# Theory of the magnetoresistance in magnetic multilayers: Analytical expressions from a semiclassical approach

A. Barthélémy and A. Fert
*Laboratoire de Physique des Solides, Bâtiment 510, Université Paris XI, 91405 Orsay CEDEX, France*
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We use the semiclassical approach of Camley and Barnas to derive analytical expressions for the giant magnetoresistance of magnetic multilayers in several simple cases. We discuss the main features of our results and apply our expressions of the magnetoresistance to the interpretation of experimental data on Fe/Cr and Co/Ru multilayers. Finally we compare our results with those obtained in quantum models.

Very large magnetoresistance (MR) effects have recently been observed in several multilayered structures, Fe/Cr,[1–5] Co/Ru,[3] Au/Co,[2,6] Ag/Co Cu/Co,[7,8] *Ni*Fe/Cu,[8] *Ni*Fe/Cu/Co.[9] One finds that the resistivity is high when the magnetizations of neighboring magnetic layers are antiparallel, and drops when they are brought into parallel alignment. For Fe(001)/Cr(001) superlattices, the resistivity decrease can amount to about 50%, i.e., $\Delta R/R_{\max} \sim 0.5$, where $\Delta R$ is the maximum resistivity change and $R_{\max}$ is the maximum resistivity. In Fe/Cr,[1–5] Co/Ru,[3] and probably Cu/Co,[7] the antiparallel alignment of the high resistivity state is due to antiferromagnetic exchange interactions through the Cr, Ru, or Cu layers. In other systems, the antiparallel alignment is obtained by introducing different pinning forces in the odd and even magnetic layers.

The "giant magnetoresistance" of the magnetic multilayers is generally ascribed to an interplay between the spin dependence of the scattering and the relative orientation of the magnetization in neighbor layers. The first theoretical model has been worked out by Camley and co-workers[10,11] and uses a semiclassical approach of the Fuchs-Sondheimer type.[12] This model assumes that the conduction electrons have spin dependent coefficients of coherent transmission through the interfaces and also spin dependent relaxation rates within the magnetic layers (spin dependent meaning dependent on the orientation of the electron spin relative to the magnetization of the magnetic layer). Recently Levy and co-workers[13,14] have developed a quantum model also based on the existence of spin dependent scattering. In this paper we come back to the semiclassical approach. However, instead of working out a numerical solution like in the work of Camley *et al.*[10,11] or Trigui *et al.*,[15] we use appropriate approximations for some practical cases and we derive *simple analytical expressions of the MR*. These expressions are interesting *to analyze the respective role of the main parameters* (thicknesses, mean free path, etc.), *to derive scaling laws and to present a physical picture of the MR mechanism.* We will consider the cases of the two most investigated structures, the periodic multilayers and the structures with only two magnetic layers, respectively. However, for simplicity, we consider systems with spin dependent scattering only at the interfaces and we do not treat cases with an additional spin dependence for the scattering within the magnetic layers. After discussing the main features of our results, we apply our expressions of the MR to the interpretation of experimental data on Fe/Cr and Co/Ru multilayers. We also compare our results with those obtained in the quantum model of Levy and co-workers.[13,14]

## CALCULATIONS

First we consider the case of a periodic structure, for example, a Fe/Cr multilayer with Fe layers of thickness $t_{\mathrm{Fe}}$, Cr layers of thickness $t_{\mathrm{Cr}}$, and a very large number of periods. The approach of Camley and Barnas[11] is based on the Boltzmann equation. For layers perpendicular to the $z$ axis and an electric field $\mathbf{E}$ along the $x$ axis, it can be written as

$$\frac{\partial g}{\partial z}+\frac{g_\sigma}{\tau v_z}=\frac{eE}{m v_z}\frac{\partial f_0}{\partial v_x} . \tag{1}$$

$f_0$ is the equilibrium distribution function, $g_\sigma(\mathbf{v},z)$ is the correction to the distribution function induced by the electric field for electrons with spin $\sigma$, and $\tau$ is the relaxation time. The diffusion term $\partial g/dz$ cancels out in homogeneous conductors but, in multilayers, it drives the dependence of $g$ on $z$. The relaxation rate $1/\tau$ appearing in Eq. (1) expresses only the scattering by defects or impurities within the Fe layers, and for simplicity, is supposed to be the same in Fe and Cr. In the semiclassical models of the Fuchs-Sondheimer type, the scattering by the interfaces is expressed by boundary conditions. For electrons (+) with a positive $v_z$ and spin $\sigma$, the boundary condition at the interface $z=0$ is written

$$g_\sigma^+(\mathbf{v},z=0^+)=T_\sigma g_\sigma^+(\mathbf{v},z=0^-) , \tag{2}$$

and, similarly for electrons with a negative $v_z$,

$$g_\sigma^-(\mathbf{v},z=0^-)=T_\sigma g_\sigma^-(\mathbf{v},z=0^+) . \tag{3}$$

The boundary conditions mean that, because a fraction of the electrons $T_\sigma$ is coherently transmitted while a fraction $(1-T_\sigma)$ is diffusely scattered by the interface roughness, the departure from the equilibrium distribution is reduced by the factor $T_\sigma$ just after an interface crossing. The dependence of $T_\sigma$ on the spin $\sigma$ gives rise to the magnetoresistance.

The general solution of Eq. (1) is of the form

$$g_\sigma^{\pm}(\mathbf{v},z)=e\tau E v_x \frac{\partial f_0}{\partial \epsilon} G_\sigma^{\pm}(v_z,z) \tag{4}$$

with

$$G_\sigma^{\pm}(v_z,z)=1-A^{(\pm)}\exp\left[\mp\frac{z}{\tau|v_z|}\right], \tag{5}$$

where $A$ is an integration constant.

In an infinite material without interface $G_\sigma^{(\pm)}=1$ uniformly. For an infinite material with one interface at $z=0$, the solution for $G_\sigma^+$ is $G_\sigma^+=1$ for $z<0$, $G_\sigma^+=T_\sigma$ for $z=0^+$ and, for $z>0$, $G_\sigma^+$ returns exponentially to its asymptotic value 1. The characteristic length of the relaxation to 1 is $\tau v_z \le \lambda=\tau v_F$, where $v_F$ is the Fermi velocity and $\lambda$ the mean free path (MFP).

Now, in the case of a multilayer with the magnetizations of all the Fe layers in the same direction, we have to express $G_\sigma^{(\pm)}(z,v_z)$ in the unit cell of Fig. 1(a), with interfaces at $z=0$, $z=t_{\mathrm{Cr}}$, and $z=t_{\mathrm{Cr}}+t_{\mathrm{Fe}}$. First we suppose that $t_{\mathrm{Fe}}$ is much larger than $t_{\mathrm{Cr}}$ and $\lambda$, which is a frequent case in the experiments on Fe/Cr.[16] Because $t_{\mathrm{Fe}} \gg \lambda$, $G_\sigma^+=1$ at the right edge of the Fe layers:

$$G_\sigma^+(v_z,z=0^-)=1 . \tag{6}$$

Applying the boundary condition, Eq. (2), at the interface $z=0$ and Eq. (5) for the variation in the Cr layer between $z=0$ and $z=t_{\mathrm{Cr}}$, leads to

$$G_\sigma^+(v_z,z=0^+)=T_\sigma , \tag{7}$$

$$G_\sigma^+(v_z,0<z<t_{\mathrm{Cr}})=1-(1-T_\sigma)\exp\left[-\frac{z}{\tau v_z}\right]. \tag{8}$$

Continuing in the same way we obtain

$$G_\sigma^+(v_z,z=t_{\mathrm{Cr}}^-)=(1-T_\sigma)\exp\left[-\frac{t_{\mathrm{Cr}}}{\tau v_z}\right], \tag{9}$$

$$G_\sigma^+(v_z,z=t_{\mathrm{Cr}}^+)=T_\sigma\left[1-(1-T_\sigma)\exp\left[-\frac{t_{\mathrm{Cr}}}{\tau v_z}\right]\right], \tag{10}$$

$$G_\sigma^+(v_z,t_{\mathrm{Cr}}<z<t_{\mathrm{Cr}}+t_{\mathrm{Fe}}) =1-\left\{1-T_\sigma\left[1-(1-T_\sigma)\exp\left[-\frac{t_{\mathrm{Cr}}}{\tau v_z}\right]\right]\right\} \times\exp\left[-\frac{z-t_{\mathrm{Cr}}}{\tau v_z}\right], \tag{11}$$

$$G_\sigma^+(v_z,z=t_{\mathrm{Cr}}+t_{\mathrm{Fe}})=1 . \tag{12}$$

The variation of $G_\sigma^+(z,v_z)$ as a function of $z$ for a fixed

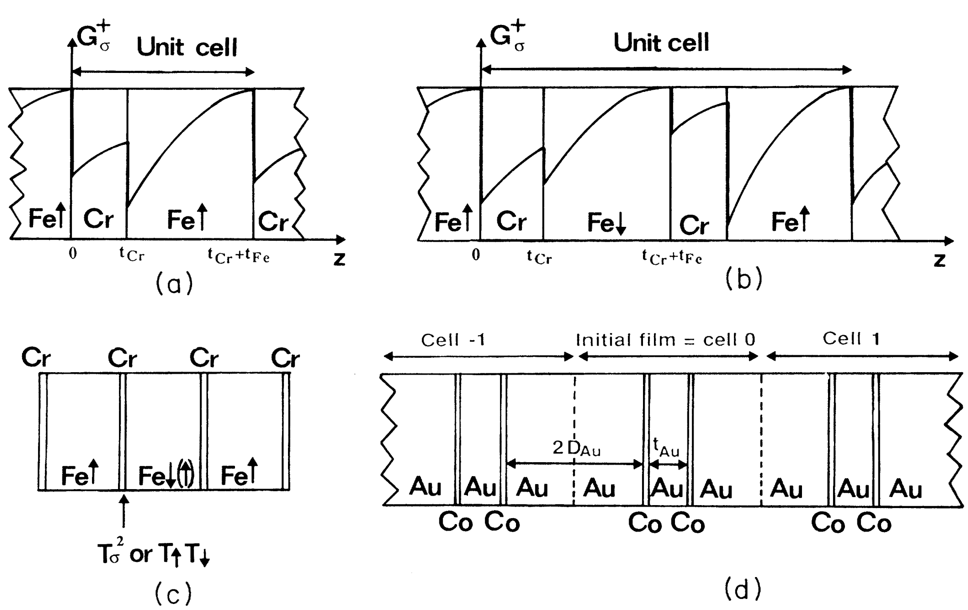


FIG. 1. (a) The function $G_\sigma^+$, that is the normalized departure from the equilibrium distribution function for electrons with spin $\sigma$ and $v_z$ positive, is plotted vs $z$ (schematically) in a Fe/Cr unit cell corresponding to a ferromagnetic arrangement of the Fe layers (limit $t_{\mathrm{Fe}} \gg \lambda$). (b) As in (a), but for the unit cell Cr/Fe$_\downarrow$/Cr/Fe$^\uparrow$ of an antiferromagnetic arrangement. (c) Scheme of a Fe/Cr multilayer structure in the limit $t_{\mathrm{Cr}} \to 0$. The transmission coefficient of a Cr layer is $T_\uparrow^2$ or $T_\downarrow^2$ for a F arrangement or $T_\uparrow T_\downarrow$ for an AF one. (d) Scheme for the five-layer structure Au/Co/Au/Co/Au studied by Dupas *et al.* (Ref. 6). When the scattering by the surfaces of the film (dotted line) are specular, the resistivity is not changed if one considers a periodic structure built by adding adjacent five-layer structures (with perfect transmission through the dotted lines).

value of $v_z$ is shown in Fig. 1(a). The extension to $G_\sigma^-$ is straightforward. The conductivity can be calculated by introducing $G_\sigma^{(\pm)}(z)$ into $g_\sigma^{(\pm)}$, Eqs. (2) and (3), integrating over **v** and $z$ and adding the spin ↑ and spin ↓ contribution to derive the current for one unit cell

$$J=\sum_\sigma \int v_x g_\sigma(v_z,z)d^3v\,dz\ . \tag{13}$$

This calculation has already been performed by Carcia and Suna[17] in the case without spin dependence. By using their Eq. (10) in Ref. 17 with $p=T_\uparrow$ and $p=T_\downarrow$ for electrons with spin ↑ and spin ↓, respectively, and adding the spin ↑ and the spin ↓ contributions to the conductivity, we obtain for the conductivity of the ferromagnetic configuration

$$\sigma_F=\frac{ne^2}{\hbar k_F}\lambda_{\rm eff}^F\ , \tag{14}$$

where $n$ is the number of electrons per volume unit (both spin directions), $k_F$ is the Fermi wave vector, and $e$ is the electron charge:

$$\lambda_{\rm eff}^F=\lambda-\frac{\lambda^2}{t_{\rm Fe}+t_{\rm Cr}}\left[(1-T_\uparrow)I_\uparrow+(1-T_\downarrow)I_\downarrow +(1-T_\uparrow^2)\frac{J_{1\uparrow}+J_{2\uparrow}}{2} +(1-T_\downarrow^2)\frac{J_{1\downarrow}+J_{2\downarrow}}{2}\right]\ . \tag{15}$$

with

$$I_\sigma=\tfrac{3}{2}\int_0^1 d\mu\,\mu(1-\mu^2)(1-e_1)(1-e_2)/(1-T_\sigma^2 e_1e_2)\ , \tag{16}$$

$$J_{1\sigma}=\tfrac{3}{2}\int_0^1 d\mu\,\mu(1-\mu^2)(1-e_1)(e_2)/(1-T_\sigma^2 e_1e_2)\ , \tag{17}$$

$$J_{2\sigma}=\tfrac{3}{2}\int_0^1 d\mu\,\mu(1-\mu^2)(1-e_2)(e_1)/(1-T_\sigma^2 e_1e_2)\ , \tag{18}$$

$$e_1=\exp\left[\frac{-t_{\rm Cr}}{\lambda\mu}\right],\quad e_2=\exp\left[\frac{-t_{\rm Fe}}{\lambda\mu}\right]\ .$$

In the limit we are presently considering, i.e., $t_{\rm Fe}\gg\lambda$, $t_{\rm Cr}$, we can use the following approximations:

$$J_{1\uparrow}=J_{1\downarrow}=0\ ,$$
$$J_{2\uparrow}=J_{2\downarrow}=J_2=\tfrac{3}{2}\int_0^1 d\mu\,\mu(1-\mu^2)\exp\left[-\frac{t_{\rm Cr}}{\lambda\mu}\right]\ , \tag{19}$$

so that Eq. (5) becomes

$$\lambda_{\rm eff}^F\approx\lambda-\frac{\lambda^2}{t_{\rm Fe}}\left[(2-T_\uparrow-T_\downarrow)I+\left[1-\frac{T_\uparrow^2+T_\downarrow^2}{2}\right]J_2\right]. \tag{20}$$

$I$ and $J_2$ can be expressed as

$$I=\tfrac{3}{8}-J_2\ , \tag{21}$$

$$J_2=\frac{3}{2}\int_1^\infty d\nu\left[\frac{1}{\nu^3}-\frac{1}{\nu^5}\right]\exp\left[-\frac{\nu t_{\rm Cr}}{\lambda}\right]\ , \tag{22}$$

or alternatively

$$J_2=\frac{3}{2}\left[E_3\left[\frac{t_{\rm Cr}}{\lambda}\right]-E_5\left[\frac{t_{\rm Cr}}{\lambda}\right]\right]\ , \tag{23}$$

where $E_n$ is the exponential integral function tabulated, for example, by Abramowitz and Stegun.[18]

For the conductivity of the antiferromagnetic (AF) configuration we have to consider the unit cell including two bilayers and represented in Fig. 1(b). By the same procedure as above we obtain

$$G_\uparrow^+(v_z,z=0^-)=1\ , \tag{24}$$

$$G_\uparrow^+(v_z,z=0^+)=T_\uparrow\ , \tag{25}$$

$$G_\uparrow^+(v_z,0<z<t_{\rm Cr})=1-(1-T_\uparrow)\exp\left[-\frac{z}{\tau v_z}\right]\ , \tag{26}$$

$$G_\uparrow^+(v_z,z=t_{\rm Cr}^+)=T_\downarrow\left[1-(1-T_\uparrow)\exp\left[-\frac{t_{\rm Cr}}{\tau v_z}\right]\right]\ , \tag{27}$$

$$G_\uparrow^+(v_z,t_{\rm Cr}<z<t_{\rm Cr}+t_{\rm Fe}) =1-\left\{1-T_\downarrow\left[1-(1-T_\uparrow)\exp\left[-\frac{t_{\rm Cr}}{\tau v_z}\right]\right]\right\} \times\exp\left[-\frac{z-t_{\rm Cr}}{\tau v_z}\right]\ , \tag{28}$$

$$G_\uparrow^+(v_z,z=t_{\rm Cr}+t_{\rm Fe})=1\ . \tag{29}$$

For the second half of the period, the expressions of$G_\uparrow^+$ are obtained from the preceding ones by inverting the order of $T_\uparrow$ and $T_\downarrow$. $G_\downarrow^-$ is easily obtained from $G_\downarrow^+$. Compared to the previous expressions for the ferromagnetic (F) configuration, the only difference is that $T_\uparrow T_\downarrow$ replaces $T_\uparrow^2$ (or $T_\downarrow^2$ for the opposite spin direction). After averaging the contributions from the spin↑ and spin↓ electrons and the first and second halves of the unit cell of Fig. 1(b), we finally find the effective mean free path of the AF configuration by replacing $(T_\uparrow^2+T_\downarrow^2)$ in Eq. (20) by $T_\uparrow T_\downarrow$:

$$\lambda_{\rm eff}^{\rm AF}=\lambda-\frac{\lambda^2}{t_{\rm Fe}}[(2-T_\uparrow-T_\downarrow)I+(1-T_\uparrow T_\downarrow)J_2]\ . \tag{30}$$

The resistivity change $\Delta R=R({\rm AF})-R({\rm F})$ is derived by comparing Eqs. (20) and (30) and using Eq. (23) for $J_2$:

$$\frac{\Delta R}{R({\rm AF})}=\frac{3}{4}(T_\downarrow-T_\uparrow)^2\frac{\lambda}{t_{\rm Fe}}\left[E_3\left[\frac{t_{\rm Cr}}{\lambda}\right]-E_5\left[\frac{t_{\rm Cr}}{\lambda}\right]\right]\ . \tag{31}$$

In the limit $t_{\rm Cr}\ll\lambda$, by using approximate expressions of $E_n(x)$ for $x\ll1$,[18] we obtain from Eq. (31)

$$\frac{\Delta R}{R(\mathrm{AF})}=\frac{3}{16}(T_\downarrow - T_\uparrow)^2\frac{\lambda}{t_{\mathrm{Fe}}}\left[1-\frac{8}{3}\frac{t_{\mathrm{Cr}}}{\lambda}\right] . \tag{32}$$

In the limit $t_{\mathrm{Cr}} \gg \lambda$, $E_n(x)$ for $x \gg 1$ can be approximated by $[1/x - n/x^2]\exp(-x)$,[18] so that Eq. (31) is written as

$$\frac{\Delta R}{R(\mathrm{AF})}=\frac{3}{2}(T_\downarrow - T_\uparrow)^2\frac{\exp\left[-\frac{t_{\mathrm{Cr}}}{\lambda}\right]}{\left[\frac{t_{\mathrm{Fe}}}{\lambda}\right]\left[\frac{t_{\mathrm{Cr}}}{\lambda}\right]^2} . \tag{33}$$

Curves 1, 2, and 3 in Fig. 2 represent the steep decrease of $\Delta R/R$ as a function of $t_{\mathrm{Cr}}$ for different values of $t_{\mathrm{Fe}}$ predicted by Eq. (31). The amplitude of the curves is inversely proportional to $t_{\mathrm{Fe}}$.

Curve 5 shows the variation of $\Delta R/R$ as a function of $t_{\mathrm{Fe}}$ in the limit $t_{\mathrm{Cr}}=0$. This limit corresponds to the scheme of Fig. 1(c) representing equal Fe layers separated by Cr planes with, for transmission coefficient, either $T=T_\uparrow^2$ or $T_\downarrow^2$ (for an F configuration) or $T=T_\uparrow T_\downarrow$ (for an AF configuration). The conductivity for both configurations can be derived from the general expressions of Carcia and Suna[17] for multilayers or, in an equivalent way, by replacing the transmissions by reflections and using the classical Fuchs-Sondheimer results for thin films.[12] The variation as $t_{\mathrm{Fe}}^{-1}$ already expressed in Eq. (31) holds only for the range $t_{\mathrm{Fe}} \gg \lambda$. For $t_{\mathrm{Fe}}$ decreasing in the range $t_{\mathrm{Fe}} \sim \lambda$, one obtains that the MR continues to increase (but its analytical expression is too heavy to be of interest here). The equations of the conductivity in the model of Carcia and Suna[17] or Fuchs-Sondheimer[12] become again simple in the limit $t_{\mathrm{Fe}} \ll \lambda$:

$$\sigma=\frac{3}{4}\frac{ne^2}{\hbar k_F}(1+2T)t_{\mathrm{Fe}}\ln\left[\frac{\lambda}{t_{\mathrm{Fe}}}\right] . \tag{34}$$

By comparing the conductivity obtained for the F ($T=T_\uparrow^2$ or $T_\downarrow^2$) and AF ($T=T_\uparrow T_\downarrow$) configurations, we

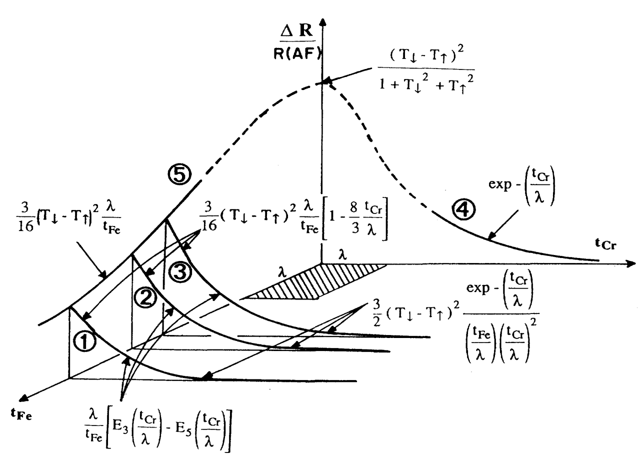

FIG. 2. Summary of our results for the variation of the MR of the Fe/Cr multilayers as a function of the thickness of Fe and Cr.

obtain for the MR for $t_{\mathrm{Cr}}=0$ and $t_{\mathrm{Fe}} \ll \lambda$:

$$\frac{\Delta R}{R(\mathrm{AF})}=\frac{(T_\downarrow - T_\uparrow)^2}{1+T_\downarrow^2+T_\uparrow^2} . \tag{35}$$

We however point out that a semiclassical approach is not really appropriate in the limit where the thicknesses are much smaller than the mean free path, as this has been emphasized by Tesanovic *et al.*[22] Consequently the predictions of Eq. (35) are more questionable than those of Eqs. (31)–(33) obtained in the regime where our semiclassical approach is appropriate. In the limit of Eq. (35), a quantum model[14] is more relevant.

Finally curve 4 in Fig. 2 represents the MR as a function of $t_{\mathrm{Cr}}$ for $t_{\mathrm{Fe}} \to 0$. We do not present its calculation and simply mention that, like curves 1, 2, and 3, curve 4 goes to zero as $\exp(-t_{\mathrm{Cr}}/\lambda)$ for $t_{\mathrm{Cr}} \gg \lambda$.

The calculations above can be extended to structures with only two magnetic layers, like the Au/Co systems studied by Binash *et al.*[2] or Dupas and co-workers.[6,15] In this paper we consider the typical structures by Dupas *et al.*,[6] with two very thin cobalt layers ($6.5\ \text{Å} \le t_{\mathrm{Co}} \le 10\ \text{Å}$) separated by a gold layer of thickness $t_{\mathrm{Au}}$ and sandwiched between thick ($D_{\mathrm{Au}} \sim 250\ \text{Å}$) seed and protection layers of gold. We calculate the MR as a function of $t_{\mathrm{Au}}$ and $D_{\mathrm{Au}}$ in the limit $t_{\mathrm{Co}} \ll \lambda$ and $D_{\mathrm{Au}} \gg \lambda$. As the analysis of Ref. 15 suggests that the specularity parameter for the reflections by the surfaces is close to one, we assume perfectly specular reflections. The resistivity of this five-layer structure is not changed if one includes it in a periodic multilayer built by stacking similar five-layer structures and replacing the specular reflections at the surfaces by a transmission without scattering to the adjacent structure. Therefore we are led to consider the multilayer of Fig. 1(d) composed by the succession of a Au layer of thickness $2D_{\mathrm{Au}}$, a Co layer of thickness $t_{\mathrm{Co}}$, a Au layer of thickness $t_{\mathrm{Au}}$, a second Co layer of thickness $t_{\mathrm{Co}}$, and again, a Au layer of thickness of $2D_{\mathrm{Au}}$, etc. With $t_{\mathrm{Co}} \ll \lambda$, the effect of a Co layer is that of an interface between Au layers with a transmission coefficient $T_\uparrow^2$ or $T_\downarrow^2$. We are back to the above Fe/Cr case except that $T_\uparrow$, $T_\downarrow$, $t_{\mathrm{Cr}}$, and $t_{\mathrm{Fe}}$ are replaced by $T_\uparrow^2$, $T_\downarrow^2$, $t_{\mathrm{Au}}$, and $2D_{\mathrm{Au}}$, respectively. From Eq. (31) we derive the general expression of the resistivity difference between the AF and F arrangements of the Co magnetizations

$$\frac{\Delta R}{R(\mathrm{AF})}=\frac{3}{8}(T_\downarrow^2 - T_\uparrow^2)^2\frac{\lambda}{D_{\mathrm{Au}}}\left[E_3\left[\frac{t_{\mathrm{Au}}}{\lambda}\right]-E_5\left[\frac{t_{\mathrm{Au}}}{\lambda}\right]\right] . \tag{36}$$

In the same way approximate expressions for the limits $t_{\mathrm{Au}} \ll \lambda$ and $t_{\mathrm{Au}} \gg \lambda$ can be derived from Eqs. (32) and (33).

## DISCUSSION

The main features of our results for the magnetoresistance of multilayers are summarized in Fig. 2 and can be understood as follows.

(a) The MR decreases steeply as $t_{\mathrm{Cr}}$ increases, see curves 1–4 for which, when $t_{\mathrm{Cr}}$ is larger than the mean free path, the MR vanishes as $\exp(-t_{Cr}/\lambda)$. This variation is easy to understand: the characteristic length for the damping of the electron distribution function is $\lambda$. Consequently an interface does not affect the electron distribution farther than $\lambda$ and increasing $t_{Cr}$ above $\lambda$ decouples the scattering effects of the two interfaces of a Cr layer. This decoupling makes the resistivity independent of the relative orientation of the magnetizations of the Fe layers and cancels out the MR.

(b) The MR decreases as $t_{Fe}$ increases but in a less pronounced way than as a function of $t_{Cr}$. Its variation as $\lambda/t_{Fe}$ for $t_{Fe} > \lambda$ means that there is only a depth $\lambda$ in a Fe layer (along a Fe/Cr interface) in which the electron distribution is affected by interface scattering. In our calculation we have assumed that only the interface scattering is spin dependent, as this turns out to be approximately the case in Fe/Cr.[20] From previous theoretical works[10,11,13,14] we know that introducing spin dependent bulk scattering does not practically change the dependence on $t_{Cr}$ but makes a maximum appear in the dependence on $t_{Fe}$.

(c) The MR depends on $T_\uparrow$ and $T_\downarrow$ via $(T_\uparrow - T_\downarrow)^2$ in agreement with the results of Refs. 10, 11, and 15.

The quantum model of Levy *et al.*[13,14] also finds a rapid decrease of the MR for increasing $t_{Cr}$ and a less pronounced decrease for increasing $t_{Fe}$, at least when only the interface scattering is spin dependent. A comparison between our results and those of Ref. 14 is not easy because the latter presents only results calculated with spin dependence for both the interface and bulk scattering. However, Ref. 14 presents results obtained with spin dependence only for interface scattering and there is a clear resemblance between the curves of Fig. 4(a) in Ref. 14 and curves 1, 2, and 3 of our Fig. 2.

The main features of the curves in Fig. 2 are also in qualitative agreement with the experimental data on Fe/Cr (or Co/Ru) superlattices.[1,3] To test our expressions more quantitatively we have fitted experimental data on Fe/Cr and Co/Ru[1,3] with Eq. (31) written as

$$\frac{\Delta R}{R} = A\left[E_3\left[\frac{t_{NM}}{\lambda}\right] - E_5\left[\frac{t_{NM}}{\lambda}\right]\right] . \tag{37}$$

The prefactor $A$ depends on $\lambda$, $T_\uparrow$, $T_\downarrow$, $t_{Fe}$ (or $t_{Co}$) but not on the thickness of the nonmagnetic layers, $t_{NM} = t_{Cr}$ or $t_{Ru}$. Figure 3 shows that series of experimental data on Fe/Cr and Co/Ru at 4.2 K can be nicely put on the same curve $E_3(t_{NM}/\lambda) - E_5(t_{NM}/\lambda)$ when the normalized MR, $\Delta R/R$ divided by a normalization factor $A$, is plotted versus $t_{NM}/\lambda$ [$t_{NM} = t_{Cr}$ or $t_{Ru}$, $\lambda = \lambda$ (Fe/Cr) or $\lambda$ (Co/Ru)]. The fit of Fig. 3 is obtained with $\lambda$ (Fe/Cr) = 20 Å and $\lambda$ (Co/Ru) = 15 Å. The values of the mean free path $\lambda$ are in reasonable agreement with the experimental values of the resistivity at 4.2 K. For Fe/Cr with $n = 0.6$ el/atom and $\lambda = 20$ Å, we obtain $\rho(\text{bulk}) = \hbar k_F / ne^2\lambda \sim 36$ $\mu\Omega$ cm, and, when we use Eq. (30) with $T_\uparrow \sim 0$ and $T_\downarrow \sim 1$ (Ref. 21) to calculate the resistivity enhancement by interface scattering for a (Fe 30 Å/Cr 9 Å) multilayer, we obtain $\rho \sim 48$ $\mu\Omega$ cm for the zero field resistivity. This is in the range 40–80 $\mu\Omega$ cm of the samples of Ref. 1. *The major difficulty is encountered when one tries to account for the large absolute values of the MR in Fe/Cr.* For the fit with Fe/Cr in Fig. 3, it is necessary to assume a value of $(T_\uparrow - T_\downarrow)^2$ as large as 5.04, whereas $T_\uparrow$ and $T_\downarrow$ cannot exceed 1. The only way to account for the experimental values of the MR with $(T_\uparrow - T_\downarrow)^2 \approx 1$, is to increase $\lambda$ by a factor of about 3. In return this deteriorates this fit with the experimental thickness dependence and lowers the zero field resistivity $\rho$ to about 28 $\mu\Omega$ cm, that is definitely below the experimental range. We believe that this difficulty arises from the underestimate of the interface contribution to the resistivity and the MR by the semiclassical approach when the thickness is smaller than the mean free path.[19] This underestimate of the interface contribution has already been emphasized by Tesanovic *et al.*[22] In contrast, in quantum models of the MR,[13,14] the interface and bulk scattering are treated in the same way, the proportion of interface scattering can be tuned more freely, and a better quantitative agreement with the experimental results of the MR can be obtained.

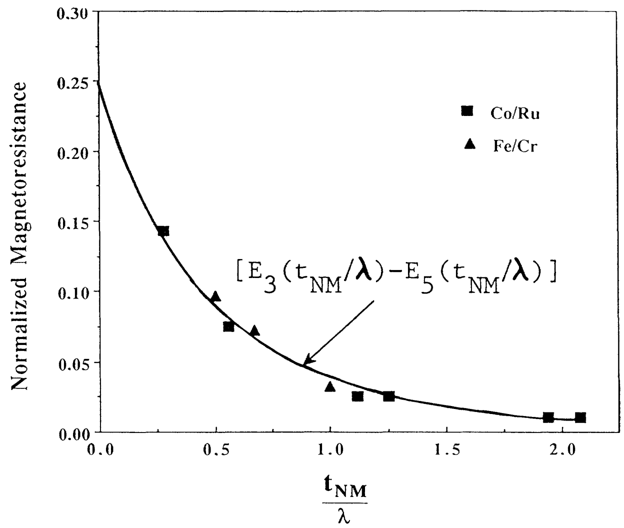


FIG. 3. The solid line represents the function $[E_3(t_{NM}/\lambda) - E_5(t_{NM}/\lambda)]$ plotted as a function of $t_{NM}/\lambda$ where $t_{NM}$ is the thickness of the nonmagnetic layer (Cr or Ru). The symbols correspond to normalized experimental values of the MR for Fe/Cr or Co/Ru, that is $\Delta R/R_{AF}$ divided by $A$, plotted as a function of $t_{Cr}/\lambda$(Fe/Cr) (triangles) or $t_{Ru}/\lambda$(Co/Ru) (squares). For Fe/Cr, the fit is obtained with $\lambda = 20$ Å, $A = 4.5$ for experimental values of $\Delta R/R$ from Ref. 1. For Co/Ru, the fit is obtained with $\lambda = 15$ Å, $A = 0.3$ and we have taken in Fig. 3(a) of Ref. 3 the experimental data in the thickness ranges where the interlayer coupling is antiferromagnetic (for multilayers deposited at 125 °C).

## CONCLUSION

We have used the semiclassical approach of Camley and co-workers[10,11] to derive analytical expressions of the magnetoresistance of magnetic multilayers in several simple cases. These expressions appear of great interest to analyze the influence of the main parameters—thickness of the magnetic and the nonmagnetic layers, mean free path, etc.—and to discuss the physical mechanism of the MR. The main features of our results are in agreement with those obtained in quantum models. There are also in qualitative agreement with series of experimental data on Fe/Cr and Co/Ru multilayers. However, a complete quantitative fit is impossible to obtain for multilayers with very large MR, such as Fe/Cr. This is probably due to the difficulty to balance the interface and bulk scattering in a semiclassical approach.

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