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Entropy transport in high-T_c superconductors in the fluctuation regime

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Making use of the expression for the heat current associated with the space-time-dependent order parameter for the *s*-wave superconductor, in the clean limit, we calculate the heat current induced by an electric field. In the absence of a magnetic field we find an extra Peltier coefficient associated with the fluctuations, which diverges logarithmically as the temperature T approaches the transition temperature T_c. In the presence of a magnetic field perpendicular to the *ab* plane, the fluctuation gives rise to the Ettingshausen effect. In a small magnetic field, the corresponding entropy transported by magnetic flux is given by $\langle S_\phi \rangle_f = [2\pi^3\tau T/21\zeta(3)d](h/\varepsilon)(1+2\alpha)^{-1/2}$, where τ and d are the transport lifetime and the interlayer spacing, $\zeta(3)=1.202\ldots$, $h=2e\xi_a^2 B$, $\varepsilon=\ln(T/T_c)$, and $\alpha=2(\xi_c/d)^2\varepsilon^{-1}$. The result is compared with a recent observation of the Ettingshausen effect in a single crystal of $YBa_2Cu_3O_7$.

The fluctuation contribution to the transport properties above the superconducting transition tells a great deal about the nature of the superconductivity. For example, the presence of the anomalous contribution[1] to the electric conductivity will eliminate all but the *s*-wave pairing.[2] Further, the rather short dephasing lifetime τ_ϕ found by analyzing the magnetoconductance[3,4] in the fluctuation regime of single-crystal $YBa_2Cu_3O_7$ compounds will tell much about the possible pairing mechanisms.

In this paper we consider the entropy transport associated with superconducting fluctuations. For simplicity, we study a superconductor in the clean limit. For example, assuming the relation $\tau T=1.35$ found for $YBa_2Cu_3O_7$ near the transition temperature,[5] we obtain $l/\xi_a=4\pi\tau T[7\zeta(3)/2]^{-1/2}\simeq 8.27\gg 1$. This means clearly that the single crystal $YBa_2Cu_3O_7$ is in the clean limit. Further we incorporate in the theory the layered structure of the copper oxides within a model used by Lawrence and Doniach.[6]

In the clean limit, the heat current in the Ginzburg-Landau regime is given by[7]

$$\mathbf{j}^h=\frac{\pi}{8T}N_0 v^2\tau(\omega_1\mathbf{q}_2+\omega_2\mathbf{q}_1)\Delta_1^*\Delta_2\,, \tag{1}$$

where N_0, v, τ, and T are the electron density of states per spin at the Fermi level, the Fermi velocity in the *ab* plane, the scattering lifetime, and the temperature, respectively.

Here ω_i and $\mathbf{q}_i$ are given by

$$\omega_i=i\frac{\partial}{\partial t_i},\quad \mathbf{q}_i=-i\frac{\partial}{\partial x}\pm 2e\mathbf{A}(x_i)\,. \tag{2}$$

On the other hand, the corresponding electric current is given by[8]

$$\mathbf{j}^e=\frac{7\zeta(3)}{(4\pi T)^2}N_0 v^2 e(\mathbf{q}_2-\mathbf{q}_1)\Delta_1^*\Delta_2\,. \tag{3}$$

Here we introduced a proper generalization for the layered compound. Both $\mathbf{j}^h$ and $\mathbf{j}^e$ are assumed to flow in the *ab* plane.

Before going into analysis of the heat current in the fluctuation regime, let us summarize heat currents in the vortex state.

In the vortex state, in an ideal situation, the vortices move perpendicular to the electric field E with velocity $v=E/B$, where B is the magnetic field perpendicular to the *ab* plane. These uniformly moving vortex lines carry entropy S_ϕ per line, giving rise to the Ettingshausen effect.[7]

In the vicinity of $T\simeq T_c$, the corresponding entropy is given by[7]

$$S_\phi=\phi_0^2\sigma|\Delta|^2/2T^2\,, \tag{4}$$

where σ is the electric conductivity in the *ab* plane and Δ is the superconducting order parameter in the vortex state;

$$|\Delta|^2=\left(\frac{7\zeta(3)\phi_0\sigma}{(2\pi T)^2\tau}\right)^{-1}\frac{H_{c2}(T)-B}{\beta_A(2\kappa^2-1)+1}\,, \tag{5}$$

where $\phi_0=\pi/e$ is the flux unit, $\beta_A=1.16$, κ is the Ginzburg-Landau parameter, and H_{c2} (T) is the upper critical field.

Combining Eq. (4) together with Eq. (5), we obtain

$$S_\phi=\frac{2\pi^3\tau}{7\zeta(3)e}\frac{H_{c2}-B}{\beta_A(2\kappa^2-1)+1}\,. \tag{6}$$

The entropy increases linearly with $H_{c2}(T)-B$ as observed experimentally.[9] Further, the transport specific heat per line is

$$C_p=-T\partial S_\phi/\partial T\simeq\frac{2\pi^3\tau T}{7\zeta(3)e}\frac{-dH_{c2}(T)/dT}{\beta_A(2\kappa^2-1)+1}\,. \tag{7}$$

Comparing this value with the observed value $-dH_{c2}/dT=7$ T/K and $C_p\simeq 0.5\times 10^{-12}$ J/Km, we obtain $\kappa\simeq 600$. On the other hand, from $\lambda_a=1400$ Å (Refs. 10-12) and $\xi_a=16$ Å (Ref. 13) we obtain $\kappa\simeq 88$. Therefore, it appears that the observed entropy in a single crystal $YBa_2Cu_3O_7$ is approximately a factor of 50 smaller than that expected from the clean limit theory. A possible resolution of this discrepancy is that the system is not in the clean limit.

Now in the fluctuation regime let us first consider the heat current parallel to the electric field. The excess heat current due to the fluctuation is given by

$$j_x^h/E = \frac{\pi}{4T} N_0 v^2 \tau \left(\frac{14\zeta(3) N_0 v^2 e T}{(4\pi T)^2} \right)^{-1} \int_0^{2\pi d^{-1}} \frac{dk}{2\pi} \int \frac{d^2 q}{(2\pi)^2} q_x^2 N_0^{-2} \{\varepsilon + \xi_a^2 [q^2 + K(1 - \cos kd)]\}^{-2}$$
$$= \frac{(2\pi T)^2 \tau e}{7\zeta(3) d} \ln[2/\varepsilon(1+\alpha+\sqrt{1+2\alpha})] , \quad (8)$$

where $\varepsilon = \ln(T/T_c)$, $K = 2(\xi_c/\xi_a d)^2$, $\alpha = 2(\xi_c/d)^2 \varepsilon^{-1}$, and $\zeta(3) = 1.202\ldots$. Here d is the interlayer distance and ξ_a and ξ_c are the coherence lengths in the plane and perpendicular to the plane, respectively. In deriving Eq. (8) we neglected j_x^h associated with the anomalous term, since this term scales exactly with the one in the electric conductivity.

The resulting Peltier coefficient is given by

$$\Pi_f(\varepsilon, 0) = \frac{(2\pi T)^2 \tau e}{7\zeta(3) d\sigma} \ln[2/\varepsilon(1+\alpha+\sqrt{1+2\alpha})] , \quad (9)$$

where σ is the electrical conductivity in the ab plane. Compared with the excess electrical conductivity, the present contribution is much smaller but perhaps it is observable in high-T_c oxides.

In the presence of a magnetic field perpendicular to the ab plane, the integral over $\mathbf{q}$ has to be replaced by a sum over the Landau levels. After summation,

$$\Pi_f(\varepsilon, B) = \frac{(2\pi T)^2 \tau e}{7\zeta(3) d\sigma} \int_0^{2\pi} \frac{d\phi}{2\pi} [-\ln(2h) - \psi(\tfrac{1}{2} + A)] \quad (10)$$

where

$$A = \frac{\varepsilon}{2h}[1 + \alpha(1 - \cos\phi)] , \quad (11)$$

$h = 2e\xi_a^2 B$, and $\psi(z)$ is the digamma function. Equation (10) has following asymptotic expressions

$$\Pi_f(\varepsilon, B) = \frac{(2\pi T)^2 \tau e}{7\zeta(3) d\sigma} \times \begin{cases} \left[\ln[2/\varepsilon(1+\alpha+\sqrt{1+2\alpha})] - \frac{1}{6}\left(\frac{h}{\varepsilon}\right)^2 (1+2\alpha)^{-3/2} \right. \\ \left. \quad + \frac{7}{120}\left(\frac{h}{\varepsilon}\right)^4 (1+\alpha)[2(1+\alpha)^2 + 3\alpha^2](1+2\alpha)^{-7/2} \right] & \text{for } h/\varepsilon \ll 1 , \\ \left[-\ln(2h) - \psi\left(\frac{1}{2} + \frac{\varepsilon}{2h}(1+\alpha)\right) - \frac{1}{4}(\alpha\varepsilon/2h)^2 \psi^{(2)}\left(\frac{1}{2} + \frac{\varepsilon}{2h}(1+\alpha)\right) \right] & \text{for } \varepsilon/h \ll 1 . \end{cases} \quad (12)$$

Second, the fluctuating order parameter gives rise to the Ettingshausen effect. When the electric field is applied in the x direction, we obtain

$$j_y^h = \frac{\pi}{8T} N_0 v^2 \tau \langle (\omega_1 \mathbf{q}_2 + \omega_2 \mathbf{q}_1) \Delta_1^* \Delta_2 \rangle = \frac{8\pi^2 \tau T e^2 E}{7\zeta(3) d} \int_0^{2\pi} \frac{d\phi}{2\pi} A[\psi(\tfrac{1}{2} + A) - \ln A] \quad (13)$$

or the entropy per flux given by

$$(S_\phi)_f = \frac{2\pi^3 \tau T}{21\zeta(3) d} \left(\frac{h}{\varepsilon}\right) (1+2\alpha)^{-1/2} \left[1 - \frac{7}{10}\left(\frac{h}{\varepsilon}\right)^2 [(1+\alpha)^2 + \tfrac{1}{2}\alpha^2](1+2\alpha)^{-2} + \cdots \right] . \quad (14)$$

The fluctuation-induced entropy is proportional to h as observed experimentally.[9] The divergence in ε should show the crossover from two-dimensional (2D) behavior at $T \gg T_c$ to 3D behavior in the vicinity of $T \simeq T_c$, around $\varepsilon = 2(\xi_c/d)^2 \sim 0.1$. However, from experimental data[9] it is difficult to see this crossover. Furthermore the observed entropy varies smoothly from the fluctuation regime to the flux-flow regime [Eq. (6)] without any divergence at $\varepsilon = 0$. Perhaps with the help of more detailed data it will be possible to analyze the fluctuation-induced Ettingshausen effect quantitatively.

In summary, we obtain expressions for both the fluctuation-induced Peltier coefficient and the Ettingshausen coefficient in layered superconductors. The present result describes semiquantitatively the observed Ettingshausen effect in a single crystal of $YBa_2Cu_3O_7$ both above the superconducting transition and in the vortex state.

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