Collective excitations in the A phase of ^{3}He

P. N. Brusov
Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208
(Received 19 March 1990; revised manuscript received 9 April 1991)

A microscopic theory of the collective excitations (CE) in the A phase of ^{3}He is constructed with the use of a path-integration method. The whole CE spectrum, taking damping into account, is calculated. The cause of additional Goldstone modes in the weak-coupling approximation and their analogy to W bosons in the weak-interaction theory are discussed. The whole set of equations, which describe the CE in arbitrary magnetic fields, is obtained. They are solved for small magnetic fields and the linear Zeeman effect for clapping and pair-breaking modes is obtained.

The A phase is perhaps the most interesting object in superfluid ^{3}He. It gives us an example of an anisotropic, superfluid quantum liquid. The main features of the A phase of ^{3}He are connected to the existence on the Fermi surface of two nodes in the gap of a single-particle spectrum. This leads to chiral fermions, gauge fields, W and Z bosons, zero-charge phenomenon, the damping of collective excitations (CE) at zero momentum, and to many other consequences for the system.

In this paper we construct a microscopic theory of CE in the A phase of ^{3}He and investigate the influence of a magnetic field on CE, using the path-integration (PI) method.

The most popular method which is used to investigate the collective excitations in superfluid He^3 is the kinetic-equation (KE) method. The main advantages of the path-integral method over the kinetic equation is the increased accuracy in calculating the collective mode frequencies. For example, in the B phase of ^{3}He, the first collective-mode (CM) dispersion laws for the whole spectrum have been calculated by Brusov and Popov.[1] The investigation of the stability of the Goldstone modes, which requires a calculation of the corrections of order k^4 in the general case, have also been made.[2] The whole CM spectrum has been calculated by taking CM damping into account in the A phase of ^{3}He^3 and recent experiments[4] show excellent agreement with these results in opposition to those obtained by KE.

The main advantages of the KE method are connected with the calculation of the coupling strength between zero sound and the CM. A fine example of this is the calculation by Koch and Wölfle[5] of the coupling strength between the real squashing mode and zero sound, which exists only via very small particle-hole asymmetry.

The cause of such a situation is as follows. The application of the path integral method to superfluid ^{3}He was developed by Brusov and Popov[6] especially to investigate the Bose spectrum. In this way they integrated over all Fermi degrees of freedom and derived the Bose fields, describing the Cooper pairs near the Fermi surface only. This made the formalism simpler and raised the possibility of moving closer to the solution of the problem of the CM eigenfrequencies. But such simplification does not allow one to investigate the interaction between Fermi and Bose degrees of freedom. Our next work will modify our procedure to include some Fermi fields. We show the possibility of including the coupling between zero sound and the CM. Inclusion of the Fermi-liquid correction leads to complications in our scheme. However, until now, the KE method which considered both the Fermi and Bose fields was more complicated and has not been very successful in calculating the CM spectrum. In our opinion, both of these methods, KE and PI, are equivalent. A good example of this is due to Combescot,[7] who subsequent to Brusov and Popov[1] obtained the same set of equations for the Bose spectrum of He^3-B by using the KE method instead of the path-integral method.

MODEL OF He^3

The model of He^3 that we describe below was first suggested by Alonso and Popov[8] and developed by Brusov and Popov.[6] We describe the He^3 system by the anticommuting functions $\chi_s(\mathbf{x},\tau)$ and $\bar{\chi}_s(\mathbf{x},\tau)$ with the Fourier expansion

$$\chi_s(\mathbf{x},\tau)=\frac{1}{(\beta V)^{1/2}}\sum_{\mathbf{k},\omega} a_s(\mathbf{k},\omega)\exp[i(\omega\tau-\mathbf{k}\cdot\mathbf{x})] \; . \qquad (1)$$

Here $s=\pm$ is the spin index, $\mathbf{x}\in V=L^3$, $\tau\in[0,\beta]$, $\beta=T^{-1}$ (in units $\hbar=k_B=1$), $k_i=2\pi n_i/L, \omega=(2n+1)\pi/\beta$; n and n_i are integers. Let us consider the statistical sum for this system $\int \exp S\, d\bar{\chi}\, d\chi$, where the functional

$$S=\int_0^\beta d\tau \int d\chi \sum_s \bar{\chi}_s(\mathbf{x},\tau)\partial_\tau \chi_s(\mathbf{x},\tau)-\int_0^\beta H'(\tau)d\tau \qquad (2)$$

is the action corresponding to the Hamiltonian

$$H'(\tau)=\int d\chi \sum_s \left[\frac{1}{2m}\nabla\bar{\chi}_s(\mathbf{x},\tau)\nabla\chi_s(\mathbf{x},\tau)-(s\mu_0 H+\lambda)\bar{\chi}_s(\mathbf{x},\tau)\chi_s(\mathbf{x},\tau)\right]$$
$$+\tfrac{1}{2}\int dx\, dy u(\mathbf{x}-\mathbf{y})\sum_{s,s'}\bar{\chi}_s(\mathbf{x},\tau)\bar{\chi}_{s'}(\mathbf{y},\tau)\chi_{s'}(\mathbf{y},\tau)\chi_s(\mathbf{x},\tau) \; , \qquad (3)$$

in which λ is the chemical potential, μ_0 is the magnetic moment of the Fermi particle, and H is the magnetic field.

Using the idea that only fermions in the vicinity of the Fermi surface are important for the superfluidity, we separate the Fermi fields (1) into two parts: "fast" fields and "slow" fields. "Fast" fields χ_1 and $\bar{\chi}_1$ are determined by the part of expansion (1) with $|k-k_F|>k_0$ or $|\omega|>\omega_0$, and the "slow" ones χ_0 and $\bar{\chi}_0$ are equal $\chi_0=\chi-\chi_1, \bar{\chi}_0=\bar{\chi}-\bar{\chi}_1$.

The auxiliary parameters k_0 and ω_0 are defined only to an order of magnitude, and the physical results should not depend on their concrete choice.

We integrate first over the "fast" Fermi fields and then over the "slow" ones, using different perturbation-theory schemes in each of these two stages. The integral over the fast fields χ_1 and $\bar{\chi}_1$ will be written in the form

$$\int \exp S \, d\bar{\chi}_1 d\chi_1 = \exp\tilde{S}[\bar{\chi}_0, \chi_0] \; . \tag{4}$$

The functional $\tilde{S}$ is the action of the "slow" fields χ_0 and $\bar{\chi}_0$, for which $|k-k_F|<k_0$ and $|\omega|<\omega_0$. The general form of the functional $\tilde{S}$ is a sum of functionals of the even powers in the fields χ_0 and $\bar{\chi}_0$:

$$\tilde{S} = \sum_{n=0} \tilde{S}_{2n} \; . \tag{5}$$

Neglecting the higher-order functionals $\tilde{S}_6, \tilde{S}_8, \ldots$, and omitting the constant $\tilde{S}_0$, which is no longer significant, we examine the forms of $\tilde{S}_2$ and $\tilde{S}_4$. The form of $\tilde{S}_2$ corresponds to noninteracting quasiparticles near the Fermi surface, and is given by

$$\sum_{\mathbf{k},\omega,s} \varepsilon_s(\mathbf{k},\omega) a_s^\dagger(\mathbf{k},\omega) a_s(\mathbf{k},\omega) \; ,$$

$$|k-k_F|<k_0, \quad |\omega|<\omega_0 \; , \tag{6}$$

with

$$\varepsilon_s(\mathbf{k},\omega) \approx Z^{-1}[i\omega - c_F(k-k_F) + s\mu H] \; . \tag{7}$$

ε_s is given in powers of ω, $k-k_F$, and H, but only the linear terms retained. The coefficient c_F is the velocity at the Fermi surface, μ is the magnetic moment of the quasiparticle, and Z is a normalization constant.

The form of $\tilde{S}_4$ describes the interaction of the quasiparticles and is given by

$$-\frac{1}{\beta V} \sum_{p_1+p_2=p_3+p_4} t_0(p_1,p_2,p_3,p_4) a_+^\dagger(p_1) a_-^\dagger(p_2) a_-(p_4) a_+(p_3)$$
$$-\frac{1}{2\beta V} \sum_{p_1+p_2=p_3+p_4} t_1(p_1,p_2,p_3,p_4)[2a_+^\dagger(p_1)a_-^\dagger(p_2)a_-(p_4)a_+(p_3)$$
$$+a_+^\dagger(p_1)a_+^\dagger(p_2)a_+(p_4)a_+(p_3)+a_-^\dagger(p_1)a_-^\dagger(p_2)a_-(p_4)a_-(p_3)] \; . \tag{8}$$

Here $p=(\mathbf{k},\omega)$ is the four-momentum; $t_0(p_i)$ and $t_1(p_i)$ are, respectively, the symmetrical and antisymmetrical scattering amplitudes under the permutations $p_1 \rightleftarrows p_2$ and $p_3 \rightleftarrows p_4$. In the vicinity of the Fermi sphere we can put $\omega_i=0$, $\mathbf{k}_i=k\mathbf{n}_i$ $(i=1,2,3,4)$, where $\mathbf{n}_i$ are unit vectors such that $\mathbf{n}_1+\mathbf{n}_2=\mathbf{n}_3+\mathbf{n}_4$. The amplitudes t_0 and t_1 should depend only on two invariants, for example, on $(\mathbf{n}_1,\mathbf{n}_2)$ and $(\mathbf{n}_1-\mathbf{n}_3,\mathbf{n}_3-\mathbf{n}_4)$ with t_0 even and t_1 odd in the second invariant. We therefore have the expressions

$$t_0 = f((\mathbf{n}_1,\mathbf{n}_2);\ (\mathbf{n}_1-\mathbf{n}_2,\mathbf{n}_3-\mathbf{n}_4)) \; ,$$
$$t_1 = (\mathbf{n}_1-\mathbf{n}_2,\mathbf{n}_3-\mathbf{n}_4) g((\mathbf{n}_1,\mathbf{n}_2),(\mathbf{n}_1-\mathbf{n}_2,\mathbf{n}_3-\mathbf{n}_4)) \; . \tag{9}$$

Here t_0 and t_1 are expressed in terms of the functions f and g, which are even in the second argument.

The functional $\tilde{S}_2+\tilde{S}_4$, defined by formulas (6)–(9) is the most general expression describing Fermi quasiparticles and their pair interaction near the Fermi sphere. The method of obtaining this functional in the path-integral formalism, and its investigation that follows below, constitute an alternative approach to that developed in the Landau theory of the Fermi liquid.

The functions f and g can easily be calculated for the gas model. For high-density systems they must be determined from experiment.

We consider hereafter a model with

$$f=0, \quad g=\text{const}<0 \tag{10}$$

as the simplified model of He^3 with pairing in the p state.

Using the Fermi-fields (1) for the description of He^3 is the most logical, however significant difficulties appear when we use them to describe low-energy (infrared) phenomena in superfluid He^3. This is due to the absence of singularities in the single-particle Green function $\langle 0|T[\chi(\mathbf{x},\tau)\bar{\chi}(\mathbf{y},\tau_1)]|0\rangle$ below $E=\Delta$, where Δ is a gap in a single-particle spectrum $E(\mathbf{k})=[\xi^2(\mathbf{k})+\Delta^2]^{1/2}$, $\xi(\mathbf{k})=c_F(k-k_F)$. Thus the description of infrared phenomena (with $E \ll \Delta$) such as zero sound, spin waves, and so on, is complicated in terms of Fermi-fields. One needs to sum an infinite set of Feynman diagrams to gain a simple understanding of these phenomena. But there are Green's functions which describe such excitations directly; they are

$$\langle 0|T[\bar{\chi}(\mathbf{x},\tau)\chi(\mathbf{x},\tau);\bar{\chi}(\mathbf{y},\tau_1)\chi(\mathbf{y},\tau_1)]|0\rangle \; ,$$
$$\left\langle 0 \left| T \left[\bar{\chi}(\mathbf{x},\tau)\frac{\sigma_a}{2}\chi(\mathbf{x},\tau);\bar{\chi}(\mathbf{y},\tau_1)\frac{\sigma_a}{2}\chi(\mathbf{y},\tau_1) \right] \right| 0 \right\rangle \; . \tag{11}$$

Singularities, which appear in such complex functions and are absent in single-particle ones are called CE. The most logical method for their description is the passage from Fermi to Bose fields which describe the Cooper pairs of quasifermions.

To make this we introduce under the integral over the Fermi field a Gaussian integral of $\exp(\bar{c}\hat{A}c)$ with respect to the Bose field c, where $\bar{c}\hat{A}c$ is a quadratic form with a certain operator $\hat{A}$. We then shift the Bose field by a quadratic form of the Fermi fields, so as to annihilate the form $\tilde{S}_4$ of the fourth degree in the Fermi fields. The integral over the Fermi fields is then transformed into a Gaussian integral and is equal to the determinant of the operator $\hat{M}(c,\bar{c})$ that depends on the Bose fields c and $\bar{c}$. We arrive at the functional

$$S_h=\bar{c}\hat{A}c+\ln\det[\hat{M}(c,\bar{c})/\hat{M}(0,0)] , \tag{12}$$

in which the ln det has been regularized by dividing $\hat{M}(c,\bar{c})$ by the operator $\hat{M}(0,0)=\hat{M}(c,\bar{c})|_{c=\bar{c}=0}$.

The functional S_h is called the "hydrodynamic action functional." It defines the point of the phase transition of the initial Fermi system as a Bose condensation of the fields c and $\bar{c}$, and determines the density of the condensate at $T<T_c$ and the spectrum of the collective excitations.

In case of p_- pairing one needs to introduce under the integral over the Fermi fields a Gaussian integral over the complex functions $c_{ia}(x,\tau)$ and $\bar{c}_{ia}(x,\tau)$ with the vector index i and the isotopic index a ($i,a=1,2,3$). The Gaussian integral is of the form

$$\int d\bar{c}_{ia}dc_{ia}\exp\left[\frac{1}{g}\sum_{p,i,a}c_{ia}^{\dagger}(p)c_{ia}(p)\right] , \tag{13}$$

where g is the constant (10). It is easily verified that the shift

$$\begin{aligned}
c_{i1}(p)&\rightarrow c_{i1}(p)+\frac{g}{2(\beta V)^{1/2}}\sum_{p_1+p_2=p}(n_{1i}-n_{2i})[a_+(p_2)a_+(p_1)-a_-(p_2)a_-(p_1)] ,\\
c_{i2}(p)&\rightarrow c_{i2}(p)+\frac{gi}{2(\beta V)^{1/2}}\sum_{p_1+p_2=p}(n_{1i}-n_{2i})[a_+(p_2)a_+(p_1)+a_-(p_2)a_-(p_1)] ,\\
c_{i3}(p)&\rightarrow c_{i3}(p)+\frac{g}{(\beta V)^{1/2}}\sum_{p_1+p_2=p}(n_{1i}-n_{2i})a_-(p_2)a_+(p_1) ,
\end{aligned} \tag{14}$$

does indeed eliminate the form $\tilde{S}_4$.

To calculate the Gaussian integral over the Fermi fields, we introduce a column vector $\psi_a(p)$ with elements

$$\psi_1(p)=a_+(p),\quad \psi_2(p)=-a_-(p),\quad \psi_3(p)=a_-^{\dagger}(p),\quad \psi_4(p)=a_+^{\dagger}(p) , \tag{15}$$

and write down a quadratic form in the Fermi fields

$$K=\tfrac{1}{2}\sum_{p_1,p_2,a,b}\psi_a^{\dagger}(p_1)M_{a,b}(p_1,p_2)\psi_b(p_2) . \tag{16}$$

The fourth-order matrix $M(p_1,p_2)$ with elements $M_{ab}(p_1,p_2)$ is given by

$$M=\begin{vmatrix} Z^{-1}[i\omega-\xi+\mu(\mathbf{H}\cdot\boldsymbol{\sigma})]\delta_{p_1p_2} & \frac{1}{(\beta V)^{1/2}}(n_{1i}-n_{2i})c_{ia}(p_1+p_2)\sigma_a \\ -\frac{1}{(\beta V)^{1/2}}(n_{1i}-n_{2i})c_{ia}^{+}(p_1+p_2)\sigma_a & Z^{-1}[i\omega+\xi+\mu(\mathbf{H}\cdot\boldsymbol{\sigma})]\delta_{p_1p_2}\end{vmatrix} , \tag{17}$$

where σ_a ($a=1,2,3$) are 2×2 Pauli matrices.

Integrating over the Fermi fields

$$\int e^K d\bar{\chi}_0 d\chi_0=(\det\hat{M})^{1/2} , \tag{18}$$

we arrive at the "hydrodynamic action" functional

$$S_h=\frac{1}{g}\sum_{p,i,a}c_{ia}^{\dagger}(p)c_{ia}(p)+\frac{1}{2}\ln\det\frac{\hat{M}(c,\bar{c})}{\hat{M}(0,0)} . \tag{19}$$

This functional contains all the information on the physical properties of the system. In particular it determines the transition temperature into the superfluid state, the order parameters (OP) of the superfluid states, the gap equation, the CE spectrum, and many others. We will use this to investigate the CE spectrum.

THE CE SPECTRUM IN THE ABSENCE OF MAGNETIC FIELDS

The collective modes (CM) in He^3-A describe the oscillations $\delta A_{i\alpha}$ of the OP (complex 3×3 matrix) around its equilibrium state

$$A_{i\alpha}^{(0)}=\Delta_0(e_1^i+ie_2^i)d_\alpha . \tag{20}$$

The unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ describe the orbital part of the order parameter; their vector product determines the orbital anisotropy vector $\mathbf{l}=[e_1,\mathbf{e}_2]$; and the unit vector $\mathbf{d}$ specifies the spin axis, i.e., the axis of the magnetic anisotropy.

The number of CM in He^3-A as well as in any other phase is equal to 18 ($3\times3\times2$). In principle all of these modes could be observed either in NMR or in zero-sound experiments.

The classification of the CM in the A phase of ^{3}He has been done by Volovik and Khazan[9] in terms of the irreducible representations of the symmetry group H of the equilibrium state Eq. (20). In contrast to the modes in the B phase of ^{3}He which are characterized by one quantum number J and a single parity (with respect to complex conjugation), the modes in He^3-A are characterized by two quantum numbers: Q and S_z, and two parities P^1 and P^2. The charges Q and S_z assume the values 0, ±1, ±2, and 0, ±1, respectively. Owing to the parities P^1 and P^2 those modes, which differ in the sign of either S_z or Q, turn out to be degenerate. Consequently, if the wave vector $\mathbf{k}$ is parallel to the orbital anisotropy axis the spectrum of modes will consist of two fourfold degenerate branches, four twofold-degenerate branches, and two nondegenerate branches (see Table I).

An additional degeneracy of the spectrum of modes is exhibited in the weak-coupling limit, on account of an enlargement of the group H owing to hidden symmetry. This leads in particular to the appearance of four additional Goldstone modes, which were first obtained by Brusov and Popov.[3]

The calculation of the CE spectrum has been made in numerous papers (see, for example, Ref. 10), where the energies of clapping $E_{\text{cl}}=1.22\Delta_0$, flapping $E_{\text{fl}}=1.56\Delta_0$, and pair-breaking $E_{\text{pb}}=2\Delta_0$ modes were obtained (here Δ_0 is the maximum value of gap $\Delta=\Delta_0(\sin\theta)$. These energy values were obtained without taking any CE damping into account. But it is clear that the vanishing of the gap along the orbital anisotropy axis $\mathbf{l}$ leads to CE damping due to decay into two fermions, because CE with nonzero energy and small momentum can always decay kinematically into two fermions whose momenta are almost opposite and close to $\mathbf{l}$. The whole CE spectrum taking into account this damping was obtained first by Brusov and Popov.[6]

Following their paper we describe below the whole CE spectrum in He^3-A without magnetic fields. For calculation of the CE spectrum in the region $T_c-T\sim T_c$, we expand the functional $\frac{1}{2}\ln\det[\hat{M}(c,\bar{c})/\hat{M}(0,0)]$ in Eq. (19) in powers of the deviation $c_{ia}(p)$ from the condensate value $c_{ia}^{(0)}(p)$, which is different for different phases. We apply the shift $c_{ia}(p)\rightarrow c_{ia}^{(0)}(p)+c_{ia}(p)$ and separate from S_h the quadratic form

TABLE I. Various modes and their respective quantum numbers.

		Quantum numbers										
		In the absence of dipole interaction and magnetic field				Taking into account dipole interaction		In magnetic field			In weak coupling	
Modes	Variables	Q	P^2	S_z	P^1	$\tilde{Q}$	$\tilde{P}^2$	Q	P^2	P^1	$\tilde{S}$	Q
Sound	$u_{23}-v_{12}$	0	-1	0	$+1$	0	-1	0	-1	$+1$		
								0	$+1$	$+1$		
	$u_{11}+v_{21}$,								$u_{11}+v_{21}$		1	0
Spin waves	$u_{12}+v_{23}$	0	$+1$	±1	$-$	±1	$-$					
								0	$+$	-1		
									$u_{11}+v_{22}$			
Orbital modes	u_{33},v_{33}	±1	$-$	0	$-$	±1	$-$	±1	$-$	-1		
						0	-1					
	u_{31},v_{31}						$u_{31}+v_{32}$	±1	$-$	$+1$		
Spin-orbit modes	u_{32},v_{32}	±1	$-$	±1	$-$	0	$+1$				1	±1
							$v_{31}-u_{32}$		u_{32},v_{32}			
						±2	$-$	±1	$-$	-1		
							$u_{32}+v_{31},u_{31}-v_{32}$		u_{31},v_{31}			
Pseudosound	$u_{13}+v_{23}$	0	$+1$	0	$+1$	0	$+1$	0	$+1$	$+1$		
								0	-1	$+1$		
	$u_{23}-v_{12}$,								$v_{11}-u_{21}$		1	0
Pseudospin modes		0	-1	±1	$-$	±1	$-$					
								0	-1	-1		
	$v_{11}-u_{21}$								$u_{22}-v_{12}$			
	$u_{23}+v_{13}$,	±2	$-$	0	$+1$	±2	$-$	±2	$-$	$+1$		
	$u_{13}-v_{23}$											
						±1	$-$	±2	$-$	$+1$		
	$u_{11}-v_{21}$,						$u_{11}-v_{21}+u_{22}+v_{12}$,		$u_{11}-v_{21}$,			
Clapping modes	$u_{21}+v_{11}$,						$u_{21}+v_{11}-u_{12}+v_{22}$		$u_{21}+v_{11}$		1	±2
		±2	$-$	±1	$-$							
	$u_{12}-v_{22}$,					±3	$-$	±2	$-$	-1		
							$u_{11}-v_{21}-u_{22}-v_{12}$,		$u_{12}-v_{22}$,			
	$u_{22}+v_{12}$						$u_{21}+v_{11}+u_{12}-v_{22}$		$u_{22}+v_{12}$			

$$\sum_p c_{ia}^\dagger(p)c_{jb}(p)A_{ijab}(p)+\tfrac{1}{2}\sum_p [c_{ia}(p)c_{jb}(-p)+c_{ia}^\dagger(p)c_{jb}^\dagger(-p)]B_{ijab}(p) .$$

This form determines, in first approximation, the Bose spectrum obtained from the equation

$$\det Q=0 . \tag{21}$$

Here Q is a matrix of quadratic form, determined by the tensor coefficients A_{ijab}, B_{ijab} in (20). These quantities are proportional to the integrals of the products of the Green's functions of the fermions. Most effective in the calculation of these integrals is the Feynman procedure customarily used in relativistic quantum theory. In the present case the procedure is based on the identity

$$\frac{1}{(\omega_1^2+\xi_1^2+\Delta^2)(\omega_2^2+\xi_2^2+\Delta^2)}=\int \frac{d\alpha}{[\alpha(\omega_1^2+\xi_1^2+\Delta^2)+(1-\alpha)(\omega_2^2+\xi_2^2+\Delta^2)]^2} . \tag{22}$$

It is easy to evaluate by this procedure the integrals with respect to the variables ω and ξ, and then with respect to the angle variables and the parameter α. The quadratic part of S_h for the A phase of the model is a sum of three quadratic forms, the first of which depends on the variables c_{i1}, the second on c_{i2}, and the third on c_{13}. The second and third forms are transformed into the first by the substitutions $c_{i2}\to c_{i1}$ and $c_{i3}\to ic_{i2}$. The quadratic form of the variables c_{i1} is

$$\sum_p \left[c_{i1}^\dagger(p)c_{j1}(p)\left[\frac{\delta_{ij}}{g}+\frac{4Z^2}{\beta V}\sum_{p_1+p_2=p} n_{1i}n_{1j}G_1G_2(\xi_1+i\omega_1)((\xi_2+i\omega_2)\right] \right.$$
$$\left. +[c_{i1}^\dagger(p)c_{j1}^\dagger(-p)+c_{i1}(p)c_{j1}(-p)]\frac{2\Delta_0^2Z^2}{\beta V}\sum_{p_1+p_2=p}(n_1\pm in_2)^2 n_{1i}n_{1j}G_1G_2 \right] , \tag{23}$$

where

$$G(p)=(\omega^2+\xi^2+\Delta^2)^{-1},\quad \Delta^2=\Delta_0^2(n_1^2+n_2^2)=\Delta_0^2\sin^2\theta . \tag{24}$$

Here Δ_0 is the maximum value of the energy gap of the Fermi spectrum. In the term $(n_1\pm in_2)^2$ of (23) the upper and lower signs correspond to multiplication with respect to $c_{i1}^\dagger c_{j1}^\dagger$ and $c_{i1}c_{j1}$, respectively.

We now investigate all the Bose-spectrum branches defined by (23) at zero momentum $\mathbf{k}$. At $\mathbf{k}=0$ the form of the variables $c_{i1}(\omega,\mathbf{k}=0)$, and $c_{i1}^\dagger(\omega,\mathbf{k}=0)$ are a sum of a form of c_{11}, $c_{11}^\dagger$, c_{21}, $c_{21}^\dagger$, and a form of c_{31}, $c_{31}^\dagger$. The coefficients of $c_{i1}^\dagger c_{j1}$, $c_{i1}c_{j1}^\dagger$, and $c_{i1}c_{j1}$ $(i,j=1,2)$ can be expressed as

$$\frac{\delta_{ij}}{g}+\frac{4Z^2}{\beta V}\sum_{p_1+p_2=p} n_{1i}n_{1j}(\xi_1+i\omega_1)(\xi_2+i\omega_2)G_1G_2$$
$$=\frac{2\delta_{ij}Z^2}{\beta V}\sum_{p_1+p_2=p}(n_1^2+n_2^2)[(\xi_1+i\omega_1)(\xi_2+i\omega_2)G_1G_2-G_1] ,$$
$$=\frac{2\Delta_0^2Z^2}{\beta V}\sum_{p_1+p_2=p}(n_1\pm in_2)^2n_{1i}n_{1j}G_1G_2=b_{ij}\frac{Z^2\Delta_0^2}{2\beta V}\sum_{p_1+p_2=p}(n_1^2+n_2^2)G_1G_2 . \tag{25}$$

Here b_{ij} $(i,j=1,2)$ are the elements of the matrix

$$\begin{bmatrix} 1 & \pm i \\ \pm i & -1 \end{bmatrix} \tag{26}$$

in which the minus sign corresponds to the variables $c_{1i}c_{1j}$, and the plus sign to $c_{1i}^\dagger c_{1j}^\dagger$.

On going from the left to the right sides of the formulas in (25) we averaged (at $\mathbf{k}=0$) over the azimuthal angle, on which the functions G_1, G_2, ξ_1, and ξ_2 are independent. We have used the equality

$$\frac{\delta_{ij}}{g}+\frac{Z^2}{\beta V}\sum_{p_1} n_{1i}n_{1j}G_1=0_1 \tag{27}$$

which determines the value of the gap that enters in $G_1=(\omega_1^2+\xi_1^2+\Delta_1^2\sin^2\theta_1)^{-1}$.

We denote the coefficient of δ_{ij} in (25) by $f(\omega)$, and the coefficient of b_{ij} by $g(\omega)$. We also put

$$u_1=\mathrm{Re}c_{11},\quad v_1=\mathrm{Im}c_{11} ,$$
$$u_2=\mathrm{Re}c_{21},\quad v_2=\mathrm{Im}c_{21} . \tag{28}$$

The quadratic form of the variables u_1, u_2, v_1, v_2 $(\mathbf{k}=\mathbf{0})$ can then be taken as a sum of two forms:

$$\{[f(\omega)+g(\omega)](u_1^2+v_2^2)-2g(\omega)u_1v_2\}$$
$$+\{[f(\omega)-g(\omega)](v_1^2+u_2^2)-2g(\omega)v_1u_2\} . \tag{29}$$

These forms correspond to the matrices

$$\begin{bmatrix} f(\omega)+g(\omega) & -g(\omega) \\ -g(\omega) & f(\omega)+g(\omega) \end{bmatrix} ,$$
$$\begin{bmatrix} f(\omega)-g(\omega) & -g(\omega) \\ -g(\omega) & f(\omega)-g(\omega) \end{bmatrix} . \quad (30)$$

Equating their determinants to zero, we obtain the equations

$$f(\omega)[f(\omega)+2g(\omega)]=0 ,$$
$$f(\omega)[f(\omega)-2g(\omega)]=0 ,$$

or

$$f(\omega)=0,\quad f(\omega)+2g(\omega)=0 ,$$
$$f(\omega)-2g(\omega)=0 . \quad (31)$$

We add to (31) the equation obtained from an examination of the terms with c_{31} and $c_{31}^\dagger$:

$$h(\omega)=g^{-1}+\frac{4Z^2}{\beta V}\sum_{p_1+p_2=p} n_3^2(\xi_1+i\omega_1)(\xi_2+i\omega_1)G_1G_2$$
$$=\frac{2Z^2}{\beta V}\sum_{p_1+p_2=p}[2n_3^2(\xi_1+i\omega_1)(\xi_2+i\omega_1)G_1G_2-(n_1^2+n_2^2)G_1]=0 . \quad (32)$$

The three equations of (31) can be combined into one:

$$\frac{2Z^2}{\beta V}\sum_{p_1+p_2=p}(n_1^2+n_2^2)\{[(\xi_1+i\omega_1)(\xi_2+i\omega_2)\pm(1,0)\Delta^2]G_1G_2-G_1\}=0 , \quad (33)$$

in which $\pm(1,0)\Delta^2$ denotes either Δ^2 or $-\Delta^2$ or 0.

Changing from summations to integrals in (32) and (33) (at $T\to 0$) and substituting the expressions for G_1 and G_2, we can write

$$\frac{2Z^2k_F^2}{(2\pi)^4c_F}\int d\Omega\, d\omega_1 d\xi_1\left[\frac{2\cos^2\theta(\xi_1+i\omega_1)(\xi_2+i\omega_2)}{(\omega_1^2+\xi_1^2+\Delta^2)(\omega_2^2+\xi_2^2+\Delta^2)}-\frac{\sin^2\theta}{(\omega_1^2+\xi_1^2+\Delta^2)}\right]=0 ,$$
$$\frac{2Z^2k_F^2}{(2\pi)^4c_F}\int \sin^2\theta\, d\Omega\, d\omega_1 d\xi_1\left[\frac{(\xi_1+i\omega_1)(\xi_2+i\omega_2)\pm(1,0)\Delta^2}{(\omega_1^2+\xi_1^2+\Delta^2)(\omega_2^2+\xi_2^2+\Delta^2)}-\frac{1}{(\omega_1^2+\xi_1^2+\Delta^2)}\right]=0 . \quad (34)$$

Integrating with respect to ω_1 and ξ_1 with the help of the Feynman procedure, we get

$$\frac{Z^2k_F^2}{4\pi^3c_F}\int_0^1 d\alpha\int\cos\theta\, d\Omega\left[\ln\left[\frac{\Delta^2}{\Delta^2+\alpha(1-\alpha)\omega^2}\right]-\frac{\alpha(1-\alpha)\omega^2}{\Delta^2+\alpha(1-\alpha)\omega^2}\right]=0 ,$$
$$\frac{Z^2k_F^2}{4\pi^3c_F}\int_0^1 d\alpha\int\sin^2\theta\, d\Omega\left[\ln\left[\frac{\Delta^2}{\Delta^2+\alpha(1-\alpha)\omega^2}\right]-\frac{2\alpha(1-\alpha)\omega^2+\Delta^2\mp(1,0)\Delta^2}{\Delta^2+\alpha(1-\alpha)\omega^2}\right]=0 . \quad (35)$$

Calculating the integrals with respect to α, substituting $\omega\to\Delta_0\omega$, and putting $\cos\theta=x$, we arrive at the equations

$$\int_0^1 dx(1-x^2)\frac{\omega^2+4(1-x^2)}{\omega[\omega^2+4(1-x)^2]^{1/2}}\ln\frac{[\omega^2+4(1-x^2)]^{1/2}+\omega}{[\omega^2+4(1-x^2)]^{1/2}-\omega}=0 ,$$
$$\int_0^1 dx(1-x^2)\frac{\omega^2+2(1-x^2)}{\omega[\omega^2+4(1-x^2)]^{1/2}}\ln\frac{[\omega^2+4(1-x^2)]^{1/2}+\omega}{[\omega^2+4(1-x^2)]^{1/2}-\omega}=0 ,$$
$$\int_0^1 dx(1-x^2)\frac{\omega}{[\omega^2+4(1-x^2]^{1/2}}\ln\frac{[\omega^2+4(1-x^2)]^{1/2}+\omega}{[\omega^2+4(1-x^2)]^{1/2}-\omega}=0 ,$$
$$\int_0^1 dx\, x\frac{\omega^2+2(1-x^2)}{\omega[\omega^2+4(1-x^2)]^{1/2}}\ln\frac{[\omega^2+4(1-x^2)]^{1/2}+\omega}{[\omega^2+4(1-x^2)]^{1/2}-\omega}=0 . \quad (36)$$

The first of these equations is the equation $f-2g=0$, the second is $f=0$, and the third is $f+2g=0$, and the fourth is $h=0$. They determine the Bose spectrum at $\mathbf{k}=0$ following the analytic continuation $i\omega\to E$. The spectrum branches corresponding to the second and fourth equations are doubly degenerate. To take into account the forms of the variables c_{i2} and c_{i3} which lead to similar equations for the spectrum, it is necessary to multiply by 3, the multiplicity of each branch in the considered model.

The third and fourth equations in (36) have roots $\omega=0$ and correspond to the Goldstone modes. From the first and second equations we obtain the complex energies of the nonphonon modes $E_1(\mathbf{k}=0)$ and $E_2(\mathbf{k}=0)$.

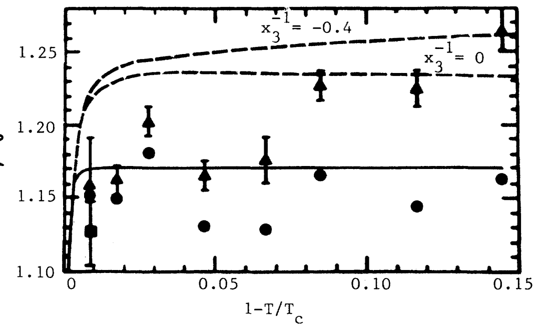

FIG. 1. The normalized clapping-mode resonances from Ref. 4 for two choices of $A=2.03$ (▲) and 2.64 (●) [$A=\Delta_0(0)/k_B T_c$]. Two upper curves follow from KE theory by using the formula $E_{\text{clapping}}=1.23\Delta_0(T)$ $(1-(0.005-0.106x_3^{-1}-0.052F_2^2)$ $[\Delta_0(T)/K_B T])$ at $x_3^{-1}=0$ and $x_3^{-1}=-0.4$, respectively. Solid curve is the result of Ref. 3.

The resulting CE energies are

$$E_1(0)=\Delta_0(1.96-i0.31)\ ,$$
$$E_2(0)=\Delta_0(1.17-i0.13)\ , \tag{37}$$

the second of the modes being doubly degenerate.

The difference between ReE here[3] and in Ref. 10 is due to the fact that taking CE damping into account (Im$E\neq0$) leads via dispersion relations to renormalization of ReE. Dobbs *et al.*[4] have made precise measurements of the clapping-mode frequency. They obtained $E_{\text{clapping}}=(1.15\pm0.01)\Delta_0(T)$, which is in excellent agreement with the results of Brusov and Popov[3] (Fig. 1). This shows mainly the CM damping is significant in obtaining the right value for the clapping-mode frequency.

The Brusov and Popov theory[3] does not require taking high pairing corrections into account, used in KE theory to explain the discrepancy between the KE value of the clapping-mode frequency $E_{\text{clapping}}=1.23\Delta_0(T)$ and the experimental data.[17] Note that a 6% difference remains between experimental data of Ref. 4 and KE theory (Fig. 1) in spite of taking higher pairing and Fermi-liquid corrections into account.

The other interesting fact, obtained first in Ref. 3, is that the number of Goldstone modes in the weak-coupling approximation is equal to 9 rather than 5, which takes place in real He^3-A. The existence of four additional quasi-Goldstone spin-orbit modes is a consequence of the *latent* symmetry of the system and we investigate this equation below in more detail.

THE LATENT SYMMETRY, ADDITIONAL GOLDSTONE MODES, *W*-BOSONS

We shall show that taking into account strong-coupling effects decreases the number of phonon modes from 9 to 5, and that turning on a magnetic field decreases the number of phonon modes from 9 to 6 for weak coupling and from 5 to 4 when strong-coupling effects are taken into account.

We consider, in the Ginzburg-Landau region $|T-T_c|\ll T_c$, that part F of the action which is independent of the gradients. In the weak-coupling model we have

$$\begin{aligned}F=&-\operatorname{tr}AA^\dagger+v\operatorname{tr}A^\dagger AP+(\operatorname{tr}AA^\dagger)^2\\&+\operatorname{tr}AA^\dagger AA^\dagger+\operatorname{tr}AA^\dagger A^*A^T-\operatorname{tr}AA^TA^*A^\dagger\\&-\tfrac{1}{2}\operatorname{tr}AA^T\operatorname{tr}A^\dagger A^*\ ,\end{aligned}\tag{38}$$

where A (the order parameter) is a complex matrix with elements A_{ia}. The A phase in the weak coupling model is described by the order parameter

$$\frac{1}{2}\begin{bmatrix}1&0&0\\i&0&0\\0&0&0\end{bmatrix}\ ,\tag{39}$$

and the phonon variables are

$$u_{21}-v_{11},\ u_{12}+v_{22},\ u_{13}+v_{23},\ u_{31}\ ,$$
$$v_{31},\ u_{32},\ v_{32},\ u_{33},\ v_{33}\ ,\tag{40}$$

where $u_{ia}=\operatorname{Re}A_{ia}$ and $v_{ia}=\operatorname{Im}A_{ia}$. These variables correspond to the Goldstone modes of the spectrum not only in the Ginzburg-Landau region, but also at all $T<T_c$. In the limit as $T\to0$, the first three of the variables in (40) correspond to sound waves with $c_Fk/\sqrt{3}$, and the six remaining ones to orbital waves $c_Fk_\parallel$. The phonon spectrum is thus degenerate in the spin index.

To take into account the strong-coupling effects, we consider F with arbitrary coefficients of the fourth-order terms:

$$\begin{aligned}F=&-\operatorname{tr}A^\dagger A+v\operatorname{tr}A^\dagger AP+a(\operatorname{tr}A^\dagger A)^2\\&+b\operatorname{tr}AA^\dagger AA^\dagger+c\operatorname{tr}AA^\dagger A^*A^T\\&+d\operatorname{tr}AA^TA^*A^\dagger+e\operatorname{tr}AA^T\operatorname{tr}A^\dagger A^*\ .\end{aligned}\tag{41}$$

The condition $\delta F=0$ yields in the A phase an order parameter in the form

$$\frac{1}{2}(a+b+d)^{1/2}\begin{bmatrix}1&0&0\\i&0&0\\0&0&0\end{bmatrix}\ .\tag{42}$$

To find the phonon variables we calculate the second variation δ^2F

$$\begin{aligned}\delta^2F=&-\operatorname{tr}AA^\dagger+v\operatorname{tr}A^\dagger AP+a\operatorname{tr}[(A^\dagger C)^2+(C^\dagger A)^2+2A^\dagger AC^\dagger C+2A^\dagger CC^\dagger A]\\&+b\operatorname{tr}[2AA^\dagger CC^\dagger+2A^\dagger AC^\dagger C+AC^\dagger AC^\dagger+A^\dagger CA^\dagger C]\\&+c\operatorname{tr}[AA^\dagger CC^T+A^\dagger A^*C^TC+A^*A^TCC^\dagger+A^TAC^\dagger C^*+AC^\dagger A^*C^T+A^TCA^\dagger C^*]\\&+d\operatorname{tr}[AA^TC^*C^\dagger+A^TA^*C^\dagger C+A^*A^\dagger CC^\dagger+A^\dagger AC^TC^*]+4e|\operatorname{tr}C^TA|^2\ ,\end{aligned}\tag{43}$$

where C is the matrix (42) and A is a variable matrix. Substituting the values of C, $C^\dagger$, C^*, and C^T, we get

$$\delta^2F=\nu(a+b+d)(u_{13}^2+v_{13}^2+u_{23}^2+v_{23}^2+u_{33}^2+v_{33}^2)+4a(u_{11}+v_{21})^2$$
$$+2b[2(u_{11}+v_{21})^2-(u_{13}-v_{23})^2-(u_{12}-v_{22})^2-(u_{22}+v_{12})^2-(u_{23}+v_{13})^2-2(u_{32}^2+v_{32}^2+u_{33}^2+v_{33}^2)]$$
$$+2c[2(u_{11}-v_{21})^2+(u_{23}+v_{13})^2+2(u_{21}+v_{11})^2+(u_{21}-v_{22})^2+(u_{22}+v_{12})^2+(u_{13}-v_{23})^2]$$
$$+2d[2(u_{11}+v_{21})^2-(u_{12}-v_{22})^2-(u_{22}+v_{12})^2-(u_{13}-v_{23})^2-2(u_{22}-v_{12})^2-2(u_{23}-v_{13})^2$$
$$-(u_{23}+v_{13})^2-2(u_{32}^2+v_{32}^2)+u_{33}^2+v_{33}^2]+4e[(u_{11}-v_{21})^2+(u_{21}+v_{11})^2]\ . \qquad (44)$$

We consider first the system in a zero magnetic field ($\nu=0$). Equation (44) is the sum of five quadratic forms multiplied by the independent coefficients a, b, c, d, and e. The variables

$$u_{12}+v_{22},\ u_{13}-v_{23},\ u_{21}-v_{11},\ u_{31},\ v_{31}\ , \qquad (45)$$

do not enter in any of these forms, which therefore correspond to Goldstone modes. Thus, allowance for the strong-coupling effects decreases the number of phonon branches from 9 to 5. The modes u_{32}, v_{32}, u_{33}, and v_{33}, which correspond in the weak-coupling approximation to orbital waves, become nonphonon modes when the strong-coupling effects are taken into account.

Expression (44) at $\nu\neq 0$ describes the system in a magnetic field. In the weak-coupling approximation the number of phonon modes decreases from 9 to 6, and the variables $u_{13}+v_{23}$, u_{33}, and v_{33} become nonphonon because of the appearance of the gap $\sim\mu H$ in the spectrum. In a system with a strong coupling, the mode that becomes nonphonon upon application of a magnetic field is $u_{13}+v_{22}$ (the modes u_{33} and v_{33} in the case of strong coupling remain Goldstone modes at $v=0$), and the number of Goldstone modes decreases from 5 to 4.

To gain an idea of the total Bose spectrum (including the Goldstone branches) when strong-coupling effects are taken into account, we write (44) at $H=0(\nu=0)$ in the form

$$\delta^2F=4(a+b+d)(u_{11}+v_{21})^2+4(c+e)[(u_{11}-v_{21})^2+(u_{21}+v_{11})^2]$$
$$+2(c-b-d)[(u_{13}-v_{23})^2+(u_{12}-v_{22})^2+(u_{22}+v_{12})^2+(u_{23}+v_{13})^2]$$
$$-4d(u_{23}-v_{13})^2+(u_{22}-v_{12})^2]-4(b+d)(u_{32}^2+v_{33}^2+u_{33}^2+v_{33}^2)\ . \qquad (46)$$

For comparison we write down δ^2F in the weak-coupling approximation, using $a=b=c=-d=-2e=1$ in (46):

$$\delta^2F=4[(u_{11}+v_{21})^2+(u_{23}+v_{13})^2+(u_{22}-v_{12})^2]$$
$$+2[(u_{11}-v_{21})^2+(u_{21}+v_{11})^2+(u_{13}-v_{23})^2+(u_{12}-v_{22})^2+(u_{22}+v_{12})^2+(u_{23}+v_{13})^2]+0[u_{32}^2+u_{33}^2+v_{32}^2+v_{33}^2]\ . \qquad (47)$$

The form (47) has three eigenvalues equal to 4, corresponding to the variables $u_{11}+v_{21}$, $u_{22}-v_{12}$, and $u_{23}-v_{13}$. The branches E_1 correspond, as $T\to 0$, to these variables. The other nonzero eigenvalue equal to 2 corresponds to six variables: $u_{21}+v_{11}$, $u_{12}-v_{22}$, $u_{13}-v_{23}$, $u_{11}-v_{21}$, $u_{22}+v_{12}$, $u_{23}+v_{13}$, and six E_2 branches as $T\to 0$.

The calculation of the Bose spectrum in Ref. 10 yields 6 clapping modes and three $2\Delta_0$ modes, i.e., as many as in the weak-coupling case considered here. Formula (46) shows that in the general case allowance for the strong-coupling effects leads to splitting. The clapping modes break up into two groups—two branches correspond to the eigenvalue $4(c+e)$ and four correspond to the number $2(c-b-d)$. The three $2\Delta_0$ branches also break up into one branch with eigenvalue $4(a+b+d)$ and two branches with eigenvalue $-4d$. We note that no conclusion can be drawn from the data of Ref. 10 concerning the splitting of the branches.

The branches u_{32}, u_{33}, v_{32}, and v_{33}, which in the weak-coupling approximation are orbital waves, become the normal flapping modes and the superflapping modes when the strong-coupling effects are taken into account, as shown by comparison with the data of Ref. 10.

Volovik[11] first showed that the fermions in the He^3-A are chiral and a field theory in superfluid ^{3}He-A which describes the dynamics of chiral fermion excitations interacting with the order-parameter collective boson modes is similar to the theory of the electroweak interaction. The roles of photons and W bosons are played by orbital waves and four quasi-Goldstone spin-orbit modes, which we obtained above, respectively.

An equation of the Dirac type for fermions in ^{3}He-A near the poles of the Fermi sphere can be derived from the Bogoliubov equation, in which it is necessary to take into account the fluctuations δA_{ai} of the order parameter A_{ai} around its equilibrium value

$$A_{i\alpha}^{(0)}=\Delta_0 d_\alpha(e_1^i+ie_2^i)\ .$$

Only certain combinations of fluctuations act on the fermions. These combinations form a "photon" field and W field:[11]

$$\begin{aligned} A_1+iA_2 &= -\frac{k_F}{\Delta_0} d_\alpha l^i \delta A_{i\alpha} , \\ W_1^\alpha + iW_2^\alpha &= \frac{k_F}{i\Delta_0} e^{\alpha\beta\gamma} d_\beta l^i \delta A_{i\gamma} , \\ A_3=\delta k_F, \quad W_3^\alpha &= \frac{1}{2k_F} e^{\alpha\beta\gamma} d^\beta \frac{\partial}{\partial t} d^\gamma \\ A_0 = k_F \mathbf{l}\cdot\mathbf{v}_s, \quad W_0^\alpha &= \tfrac{1}{2} k_F e^{\alpha\beta\gamma} d_\beta (\mathbf{l}\cdot\nabla) d_\gamma . \end{aligned} \tag{48}$$

Equations (48) also incorporate the effect of the fluctuational spin $\mathbf{S}\sim[\mathbf{d}\partial\mathbf{d}/\partial t]$, which accounts for a third component of the W field, in precisely the same way as density fluctuations account for a third component, along $\mathbf{l}$, of the "photon" field $\mathbf{A}$.

In terms of the fields in (48), the Bogoliubov equation for the Bogoliubov spinor ψ takes the following form near the poles of the Fermi sphere:

$$\left[\left[i\frac{\partial}{\partial t} - eA_0 - e\sigma^\alpha W_0^\alpha \right] + e[c_\parallel l^i \tau_3 + c_\perp (e_1^i \tau_1 + e_2^i \tau_2)] \times \left[\frac{1}{i}\nabla_i - eA_i - e\sigma^\alpha W_i^\alpha \right] \right], \quad \psi = 0 . \tag{49}$$

Here τ_i and σ^α are the Pauli matrices corresponding to the Bogoliubov isospin and ordinary spin. This equation is reminiscent of the Dirac equation for massless chiral fermions in the Weinberg-Salam theory. The primary distinction is in the anisotropy of He^3-A along the $\mathbf{l}$ and $\mathbf{d}$ axes. The velocity $c_\parallel = v_F$ along $\mathbf{l}$ is far greater than the transverse velocity $c_\perp = \Delta_0/k_F$, and we have $W^\alpha d^\alpha = 0$; i.e., there are no Z bosons. The charge $e = \mathbf{k}\cdot\mathbf{l}/k_F$ takes on the values $+1$ and -1 for fermions near the upper and lower poles, respectively.

An important point is that in the weak-coupling approximation (in which the Fermi spheres with different spin projections do not interact with each other) there is an additional SO(3) symmetry, which combines the "photons" and the W bosons in a single triplet (more precisely, a sextet, when we take into account the polarization of the collective modes). In this approximation, the W bosons, like the Goldstone orbital waves (or "photons"), have no mass in consequence with results obtained above. But as Brosov and Popov showed first[3] (see above) four additional Goldstone modes become nonphonon if we turn on the strong-coupling corrections. It means that the W bosons acquire a mass via the strong-coupling corrections. Consequently, and in contrast with the Weinberg-Salam theory, the Higgs phenomenon is not required for the appearance of massive W bosons in He^3-A.

THE LINEAR ZEEMAN EFFECT FOR CLAPPING AND PAIR-BREAKING MODES

Below we consider the influence of magnetic fields on the CE spectrum in He^3-A.[12]

In accordance with Nasten'ka and Brusov's idea[13] for the investigation of the CE spectrum in the presence of a magnetic field, we must take into account both the additional term in S_h and the distortion of the order parameter. The latter in our case is equal to[14]

$$c_{ia}(p) = c\sqrt{\beta V}\, \delta_{p0}(\delta_{a1}\alpha_+ + i\delta_{a2}\alpha_-)(\delta_{i1}+i\delta_{i2}) . \tag{50}$$

Here

$$\begin{aligned} \alpha_\pm &= \frac{\Delta_\uparrow \pm \Delta_\downarrow}{2\Delta}, \quad \Delta^2_{\uparrow\downarrow} = N(0)(\tau \pm \eta h)/2\beta_{245} , \\ \eta &= (N'(O)/N(O)) T_c \ln(1.14\varepsilon_0/T_c) , \\ h &= \frac{\mu_0 H}{T_c}, \quad \Delta = 2cZ - , \end{aligned}$$

is a single fermion spectrum gap, determined by the gap equation

$$\frac{1}{g} = -\frac{Z^2}{\beta V}\frac{1}{(\alpha_+^2+\alpha_-^2)} \sum_p \left[\frac{(\alpha_+ + \alpha_-)^2 \sin^2\theta}{\omega^2 + (\xi - \mu H)^2 + \Delta^2 \sin^2\theta(\alpha_+ + \alpha_-)^2} + \frac{(\alpha_+ - \alpha_-)^2 \sin^2\theta}{\omega^2 + (\xi + \mu H)^2 + \Delta^2 \sin^2\theta(\alpha_+ - \alpha_-)^2} \right] . \tag{51}$$

We could calculate the CE spectrum of our system in the presence of a magnetic field by the techniques developed above (see, Appendix B, part II) but using the deformed OP (50). Making these calculations in Appendix A, we obtained 18 equations, which completely determine the CE spectrum in ^3He-A in an arbitrary magnetic field $\mathbf{H}$ and with arbitrary CE momenta $\mathbf{k}$. In Appendix B we consider the case of small $\mathbf{H}$ and zero momentum of CE, $\mathbf{k}=0$, and calculate the linear corrections to the CE spectrum. We obtained for the energies of clapping and pair-breaking modes:

$$\begin{aligned} &\text{clapping:} \quad E_1 = (1.17 - i0.13)\Delta_0 , \\ &\qquad E_{2,3} = (1.17 \pm 1.70\gamma H)\Delta_0 - i(0.13 \pm 1.20\gamma H)\Delta_0 ; \\ &\text{pair breaking:} \quad E_1 = (1.96 - i0.31)\Delta_0 , \\ &\qquad E_{2,3} = (1.96 \pm 2.04\gamma H)\Delta_0 - i(0.31 \mp 0.06\gamma H)\Delta_0 . \end{aligned} \tag{52}$$

So for small $\mathbf{H}$ we have threefold splitting of the clapping and pair-breaking modes; i.e., we have a linear Zeeman effect for these modes. Magnetic fields lift the degeneracy of pair-breaking modes completely, and lift the degeneracy of clapping modes particularly, each branch of the latter modes remaining twice degenerate.

Note that the magnetic field changes both the real and imaginary parts of the CE energies, i.e., it changes the CE frequencies and damping even in the linear approximation. The damping of some of the modes increases while others decrease with a magnetic field.

We can compare our results with those from Refs. 15 and 16. Our CE spectrum differs via the existence of four additional quasi-Goldstone modes. Moreover, our results for CE energies are more precise because we take into account the CE damping via the process of Cooper pair breaking.

In Ref. 15 some modes remain unchanged while the frequencies of others are shifted from ω_i to $(\omega_i^2+\Omega^2)^{1/2}$ (here Ω is the effective Larmor frequency). This expression does not take into account the gap distortion. Our results for the real parts of the CE energies are closer to those found in Ref. 16. (There however the CE damping is not taken into account and the influence of a magnetic field on it is not investigated.) The linear splitting of some CE frequencies via gap distortion was also found:

$$\omega_{i\sigma}\left[\frac{T}{T_{c\sigma}}\right]=\frac{T_{c\sigma}}{T_{c0}}\omega_i\left[\frac{T}{T_{c0}}\right],$$

i = normal flapping, clapping, super flapping, super clapping (53)

$\sigma=\uparrow,\downarrow$

The frequencies of other modes are shifted from the zero-field values ω_i to $(\omega_i^2+\Omega^2)^{1/2}$, i.e., remain unchanged in linear field approximation.

Later, we concluded that the linear Zeeman effect for clapping and pair-breaking modes takes place via the distortion of the order parameter only (or particle-hole asymmetry) in the case of zero $\mathbf{k}$. For nonzero $\mathbf{k}$ there is in principle a possibility that an additional term in the action (without gap distortion) will lead to linear field corrections to the mode energies.

As follows from (53), to obtain the collective-mode frequencies in He^3-*A* in a magnetic field one needs to make the substitution $\Delta\rightarrow\Delta\pm\gamma H$. From our data (52) it follows that this conclusion cannot be applied directly. Instead of this principle, we obtained the next one: To get the collective-mode frequencies in applied magnetic field one must make the substitution $\Delta\rightarrow\Delta\pm\alpha_i\gamma H$, where α_i depends on type of collective mode (clapping or pair breaking) and is different for the real and the imaginary parts of frequency for the same mode. The cause of this difference is connected with the fact that in this paper, in opposition to Ref. 16, we take into account the damping of collective excitations via the existence of a gap in the single-particle spectrum.

We mentioned above that threefold splitting of the clapping and pair-breaking modes could be observed in zero-sound experiments. Note that in the case of the *A* phase, in contrast to the case of the *B* phase, the gap distortion is linear in the field. This leads to the possibility of observing this effect in moderate fields.

ACKNOWLEDGMENTS

I wish to thank M. Y. Nasten'ka for supporting me during my stay at Northwestern. I thank T. V. Filatova-Novoselova, V. N. Popov and M. V. Lomakov for taking part in the investigation of CE in ^{3}He-*A*, G. E. Volovik, J. B. Ketterson, W. P. Halperin, J. A. Sauls, and Z. Tešanovic for helpful discussions, and Northwestern University, I. Weil, and IREX for their hospitality.

APPENDIX A: THE EQUATIONS FOR CE SPECTRUM IN He^3-*A* IN ARBITRARY MAGNETIC FIELD AND AT ARBITRARY CE MOMENTA

In first approximation the CE spectrum is determined by the quadratic part of S_h. To obtain it we need to calculate G. One has for G^{-1}

$$G^{-1}=\begin{bmatrix} Z^{-1}(i\omega-\xi+\mu H\sigma_3)I_+ & 2c(n_1+in_2)(\alpha_+\delta_1+i\alpha_-\sigma_2)I_- \\ -2c(n_1-in_2)(\alpha_+\xi_1-i\alpha_-\sigma_2)I_- & Z^{-1}(-i\omega+\xi+\mu H\sigma_3)I_+ \end{bmatrix} \tag{A1}$$

Inverting G^{-1} one gets

$$G=\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22}\end{bmatrix},\ \text{where } G_{11}=\begin{bmatrix} \frac{a^\dagger+b}{d_1} & 0 \\ 0 & \frac{a^\dagger-b}{d_2}\end{bmatrix} I_+,\quad G_{12}=\begin{bmatrix} 0 & \frac{q_1-iq_2}{d_1} \\ \frac{q_1+q_2}{d_2} & 0\end{bmatrix} I_-,$$

$$G_{21}=\begin{bmatrix} 0 & \frac{-q_1^\dagger+iq_2^\dagger}{d_2} \\ \frac{-q_1^\dagger-iq_2^\dagger}{d_1} & 0\end{bmatrix} I_+,\quad G_{22}=\begin{bmatrix} \frac{-a^\dagger+b}{d_2} & 0 \\ 0 & \frac{-a^\dagger-b}{d_1}\end{bmatrix} I_-,$$

where $a=Z^{-1}(i\omega-\xi)$, $b=Z^{-1}\mu H$, $q_1=Z^{-1}\Delta(n_1+in_2)\alpha_+$, $q_2=iZ^{-1}\Delta(n_1+in_2)\alpha_-$,

$$d_{1,2}=Z^{-2}[\omega^2+(\xi\mp\mu H)^2]+\Delta^2\sin^2\theta(\alpha_+\pm\alpha_-)^2 .$$

Using the expression (A1) for G and the following expression for u:

$$u=(\beta V)^{-1/2}\begin{bmatrix} 0 & (n_{1i}-n_{2i})c_{ia}(p_1+p_2)\sigma_a \\ -(n_{1i}-n_{2i})c_{ia}(p_1+p_2)\sigma_a & 0\end{bmatrix}$$

one can obtain the quadratic part of S_h

$$S_h=g^{-1}\sum_{p,i,a}c_{ia}^{\dagger}c_{ia}+\frac{Z^2}{\beta V}\sum_{p_1+p_2=p}n_{1i}n_{1j}(\{-\Delta^2(n_1+in_2)^2[-\eth_3c_{i3}^{\dagger}c_{j3}^{\dagger}+\eth_1(c_{i1}^{\dagger}c_{j1}^{\dagger}-c_{i2}^{\dagger}c_{j2})+i\eth_2(c_{i1}^{\dagger}c_{j2}^{\dagger}+c_{i2}^{\dagger}c_{j1}^{\dagger})]+\text{H.c.}\}$$
$$+D_1(c_{i1}^{\dagger}c_{j1}+c_{i1}c_{j1}^{\dagger}+c_{i2}^{\dagger}c_{j2}+c_{i2}c_{j2}^{\dagger})+D_3(c_{i3}^{\dagger}c_{j3}+c_{i3}c_{j3}^{\dagger})$$
$$+iD_2(c_{i2}^{\dagger}c_{j1}+c_{i1}c_{j2}^{\dagger}-c_{i2}c_{j1}^{\dagger}-c_{i1}^{\dagger}c_{j2}))\ . \qquad \text{(A2)}$$

Here

$$\eth_{1,2}=\frac{(\alpha_++\alpha_-)^2}{d_1(1)d_1(2)}\pm\frac{(\alpha_+-\alpha_-)^2}{d_2(1)d_2(2)},\quad D_{1,2}=\frac{(a^{\dagger}(1)+b)(a^{\dagger}(2)+b)}{d_1(1)d_1(2)}\pm\frac{(a^{\dagger}(1)-b)(a^{\dagger}(2)-b)}{d_2(1)d_2(2)},$$

$$\eth_3=(\alpha_+^2-\alpha_-^2)\left[\frac{1}{d_1(2)d_2(1)}+\frac{1}{d_1(1)d_2(2)}\right],\quad D_3=\frac{(a^{\dagger}(1)+b)(a^{\dagger}(2)-b)}{d_1(1)d_2(2)}+\frac{(a^{\dagger}(1)-b)(a^{\dagger}(2)+b)}{d_2(1)d_1(2)}\ .$$

By diagonalizing the quadratic form we get its canonical form (here $u_{ia}=\text{Re}c_{ia}$, $v_{ia}=\text{Im}c_{ia}$, and the summation is with respect to $p_1+p_2=p$):

$$S_h=\left[g^{-1}+\frac{2Z^2}{\beta V}\sum D_3\cos^2\theta\right](u_{33}^2+v_{33}^2)+\left[g^{-1}+\frac{2Z^2}{\beta V}\sum(D_1+D_2)\cos^2\theta\right][(u_{31}+v_{32})^2+(u_{32}+v_{31})^2]$$
$$+\left[g^{-1}+\frac{2Z^2}{\beta V}\sum(D_1-D_2)\cos^2\theta\right][(u_{31}-v_{32})^2+(u_{32}-v_{31})^2]$$
$$+\left[g^{-1}+\frac{Z^2}{\beta V}\sum D_3\sin^2\theta\right][(v_{31}+u_{23})^2+(u_{13}-v_{23})^2]+\left[g^{-1}+\frac{Z^2}{\beta V}\sum(-\Delta^2\sin^2\theta\eth_3+D_3)\sin^2\theta\right](u_{13}+v_{23})^2$$
$$+\left[g^{-1}+\frac{Z^2}{\beta V}\sum(\Delta^2\sin^2\theta\eth_3+D_3)\sin^2\theta\right](v_{13}-u_{23})^2$$
$$+\left[g^{-1}+\frac{Z^2}{\beta V}\sum(D_1+D_2)\sin^2\theta\right][(u_{12}+v_{11}+u_{21}+v_{21})^2+(u_{11}+v_{12}-u_{22}-v_{21})^2]$$
$$+\left[g^{-1}+\frac{Z^2}{\beta V}\sum(D_1-D_2)\sin^2\theta\right][(u_{12}-v_{11}-u_{21}+v_{22})^2+(u_{11}-v_{12}+u_{22}-v_{21})^2]$$
$$+\left[g^{-1}+\frac{Z^2}{\beta V}\sum\sin^2\theta[D_1+D_2-\Delta^2\sin^2\theta(\eth_1+\eth_2)]\right](u_{12}+v_{11}-u_{21}-v_{22})^2$$
$$+\left[g^{-1}+\frac{Z^2}{\beta V}\sum\sin^2\theta[D_1-D_2-\Delta^2\sin^2\theta(\eth_1-\eth_2)]\right](u_{11}-v_{11}+u_{21}-u_{22})^2$$
$$+\left[g^{-1}+\frac{Z^2}{\beta V}\sum\sin^2\theta[D_1+D_2+\Delta^2\sin^2\theta(\eth_1+\eth_2)]\right](u_{11}+v_{12}+u_{22}+v_{21})^2$$
$$+\left[g^{-1}+\frac{Z^2}{\beta V}\sum\sin^2\theta[D_1-D_2+\Delta^2\sin^2\theta(\eth_1-\eth_2)]\right](u_{11}-v_{12}-u_{22}+v_{21})^2\ . \qquad \text{(A3)}$$

The equation $\det Q=0$, where Q is the matrix of quadratic form (A3), gives us 18 equations which completely determine 18 collective modes in He^3-A in an arbitrary magnetic field and at arbitrary CE momenta.

APPENDIX B: THE CE SPECTRUM FOR SMALL FIELDS AND ZERO CE MOMENTA

Below we consider the case of small $\mathbf{H}$ and $\mathbf{k}=0$, and calculate the linear correction to the CE spectrum. Setting $\mathbf{k}=0$ and retaining the first-order terms in the field we obtain

$$
\begin{aligned}
&(\mathrm{I})\quad \int_0^\pi (1-(1+2c)I)\cos^2\theta \sin\theta\, d\theta=0,\quad u_{33},\ v_{33}\ ,\\
&\int_0^\pi (-1+(1+2c)I)\cos^2\theta \sin\theta\, d\theta \pm \gamma H \int_0^\pi \frac{4c}{1+4c}(1+2cI)\cos^2\theta \sin^3\theta\, d\theta=0\ ,\\
&u_{31}+u_{32}, u_{32}+v_{31}(u_{31}-v_{32}, u_{32}-v_{31})\ ;\\
&(\mathrm{II})\quad \int_0^\pi I \sin^3\theta\, d\theta=0,\quad u_{23}-v_{13}\ ,\\
&\int_0^\pi I\sin^3\theta\, d\theta \pm \gamma H \int_0^\pi \frac{4c}{1+4c}(2-I)\sin^3\theta\, d\theta=0,\quad u_{11}+v_{12}+u_{22}+v_{21}(u_{11}-v_{12}-u_{22}+v_{21})\ ;\\
&(\mathrm{III})\quad \int_0^\pi (1+2c)I\sin^3\theta\, d\theta=0,\quad v_{13}+u_{23}, u_{13}-v_{23}\ ,\\
&\int_0^\pi (1+2c)I\sin^3\theta\, d\theta \pm \gamma H \int_0^\pi \frac{4c}{1+4c}(1+2cI)\sin^3\theta\, d\theta=0\ ,\\
&u_{12}+v_{11}+u_{21}+v_{22},\quad u_{11}+v_{12}-u_{22}-v_{21}(u_{12}-v_{11}-u_{21}+v_{22},\quad u_{11}-v_{12}+u_{22}-v_{21})\ ;\\
&(\mathrm{IV})\quad \int_0^\pi (1+4c)I\sin^3\theta\, d\theta=0,\quad u_{13}+v_{23}\ ,\\
&\int_0^\pi (1+4c)I\sin^3\theta\, d\theta \pm \gamma H \int_0^\pi 4cI\sin^3\theta\, d\theta=0,\quad u_{12}+v_{11}-u_{21}-v_{22}(u_{12}-v_{11}+u_{21}-v_{22})\ ,
\end{aligned}
\tag{B1}
$$

where

$$I=\frac{1}{\sqrt{1+4c}}\ln\frac{\sqrt{1+4c}+1}{\sqrt{1+4c}-1}\ ,$$

$$c=\frac{\Delta^2\alpha_+^2\sin^2\theta}{\omega^2},\quad \gamma H=\frac{\alpha^-}{\alpha^+}\ .$$

We have four groups of three or six equations. The I and II groups describe the Goldstone modes. For these modes we need to take into account the quadratic field corrections. In Ref. 4 (see part III above) the conclusion has been made that in the presence of a magnetic field three out of nine Goldstone modes become nonphonon because of the appearance of the gap $\sim\mu H$ in their spectrum.

The III and IV groups of equations describe the clapping and pair-breaking modes. If we write these equations as $F_0(E)\pm\gamma H F_1(E)=0$ and try to express E as $E=E_0\pm\gamma H E_1$, we obtain

$$E=E_0\pm\gamma H E_1=E_0\left[1\mp\frac{F_1(E_0)}{\frac{8}{3}-F_1(E_0)}\right]\ . \tag{B2}$$

Using the values of E_0 obtained by Brusov and Popov earlier[2] we obtain for the energies of clapping and pair-breaking modes:

$$
\begin{aligned}
\text{clapping:}\quad & E_1=(1.17-i0.13)\Delta_0\ ,\\
& E_{2,3}=(1.17\pm1.70\gamma H)\Delta_0\\
&\qquad -i(0.13\pm1.20\gamma H)\Delta_0\ ;\\
\text{pair breaking:}\quad & E_1=(1.96-i0.31)\Delta_0\ ,\\
& E_{2,3}=(1.96\pm2.04\gamma H)\Delta_0\\
&\qquad -i(0.31\mp0.06\gamma H)\Delta_0\ .
\end{aligned}
\tag{B3}
$$

[1]P. N. Brusov and V. N. Popov. Zh. Eksp. Teor. Fiz. **78**, 2419 (1980) [Sov. Phys. JETP **51**, 1217 (1980)].

[2]P. N. Brusov and V. N. Popov, Zh. Eksp. Teor. Fiz. **78**, 234 (1980) [Sov. Phys. JETP **51**, 117 (1980)].

[3]P. N. Brusov and V. N. Popov. Zh. Eksp. Teor. Fiz. **79**, 1871 (1980) [Sov. Phys. JETP **52**, 945 (1980)].

[4]E. R. Dobbs, R. Ling, J. Saunders, and W. Wojtanowski, in *Symposium on Quantum Fluids and Solids* (University of Gainesville, Florida, 1989), Proceedings of the Symposium on Quantum Fluids and Solids, AIP Conf. Proc. No. 194, edited by Gary G. Ihas and Uasumasa Takano (AIP, New York, 1989); R. Ling, W. Wojtanowski, J. Saunders, and E. R. Dobbs, J. Low Temp. Phys. **78**, 187 (1990).

[5]V. E. Koch and P. Wölfe, J. Phys. (Paris) Colloq. Suppl. 8, **39**, C6-6 (1978).

[6]P. N. Brusov and V. N. Popov, *The Superfluidity and Collective Properties of Quantum Liquids* (Nauka, Moskow, 1988), p. 215.

[7]R. Combescot, J. Low Temp. Phys. **49**, 295 (1982).

[8]V. Alonso and V. N. Popov, Zh. Eksp. Teor. Fiz. **73**, 1445 (1977) [Sov. Phys. JETP **46**, 760 (1977)].

[9]G. E. Volovik and M. V. Khazan, Zh. Eksp. Teor. Fiz. **85**, 948 (1983) [Sov. Phys. JETP **58**, 551 (1983)].

[10]P. Wölfe, Physica **90B**, 96 (1977).

[11]G. E. Volovik, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 535 (1986) [JETP Lett. **43**, 693 (1986)].

[12]P. N. Brusov, M. Y. Nasten'ka, T. V. Filatova-Novoselova, M. V. Lomakov, and V. N. Popov, in *Symposium on Quantum Fluids and Solids* (Ref. 4), p. 111; Physica B **165-166**, 625 (1990).

[13]M. Y. Nasten'ka and P. N. Brusov, Phys. Lett. **136A**, 321 (1989).

[14]G. E. Gurgenishvili and G. A. Kahradze, *Superfluid Phases of Liquid He*^{3} (Martseba, Tbilisi, 1986), p. 180.

[15]L. Tewordt and N. Schopohl, J. Low Temp. Phys. **34**, 489 (1979).

[16]N. Schopohl, W. Marquardt, and L. Tewordt, J. Low Temp. Phys. **59**, 469 (1985).

[17]P. N. Brusov, J. Low Temp. Phys. **82**, 31 (1991).