq. (2.5) to define a phase shift.

However, that is not to say that the results we derive for dephasing do not have general application. In particular, they do not depend upon our assumption that the particle moves in a harmonic well. The dephasing time does not change in the limit $\omega \to 0$, so that Eq. (3.41) is still valid. If we consider an electron moving in a square potential that simply serves to confine it to an annulus, then we expect our results to be qualitatively correct for the diffusive paths that traverse the system. In two and three dimensions, we simply obtain the square and cube of our results, so that τ_ϕ will be given by the result above [Eq. (3.40b)] multiplied by a factor that depends upon the dimensionality of the system. More problematic might be the controlled realization of having different disconnected environments on the two sides of the ring. One method would be to heat one side relative to the other, either by a laser pulse or by creating a heating element lithographically on one side of the ring. At low temperatures, we expect that the temperature difference between the two sides would play the role of T in our results. Then the dephasing time could be measured as a function of the heat deposited. Alternatively, the ring could be fabricated in such a fashion that the two "environments" are different, say of different materials, impurity concentration, or thicknesses. Again, the dephasing time could be measured as the doping or thickness are varied.

The issue of dephasing can be quite subtle. For example, consider the common assumption that the initial conditions of the bath and electron are separable [Eq. (3.1)]. This in itself depends upon how fast the quantum correlations between the two die out, which is the quantity we wish to determine. The exponential decay found in our calculations are consistent with this assumption; if they were not, we could not have chosen factorized initial conditions.

It is intriguing to compare our case to one where the electron sees a single environment. First, note that it is trivial to generalize our results to a model where the electron sees separate, disjoint environments on each side of the ring. We would simply have a dephasing rate $\Gamma_{\rm tot} = \Gamma_l + \Gamma_r$, where we calculate a dephasing rate for each side based upon Eq. (3.40a). However, this result for two *disjoint* environments is quite different from the CL result for a single environment: we find in the high-temperature, strong damping limit, a dephasing time $\tau_\phi^{-1} \propto \sqrt{T}$; whereas for the result for the single, uniform environment,[6] $\tau_\phi^{-1} \propto T$. Furthermore, in our case, α_R suppresses the interference while α_I has no effect, while in the CL result, this is reversed. But when is an environment one large environment and when is it a conglomeration of disjoint environments? The extremes are clear: in weak localization, the electrons travel in a single medium, while in recent neutron interferometry experiments the split beams may travel through environments separated by meters. But how do we pass from one limit to the other? As one considers longer and longer diffusive paths in weak-localization calculations,[10,11] such a single environment approach must break down. In mesoscopic systems, we are precisely concerned with such a crossover; how it is to be specified is still a matter for further investigation.

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