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Experimental consequences of the uniform resonating-valence-bond state

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Our recent work on the effect of gauge-field fluctuations on the uniform resonating-valence-bond state is extended to the case of strong coupling, where the Boltzmann description breaks down. The important Feynman paths are found to be modified in shape so that almost retraceable paths dominate. We find features in the resistivity and Hall constant which improve the agreement with experiments. We also discuss a variety of experimental results such as the anisotropy in magnetic susceptibility and quasielastic Raman scattering, which may provide more information on the fluctuating gauge field.

Recently, the importance of the gauge field has been recognized by several authors and the unusual properties of the high-T_c materials are analyzed from this viewpoint.[1-4] The present authors[2] discussed the normal state properties of the uniform resonating-valence-bond (RVB) state by using the following effective Lagrangian density:

$$L(r,\tau)=\sum_\sigma f_\sigma^*(r,\tau)\left(\partial_\tau-a_0-\mu_F+\frac{1}{2m_F}(-i\nabla-\mathbf{a})^2\right)f_\sigma(r,\tau)+b^*(r,\tau)\left(\partial_\tau-a_0-\mu_B+\frac{1}{2m}(-i\nabla-\mathbf{a})^2\right)b(r,\tau)\,, \tag{1}$$

where f_σ and f_σ^* are the Grassman variables for the fermions with spin σ while b and b^* are the c numbers for the slave bosons. The spatial components of the gauge field $\mathbf{a}$ come from the phase of the RVB order parameters. The flux associated with the gauge field $\mathbf{h}=\nabla\times\mathbf{a}$ is proportional to the spin chirality $\mathbf{S}_1\cdot\mathbf{S}_2\times\mathbf{S}_3$. If a gauge-charged particle goes around a closed loop, the quantum amplitude acquires the phase factor $\exp[i\Phi]$ where Φ is the flux penetrating the area enclosed by the loop. The time component a_0 comes from the Lagrange multiplier and imposes the following local constraint for each site i:

$$\sum_\sigma f_{i\sigma}^* f_{i\sigma}+b_i^* b_i=1\,. \tag{2}$$

Therefore, the gauge field represents the fluctuation of the spin chirality and the local constraint. The latter gives the composition laws for physical quantities. For example, the resistivity is given by $\rho=\rho_F+\rho_B$ where ρ_F and ρ_B are the resistivity of the fermions and bosons, respectively.[5] An external magnetic field H is screened by the gauge field to satisfy the local current constraint $J_F^d+J_B^d=0$, where J_F^d (J_B^d) is the diamagnetic current of the fermions (bosons), which is proportional to the Landau diamagnetic susceptibility χ_F (χ_B). The Hall coefficient R_H is given by

$$R_H=\frac{\chi_F R_H^B+\chi_B R_H^F}{\chi_F+\chi_B}\,, \tag{3}$$

where R_H^F (R_H^B) is the Hall coefficient of the fermions (bosons). The magnetoresistance is given similarly by

$$\Delta\rho=\frac{\chi_F^2\Delta\rho_B+\chi_B^2\Delta\rho_F}{(\chi_F+\chi_B)^2}\,. \tag{4}$$

When the normal state of the high-T_c materials is identified as the temperature region where the slave bosons are not condensed and obey the Boltzmann distribution, χ_B and ρ_B^{-1} are much smaller than χ_F and ρ_F^{-1}, so that ρ and R_H are dominated by those of the bosons. We are therefore interested in the bosons coupled with the gauge field described by the following effective action S:

$$S=\int_0^\beta d\tau\int dr\, b^*(r,\tau)\left(\partial_\tau-a_0-\mu_B+\frac{1}{2m}(-i\nabla-\mathbf{a})^2\right)b(r,\tau)+S_{\text{gauge}}\,. \tag{5}$$

The action for the gauge field S_{gauge} is derived by integrating out the fermions and bosons as follows.

$$S_{\text{gauge}}=\sum_{q,\omega_n}\left(\chi_d q^2+\frac{|\omega_n|}{q}\right)\left(\delta_{\alpha\beta}-\frac{q_\alpha q_\beta}{q^2}\right)a_\alpha(q,\omega_n)a_\beta(-q,-\omega_n)\,, \tag{6}$$

where $\chi_d=\chi_F+\chi_B$, and only the transverse part of the gauge field is taken into account. In Ref. 2, we discussed this problem in terms of the conventional Boltzmann transport theory, and the results are $\rho_B\approx T/(x\chi_d)$ and $R_H^B=x^{-1}$. The Boltzmann transport equation, however, is justified only if the mean free path is greater than the thermal length $\lambda=(2\pi/mT)^{1/2}$. In fact, the mean free path l is given by

$$l=v\tau_B\sim\left(\frac{T}{m}\right)^{1/2}\frac{m\chi_d}{T}\sim\frac{\lambda}{g}\,, \tag{7}$$

where v is the thermal velocity, τ_B is the transport time of the boson, and we have introduced the dimensionless coupling constant

$$g=(m\chi_d)^{-1}. \tag{8}$$

We estimate $\chi_F \sim (1-x)/m_F$ and $\chi_B \sim T_{\rm BE}^{(0)}/Tm$ where $T_{\rm BE}^{(0)} \approx 2\pi x/m$. In the mean-field treatment of the t-J model, $m_F^{-1} \approx J$ and $m^{-1} \approx t$, so that g is temperature dependent and is of order unity near the condensation temperature. While we expect significant renormalization of the continuum parameters m and m_F, we expect g to be ≥ 1 in the normal state so that the applicability of the Boltzmann theory is questionable. In this paper we examine the strong-coupling limit $g \gg 1$ in order to estimate the correct behavior in the intermediate coupling regime by interpolation.

We use the trajectory path integral picture as in Ref. 2. The boson polarization $\pi_B(\mathbf{r},\tau)$ is expressed by the integral over the closed path passing the two points $(\mathbf{0},0)$ and $(\mathbf{r},\tau)$, whose projection onto the real space is shown in Fig. 1.

$$\pi_B(\mathbf{r},\tau)=\int \mathcal{D}\mathbf{r}_1(t)\left\langle \exp\left[-\frac{m}{2}\int_0^{2\tau}[\dot{\mathbf{r}}_1(t)]^2dt+i\Phi[\mathbf{r}_1]\right]\right\rangle_{\rm a}, \tag{9}$$

where the path $\mathbf{r}_1(t)$ obeys $\mathbf{r}_1(0)=\mathbf{r}_1(2\tau)=0$ and $\mathbf{r}_1(\tau)=\mathbf{r}$, and

$$\Phi[\mathbf{r}_1]=\oint \mathbf{a}[r_1(t)]\cdot\dot{\mathbf{r}}_1(t)dt=\int dA\,h(\mathbf{r}) \tag{10}$$

is the flux penetrating the area A enclosed by the path $\mathbf{r}_1(t)$. In Eq. (9) $\langle\cdots\rangle_{\rm a}$ means the average over the gauge-field fluctuations.

From Eq. (6), the fluctuation of the magnetic field $h=\partial_x a_y-\partial_y a_x$ is described by[2]

$$\langle h_q h_{-q}\rangle=\int d\omega\frac{1}{e^{\beta\omega}-1}q^2\,{\rm Im}\frac{-1}{i\omega/q+\chi_d q^2}=\frac{T}{\chi_d}, \tag{11}$$

for $q<q_0$ with $q_0 \approx (T/\chi_d)^{1/3}$. When the linear dimension of the area A for the integral Eq. (10) is larger than q_0^{-1}, the integral is the sum of many independent flux fluctuations $\Phi[\mathbf{r}_1]=\sum_{i=1}^N \Phi_i$. Upon averaging over the gauge field $\mathbf{a}$, the Debye-Waller factor D for a given path $\mathbf{r}_1$ is

$$D=\langle e^{i\Phi[\mathbf{r}_1]}\rangle_{\rm a}=\exp(-\tfrac{1}{2}\langle\Phi[\mathbf{r}_1]^2\rangle_{\rm a}). \tag{12}$$

We obtain $\langle\Phi^2[\mathbf{r}_1]\rangle=\sum_{i=1}^N\langle\Phi_i^2\rangle=N\langle\Phi_i^2\rangle$ where the number of independent flux fluctuations N is roughly Aq_0^2, while $\langle\Phi_i^2\rangle$ is estimated as

$$\langle\Phi_i^2\rangle \cong q_0^{-4}\int_0^{q_0} q\,dq\,T/\chi_d=q_0^{-2}T/\chi_d.$$

The Debye-Waller factor D in Eq. (12) is estimated as

$$D\cong e^{-AT/\chi_d}\cong e^{-gA/\lambda^2}. \tag{13}$$

In summary, we have the expression for the boson polarization $\pi_B(\mathbf{r},\tau)$ as follows:

$$\pi_B(\mathbf{r},\tau)=\int \mathcal{D}\mathbf{r}_1(t)\exp\left(-\tfrac{1}{2}m\int[\dot{\mathbf{r}}_1(t)]^2dt-gA/\lambda^2\right). \tag{14}$$

The first term in the exponent of Eq. (14) expresses the entropy force which makes the particle diffuse, while the second term is an energy cost proportional to area A which the trajectory encloses. For the unperturbed trajectories [Fig. 1(a)], the first term is of the order of 1 and the area $A_0 \sim r(\tau/m)^{1/2}$. Therefore, when $g \ll 1$, the first term dominates the second term which is of order g when we put A_0 into A with $r\sim\lambda$ and $\tau\sim\beta$. In this case, the trajectories are unperturbed, and we obtain

$$\pi_B(\mathbf{r},\tau)=\pi_B^{(0)}(\mathbf{r},\tau)\exp\left[-gTmr\left(\frac{\tau}{m}\right)^{1/2}\right], \tag{15}$$

where $\pi_B^{(0)}(\mathbf{r},\tau)=\tau^{-2}\exp(-2mr^2/\tau)$. Thus the typical $r\sim(\tau/m)^{1/2}$, and upon putting this relation into Eq. (15) we obtain the mean free time τ_B of the boson as

$$\tau_B^{-1}\cong gT, \tag{16}$$

which is in agreement with the results by Boltzmann transport theory in Eq. (7).

Next we consider the opposite limit $g \gg 1$. In this case, loops with areas of order A_0 are suppressed by the factor $\exp(-g)$. Only loops with area A of order λ^2/g contribute. This reduction in area A means that the almost retracing paths give dominant contribution as shown in Fig. 1(b). Even if we restrict to the exactly retraceable path so that $\mathbf{r}_1(\tau+t)=\mathbf{r}_1(\tau-t)$ for $0<t<\tau$, we still have $\pi_B(\mathbf{r},\tau)$ given by

$$\pi_B(\mathbf{r},\tau)=\int \mathcal{D}\mathbf{r}_1(t)\exp\left[-2\frac{m}{2}\int_0^{\tau}[\dot{\mathbf{r}}_1(t)]^2dt\right],$$

where $\mathbf{r}_1(\tau)$ is subject to the condition $\mathbf{r}_1(0)=0$, $\mathbf{r}_1(\tau)=\mathbf{r}$. This is the expression for the propagator $G_B(\mathbf{r},\tau)$ of a single particle with the mass $2m$. Therefore, the characteristic length scale for $\pi_B(\mathbf{r},\tau)$ remains λ. In general, we can choose an arbitrary function $f(t)$ with $f(0)$ and $f(\tau)=\tau$, and make a path by $\mathbf{r}_1(\tau+t)=\mathbf{r}_1[\tau-f(t)]$ for $0<t<\tau$, which is also retraceable when projected onto the real space. Therefore, $\pi_B(\mathbf{r},\tau)$ is different from G_B of the free boson, but its mean free path remains of order λ. Since the area of the sausage-shaped loop in Fig. 1(b) is of order λ^2/g, its width is of order λ/g.

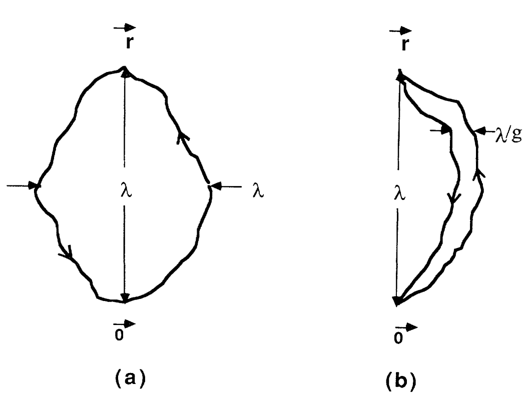

FIG. 1. Typical Feynman paths for the boson polarization π_B projected onto the two-dimensional plane for (a) weak-coupling case $g \ll 1$, and (b) strong-coupling case $g \gg 1$. λ is the thermal length.

Combining the above discussion, we now interpolate the weak- and strong-coupling limits as follows. The transport mean free time τ_B of the boson which appears in the resistivity behaves as $1/gT$ for $g \ll 1$, and remains of order $1/T$ for $g \gtrsim 1$. Therefore, the resistivity ρ_B is given by

$$\rho_B \approx \frac{mT}{x}\min(g,1) . \tag{17}$$

The saturation effect in Eq. (17) ensures the T-linear temperature dependence of the resistivity in spite of the temperature-dependent g as long as $g \gtrsim 1$. Experimentally,[6] τ_B^{-1} estimated from the infrared optical absorption is about 2 T, which suggests that $g \gtrsim 1$.

Area A enclosed by a path is $A = A_0 \sim \lambda^2$ for $g \ll 1$ and $A = A_0/g \sim \lambda^2/g$ for $g \gg 1$ which is interpolated as

$$A \sim \frac{A_0}{1+g} . \tag{18}$$

This reduction of area A reduced the response of the boson to the external magnetic field H which always appears as the flux $\phi = HA$. Therefore, the reduction of the area is taken into account by reducing H to $H_{\text{eff}} = H/(1+g)$ as a rough estimate. For example, since σ_{xy} is proportional to H, which should now be replaced by H_{eff}, we obtain

$$R_H^B = \frac{R_H^{B(0)}}{1+g} = \frac{1}{x}\frac{1}{1+g} . \tag{19}$$

Landau's diamagnetic susceptibility χ_B is also estimated as follows. The increase in the free energy $\Delta F = \frac{1}{2}\chi_B H^2 = \frac{1}{2}\chi_B^{(0)} H_{\text{eff}}^2$, which gives

$$\chi_B = \frac{\chi_B^{(0)}}{(1+g)^2} , \tag{20}$$

where $\chi_B^{(0)} \approx T_{\text{BE}}^{(0)}/mT$ is the susceptibility for noninteracting bosons. Equation (20) can be written as

$$\chi_B \approx \frac{\chi_{\text{BE}}^{(0)}}{m(1+g)^2 T} \approx \frac{T_{\text{BE}}}{mT} ,$$

which means that the Bose-Einstein temperature T_{BE} is suppressed[7] by the factor $(1+g)^2$ from the noninteracting value $T_{\text{BE}}^{(0)}$ due to the scattering by the gauge field.

The coupling constant g is inversely proportional to $\chi_d = \chi_F + \chi_B$, which is temperature dependent and should be determined self-consistently. We expect χ_B to cross over from T_{BE}/mT to $(x/m)\xi_B^2$ below T_{BE}, where ξ_B is the boson coherence length. This is because on distance scale less than ξ_B, the boson behaves like a perfect diamagnet. χ_F is expected to be T independent below a temperature scale of J. However, if pairing correlation with the order parameter $\Delta_{ij} = \langle f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger - f_{i\downarrow}^\dagger f_{j\uparrow}^\dagger \rangle$ is taken into account, χ_F will also grow as ξ_F^2, where ξ_F is the pairing coherence length below some temperature scale $T_\Delta \ll J$. As we argued in Ref. 2, if the suppression of T_Δ and T_{BE} from their bare values due to gauge-field fluctuations is strong enough, there will be a single transition where T_Δ and T_{BE} coincide with the superconductivity transition temperature T_c. This is because the onset of $\langle ff \rangle$ or $\langle b \rangle$ makes χ_d infinite which in turn suppresses the pair-breaking effect of the gauge field. In this case χ_B and χ_F both diverge with a finite ratio χ_B/χ_F as $T \to T_c$ and g scales to weak coupling. The fluctuation phenomena near T_c are then similar to conventional Ginsburg-Landau superconductivity theory.

Now we can discuss the temperature dependence of various physical quantities in the normal state. In Ref. 2, we showed that the resistivity is $\sim (x\chi_d)^{-1}T$. While the leading T dependence is linear in T, the temperature dependence of χ_d is probably sufficiently strong to spoil agreement with experiment. This deficiency is now overcome by our strong-coupling theory, where ρ is linear in T with a coefficient which is T independent even as $g(T)$ changes from strong coupling to of order unity. Near T_c, ρ approaches zero as in usual superconductors. The Hall coefficient R_H is obtained from Eqs. (3) and (19),

$$R_H = \frac{1}{x}\frac{\chi_F}{m_B^{-1} + \chi_F + \chi_B} + R_H^F \frac{\chi_B}{\chi_F + \chi_B} . \tag{21}$$

The fermion Hall coefficient R_H^F depends on the details of the band structure, but its magnitude is expected to be smaller than x^{-1} so that we may approximate R_H with only the first term in Eq. (21),

$$R_H = \frac{1}{x}\frac{\chi_F}{m_B^{-1} + \chi_F + \chi_B} .$$

For $T \gg T_c$, $\chi_B \ll \chi_F$ and we obtain $R_H \approx x^{-1}/(1+g)$, in contrast with the weak-coupling theory[2] where $R_H \to x^{-1}$ for large T. Experimentally,[8] the high temperature limit of R_H is reduced from x^{-1} by a factor of 2 to 4 depending on the material, in agreement with our expectation of intermediate coupling g. As T is reduced towards T_c, $\chi_F + \chi_B \gg m^{-1}$ and R_H approaches a finite value $x^{-1}(1 + \chi_B/\chi_F)^{-1}$. In Ref. 2 we assumed that χ_F is T independent, which lead R_H to decrease with decreasing T, in disagreement with experiment. Including the T dependence of χ_F due to pairing fluctuations may reverse this trend, but we have not attempted any concrete calculation.

As we can see the T dependence of χ_B and χ_F plays a crucial role in our theory and it will be desirable to measure them directly from experiments. So now we discuss a variety of experiments in addition to R_H where different combinations of χ_F and χ_B appear. The first one is the contribution to the physical diamagnetic susceptibility χ_d^{ph} which is obtained using arguments similar to the Ioffe-Larkin resistivity formula

$$\chi_d^{\text{ph}} = \frac{\chi_F \chi_B}{\chi_F + \chi_B} . \tag{22}$$

For $T_c \ll T \ll J$, $\chi_F \gg \chi_B$, and $\chi_d^{\text{ph}} \approx \chi_B(T)$. Near T_c, χ_d^{ph} diverges as in the usual fluctuation diamagnetism in superconductors. Experimentally, χ_d^{ph} can be extracted from susceptibility measurements by noting that χ_d^{ph} is anisotropic and depends only on the field component perpendicular to the plane whereas the usual spin contribution is isotropic. A temperature-dependent anisotropic component of χ has been observed experimentally,[9,10] which should be analyzed using Eq. (22) to provide some information about $\chi_B(T)$ and $\chi_F(T)$.

The magnetoresistance ρ given in Eq. (4) is also suppressed by the strong-coupling effect. The boson magnetoresistance $\Delta\rho_B$ is suppressed by the factor $(1+g)^2$ from its weak-coupling value, because $\Delta\rho_B$ is proportional to H^2. The coefficient of $\Delta\rho_F$, i.e., χ_B^2/χ_d^2 is also suppressed by the factor $(1+g)^4$. Then, we expect an order-of-magnitude smaller magnetoresistance than is expected from the Boltzmann transport theory for $T \gg T_c$. As $T \rightarrow T_c$, however, the suppression becomes less and less and finally $\Delta\rho$ is governed by the contribution from the fluctuation effect of the superconductivity.

In the above discussion, we concentrate on the coupling between bosons and gauge field, because many transport properties are governed mainly by the boson system at least for $T \gg T_c$. The thermal conductivity K, however, is dominated by that of the fermion system K_F. Unlike the boson case, the coupling between fermions and gauge field can be treated by the Boltzmann transport theory, but the inelasticity of the scattering becomes more important because the energy exchange is of the order of T. Ioffe and Kotliar[4] pointed out that in contrast to the current relaxation even the small momentum-transfer scattering is effective to the energy relaxation. This effect reduces the thermal conductivity from that predicted by the Wiedemann-Franz law $(\pi^2/3)(k/e)^2\sigma_F T$ for the fermion system leading to $K \propto T^{1/3}$. It is worth noting that the prefactor is

$$K_F \propto \chi_d^{-2/3} T^{1/3} , \tag{23}$$

which may provide information on the T dependence of χ_d.

We next point out that the quasielastic component of the Raman scattering provides a direct measurement of χ_d. Recently, Shastry and Shraiman[11] developed a theory of Raman scattering in the Mott-Hubbard system, and pointed out that in the B_{1g} and B_{2g} geometry, the Raman tensor includes the spin chirality operator. Then, we predict from Eq. (11) the scattering intensity to be

$$I(\omega) \propto \langle h_{q,\omega} h_{-q,-\omega} \rangle = \frac{Tq^3}{\omega^2 + (\chi_d q^3)^2} ,$$

where q is the momentum difference of the incident and scattered light, and is of the order of 10^4 cm^{-1}. The characteristic Stokes shift ω is about $\chi_d q^3$, which is estimated to be around 2×10^4 s^{-1}, too small to be resolved experimentally. It may be possible to measure the frequency integrated scattered light, in which case we predict

$$\int d\omega\, I(\omega) \propto \frac{T}{\chi_d} . \tag{24}$$

From Eq. (24), deviation from the T-linear behavior can give the temperature dependence of χ_d.

Finally, we mention that the fluctuating chirality produces fluctuating transverse current in the layers. The resulting magnetic fields can relax the nuclear spin with interesting temperature dependence related to χ_F and χ_B.[12]

In summary, we have discussed the temperature dependence of various physical quantities taking into account the strong-coupling effect of the bosons and gauge field. The experiments thus far seem to be consistent with the predictions, but further experimental studies are desired to determine χ_F and χ_B.

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