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Theory of charge-density-wave tunneling

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The quantized sine-Gordon system is utilized to examine collective charge transport by moving charge-density waves (CDW's). The presence of an energy gap, or pinning gap, in the collective excitation (quantum soliton) spectrum of the quantized system implies that creation of quantum soliton-antisoliton pairs by Zener tunneling is required in order to induce CDW depinning by an applied electric field, thus providing a justification for the semiconductor model originally proposed by Bardeen. It is argued that the three-dimensional coherence of a real CDW prevents thermal excitations across the pinning gap, but allows Zener tunneling to take place. Finally, the striking similarities between the current-voltage characteristics and coherent oscillations observed in CDW's and the phenomena of Coulomb blockade and time-correlated tunneling events observed in series arrays of small tunnel junctions are discussed.

I. INTRODUCTION

The remarkable non-Ohmic transport properties of inorganic linear chain compounds, due to collective charge-density-wave (CDW) depinning,[1–3] has eluded complete understanding because of unresolved fundamental issues. One extremely important issue is whether a completely classical description suffices to fully characterize CDW depinning and dynamics,[4–9] or whether an intrinsically quantum-mechanical description is required,[10] as suggested by a number of experiments.[11–16] Another question is whether CDW pinning is more accurately described as weak pinning,[17] where fluctuations in the impurity concentration pin the CDW phase over distances that are long compared to the amplitude coherence length, as indicated by recent experiments on $NbSe_3$,[18] or strong pinning,[19] where a single impurity is sufficient to pin the CDW phase at a given point.

In this paper we plan to address the first issue by examinging the well-known relationship between the quantum sine-Gordon system and the massive Thirring model,[20] which describes a $(1+1)$-dimensional system of interacting fermions. We will show that, for this idealized case chosen for its mathematical tractability, the existence of an energy gap, or pinning gap, in the spectrum of charged collective excitations (quantum solitons) implies that the creation of quantum soliton-antisoliton pairs by Zener tunneling is required in order to induce electrical charge transport. It will be shown that the quantum solitons of the CDW system are essentially dressed electrons that incorporate the strong coupling of the bare electrons to the CDW lattice distortion, and that CDW pinning cannot be simply treated as a perturbation, as implied by purely classical models, but must be included to zero order in the model Hamiltonian. We will then present heuristic arguments that the three-dimensional phase coherence of a real CDW prevents thermal excitations across the pinning gap, but still allows Zener tunneling to take place. However, we believe that a complete, quantum-mechanical treatment of the dynamics of a three-dimensional CDW system is an extremely important topic of future theoretical work. Finally, we will discuss a possible analogy between CDW tunneling and time-correlated tunneling events in series arrays of small tunnel junctions exhibiting Coulomb blockade.

II. QUANTIZATION OF THE PHASE HAMILTONIAN

The theoretical description adopted here will be based upon the formulation originally proposed by Rice, Bishop, Krumhansl, and Trullinger,[21] and further developed by Maki,[22] Kurihara and Furuya,[23] and Tucker and Miller.[24] The Hamiltonian for the CDW sine-Gordon system corresponding to a single chain may be written as

$$H=\int_0^L dx\left[\frac{1}{2D}\Pi^2(x)+\tfrac{1}{2}Dc_0^2\phi_x^2(x)+D\omega_p^2[1-\cos\phi(x)]\right]. \tag{2.1}$$

Here the phase $\phi(x,t)$ is measured relative to the undistorted CDW, $\Pi(x,t)=D\partial\phi/\partial t$ is the canonical momentum density, $c_0=(m/M^*)^{1/2}v_F$ is the phason velocity, and ω_p is the pinning frequency. The parameter $D=(\hbar/4\pi v_F)(M^*/m)$ corresponds to a single chain and counts only one spin. We shall treat the two spin components as independent throughout the remainder of this discussion. The Frohlich mass $M^*=M_F+m$ reflects the fact that the electrons are tightly coupled to the macroscopically occupied phonons of the lattice distortion in a CDW system. The charge density $\rho(x)$, averaged over several CDW wavelengths, and current density $j(x)$ are given, respectively, by

$$\rho(x)=\frac{e}{2\pi}\frac{\partial\phi}{\partial x} \tag{2.2}$$

$$j(x)=\frac{e}{2\pi D}\Pi(x) . \tag{2.3}$$

A uniform applied electric field E couples to the charge density $\rho(x)$, resulting in an additional contribution to the Hamiltonian of the form $H'=Ex\rho(x)=(eEx/2\pi)\partial\phi/\partial x$. One can derive an equation of motion for the phase operator simply by employing the Heisenberg equation $\partial\Pi/\partial t=D\partial^2\phi/\partial t^2=(1/i\hbar)[\Pi,H]$, with the result

$$\frac{\partial^2\phi}{\partial t^2}-c_0^2\frac{\partial^2\phi}{\partial x^2}+\omega_p^2\sin\phi=\frac{e}{D}E . \tag{2.4}$$

The above equation is formally identical to the classical equation of motion in the absence of dissipation, as is the case with *any* operator equation of motion derived within the Heisenberg picture. Nevertheless, it is not possible to solve (2.4) simply by treating the phase as a c number, because the phase operator does not commute with itself for unequal times, i.e., $[\phi(t),\phi(t')]\neq 0$ for $t\neq t'$. This is one reason why it is difficult, in general, to solve quantum-mechanical problems simply by solving the equations of motion for the operators.

The Hamiltonian of (2.1) does not include amplitude fluctuations in the CDW order parameter, and is thus valid only for phase variations that are small within an amplitude coherence length. We therefore employ the following commutation relations for the quantized phase and momentum operators:

$$[\phi(x),\Pi(x')]=\frac{i\hbar}{\pi}\frac{\epsilon}{\epsilon^2+(x-x')^2} , \tag{2.5}$$

$$[\phi(x),\phi(x')]=[\Pi(x),\Pi(x')]=0 , \tag{2.6}$$

where the parameter ϵ represents the CDW amplitude coherence length. The term on the right-hand side of (2.5) reduces to $\delta(x-x')$ in the limit as $\epsilon\to 0$. The self-consistency of the definition of the momentum operator can be checked with use of the Heisenberg equation

$$D\frac{\partial\phi}{\partial t}=D\frac{1}{i\hbar}[\phi(x),H]=\int_0^L dx'\Pi(x')\frac{1}{\pi}\frac{\epsilon}{\epsilon^2+(x-x')^2} , \tag{2.7}$$

which reduces to $D\partial\phi/\partial t=\Pi(x)$ in the limit $\epsilon\to 0$. Equations (2.5) and (2.7) thus imply that our quantization procedure will be valid over length scales that are long compared to ϵ.

Following Takada and Misawa,[25] the phase operator is expressed in terms of boson creation and annihilation operators:

$$\phi(x)=i\sum_{k>0}f_k[(b_{1,k}^\dagger+b_{2,-k})e^{-ikx}-(b_{1,k}+b_{2,-k}^\dagger)e^{ikx}]e^{-\epsilon k/2} \tag{2.8}$$

where

$$[b_{1,k},b_{1,k}^\dagger]=1=[b_{2,-k},b_{2,-k}^\dagger] , \tag{2.9}$$

all other commutators being zero.

It is convenient to separate the Hamiltonian into two contributions $H=H_0+H_1$, where H_0 is chosen to be diagonalizable in the Bosonic representation:

$$:H_0:=\sum_{k>0}\hbar\omega_k(b_{1,k}^\dagger b_{1,k}+b_{2,-k}^\dagger b_{2,-k})e^{-\epsilon k} , \tag{2.10}$$

and the colons : : denote normal ordering with respect to the boson operators. We will consider two different choices for H_0: (i) the unpinned Hamiltonian, and (ii) the unpinned Hamiltonian plus the ϕ^2 contribution to the $(1-\cos\phi)$ pinning term. In either case, the phase operator commutes with H_1, so that

$$\frac{\partial\phi}{\partial t}=\frac{1}{i\hbar}[\phi(x),H_0] . \tag{2.11}$$

Combining (2.7), (2.8) and (2.11) yields an expression for the momentum operator:

$$\Pi(x)=-\sum_{k>0}f_k[(b_{1,k}^\dagger-b_{2,-k})e^{-ikx}-(b_{1,k}-b_{2,-k}^\dagger)e^{ikx}]e^{-\epsilon k/2} . \tag{2.12}$$

The values of the coefficients f_k which make H_0 diagonal are found to be given by

$$f_k=\sqrt{\hbar/2DL\omega_k} . \tag{2.13}$$

III. BOSONIC AND FERMIONIC REPRESENTATIONS

The quantum sine-Gordon system is well known to be equivalent to the massive Thirring model,[20] which describes a system of interacting fermions. For the case of a CDW these fermions represent dressed electrons incorporating the motion of the lattice distortion. In the bosonic representation, the diagonal form of the unpinned Hamiltonian, obtained from the first two terms of (2.1), is given by (2.10). The excitation energies are $\hbar\omega_k=\hbar c_0 k$, representing the energies of phase excitations, or phasons, originally proposed by Overhauser,[26] and subsequently formulated by Lee, Rice, and Anderson.[27] In this case the phasons are the Goldstone bosons of the system. Substituting $\omega_k=c_0k$ into (2.13) yields $f_k=(m/M^*)^{1/4}\sqrt{2\pi/Lk}$.

The equivalence between the bosonic and fermionic representations for a one-dimensional electron gas was originally demonstrated by Mattis and Lieb.[28] An explicit Bose-Fermi transformation was first developed by Mattis[29] and by Luther and Peschel.[30] Their work caused the genesis of the field of "g-ology," which describes the various scattering processes (forward, backward, umklapp, etc.) in a one-dimensional electron gas.[31] The Bose-Fermi transformation was subsequently applied in the context of field theory by Mandelstam.[32]

Here we follow the procedure outlined by Takada and Misawa[25] in order to convert from the bosonic to the fermionic representation. The calculations are rather lengthy, and are treated in greater detail in Appendix A. It is found that the unpinned Hamiltonian can be expressed in the diagonal form

$$:H_0:=\sum_k{}' \hbar c_0 k(a_{1k}^\dagger a_{1k} - a_{2k}^\dagger a_{2k})e^{-\epsilon|k|} , \quad (3.1)$$

where $\sum'$ signifies that the $k=0$ term is to be excluded from the summation and the fermion operators satisfy the anticommutation rules

$$\{a_{nk}, a_{n'k'}^\dagger\} = \delta_{nn'}\delta_{kk'} ,$$
$$\{a_{nk}, a_{n'k'}\} = \{a_{nk}^\dagger, a_{n'k'}^\dagger\} = 0 . \quad (3.2)$$

In the treatment by Takada and Misawa,[25] a_{1k} and a_{2k} were found to commute rather than anticommute, but it is straightforward to construct anticommuting operators with use of an appropriate transformation, as discussed in Appendix D. The physical interpretation of the fermion operators is clear in the limit $m/M^* \to 1$. In this case the unpinned Hamiltonian becomes

$$:H_0:(M^* \to m) = \sum_k{}' \hbar v_F k(a_{1k}^\dagger a_{1k} - a_{2k}^\dagger a_{2k})e^{-\epsilon|k|} . \quad (3.3)$$

This is simply the Hamiltonian for a free-electron gas, where the "1" and "2" branches refer to electron momentum states $k \pm k_F$ on the right and left sides of the Fermi sea, respectively, and the energies are measured with respect to the Fermi energy. Unfortunately, no such simple correspondence can be established between the fermion operators and the bare electrons for $M^* \gg m$ in the CDW system. Nevertheless, it is possible to obtain physical insight by examining the charge, current, and momentum carried by each fermion. The total charge Q, current I, and momentum P of the system are obtained by integrating over the sample length, and, in the fermionic representation, are found to be given, respectively, by

$$Q = \int_0^L \rho(x)dx = e^* \sum_k{}'(a_{1k}^\dagger a_{1k} + a_{2k}^\dagger a_{2k})e^{-\epsilon|k|} , \quad (3.4)$$

$$I = \int_0^L j(x)dx = e^* c_0 \sum_k{}'(a_{1k}^\dagger a_{1k} - a_{2k}^\dagger a_{2k})e^{-\epsilon|k|} , \quad (3.5)$$

$$P = \hbar k_F \sum_k{}'(a_{1k}^\dagger a_{1k} - a_{2k}^\dagger a_{2k})e^{-\epsilon|k|} , \quad (3.6)$$

where $e^* = (m/M^*)^{1/4}e$. Thus the fermions are particles of effective charge $e^* = (m/M^*)^{1/4}e$, velocity c_0, mass $M' = (M^*/m)^{1/2}m$, and momentum $\pm\hbar k_F$. The ground state of (3.1) or (3.3) is obtained by filling up all the negative-energy eigenstates, corresponding to a filled Fermi (or Dirac) sea. A Fröhlich state moving to the right can be constructed simply by removing $2,+k$ particles and adding $1,+k$ particles. The energy of the moving ground state is then found to be $\frac{1}{2}N\hbar^2 q^2/2M'$, where $N = k_F L/\pi$ is the total number of particles and q is the displacement of the Fermi sea, as expected. In the limit $\epsilon \to 0$, the boson (phason) operators b_{nk} are found to be related to the fermion operators a_{nk} by an expression originally derived by Mattis and Lieb:[28]

$$b_{nk} = \left[\frac{2\pi}{Lk}\right]^{1/2} \rho_n(k) , \quad (3.7)$$

where $\rho_n(k) = \sum_q a_{nq}^\dagger a_{nq+k}$ is the fermion density operator.

We will now include the ϕ^2 contribution to the pinning energy by defining

$$H_0' = \int_0^L dx \left[\frac{1}{2D}\Pi^2(x) + \tfrac{1}{2}Dc_0^2\phi_x^2(x) + \tfrac{1}{2}D\omega_p^2\phi^2(x)\right] . \quad (3.8)$$

This Hamiltonian is again diagonalizable in both the bosonic and fermionic representations. In the bosonic representation, the Hamiltonian is given by (2.10), where the excitation energies $\hbar\omega_k = \hbar(\omega_p^2 + c_0^2 k^2)^{1/2}$ and the terms f_k in (2.13) are modified by pinning frequency ω_p. In the fermionic representation, the Hamiltonian H_0' is found in Appendix A to be given by

$$:H_0':=\sum_k{}' \hbar c_0(2k - k^*)(a_{1k}^\dagger a_{1k} - a_{2k}^\dagger a_{2k})e^{-\epsilon|k|} , \quad (3.9)$$

where $k^* = \text{sgn}(k)(k^2 + k_0^2)^{1/2}$ and $k_0 = \omega_p/c_0$. The expressions for Q, I, and P essentially remain unaltered from the unpinned case considered earlier when $k \gg k_0$. When $k \le k_0$, the expressions (3.4)–(3.6) become modified by the presence of higher-order scattering terms, which may contribute to the dissipation discussed in Sec. IV. In addition, the effective group velocity and current per particle are likely to be smaller than c_0 and $e^* c_0$, respectively, for $k < k_0$, because of the reduced slope in the energy dispersion relation in this region.

Thus far we have considered the bosonic and fermionic representations of the unpinned Hamiltonian, and have also considered the case where the ϕ^2 contribution to the pinning energy is included to zero order in the model Hamiltonian. When the full sinusoidal pinning potential is included, the low-lying excitation energies in the bosonic representation will only be slightly altered from $\hbar\omega_p$, provided the number of bosons (phasons) is sufficiently small that their total energy $n\hbar\omega_p \ll E_\phi = (2/\pi)(M^*/m)^{1/2}\hbar\omega_p$ is substantially smaller than the soliton energy. When the total phason energy is comparable to the soliton energy, $n\hbar\omega_p \sim E_\phi$, the phason-phason interactions, as represented by the higher-order terms in the series expansion of the pinning potential, become substantial,[33] causing shifts in the phason energies and possibly leading to finite lifetimes.

The situation for the fermionic spectrum is complicated by the fact that coefficients containing $(2 - k^*/k)^{1/2}$ appear to all orders in the expansion of $[1 - \cos\phi]$ in terms of the fermion operators. These coefficients become large for $k \le (m/M^*)^{1/2}k_0$, so that the fermion expansion no longer forms a perturbatively converging series. However, it is possible to gain insight about the excitation spectrum by examining the model pinning potential $V(\phi) = D\omega_p^2\{1 - \cos[(M^*/m)^{1/4}\phi]\}$, hereafter referred to as $(m/M^*)^{1/4}$ commensurability. In this case the full fermionic (massive Thirring) Hamiltonian can be diagonalized exactly, as outlined in Appendix C, with the result that the fermion energies are given by $E(k) = \pm[\Delta_p^2 + (\hbar c_0 k)^2]^{1/2}$, where $\Delta_p = \pi D\omega_p^2\epsilon$. In the ground state, all the negative-energy states are filled, and the minimum excitation energy is $E_G' = 2\Delta_p$. The physical interpretation of this "pinning energy gap" is clear in the limit $M^*/m \rightarrow 1$ of an electron gas. In this case, the gap results from mixing of the "1 branch" electrons of wave vector k_F+k near the right-hand Fermi surface with the "2 branch" electrons near the left-hand Fermi surface. The implication is that the $2k_F$ component of the pinning potential resulting from impurities or commensurability cannot be treated as a perturbation, but must be included to zero order in the model Hamiltonian.

When $M^*/m \gg 1$ and the potential is taken to be $D\omega_p^2[1-\cos\phi]$, the gap energy is expected to be comparable to twice the energy of a quantum soliton, which has been calculated using nonperturbative techniques by Dashen, Hasslacher, and Neveau[34] and others.[24,33] The energy of a quantum soliton of charge e per chain per spin is found to be equal to the classical soliton energy $E_\phi=(2/\pi)(M^*/m)^{1/2}\hbar\omega_p$, within corrections of order $(m/M^*)^{1/2}E_\phi$, provided soliton-soliton and soliton-antisoliton interactions are neglected. In Sec. II we found that the effective charge of a dressed fermion, which fully incorporates interactions and corresponds to the unpinned Hamiltonian, is given by $e^*=(m/M^*)^{1/4}e$, and this effective charge may be applicable to the pinned system, provided excitations are created in pairs so that there are no topological "kinks" existing in the absence of "antikinks." The gap for the interacting system may therefore be scaled down by about $(m/M^*)^{1/4}$ from twice the classical soliton energy, resulting in the expression

$$E_G=\lambda\left[\frac{m}{M^*}\right]^{1/4}2E_\phi=\frac{2\lambda}{\pi}\left[\frac{m}{M^*}\right]^{1/4}\hbar\omega_p\ , \tag{3.10}$$

where λ is a dimensionless parameter of order unity. The energies of the dressed fermions would then be given by

$$E(k)=\pm[(\hbar c_0 k)^2+(E_G/2)^2]^{1/2}\ . \tag{3.11}$$

It should be pointed out, however, that this issue of whether the appropriate charge is the "dressed" charge e^* or the topological charge e is still an open question.

IV. ZENER TUNNELING IN AN ELECTRIC FIELD

In the model presented here the current carriers in the CDW system are considered to be dressed electrons (quantum solitons) each carrying an effective charge $e^*=(m/M^*)^{1/4}e$, velocity $c_0=(m/M^*)^{1/2}v_F$, momentum $\hbar k_F$, and energy $\pm[(\hbar c_0 k)^2+(E_G/2)^2]^{1/2}$. In the absence of any external field, the fermions will be in the ground-state configuration in which all the $1,-k$ and $2,k$ states are occupied while the $1,k$ and the $2,-k$ states are empty ($k>0$). Excitations above this ground-state configuration are represented by particle-hole (quantum soliton-antisoliton) pairs above and below the pinning gap, respectively. These are collective excitations of the CDW system, which are distinct from the single-particle excitations above the Peierls gap. Thermal excitation of these collective entities is suppressed by the three-dimensional coherence resulting from interchain coupling and delocalization of the dressed electrons in the transverse direction, as discussed in Sec. V. When a uniform electric field E is applied along the length of the sample, the particles are accelerated in k a space against a damping term, due to scattering, which is governed by the mean relaxation time of the system. When a $1,-k$ particle reaches the Fermi surface, it either Zener tunnels through the pinning energy gap E_G into the $1,k$ states with probability $P(E)$, or is Bragg reflected into the $2,k$ states with probability $1-P(E)$. The Zener tunneling probability is given by[35,36]

$$P(E)=e^{-E_0/E}\ , \tag{4.1}$$

where

$$E_0=\frac{E_G^2}{\pi\hbar e^* c_0}=\left[\frac{M^*}{m}\right]^{5/4}\frac{4\lambda\hbar\omega_p^2}{\pi e v_F}\ . \tag{4.2}$$

Note that if we scale the soliton energy and charge by $(m/M^*)^{1/4}$, as indicated in Sec. III, then our expression for E_0 will differ from Bardeen's by a factor $\lambda(M^*/m)^{1/4}$.

The equation of motion for the displacement q of the Fermi sea of dressed fermions with respect to their ground-state configuration is given by, in the absence of dissipation and polarization effects,

$$\hbar\frac{dq}{dt}=e^*EP(E)\ . \tag{4.3}$$

When integrated over an effective relaxation time τ, the steady-state displacement will be $q=e^*E\tau/\hbar$. The CDW contribution to the current is obtained by setting $J_{\mathrm{CDW}}=ne^*v_d=ne^*\hbar q/M'$, with the result

$$J_{\mathrm{CDW}}=\sigma_b E e^{-E_0/E}\ , \tag{4.4}$$

where the limiting high-field CDW conductivity is given by

$$\sigma_b=\frac{ne^2\tau}{M^*}\ . \tag{4.5}$$

A quantum-mechanical treatment of the forced sine-Gordon theory has recently been used by Widom and Srivastava[37] to derive an expression for the transition rate for the creation of soliton-antisoliton pairs by Zener tunneling. In their treatment, using the boson-fermion $(1+1)$-dimensional map for one particular value of the coupling strength, they employed Swinger's proper time method[38] to calculate the *V-I* characteristic for a model long, thin Josephson junction. Each soliton in a long Josephson junction is a fluxon, carrying one magnetic flux quantum, and the relationship between voltage and current is inverted as compared to the CDW system.

It has been proposed[36] that the effective mean free path $l\equiv 2v_F\tau$ for the CDW system ought to be comparable to the distance over which the phase is correlated $\sim c_0/\omega_p$, leading to a relaxation frequency $\tau^{-1}\sim(M^*/m)^{1/2}\omega_p$ directly proportional to the pinning frequency. However, millimeter wave measurements on $NbSe_3$ (Ref. 39) and TaS_3 (Ref. 40) demonstrate that the relaxation frequency is essentially independent of impurity concentration and pinning frequency, suggesting that other mechanisms besides impurity scattering are dominant. Furthermore, the Arrhenius behavior of the CDW current and dielectric relaxation frequency observed in $K_{0.3}MoO_3$ (blue bronze),[41] $(TaSe_4)_2I$,[42] and orthorhombic TaS_3 (Ref. 43) at low temperatures provides compelling evidence that screening by normal carriers plays a key role in the dissipation at relatively low fields and frequencies in these materials.[44] However, at high electric fields, dramatic evidence for dissipationless CDW conduction has been observed in blue bronze at low temperatures (~ 4 K),[45] where the differential conductance dI/dV has been found to diverge at a well-defined threshold voltage. In this case, the entire CDW appears to be moving with little or no dissipation, where screening by normal carriers is no longer involved.

V. EFFECT OF THREE-DIMENSIONAL COHERENCE

One of the main objections to the tunneling hypothesis is the fact that the pinning gap energy $E_G \sim 10^{-4} k_B T$ must be substantially smaller than the thermal energy $k_B T$, in order for the predicted values of E_0 to agree with experiment at temperatures $T \sim 100$ K at which non-Ohmic CDW transport is typically observed. It must be emphasized, however, that E_G refers to the gap energy *per chain*. In reasonable quality crystals, the CDW motion is coherent across $N \sim 10^7$ parallel chains, as pointed out by Bardeen.[10] The three-dimensional coherence resulting from electron delocalization and interchain coupling is, in fact, an important prerequisite for the occurrence of a Peierls transition. If the CDW phase is assumed to be correlated across N parallel chains, then the energy required to thermally excite a collective soliton-antisoliton pair will be NE_G, which is substantially larger than $k_B T$, so that the probability for thermal excitation $\exp(-NE_B/k_B T)$ will be extremely small. On the other hand, the Zener tunneling probability will be unaffected by this rescaling, as can be seen by examining the various quantities which go into (4.2). If we treat the tunneling events involving N parallel chains (or, in *k space*, N transverse wave vectors $\mathbf{k}_\perp$) within a transverse coherence distance as being statistically correlated, then the effective charge $\overline{e^*} = Ne^*$ and energy $\overline{E(k)} = NE(k)$ will each be scaled up accordingly. The implication of this rescaling is that *both* the effective energy gap $\overline{E_G} = NE_G$ and slope $\overline{\hbar c_0} = N\hbar c_0$ of the energy dispersion relation in the large-k limit will be scaled up by N. By inserting these quantities into (4.2) the factors of N are found to cancel out, yielding the same value of E_0 as before.

The heuristic argument presented above is undoubtedly overly simplistic, and substantial further theoretical work is clearly needed in order to address this extraordinarily difficult, and important issue of three-dimensional CDW coherence. Alternative heuristic arguments about the three-dimensional coherence have been made by Bardeen *et al.*[46] and by Tucker and Miller,[24] utilizing an analogy to Josephson tunneling. The tunneling of paired electrons between two superconductors constitutes an important precedent for coherent tunneling of individual entities within a many-body ground state. In this case the relevant tunneling probability amplitude is that of a single Cooper pair, despite the fact that there is only one thermal degree of freedom for the entire system of N pairs. Furthermore, recent theoretical work[47,48] on other systems where evidence for tunneling has been observed, including phase slip at low temperatures in one-dimensional superconducting wires[49] and quantum evaporation of liquid helium, have found that semiclassical calculations involving a single "saddle point" for the macroscopic system yield theoretical WKB tunneling probabilities which are far too low in these systems, indicating that the semiclassical WKB approach, which ignores the individual microscopic quantum degrees of freedom, is inadequate.

Finally, it should be pointed out that there is compelling evidence suggesting that a quantum mechanism may play an important role in magnetic relaxation by flux creep in high T_c, and other type-II superconductors. Several experiments at millikelvin temperatures have demonstrated that the magnetic relaxation rate $dM/d\ln t$ extrapolates to a large finite value as the temperature approaches zero in Y-Ba-Cu-O,[50] ternary molybdenum sulfide crystals,[51] and UPt_3 (Ref. 52)—a result that contradicts the prediction $dM/d\ln t \sim k_B T/U_0$ of the Anderson-Kim model[53] of thermally activated flux creep and its variants. If a quantum mechanism does indeed play an important role in these systems, then it is very likely that the internal quantum degrees of freedom are crucial in allowing tunneling to take place. Taken together with the evidence[11–16] for a quantum mechanism of CDW depinning, these results suggest that the concept of coherent, or collective, tunneling of particles with individual quantum degrees of freedom within a condensate ought to be generalized beyond Josephson tunneling of paired electrons between two superconductors. One might further speculate on whether coherent quantum effects play an important role in the cooperative behavior of biological macromolecules.

VI. ANALOGY BETWEEN CDW TUNNELING AND COULOMB BLOCKADE

An additional effect of three-dimensional coherence is that the CDW order parameter will have a well-defined expectation value $n_c^{1/2}\langle e^{i\phi}\rangle$ proportional to the square root of the density n_c of condensed CDW electrons. One might therefore be tempted to interpret many aspects of CDW dynamics, such as the observed threshold field and coherent oscillations,[2] on a semiclassical basis. Note, however, that, in general, $\langle e^{i\phi}\rangle \neq e^{i\langle\phi\rangle}$, and we believe that this fact precludes simply rewriting Eq. (2.4) as a classical equation of motion for the expectation value of the phase $\langle\phi\rangle$. We believe that important clues to the origin of the sharp threshold field, coherent oscillations, "narrow-band noise," and mode locking in CDW's can be found by examining a striking similarity of these phenomena to the remarkable phenomena of Coulomb blockade and time-correlated tunneling events in small tunnel junctions.[54,55] In a pioneering paper, Fulton and Dolan[56] confirmed experimentally the existence of charging effects in small circuits of planar tunnel junctions. In linear arrays of small tunnel junctions charge is transferred by mutually repulsive charge solitons,[57] resulting in time-correlated tunneling events with frequency I/e. Delsing *et al.*[58] demonstrated this effect by superimposing onto the dc bias an ac signal of frequency f, resulting in partial mode locking at current levels $I=ef$ and $2ef$. More recently, complete mode locking has been observed in a frequency-locked "turnstile device" for single electrons by Geerligs *et al.*[59]

An ideal CDW with a complete Peierls gap at low temperatures exhibits a similar scaling of CDW current with frequency of the form $I_{\rm CDW}=2Nef_D$, where N is the total number of parallel CDW chains, f_D represents the drift, or "washboard" (we believe this term is a misnomer) frequency, and the factor of 2 counts both spins. Moreover, in $NbSe_3$ samples of high purity, the bias voltage-dependent differential resistance dV/dI has a sharp discontinuity at the threshold voltage,[16] and the high-current asymptote of the CDW current-voltage (I-V) characteristic extrapolates through the threshold voltage (V_T) rather than through the origin, which is nearly the "ideal" behavior expected for Coulomb blockade in a small tunnel junction at low temperatures. It should be further noted that both the rigid overdamped oscillator model[4,5] and the classical deformable model[6–9] of CDW depinning predict that the asymptote of the I-V curve should extrapolate through the *origin*.

In order to make the analogy between CDW tunneling and Coulomb blockade more concrete, one must examine the origin of the threshold voltage in each case. In the case of a small tunnel junction, Coulomb blockade results from the fact that an electron tunneling through the junction with a bias voltage $V_{\rm bias}$ must gain enough energy $eV_{\rm bias}$ to overcome the "charging" energy $e^2/2C$, resulting in a threshold voltage $V_t=e/2C$ and offset voltage in the current-voltage characteristic $V_{\rm off}=\pm e/2C$. For the case of charge-density waves, the fact that pinning results largely from randomly distributed impurities implies that the CDW phase is correlated only over a Fukuyama-Lee-Rice domain length L, which is comparable to the length c_0/ω_p of a classical soliton. Suppose we consider a CDW "tunnel junction" consisting of N parallel chains in a domain of length L and cross-sectional area A. The capacitance will be given by

$$C=\frac{\epsilon(0)A}{L}\ , \tag{6.1}$$

where $\epsilon(0)\sim 10^7\epsilon_0$ is the dielectric constant at zero frequency. Assuming that N tunnel events are statistically correlated, then the threshold voltage across this region can be estimated by using the analogy to Coulomb blockade:

$$V_T=E_TL=\frac{n_c}{n}\frac{Ne'}{2C}=\frac{n_c}{n}\frac{Ne'L}{2\epsilon(0)A}\ , \tag{6.2}$$

so that

$$\epsilon(0)E_T=\frac{1}{2}\frac{n_c}{n}n_{\rm ch}e'\ , \tag{6.3}$$

where n_c/n is the fraction of condensed electrons, $n_{\rm ch}$ is the number of spin chains per unit area, and e' is an effective charge.

The experimental situation is ambiguous as to whether the relevant charge e' should be the topological charge e or the "dressed" charge e^*. A reversible threshold polarization $\epsilon(0)E_T$ of about $0.5e$ per conducting chain has been found emperically by Wu, Jánossy, and Grüner[60] in TaS_3 and alloys with Nb. About the same value has been found in other CDW compounds[61] measured at temperatures sufficiently high for complete Coulomb screening by normal carriers, and a similar relation has been discussed theoretically by Bardeen.[62] However, Mihaly and coworkers[63–65] find an effective polarization per chain of about 0.16–$0.24e$ in blue bronze, TaS_3, and $(TaSe_4)_2I$ at low temperatures, where the normal carriers are almost completely frozen out.

In extremely pure samples, the relevant length L may actually be the distance between contacts. The CDW in the regions under the contacts will be stationary, so these regions may be considered as CDW electrodes on the left- and right-hand sides of a "tunnel junction." The tunnelling of quantum solitons between the electrodes could then be treated within a tunneling Hamiltonian formulation,[66,67] where the polarization effects would be included with an additional contribution $Q^2/2C$ to the Hamiltonian. The theoretical treatment developed by Widom, Clark, and Megalondis,[54] Averin and Likharev,[55] and others[57] could then be applied to describe the time-correlated tunneling events.

An alternative, but related interpretation of the CDW threshold field based on the soliton tunneling mechanism has been obtained by Krive and Rozhavsky,[68] by considering the coherent Coulomb interaction between charged solitons. In their interpretation, the threshold field E_T is the deconfinement field of bound soliton-antisoliton pairs, and the universality relation $\epsilon(0)E_T=$const is also obtained.

VII. CONCLUSION

We have attempted to construct a quantum-mechanical theory of CDW dynamics by utilizing techniques of nonperturbative quantum field theory developed for the quantum sine-Gordon system. Within this context, although it is possible to derive an operator equation of motion that is formally identical to the corresponding classical equation, the noncommuting properties of the phase operator for unequal times imply that a fully quantum-mechanical treatment is required in order to correctly describe CDW dynamics. Furthermore, the existence of an energy gap in the spectrum of collective excitations, as originally proposed by Bardeen, implies that Zener tunneling is required in order to depin the CDW, at least for the idealized case of the sine-Gordon system. Although this is clearly an oversimplification, we nevertheless believe that is a useful approximation in describing highly coherent samples of $NbSe_3$ and TaS_3. We have argued that the three-dimensional coherence plays an important role in suppressing thermal excitations across the "pinning energy gap," but still allows Zener tunneling to take place. Finally, the possible analogy between CDW tunneling and the phenomena of Coulomb blockade and time-correlated tunneling in small tunnel junctions has been suggested as an interesting interpretation of the observed sharp threshold field and coherent oscillations in CDW systems.

During the past several years a number of experiments[11–16] have been performed on CDW materials in order to test the predictions of the tunneling model. In particular, the theory of photon-assisted tunneling[69] has been adapted to Bardeen's model in order to generate predictions of linear and nonlinear rf experiments. The essentially quantitative agreement obtained in these diverse experiments constitutes powerful evidence that tunneling is indeed responsible for CDW depinning in an electric field. The analysis presented here argues strongly for tunneling based on established techniques of quantum field theory. If the tunneling hypothesis is correct, then CDW transport constitutes an important cooperative quantum phenomenon.

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APPENDIX A: THE BOSE-FERMI TRANSFORMATION

Here we treat the case of a ϕ^2 potential. The results for the unpinned system can easily be obtained from this treatment simply by setting $\omega_p=0$, or equivalently, $k^*=k$. We begin by defining

$$\bar{\phi}_1(x)=i\sum_{k>0}\frac{k^*}{k}\left[\frac{2\pi}{Lk^*}\right]^{1/2}(b_{1k}^{\dagger}e^{-ikx}-b_{1k}e^{ikx})e^{-\epsilon k/2}\;, \tag{A1}$$

$$\bar{\phi}_2(x)=i\sum_{k>0}\frac{k^*}{k}\left[\frac{2\pi}{Lk^*}\right]^{1/2}(b_{2,-k}^{\dagger}e^{-ikx}-b_{2,-k}e^{ikx})e^{-\epsilon k/2}\;, \tag{A2}$$

and

$$\bar{\psi}_i(x)=\frac{1}{2\pi\epsilon}\exp[i\bar{\phi}_i(x)],\quad i=1,2\;. \tag{A3}$$

Whenever we write the product of two or more field operators ψ at the same position x we are generally interested in length scales large compared to the cutoff parameter ϵ. Effectively, we are neglecting the amplitude excitations, which require considerably more energy than the phase excitations. Therefore, following Takada and Misawa,[25] we define

$$\bar{\psi}_i{}^{\dagger}(x)\bar{\psi}_i(x)=\tfrac{1}{2}\lim_{\epsilon\ll\eta\to 0}[\bar{\psi}_i{}^{\dagger}(x)\bar{\psi}_i(x')+\bar{\psi}_i{}^{\dagger}(x')\bar{\psi}_i(x)]\;, \tag{A4}$$

where $\eta=x-x'$ and likewise for the other products. Using the identity

$$e^{A}e^{B}=e^{A+B}e^{(1/2)[A,B]}\;, \tag{A5}$$

which holds for any operators A and B provided $[A,B]$ is a c number, we obtain

$$\begin{aligned}\bar{\psi}_1{}^{\dagger}(x)\bar{\psi}_1(x')&=\frac{1}{2\pi\epsilon}e^{-i\bar{\phi}_i^{\dagger}(x)}e^{i\bar{\phi}_1(x')}\\&=\frac{1}{2\pi\epsilon}e^{[A_1^{\dagger}(x)-A_1^{\dagger}(x')]}\\&\quad\times e^{[-A_1(x)+A_1(x')]}e^{\xi(k_0,\eta)}\;,\end{aligned} \tag{A6}$$

where

$$A_1(x)=\sum_{k>0}\frac{k^*}{k}\left[\frac{2\pi}{Lk^*}\right]^{1/2}b_{1k}e^{ikx}e^{-\epsilon k/2}$$

and

$$\begin{aligned}\xi(k_0,\eta)&=\sum_{k>0}\left[\frac{k^*}{k}\right]^2\frac{2\pi}{Lk^*}(e^{ik\eta}-1)e^{-\epsilon k}\\&=\int_{k_m}^{\infty}\frac{(k^2+k_0^2)^{1/2}}{k^2}(e^{ik\eta}-1)e^{-\epsilon k}dk\;,\end{aligned}$$

where $k_m=2\pi/L$. Under normal experimental conditions (for typical parameter values refer to Appendix B), $k_m\ll k_0\sim c_0/\omega_p\ll 1/\epsilon$. Thus there exists an integer n such that $n\gg 1$ (≥ 10), $\eta n k_0\ll 1$ (≤ 0.2). We can then use the approximation $e^{i\eta n k_0}\approx 1$ and $e^{-\epsilon\eta k_0}\approx 1$. The integral in Eq. (A6) can therefore be approximated as

$$\xi(k_0,\eta)=\int_{km}^{nk_0}(\;)+\int_{nk_0}^{\infty}(\;)\;,$$

where

$$(\;)=\frac{(k^2+k_0^2)^{1/2}}{k^2}(e^{ik\eta}-1)e^{-\epsilon k}dk$$

so that

$$\begin{aligned}\xi(k_0,\eta)&\approx\int_{k_m}^{nk_0}\frac{(k^2+k_0^2)^{1/2}}{k^2}ik\eta\,dk\\&\quad+\int_{nk_0}^{\infty}\frac{1}{k}(e^{ik\eta}-1)e^{-\epsilon k}dk\\&\approx\ln\left[\frac{\epsilon}{\epsilon-i\eta}\right]+i\eta k_0\ln\left[\frac{2k_0}{k_m e}\right]\\&\approx\ln\left[\frac{\epsilon}{\epsilon-i\eta}\right]+i\eta\alpha\;,\end{aligned} \tag{A7}$$

where $e=2.718$ and $\alpha=k_0\ln(2k_0/k_m e)$. Under typical experimental conditions, $k_m\ll\alpha\ll 1/\epsilon$. One can thus expand out the exponential and keep only the terms linear in η. Using the definition (A4) and Eqs. (A6) and (A7), and expanding to terms linear in η yields

$$\bar{\psi}_1{}^{\dagger}(x)\bar{\psi}_1(x)=\lim_{\epsilon\ll\eta\to 0}\frac{1}{2\pi}\left[\frac{\epsilon}{\epsilon^2+\eta^2}+\frac{\eta^2}{\epsilon^2+\eta^2}(-\alpha+X)\right]$$
$$=\lim_{\epsilon\ll\eta\to 0}\frac{1}{2\pi}\left[\frac{\epsilon}{\epsilon^2+\eta^2}+(-\alpha+X)\right], \quad (A8)$$

where

$$X=i\sum_{k>0}\left[\frac{2\pi k^*}{L}\right]^{1/2}(b_{1k}e^{ikx}+b_{1k}^{\dagger}e^{-ikx})e^{-\epsilon k/2}. \quad (A9)$$

Similarly,

$$\bar{\psi}_1(x)\bar{\psi}_1{}^{\dagger}(x)=\lim_{\epsilon\ll\eta\to 0}\frac{1}{\pi}\left[\frac{\epsilon}{\epsilon^2+\eta^2}+(-\alpha-X)\right]. \quad (A10)$$

Adding (A8) and (A10) we obtain

$$[\bar{\psi}_1(x),\bar{\psi}_1{}^{\dagger}(x)]=\lim_{\epsilon\ll\eta\to 0}\frac{1}{\pi}\left[\frac{\epsilon}{\epsilon^2+\eta^2}-\alpha\right]. \quad (A11)$$

Similar results for the two particles are

$$\bar{\psi}_2^{\dagger}(x)\bar{\psi}_2(x)=\lim_{\epsilon\ll\eta\to 0}\frac{1}{2\pi}\left[\frac{\epsilon}{\epsilon^2+\eta^2}+(-\alpha+Y)\right], \quad (A12)$$

where

$$Y=\sum_{k>0}\left[\frac{2\pi k^*}{L}\right]^{1/2}(b_{2-k}^{\dagger}e^{ikx}+b_{2-k}e^{-ikx})e^{-\epsilon k/2}, \quad (A13)$$

$$\bar{\psi}_2(x)\bar{\psi}_2^{\dagger}(x)=\lim_{\epsilon\ll\eta\to 0}\frac{1}{2\pi}\left[\frac{\epsilon}{\epsilon^2+\eta^2}+(-\alpha-Y)\right], \quad (A14)$$

and

$$[\bar{\psi}_2(x),\bar{\psi}_2{}^{\dagger}(x)]=\lim_{\epsilon\ll\eta\to 0}\frac{1}{\pi}\left[\frac{\epsilon}{\epsilon^2+\eta^2}-\alpha\right]. \quad (A15)$$

In addition, the commutation relation $[\bar{\phi}_1(x),\bar{\phi}_2(x')]=0$ implies that

$$[\bar{\psi}_1(x),\bar{\psi}_2(x')]=[\bar{\psi}_1{}^{\dagger}(x),\bar{\psi}_2^{\dagger}(x')]=0,\ \cdots\ . \quad (A16)$$

In the limit $\epsilon\ll\eta\to 0$ as the amplitude coherence length approaches zero, the term $\epsilon/(\epsilon^2+\eta^2)$ will likewise approach zero. Therefore we can dispense with this term in Eqs. (A8), (A10), (A12), and (A14). However, we keep this term in the anticommutation relations (A11) and (A15) because, as we will see, it dramatically controls the energy spectrum of the fermions. Using the definition

$$\bar{\psi}_i{}^{\dagger}(x)\frac{\partial}{\partial x}\bar{\psi}_i(x)=\lim_{\epsilon\ll\eta\to 0}\frac{1}{2\eta}[\bar{\psi}_i{}^{\dagger}(x)\bar{\psi}_i(x+\eta)-\bar{\psi}_i{}^{\dagger}(x)\bar{\psi}_i(x-\eta)],\quad i=1,2$$

we obtain the relation

$$:\int_0^L\bar{\psi}_i{}^{\dagger}(x)\frac{\partial}{\partial x}\bar{\psi}_i(x)dx:=\pm i\sum_{k>0}k^*b_{i\pm k}^{\dagger}b_{i\pm k}e^{-\epsilon k}, \quad (A17)$$

where the upper (lower) sign is for the 1 (2) particles. With use of (2.10) and (A17), taking $\omega_k=c_0k^*$, we obtain

$$:H_0:=-i\hbar c_0:\int_0^L\left[\bar{\psi}_1{}^{\dagger}(x)\frac{\partial}{\partial x}\bar{\psi}_1(x)-\bar{\psi}_2{}^{\dagger}(x)\frac{\partial}{\partial x}\bar{\psi}_2(x)\right]dx:. \quad (A18)$$

We now come to the problem of defining the field operators $\bar{\psi}$ in terms of the fermion operators a_{ik} and $a_{ik}^{\dagger}$, $i=1,2$, such that the Eqs. (A11), (A15), and (A16) are satisfied. One can verify that we have derived the correct expressions by employing the same approximations used to derive Eq. (A8):

$$\bar{\psi}_i(x)=\frac{1}{\sqrt{L}}\sum_{k\neq 0}\left[2-\frac{k^*}{k}\right]^{1/2}a_{ik}e^{ikx}e^{-\epsilon|k|/2},\quad i=1,2 \quad (A19)$$

where the fermion operators obey the anticommutation and commutation relations

$$\{a_{ik},a_{ik'}^{\dagger}\}=\delta_{k,k'},\quad i=1,2$$
$$[a_{1k},a_{2k'}]=[a_{1k},a_{2k'}^{\dagger}]=0\ . \quad (A20)$$

It is preferable for the "1" fermions to anticommute with the "2" fermions rather than commute, especially when one is required to make a Bogoliubov transformation involving both the 1 and the 2 particles. Complete anticommutation relations can be established by performing a Klein-transformation[32] as discussed in Appendix D. A particularly appealing feature of this transformation is that all the equations written below are invariant under it, to within physically irrelevant phase factors, and henceforth we shall take it for granted that the 1 and 2 operators are anticommuting. A straightforward substitution using (A19) yields the expression

$$\bar{\psi}_i^{\dagger}(x)\bar{\psi}_i(x)=\frac{1}{L}\lim_{\epsilon\ll\eta\to 0}\sum_{k,k'\neq 0}\left[\left[2-\frac{k}{k^*}\right]\left[2-\frac{k'}{k'^*}\right]\right]^{1/2}a_{ik}^{\dagger}a_{ik'}e^{i(k-k')x}\cos(k\eta)e^{-\epsilon(|k|+|k'|)/2}, \quad (A21)$$

where $\eta = x - x'$. Comparing Eq. (A21) with (A8) for each k yields a rather complicated relation between the "b" and "a" operators that reduces to the simplified form of Eq. (3.7) in the limit $\omega_p, \epsilon \to 0$. Combining Eqs. (A18), (A19), and (A20) yields the result

$$:H_0:=\sum_k{}' \hbar c_0(2k-k^*)(a_{1k}^\dagger a_{1k}-a_{2k}^\dagger a_{2k})e^{-\epsilon|k|} . \quad (A22)$$

APPENDIX B: TYPICAL EXPERIMENTAL VALUES OF RELEVANT PARAMETERS

Here we present five typical experimental values of relevant parameters:

$$\frac{m}{M^*} \approx 10^{-3} ,$$

$$L \approx 2\text{mm} ,$$

$$\omega_p \approx 10^{10}-10^{11}\text{sec}^{-1} ,$$

$$v_F \approx 10^7-10^8 cm/\text{sec} ,$$

$$\epsilon = 1/k_c \approx 10^{-3} c_0/\omega_p .$$

APPENDIX C: EXACT SOLUTION FOR $(m/M^*)^{1/4}$ COMMENSURABILITY

We begin by taking H_0 to be the unpinned Hamiltonian and then show that the intire pinning term $\{1-\cos[(M^*/m)^{1/4}\phi]\}$ can be expressed in terms of the field operators ψ_i defined in Appendix A. Following Sec. II we recall that, for the unpinned case, the phase operator at a point x is given in the bosonic representation by

$$\phi(x)=i\left[\frac{m}{M^*}\right]^{1/4}\sum_{k>0}\left[\frac{2\pi}{Lk}\right]^{1/2}[(b_{1,k}^\dagger+b_{2,-k})e^{-ikx}-(b_{1,k}+b_{2,-k}^\dagger)e^{ikx}]e^{-\epsilon k/2} , \quad (C1)$$

where we have used Eqs. (2.8) and (2.13) and the result that $\omega_k = c_0 k$ for this case of interest. Following Appendix A we now define the field operators as

$$\psi_i(x)=\frac{1}{\sqrt{2\pi\epsilon}}\exp[i(M^*/m)^{1/4}\phi_i(x)] , \quad (C2)$$

where $(M^*/m)^{1/4}\phi_i(x)=\bar{\phi}_i(x)$, as defined in Appendix A for the case $\omega_p=0$. If one now follows the algebra of Appendix A, the integral $\xi(k_0,\eta)$ can now be evaluated in closed form, resulting in an expression which is identical to (A7) with $\alpha=0$. All of the equations given in Appendix A thus hold identically for the case under consideration simply by setting $\alpha=0$ and $\omega_p=0$ (i.e., $k^*=k$ and $k_0=0$). In particular, Eqs. (A19) and (A22), respectively, reduce to

$$\psi_i(x)=\frac{1}{\sqrt{L}}\sum_{k\neq 0} a_{ik}e^{ikx}e^{-\epsilon|k|/2}, \quad i=1,2 \quad (C3)$$

and

$$:H_0:=\sum_{k\neq 0}\hbar c_0 k(a_{1k}^\dagger a_{1k}-a_{2k}^\dagger a_{2k})e^{-\epsilon|k|} . \quad (C4)$$

However, this particular case of $(m/M^*)^{1/4}$ commensurability is unique in that the full pinning term can also be expressed in terms of the a operators as shown below:

$$\begin{aligned}:H_{\text{pin}}:&=:\int_0^L dx\, D\omega_p^2[1-\cos(M^*/m)^{1/4}\phi(x)]:\\ &=-\pi\epsilon D\omega_p^2:\int_0^L dx[\psi_1^\dagger\psi_2(x)+\psi_2^\dagger\psi_1(x)]:\\ &=-\Delta_{\text{pin}}\sum_{k\neq 0}(a_{1k}^\dagger a_{2k}+a_{2k}^\dagger a_{1k}) , \end{aligned}\quad (C5)$$

where $\Delta\text{pin}=\pi\epsilon D\omega_p^2$. Here it should be pointed out that now the order of the 1 and 2 operators is important since we have implicitly performed the Klein transformation (refer to Appendix D for more details), which makes them anticommute. The total Hamiltonian, consisting of the sum of Eqs. (C4) and (C5), can be easily diagonalized by a standard Bogoliubov transformation, yielding the quasiparticle spectrum

$$E_k=\pm[(\hbar c_0 k)^2+\Delta_{\text{pin}}^2]^{1/2} . \quad (C6)$$

APPENDIX D: THE KLEIN TRANSFORMATION

We remarked earlier that, although the ψ_i-field operators anticommute among themselves, ψ_1 and ψ_2 commute with each other rather than anticommuting. Strictly speaking, this would not allwo one to perform a Bogoliubov tranformation, which mixes the 1's with the 2's. However, the required anticommutation properties between the 1's and 2's can be obtained with use of the following transformation:

$$\psi_1 \to \psi_1^* = e^{i\alpha}\psi_1, \quad \psi_2 \to \psi_2^* = e^{i\beta}\psi_2 , \quad (D1)$$

where α and β are Hermitian operators obeying the following commutation relations:

$$[\alpha,b_{ik}]=[\beta,b_{ik}]=0, \quad [\alpha,\beta]=i(2n+1)\pi , \quad (D2)$$

where n is any integer. ONe can easily construct operators α and β from the boson operators b_0 and $b_0^\dagger$. For example, when $n=0$ one makes the following definitions:

$$\alpha=i\sqrt{\pi}\frac{b_0-b_0^\dagger}{\sqrt{2}}, \quad \beta=\sqrt{\pi}\frac{b_0+b_0^\dagger}{\sqrt{2}} , \quad (D3)$$

which satisfy all the conditions imposed in (D2) and establish the required anticommutation relations. However, although the unpinned Hamiltonian remains covarient under the transformation (D1), the pinning part transforms to

$$\begin{aligned}:H_{\text{pin}}:=i\Delta_{\text{pin}}:\int_0^L&[e^{i(\alpha-\beta)}\psi_1^{*\dagger}\psi_2(x)\\ &-e^{-i(\alpha-\beta)}\psi_2^{*\dagger}\psi_1(x)]dx: \end{aligned}\quad (D4)$$

In order to get rid of the α and β operators one introduces the fermion operators with use of the relations

$$\psi_1^*(x)=\frac{1}{i}\exp[i(\alpha-\beta)]\frac{1}{\sqrt{L}}\sum_{k\neq 0}a_{1k}^{*}e^{ikx}e^{-\epsilon|k|/2}$$

and

$$\psi_2^*(x)=\exp[-i(\alpha-\beta)]\frac{1}{\sqrt{L}}\sum_{k\neq 0}a_{2k}^{*}e^{ikx}e^{-\epsilon|k|/2}\ . \tag{D5}$$

We note that these new 1 and 2 fermion operators now anticommute as desired. We can then write the entire Hamiltonian in terms of the fermion operators, for the commensurability of $(m/M^*)^{1/4}$, as

$$:H_0:=\sum_{k\neq 0}[\hbar c_0 k(a_{1k}^{*\dagger}a_{1k}^{\dagger}-a_{2k}^{*\dagger}a_{2k}^{*}) -\Delta_{\text{pin}}(a_{1k}^{*\dagger}a_{2k}^{*}+a_{2k}^{*\dagger}a_{1k}^{*})]e^{-\epsilon|k|}\ . \tag{D6}$$

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