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Hall coefficient of the doped Mott insulator: A signature of parity violation

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We show that the Hall coefficient R_H of a doped two-dimensional Mott insulator is temperature dependent in the state where long-range magnetic order vanishes. The Hall number $n_H = R_H^{-1}$ is linear in the doping density at high temperatures and decreases monotonically with decreasing temperature. At low temperatures R_H diverges, indicating the possible onset of a phase with spontaneous chirality.

The properties of the normal phase of high-temperature superconductors are well known to be unusual for normal metals.[1,2] Among these anomalous properties is the temperature dependence of the Hall coefficient.[3] In this paper we show that this phenomenon occurs naturally in the picture of the doped Mott insulator where long-range magnetic order is absent and where strong interaction between electrons destroys the Fermi-liquid coherence. In this case the relevant low-energy excitations are described by a Hamiltonian for Bose and Fermi particles interacting with a gauge field. We shall also argue that in this picture the ubiquitous growth of R_H with decreasing temperature is a precursor of parity and time-reversal symmetry breaking.

There is little doubt that the appropriate microscopic model for high-T_c materials is that of a doped Mott insulator.[1] In the absence of long-range magnetic order the Mott insulator behaves as a system of fermions and bosons coupled by a gauge field $(\phi,\mathbf{a})$:[4,5]

$$\mathcal{H}=\frac{1}{2m_F}f^\dagger(-i\nabla+\mathbf{a})^2f+f^\dagger(\phi-\mu_F)f+\frac{1}{2m_B}b^\dagger(-i\nabla+\mathbf{a}+e\mathbf{A})^2b+b^\dagger(\phi+e\Phi-\mu_B)b\,, \tag{1}$$

where auxilary Fermi and Bose fields represent the real electron $c=b^\dagger f$, the gauge field ensures the constraint $c^\dagger c\leq 1$, and μ_F, μ_B are the chemical potentials of fermions and bosons which are related to the electron chemical potential μ by $\mu=\mu_F-\mu_B$. This Hamiltonian can be formally derived from the canonical t-J model using the slave-boson method[6,7] in the heavy-hole limit. The internal gauge field $a_\mu=(\phi,\mathbf{a})$ appears after a Hubbard-Stratonovich decoupling of the interactions between electrons. In the opposite limit of light holes the slave-boson theory predicts large renormalization of all interaction constants,[8] and the approach based on the Schwinger-boson representation[5,9] becomes more justified. In this context a_μ reproduces the Berry phase acquired by a hole moving in a slowly varying ferromagnetic background. The Hamiltonian (1) implies the existence of Fermi quasiparticles with a large Fermi surface, which is in good agreement with photoemission experiments.[10] The lattice version of the theory (1) becomes unstable in the limit of zero doping.[11,12] However, this instability is removed at a finite concentration of holes.[13]

Without loss of generality, we assume that the external electromagnetic field $A_\mu=(\Phi,\mathbf{A})$ couples only to the bosons. The assignment of the electric charge to the bosons is convenient but has no effect on the physical results.[7,14] The gauge-field interaction described by the Hamiltonian (1) yields the electrical and thermal conductivities which are in agreement with the normal-state behavior of high-T_c oxides.[15-17]

Qualitatively, the model of the doped Mott insulator defined by (1) corresponds to a system of holes moving in a disordered spin background. The gauge field describes the most important fluctuations of this background. These fluctuations provide the scattering mechanism for the charge carriers. In the presence of an external magnetic field the numbers of left-handed and right-handed fluctuations are not equal. The scattering process becomes chiral, which results in an additional (temperature-dependent) contribution to the Hall effect. We emphasize that this effect is entirely due to the transverse character of the gauge interaction between particles.

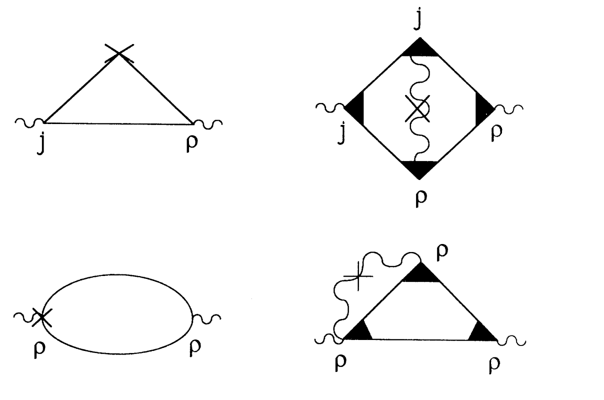
We now subject the system to a weak uniform magnetic field $\mathbf{H}=H\hat{\mathbf{z}}$ and compute the Hall coefficient $R_H=R_{yx}/H$, where $R_{yx}=\sigma_{xy}/\sigma_{xx}^2$ is the Hall resistance. It is useful to introduce the unphysical conductivities σ^F and σ^B of the Fermi and Bose subsystems, which are convenient to treat separately. The physical resistivity tensor is the sum of fermion and boson contributions;[7] thus,

$$R_{xx}=(\sigma_{xx}^F)^{-1}+(\sigma_{xx}^B)^{-1}\,,\qquad R_{yx}=\frac{\sigma_{xy}^F}{(\sigma_{xx}^F)^2}+\frac{\sigma_{xy}^B}{(\sigma_{xx}^B)^2}\,. \tag{2}$$

To first order in H, the conductivities σ_{xx} retain their zero-field values,[15,17] while the Hall conductivity σ_{xy} is proportional to H. It is related to the (real-time) current-current correlation function $\Pi_{\mu\nu}(q) = -i\langle J_\mu(-q) \times J_\nu(q)\rangle$ as follows:

$$\sigma_{xy}^{B,F} = \frac{1}{\omega}\,\mathrm{Im}\Pi_{xy}^{B,F}(\omega,\mathbf{q}) = \frac{\epsilon_{ji}q_i}{q^2}\,\mathrm{Im}\Pi_{j0}^{B,F}(\omega,\mathbf{q}) , \qquad (3)$$

where the last equality follows from current conservation. For noninteracting bosons or fermions Π_{j0} is given by the two diagrams shown in Fig. 1(a), which yields the classical result $\sigma_{xy}^{A}(\omega) = h^A\rho_A(m_A\omega)^{-2}$. Here h^A is the average field acting on the particles ($A=B$ for bosons, F for fermions), and ρ_A is their density. The interaction with the gauge field has two effects. First, it dresses the propagators and vertices in the diagrams of Fig. 1(a). This effect is described semiclassically by the Boltzmann equation and leads to the usual result $\sigma_{xy}^{A(0)} = h^A\rho_A(\tau_A/m_A)^2$ where τ_A is the relaxation time. Second, the gauge propagator itself is modified by the magnetic field and acquires a parity-odd component. The scattering by these chiral fluctuations of the gauge field gives an additional contribution to the Hall coefficient. Such behavior is in contrast with the usual electron systems where weak scattering is modified negligibly by a weak magnetic field. This effect results in the anomalous temperature dependence of R_H.



(a) (b)

FIG. 1. Diagrams for the polarization Π_{0j}: (a) Classical result. (b) Lowest-order corrections. Vector vertices are denoted by j; scalar vertices, by ρ. The wavy line with a cross stands for the parity-odd gauge propagator given by (6). Cross denotes the interaction with the external magnetic field.

The leading correction to Π_{j0} responsible for this phenomenon is given by the diagrams shown in Fig. 1(b).

The effective gauge-field Lagrangian $L_{\mathrm{eff}}(a,A)$ is given in imaginary time by

$$L_{\mathrm{eff}}(a,A) = -\tfrac{1}{2}\int\frac{d^2q}{(2\pi)^2}[a_q\Pi^F a_{-q} + (a_q+A_q)\Pi^B(a_{-q}+A_{-q})] , \qquad (4)$$

which yields the a-field propagator $\langle a_{q\mu}a_{-q\nu}\rangle = (\Pi^F + \Pi^B)^{-1}_{\mu\nu}$. In the absence of external fields, the nonzero components of $\langle a_{q\mu}a_{-q\nu}\rangle$ in the transverse gauge are

$$\langle\phi_q\phi_{-q}\rangle = (\Gamma_\parallel|\omega| + \nu)^{-1} ,$$
$$\langle a_q a_{-q}\rangle = (\Gamma_\perp|\omega| + \chi q^2)^{-1} , \qquad (5)$$

where ν is the density of states, χ is the diamagnetic susceptibility to the a field, and $\Gamma_{\parallel,\perp}$ are the Landau damping parameters for the longitudinal and transverse oscillations. Each of the quantities Γ, ν, and χ is the sum of boson and fermion parts. In a weak magnetic field, both systems develop small Hall conductivities. We define $\sigma \equiv \sigma_{xy}^F + \sigma_{xy}^B$, which should not be confused with the physical Hall conductance R_{yx}/R_{xx}^2. A nonzero value of σ adds a parity-odd component to the a-field propagator:

$$D(i\omega,q) = \langle a_{qj}\phi_{-q}\rangle = \sigma\epsilon_{jk}q_k F(\omega,q^2) , \qquad (6)$$

where

$$F(\omega,q^2) = \langle\phi_q\phi_{-q}\rangle\langle a_q a_{-q}\rangle$$
$$= \frac{1}{(\Gamma_\parallel|\omega| + \nu)(\Gamma_\perp|\omega| + \chi q^2)} . \qquad (7)$$

We determine the effect of the a field on σ_{xy} by computing corrections to the correlator Π_{j0} produced by the propagator (6).

To the leading order the corrections to Π_{j0} are given by the diagrams in Fig. 1(b). The wavy line with a cross denotes a gauge-field propagator of the form (6), which contains the fully renormalized σ. These diagrams therefore represent the summation of the ladder series for σ_{xy} which can be written as follows:

$$\sigma_{xy}^B = \sigma_{xy}^{B(0)} + \alpha_B\sigma ,$$
$$\sigma_{xy}^F = \sigma_{xy}^{F(0)} + \alpha_F\sigma . \qquad (8)$$

The quantities $\alpha_{B,F}$ describe the effect of the parity-odd gauge propagator. The bare values $\sigma_{xy}^{(0)}$ characterize a system where the influence of the magnetic field on the scattering process has been ignored, i.e., the propagator D has been turned off.

The bare Hall conductivity $\sigma_{xy}^{(0)}$ is determined by the mean magnetic field acting on the particles. The total magnetic field acting on the bosons is the sum of the applied field H and the induced mean internal field h. Fermions, on the other hand, are affected only by the internal field. By minimizing the action for the effective Lagrangian (4), it follows that $h/H = -\chi_B/(\chi_B+\chi_F)$, where $\chi_{B,F} \sim 1/m_{B,F}$. If $m_F \ll m_B$, then $h \ll H$ and $\sigma_{xy}^{F(0)}$ is negligible compared to $\sigma_{xy}^{B(0)}$. The Bose susceptibility χ_B depends on temperature, resulting in a weak temperature dependence of the physical R_{xy}, as first noted in Ref. 15. This effect is sensitive to the shape of the Fermi surface.[17] Here we neglect this small contribution to σ_{xy} and consider only the stronger temperature dependence due to parity-odd scattering processes.

We evaluate $\alpha_{B,F}$ by dropping the small vertex and self-energy corrections[18] in the diagrams in Fig. 1(b). The bare-particle Green's functions are $G(i\epsilon,p)$

$=(i\epsilon-\xi_p)^{-1}$, where $\xi_p=p^2/2m-\mu$, and the gauge propagator $D_{j0}(i\omega,q)$ is given by (6). For both fermions and bosons the square and the triangle diagrams contribute equally to Π_{j0}:

$$\Pi_{j0}^{(4)}(0,k)=-2(-1)^F N_c T^2\int\frac{d^2p\,d^2q}{(2\pi)^4}\sum_{\epsilon,\omega}G(i\epsilon,p+k/2)G(i\epsilon-i\omega,p-q+k/2)\times G(i\epsilon-i\omega,p-q-k/2)G(i\epsilon,p-k/2)D_{i0}(i\omega,q)\frac{p_j}{m}\frac{(p+k/2)_i}{m},$$

$$\Pi_{j0}^{(3)}(0,k)=-4(-1)^F N_c T^2\int\frac{d^2p\,d^2q}{(2\pi)^4}\sum_{\epsilon,\omega}G(i\epsilon,p+k/2)G(i\epsilon,p-k/2)G(i\epsilon-i\omega,p-q+k/2)D_{j0}(i\omega,q)\frac{1}{2m}. \qquad (9)$$

For fermions $F=1$ and $\epsilon=(2n+1)\pi T$; for bosons $F=0$ and $\epsilon=2n\pi T$. The multiplicity factors N_c are model dependent: in the slave-boson approach $N_c^F=2$, $N_c^B=1$, whereas in the Schwinger representation $N_c^F=1$, $N_c^B=2$. We find

$$\Pi_{j0}^{B,F}(0,k)=N_c(\sigma\epsilon_{ji}k_i/16\pi^2 m_{B,F})\int_0^1 dy(1-y)y\sum_{s=1}^{\infty}(-1)^{F(s-1)}s^2e^{s\beta\mu_{B,F}}\times\sum_\omega e^{is\beta\omega y}\int_0^\infty d(q^2)q^2F(\omega,q^2)\exp\left[-\frac{s\beta q^2}{2m_{B,F}}y(1-y)\right], \qquad (10)$$

where β is the inverse temperature and the function $F(\omega,q^2)$ is related to the parity-odd gauge propagator by (6). This expression for Π_{j0} is valid for any $F(\omega,q^2)$ which is even in ω and of order $(1/\omega)$ for large ω.

The calculation of Π_{j0}^B is simplified if we reduce the gauge propagator to the static form defined by $F(\omega,q^2)=(1/v\chi q^2)\delta_{0,\omega}$. This approximation is valid at low enough temperatures $T\ll 1/\chi^2m_B^3$. In this temperature range the one-loop expressions $\Gamma_\perp=p_F/\pi q$ and $\chi=1/12\pi m_F$ imply that, for the statistically important boson-scattering processes with $\omega\approx T$, $q\approx(m_BT)^{1/2}$, only the $\omega=0$ contribution should be retained since $\Gamma_\perp|\omega|\gg\chi q^2$. This is the same temperature range where resistivity is linear with temperature in the framework of this approach. [The longitudinal damping term in (7) is always negligible compared to $v=m_F/\pi$.] The definition $\alpha_{B,F}=\lim_{k\to 0}[\Pi_{j0}^{B,F}(0,k)/\sigma\epsilon_{ji}k_i]$ then yields

$$\alpha_B=-\frac{N_cTn_B'(0)}{8\pi^2v\chi}, \qquad (11)$$

where $n_B(\epsilon)=\{\exp[(\epsilon-\mu_B)/T]-1\}^{-1}$ is the Bose distribution function and $n_B'(\epsilon)=\partial n_B(\epsilon)/\partial\epsilon$. A similar calculation for fermions gives an exponentially small result $\alpha_F\propto\exp(-\mu_F/T)$, but higher-order corrections may be of order $(T/\mu_F)^{1/3}$, the latter being a measure of how strongly the fermions are scattered by the gauge field. In any case, the fermionic contribution is small at low temperatures and will be ignored. On the other hand, Eqs. (8) and (11) imply that σ_{xy}^B is enhanced at low temperatures and becomes very large at $T\sim T_{c0}=\rho_B/m_B$.

Combining (2), (8), and (11), we obtain the following expression for the Hall coefficient of the system:

$$R_H=R_H^{(0)}\left[\frac{1}{1-\alpha_B}\right], \qquad (12)$$

where $R_H^{(0)}=\sigma_{xy}^{B(0)}/(\sigma_{xx}^B)^2H=1/\delta$ is the Hall coefficient for a system without parity-odd gauge interactions; δ is the doping density. In deriving (12) we have neglected $\sigma_{xy}^{F(0)}$ which is small compared to $\sigma_{xy}^{B(0)}$. As discussed above, $R_H^{(0)}$ corresponds to the semiclassical value of the Hall conductivity and is independent of temperature. The only source of temperature dependence in (12) is the quantity α_B. Figure 2 shows the dependence of the Hall number $n_H=R_H^{-1}$ on temperature and doping density (equal to the density of bosons), with R_H given by (12), α_B given by (11), and $R_H^{(0)}=1/\delta$.

The results shown in Fig. 2 have two noteworthy features. First, n_H grows monotonically with increasing temperature. This is caused by an increase in the number of low-energy bosons. The solid portions of the curves denote the region where the corrections to n_H are small compared to δ, i.e., where a linear-response calculation of σ_{xy} is expected to be valid. We also note that the average slope of the curves in this region increases with increasing

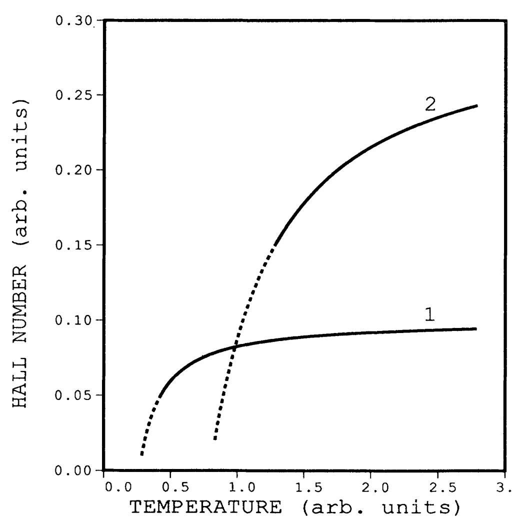


FIG. 2. Hall number n_H as a function of temperature for two doping densities, $\delta_1=0.1$ and $\delta_2=0.3$, in dimensionless units of particles per two-dimensional plaquette. The unit of temperature is $T_0=2\pi\hbar^2/(l_0^2m_B)$, where l_0 is the lattice spacing. Characteristic Bose temperature T_{c0} is taken to be $T_{c0}=2\pi\hbar^2\delta/m_B$, and free-boson temperature dependence of μ_B is assumed.

doping. These results are fully consistent with the behavior of n_H in high-T_c materials as observed in $La_{2-x}Sr_xCuO_4$ and $YBa_2Cu_3O_7$.[3] Moreover, recent measurements[19] show that the curvature of the $n_H(T)$ curves at high temperatures is negative, in agreement with the present theory.

Enhancement of the Hall coefficient can be obtained quantitatively and transparently for the high-frequency response. At high frequencies ($\omega\tau_B \gg 1$) each particle samples the local value of the fields in both time and space. The Hall conductance is then given by $\sigma_B = \langle\rho_B(\phi)(h+H)\rangle/(m_B\omega)^2$, where the average is over the distribution of internal fields h, and we have emphasized the dependence of the local density of bosons on the local value of the scalar potential. In the presence of the external field H, the internal field h becomes correlated with ϕ as follows: $\langle h\phi\rangle = -\sigma/(\chi v)$. Thus, performing the average we get an enhanced value of the Hall conductance:

$$\sigma_B = [\rho_B(0)H - (\sigma/\chi v)d\rho_B/d\phi]/(m_B\omega)^2 .$$

The parity-odd gauge field propagator can be nonzero even in the absence of an external magnetic field. This occurs when parity and time-reversal symmetries are violated spontaneously. Large renormalization of the Hall coefficient at low temperatures indicates a large susceptibility of the system to parity-violating perturbations. The results presented here predict spontaneous symmetry breaking at $T \sim T_{c0}$ where $\alpha_B = 1$. Conceivably, higher-order corrections soften this singularity, while retaining the instability.

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