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Lifetime of the Stark resonant level in double-barrier structures

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We present an exact numerical calculation for the energy level and the Stark resonance width of the lowest quasibound state in the double-barrier structure with an applied electric field by solving the Schrödinger equation directly. Our results clearly demonstrate the effect of the applied electric field on both the energy level and the resonance width, which has so far been ignored. Our results predict a rapid decrease of the resonant-level lifetime with increasing electric field. The dependence of our results on the barrier width is also discussed.

Since the pioneering work of Esaki and Tsu,[1] there has been great interest in double-barrier resonant tunneling devices. Among the important problems in the study of these devices, negative differential resistance,[2] fast response time,[3] and bistability in current-voltage response,[4] are particularly attractive for application purposes. With a simple effective-mass picture, Bahder, Morrison, and Bruno presented a quantum-mechanical calculation of the resonant energy and the resonance width as a function of barrier and well dimensions.[5] These results are qualitatively consistent with the existing experimental observation of Solner *et al.*[3] However, they have ignored the effects of a bias voltage on the potential-energy shape and have used the effective-mass theory in a rather cavalier manner, ignoring both the mixing of Γ- and *X*-point states for high aluminum[6] concentration and the details of band structure.[7]

In this paper, we present calculations for the energy level and the resonance width of the lowest quasibound states in the double-barrier structure with an applied electric field by solving the Schrödinger equation directly. We are able to obtain both the resonance position and the width from the single complex energy eigenvalue. This approach was first introduced by Ahn and Chuang to calculate the quasibound states of a quantum well in an external electric field.[8] It is found that their results are in agreement with those based on the phase-shift analysis[9,10] and the stabilization method.[11] The disadvantage of this method is that numerical subroutines of the Airy functions with complex arguments are required. Our calculations are still within the effective-mass picture and based on the assumption that both the barrier and well region have the same effective mass m^*. It is also assumed that the electric field is uniform within the double-barrier structure.

The potential-energy profile is pictorially shown in Fig. 1. We choose the origin to be at the center of the well. The Schrödinger equation of the system is given by

$$(-\hbar^2/2m^*)(d^2/dz^2)\Psi(z)+V(z)\Psi(z)=E\Psi(z) , \qquad (1)$$

where m^* is the effective mass and $V(z)$ is given by

$$V(z)=\begin{cases} eFz+V_0 & \text{within the barrier} \\ eFz & \text{otherwise ,} \end{cases} \qquad (2)$$

where V_0 and F are the barrier height and the electric field, respectively. Since the potential energy term in Eq. (2) tends to $-\infty$ as z goes to $-\infty$, the system does not, strictly speaking, have true bound states. However, as argued by Ahn and Chuang,[8] we regard the system as having quasibound states in which the particles move "inside the well" for a considerable period of time then leave through tunneling only after a fairly long time interval τ has elapsed. Instead of considering the solutions of the Schrödinger equation with a boundary condition requiring the finiteness of the wave function at infinity, we shall look for solutions that represent outgoing waves at infinity. This implies that the particle finally leaves the structure by tunneling. By solving the Schrödinger equa-

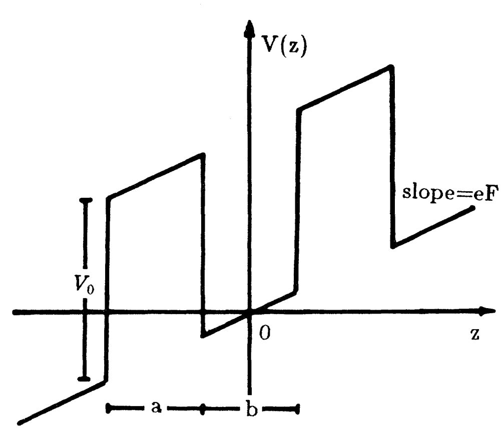

FIG. 1. Potential-energy diagram of a double-barrier structure with an applied electric field. The widths of the barrier and well are *a* and *b*, respectively.

tion, we obtain a set of complex eigenvalues, which are in the form

$$E = E_0 - i\Gamma/2 \ , \tag{3}$$

where Γ is found to be positive. E_0 and Γ correspond to the quasibound-state energy level and the resonance width, respectively. The tunneling probability per unit time is defined by

$$\omega = \Gamma/\hbar \ . \tag{4}$$

The solutions of Eq. (1) with the outgoing-wave condition are linear combinations of two independent Airy functions

$$\psi(z) = \begin{cases} a_1[\mathrm{Bi}(\eta_b) + i\mathrm{Ai}(\eta_b)], & -\infty < z < -(a+b/2) \ , \\ a_2\mathrm{Ai}(\eta_a) + a_3\mathrm{Bi}(\eta_a), & -(a+b/2) < z < -b/2 \ , \\ a_4\mathrm{Ai}(\eta_b) + a_5\mathrm{Bi}(\eta_b), & -b/2 < z < b/2 \ , \\ a_6\mathrm{Ai}(\eta_a) + a_7\mathrm{Bi}(\eta_a), & b/2 < z < b/2 + a \ , \\ a_8\mathrm{Ai}(\eta_b), & z > b/2 + a \ , \end{cases} \tag{5}$$

with

$$\eta_a = -[2m^*/(e\hbar F)^2]^{1/3}(E - V_0 - eFz) \tag{6a}$$

and

$$\eta_b = -[2m^*/(e\hbar F)^2]^{1/3}(E - eFz) \ . \tag{6b}$$

The wave function for $z < -(a+b/2)$ represents an electron traveling to $z = -\infty$ after tunneling. The complex energy E can be found by solving the secular equation obtained by matching the value of ψ and its derivative at each interface. The resulting determinant equation is

$$\begin{vmatrix} BA(\eta_b^{z_1}) & -\mathrm{Ai}(\eta_a^{z_1}) & -\mathrm{Bi}(\eta_a^{z_1}) & 0 & 0 & 0 & 0 & 0 \\ BA'(\eta_b^{z_1}) & -\mathrm{Ai}'(\eta_a^{z_1}) & -\mathrm{Bi}'(\eta_a^{z_1}) & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathrm{Ai}(\eta_a^{z_2}) & \mathrm{Bi}(\eta_a^{z_2}) & -\mathrm{Ai}(\eta_b^{z_2}) & -\mathrm{Bi}(\eta_b^{z_2}) & 0 & 0 & 0 \\ 0 & \mathrm{Ai}'(\eta_a^{z_2}) & \mathrm{Bi}'(\eta_a^{z_2}) & -\mathrm{Ai}'(\eta_b^{z_2}) & -\mathrm{Bi}'(\eta_b^{z_2}) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathrm{Ai}(\eta_b^{z_3}) & \mathrm{Bi}(\eta_b^{z_3}) & -\mathrm{Ai}(\eta_a^{z_3}) & -\mathrm{Bi}(\eta_a^{z_3}) & 0 \\ 0 & 0 & 0 & \mathrm{Ai}'(\eta_b^{z_3}) & \mathrm{Bi}'(\eta_b^{z_3}) & -\mathrm{Ai}'(\eta_a^{z_3}) & -\mathrm{Bi}'(\eta_a^{z_3}) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathrm{Ai}(\eta_a^{z_4}) & \mathrm{Bi}(\eta_a^{z_4}) & \mathrm{Ai}(\eta_b^{z_4}) \\ 0 & 0 & 0 & 0 & 0 & \mathrm{Ai}'(\eta_a^{z_4}) & \mathrm{Bi}'(\eta_a^{z_4}) & \mathrm{Ai}'(\eta_b^{z_4}) \end{vmatrix} = 0 \tag{7}$$

with $BA(\eta_b^{z_1}) = \mathrm{Bi}(\eta_b^{z_1}) + i\,\mathrm{Ai}(\eta_b^{z_1})$. $\eta_a^{z_i}$ and $\eta_b^{z_i}$ are the values of η_a and η_b evaluated at $z_i = -a - b/2$, $-b/2$, $b/2$, and $a + b/2$, respectively. If we introduce a new parameter $E^{(0)} = (\hbar^2/2m^*)(\pi/b)^2$ and define the normalized energy $\tilde{E} = E/E^{(0)}$, the normalized electric field $\tilde{F} = eFb/E^{(0)}$, the normalized barrier height $\tilde{V} = V_0/E^{(0)}$, and the normalized position $\tilde{z} = z/b$, we may express $\eta_a^{z_i}$ and $\eta_b^{z_i}$ by these normalized quantities,

$$\eta_a^{z_i} = -(\pi/\tilde{F})^{2/3}(\tilde{E} - \tilde{V} - \tilde{z}_i\tilde{F}) \ , \tag{8a}$$

$$\eta_b^{z_i} = -(\pi/\tilde{F})^{2/3}(\tilde{E} - \tilde{z}_i\tilde{F}) \ , \tag{8b}$$

with $\tilde{z}_i = -1/\delta - \frac{1}{2}$, $-\frac{1}{2}$, $\frac{1}{2}$, and $\frac{1}{2} + 1/\delta$, where $\delta = b/a$. This means that the solution of $\tilde{E}$ from the determinant equation is universal and the normalized energy can be expressed in terms of three normalized parameters $\tilde{V}$, $\tilde{F}$, and δ.

We have solved the determinant equation numerically to the desired accuracy using the series and asymptotic expressions of the Airy functions with complex argu-

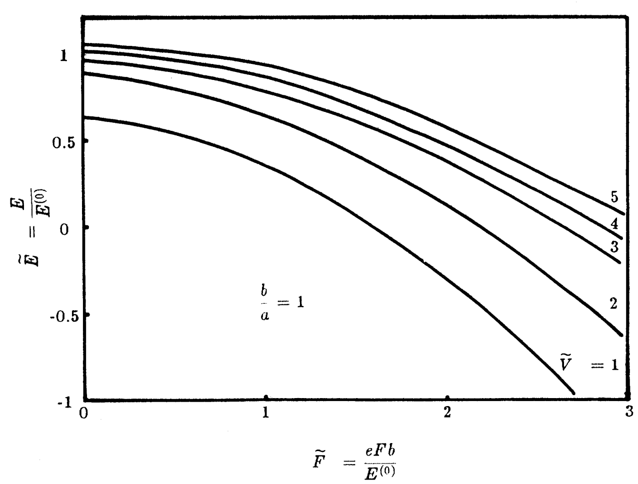

FIG. 2. The normalized energy $\tilde{E}$ of the lowest resonant state for various normalized barrier heights $\tilde{V}$ is plotted vs the normalized electric field $\tilde{F}$.

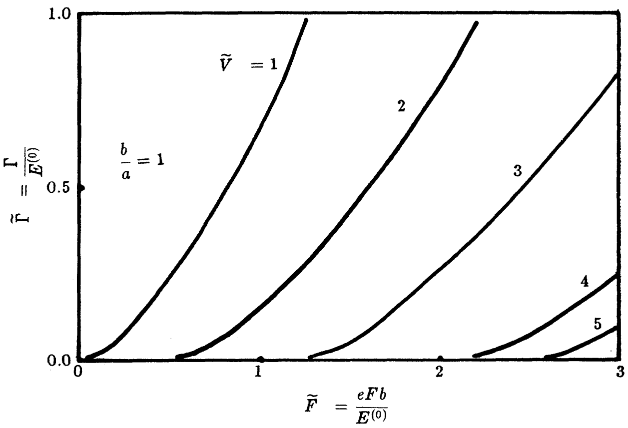

FIG. 3. The normalized resonance width $\tilde{\Gamma}$ of the lowest resonant state for various normalized barrier heights $\tilde{V}$ is plotted vs the normalized electric field $\tilde{F}$.

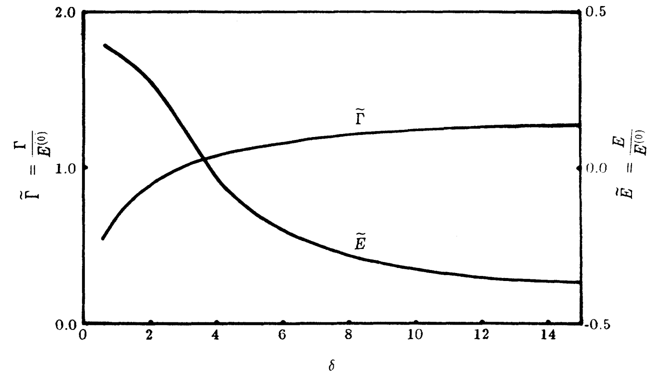

FIG. 4. The normalized energy $\tilde{E}$ and its resonance width $\tilde{\Gamma}$ of the lowest resonant state is plotted as a function of δ with $\delta = b/a$. Here we choose $\tilde{F} = 1$ and $\tilde{V} = 1$.

ments. In our computations, we have set $\delta = 1$, i.e., the width of the barrier equal to that of the well. The lowest normalized resonance energy $\tilde{E}_0 = E_0/E^{(0)}$ for various $\tilde{V}$ vs $\tilde{F}$ is plotted in Fig. 2. The resonance energy increases with barrier height, which is a simple consequence of electron confinement. As the electric field approaches zero, the resonance energy increases smoothly to a value corresponding to that obtained from the flat band model.[5]

In Fig. 3, we plot the normalized resonance width $\tilde{\Gamma} = \Gamma/E^{(0)}$ of the lowest resonant state for various $\tilde{V}$ as a function of the applied electric field $\tilde{F}$. Since the lifetime τ is defined by $\tau = \hbar/\Gamma$, our results predict a rapid decrease of the carrier lifetime with increasing applied electric field by field-enhanced tunneling. As the field is very low, the resonance width tends to be very small, which indicates a very long carrier lifetime, i.e., electronic state in the well is almost localized in it.

To show the dependence of our results on the barrier width, we plot the normalized energy and its normalized resonance width of the lowest resonant state as a function of $\delta = b/a$ in Fig. 4. Here we have assumed a constant electric field $\tilde{F} = 1$ and a constant barrier height $\tilde{V} = 1$. It can be seen that for a given well width b, the resonance width $\tilde{\Gamma}$ increases as the barrier width a is decreased, while the resonant energy $\tilde{E}$ decreases as the barrier width a is decreased. This can be understood as a result of weaker potential confinement with decreasing barrier width. For $\delta \ll 1$ the double-barrier structure reduces to a single quantum well with well width b. So the results here approach those given by Ahn and Chuang.[8] As δ tends to be large, i.e., the barrier tends to be narrow, our results indicate that the lowest resonant state tends to be located at an energy near the barrier of lower energy with a large resonance width. When δ tends to infinity, the energy difference between adjacent resonant states tends to be zero.

In conclusion, we have calculated the lowest resonant energy and its resonance width for the double-barrier structure with uniform applied electric field by solving the Schrödinger equation directly. Our results predict a rapid decrease of the resonant lifetime with increasing electric field.

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