Hydrodynamic theory of magnetoplasmons in a modulated two-dimensional electron gas

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The dielectric response properties and the collective charge-density oscillations of a two-dimensional electron gas subject to both a perpendicular magnetic field and an in-plane superlattice potential are studied with use of a hydrodynamic theory of linear response. It is demonstrated that in addition to the usual principal magnetoplasmon, an additional branch in the collective spectrum develops as a result of the resonance drift of the cyclotron-orbit center in the crossed-field configuration, which leads to regular oscillations in $1/B$ of this mode frequency. Both one-dimensional and two-dimensional superlattice potentials are considered, and the respective dielectric tensors and magnetoplasma spectra are examined.

I. INTRODUCTION

Since the magnetoresistance measurements of Weiss *et al.*[1] on the density- and potential-modulated two-dimensional electron gas (2DEG), revealing a remarkable class of oscillations in the magnetoresistivity, interest in the transport and response properties of such systems has been fast growing.[2–14] Because of the unique experimental situation, i.e., weak modulation ($eV_0/E_F<0.1$, V_0 is the strength of the potential modulation, E_F is the Fermi energy) and moderately low magnetic field ($E_F/\hbar\omega_c>10$, ω_c is the cyclotron frequency—such that many Landau levels are occupied) involved in the measurements, the problem falls into the somewhat equivocal boundary area between classical physics and quantum mechanics. Thus these magnetoresistance oscillations can be interpreted either as the manifestation of state-density oscillations, resulting from a modulation-induced oscillatory broadening of the Landau levels,[5,11] or alternatively as a resonance between the periodic cyclotron-orbit motion and the oscillating $\mathbf{E}\times\mathbf{B}$ drift of the orbit center induced by the modulation potential.[4] Both theories explain the experimental data well in regard to the commensurability oscillations, as one should expect, since the predictions of quantum mechanics and those of classical physics are the same in the *correspondence* limit.

The essential ingredients in the understanding of these magnetoresistance oscillations are the physical length scales involved in the system: the modulation period a and the cyclotron radius R_c at the Fermi level, and the interplay between them. When the two lengths are commensurable, the electronic motion acquires a resonance character, which should affect all physical quantities related to the motion of the electrons in the system. Thus in addition to the observed magnetoresistance oscillations, we have recently predicted a similar oscillation in the magnetoplasmon spectrum,[7] based on an analysis of the dielectric response function of the modulated 2DEG in the random-phase approximation (RPA). This quantum-mechanical treatment of the collective excitations devolves upon a first-order perturbation correction (in terms of the weak modulation) of the single-particle energy spectrum, which seems necessary to obtain an analytical expression for the electronic polarizability. However, the success of the semiclassical theory of the corresponding dc magnetotransport problem[4] has motivated us to seek a similar explanation of the novel magnetoplasmon oscillation in the hope of bridging the conceptual gap between the two theories. Furthermore, when a two-dimensional (2D) modulation replaces the unidirectional modulation,[8,9] a similar quantum-mechanical perturbation correction to the electron energy spectrum has proven to be highly nontrivial, due to its apparent gauge dependence and/or the necessity to invoke infinitely degenerate perturbation expansion.[15–19] On the other hand, a semiclassical theory, such as the hydrodynamic analysis of the magnetoplasmons that we present here, is not only analytically tractable, but it is also capable of achieving the same level of understanding of the commensurability oscillation as does a full-fledged quantum-mechanical theory where it exists, as in the case of a unidirectional modulation. Furthermore, the hydrodynamic theory can be applied to both a unidirectional modulation and a 2D modulation with equal ease, leading quickly to transparent solutions in both cases.

In presenting the hydrodynamic theory of magnetoplasmons in a 2DEG subject to an in-plane superlattice potential, we shall first briefly review in Sec. II the problem of a one-dimensional (1D) superlattice modulation potential. It will, therefore, be easier to introduce some of the ideas not commonly found in conventional hydrodynamic theories dealing with homogeneous systems. These ideas are crucial in incorporating the guiding-center drift into the formalism. Equally importantly, this section serves to ascertain the soundness and deficiencies of this theory by checking its conclusions against those reached quantum mechanically,[7] and to appreciate any differences therein. In Sec. III, the effect of retardation is considered. The hydrodynamic theory is then applied to the problem of a 2D superlattice modulation potential in Sec. IV, where we derive the dielectric tensor of the 2D magnetoplasma and examine the collective charge-density fluctuations.

II. ONE-DIMENSIONAL MODULATION

Central in the hydrodynamic theory of dielectric response of a many-electron system[20–23] are the equation of continuity,

$$\frac{\partial}{\partial t}\rho(\mathbf{x},t)+\nabla\cdot[\rho(\mathbf{x},t)\mathbf{v}(\mathbf{x},t)]=0\ , \tag{1}$$

and the equation of motion (Euler's equation),

$$\rho(\mathbf{x},t)\frac{\partial}{\partial t}\mathbf{v}(\mathbf{x},t)=-s^2\nabla\rho(\mathbf{x},t)-\frac{e}{m}\rho(\mathbf{x},t)[\mathbf{E}_{\rm tot}(\mathbf{x},z=0,t)+\mathbf{v}(\mathbf{x},t)\times\mathbf{B}]\ . \tag{2}$$

The two quantities describing completely the 2D motion of the electrons (of mass m and charge $-e$) confined in the x-y plane at $z=0$, in the ambient magnetic field $\mathbf{B}=B\hat{z}$ and the total electric field $\mathbf{E}_{\rm tot}(\mathbf{x},z=0,t)$, are the electron density $\rho(\mathbf{x},t)$ and the electron-drift velocity $\mathbf{v}(\mathbf{x},t)$, with $\mathbf{x}$ a 2D position vector in the x-y plane. The first term on the right-hand side of Eq. (2) arises from the hydrodynamic pressure, with s being the sound velocity. In the usual hydrodynamic theory of plasma excitations s is parametrized to match the leading RPA correction at long wavelengths. The electron density can be written as

$$\rho(\mathbf{x},t)=n_0(\mathbf{x})+n(\mathbf{x},t)\ , \tag{3}$$

where $n(\mathbf{x},t)$ is the induced density and

$$n_0(\mathbf{x})=n_0+\Delta n(\mathbf{x}) \tag{4}$$

is the unperturbed density, with n_0 the average 2D density and

$$\Delta n(\mathbf{x})=\Delta n\cos\left[\frac{2\pi}{a}x\right] \tag{5}$$

is the modulation density of period a. The total electric field also consists of two parts:

$$\mathbf{E}_{\rm tot}(\mathbf{x},z=0,t)=\mathbf{E}_0(\mathbf{x},z=0)+\mathbf{E}(\mathbf{x},z=0,t)\ , \tag{6}$$

where $\mathbf{E}(\mathbf{x},z=0,t)$ is the self-consistent electric field at the plane containing the 2DEG, while $\mathbf{E}_0(\mathbf{x},z=0)$ is a modulation-induced static electric field. Linearizing the hydrodynamic equations (1) and (2) in the induced electron density $n(\mathbf{x},t)$ and the electron-drift velocity $\mathbf{v}(\mathbf{x},t)$, which are linear in the self-consistent electric field $\mathbf{E}(\mathbf{x},z=0,t)$, we obtain

$$\frac{\partial}{\partial t}n(\mathbf{x},t)+\nabla\cdot[n_0(\mathbf{x})\mathbf{v}(\mathbf{x},t)]=0\ , \tag{7}$$

and

$$n_0(\mathbf{x})\frac{\partial}{\partial t}\mathbf{v}(\mathbf{x},t)=-s^2\nabla n(\mathbf{x},t)-\frac{e}{m}n_0(\mathbf{x})[\mathbf{E}(\mathbf{x},z=0,t)+\mathbf{v}(\mathbf{x},t)\times\mathbf{B}]-\frac{e}{m}n(\mathbf{x},t)\mathbf{E}_0(\mathbf{x},z=0)\ . \tag{8}$$

Associated with the induced electron density there arises an induced potential satisfying Poisson's equation

$$\nabla^2\phi(\mathbf{x},z,t)=\frac{4\pi e}{\kappa}n(\mathbf{x},t)\delta(z)\ , \tag{9}$$

where κ is the background dielectric constant. Equation (9) is readily solved in wave-vector ($\mathbf{q}$) and frequency (ω) representation as

$$\phi(\mathbf{q},z,\omega)=-\frac{2\pi e}{\kappa q}n(\mathbf{q},\omega)\exp(-q|z|)\ . \tag{10}$$

The modulated electron density is accompanied by a modulated static potential

$$V(\mathbf{x},z=0)=V_0\cos\left[\frac{2\pi}{a}x\right] \tag{11}$$

and the corresponding static electric field

$$\mathbf{E}_0(\mathbf{x},z=0)=\hat{\mathbf{x}}\frac{2\pi}{a}V_0\sin\left[\frac{2\pi}{a}x\right]\ , \tag{12}$$

both resulting from the spatially modulated background ion density.

The motion of an electron in the plane perpendicular to a constant magnetic field $\mathbf{B}$ follows the circular orbit prescribed by

$$x=X+R_c\cos\alpha,\quad y=Y+R_c\sin\alpha\ . \tag{13}$$

Here (X,Y) is the center of the cyclotron orbit, while the angular variable is $\alpha=\omega_c t$. The electronic motion may be decomposed into two parts: a rapid rotation of frequency ω_c about the orbit center, superposed on a slow drift of the orbit center with velocity $\mathbf{E}\times\mathbf{B}/B^2$. In the present case of a weak magnetic field, the modulation-induced electric field oscillates rapidly within one cyclotron orbit since $R_c>a$. It then follows that only the time-averaged field is important and need be retained. Integrating out the fast variable α then leads to

$$\mathbf{E}_0(X)=\frac{1}{2\pi}\int_0^{2\pi}d\alpha\,\mathbf{E}_0(X+R_c\cos\alpha)=\hat{\mathbf{x}}V_0\left[\frac{2\pi}{a}\right]J_0\left[\frac{2\pi}{a}R_c\right]\sin\left[\frac{2\pi}{a}X\right]\ , \tag{14}$$

where $J_0(x)$ is a Bessel function. Similar results are obtained for the modulation potential and electron density. Thus the oscillatory electric field leads to an oscillatory drift of the cyclotron-orbit center, which is the origin of the magnetoplasmon oscillations. Equation (14) corresponds to the guiding-center approximation in classical plasma theory, in which the particle orbit is approximated by the guiding center of the orbit, with finite-Larmor-radius corrections.[24]

When the fluctuating electric field as given by Eq. (14) and the corresponding electron density and/or the potential are substituted in the hydrodynamic equations, they give rise to both linear and bilinear terms in the sinusoidal functions. The former linear terms fluctuate about zero and are thus averaged out over one period of oscillation, while the latter fluctuate about constant values and lead to nonvanishing root-mean-square (rms) contributions upon averaging over one period of the modulations.[4] With this, the Fourier transformed Eqs. (7) and (8) may be combined to yield (suppressing the **q** and ω dependence of the drift velocity and the electric field)

$$\Omega^2\mathbf{v}=s^2\mathbf{q}\mathbf{q}\cdot\mathbf{v}+\hat{\mathbf{x}}v_x\varpi^2-i\frac{e}{m}\omega\mathbf{E}-i\omega\omega_c\mathbf{v}\times\hat{\mathbf{z}}\ , \tag{15}$$

where

$$\varpi^2=\frac{1}{2n_0m}\frac{V_0^2}{2\pi}\left[\frac{2\pi}{a}\right]^3[J_0(2\pi R_c/a)]^2\ . \tag{16}$$

Here we have introduced a phenomenological damping time τ and written $\Omega^2=\omega^2+i\omega/\tau$. Invoking the definition of the current density $\mathbf{J}(\mathbf{q},\omega)=n_0(-e)\mathbf{v}(\mathbf{q},\omega)$, along with Ohm's law $\mathbf{E}=\overleftrightarrow{\rho}\cdot\mathbf{J}$, one easily obtains the resistivity tensor

$$\overleftrightarrow{\rho}=\frac{m}{in_0e^2\omega}\begin{bmatrix}\Omega^2-\varpi^2-s^2q_x^2 & i\omega\omega_c-s^2q_xq_y\\ -i\omega\omega_c-s^2q_xq_y & \Omega^2-s^2q_y^2\end{bmatrix}\ , \tag{17}$$

which can now be inverted to yield the conductivity tensor $\overleftrightarrow{\sigma}$. The latter is related to the dielectric tensor $\overleftrightarrow{\epsilon}$ through the relation

$$\overleftrightarrow{\epsilon}=\overleftrightarrow{I}+i\frac{2\pi q}{\omega}\overleftrightarrow{\sigma}\ . \tag{18}$$

It follows that the dielectric tensor takes the form

$$\overleftrightarrow{\epsilon}=\begin{bmatrix}1-\omega_{p,2\mathrm{D}}^2(\Omega^2-s^2q_y^2)/\mathcal{D} & -\omega_{p,2\mathrm{D}}^2(s^2q_xq_y-i\omega\omega_c)/\mathcal{D}\\ -\omega_{p,2\mathrm{D}}^2(s^2q_xq_y+i\omega\omega_c)/\mathcal{D} & 1-\omega_{p,2\mathrm{D}}^2(\Omega^2-\varpi^2-s^2q_x^2)/\mathcal{D}\end{bmatrix}\ , \tag{19}$$

with

$$\mathcal{D}=\Omega^4-\Omega^2(s^2q^2+\varpi^2)-\omega^2\omega_c^2+s^2q_y^2\varpi^2 \tag{20}$$

and $\omega_{p,2\mathrm{D}}^2=2\pi e^2n_0q/\kappa m$ is the usual long-wavelength 2D plasmon.

The longitudinal dielectric function can be obtained by projecting $\overleftrightarrow{\epsilon}$ in the **q** direction as

$$\epsilon_{\mathrm{L}}(\mathbf{q},\omega)=\hat{\mathbf{q}}\cdot\overleftrightarrow{\epsilon}\cdot\hat{\mathbf{q}}\ . \tag{21}$$

Collective charge-density oscillation frequencies are then obtained by solving the longitudinal dispersion relation

$$q_x^2\epsilon_{xx}+q_y^2\epsilon_{yy}+q_xq_y(\epsilon_{xy}+\epsilon_{yx})=0\ . \tag{22}$$

This yields two normal-mode frequencies given approximately as (neglect damping)

$$\omega_+^2\equiv\omega_c^2+s^2q^2+\omega_{p,2\mathrm{D}}^2+\varpi^2 \tag{23}$$

and

$$\omega_-^2=\frac{\varpi^2\omega_{p,2\mathrm{D}}^2}{\omega_+^2}\frac{q_y^2}{q^2}\cong\frac{(eV_0)^2}{2\kappa m^2\omega_c^2}\left[\frac{2\pi}{a}\right]^3\frac{q_y^2}{q}[J_0(2\pi R_c/a)]^2\ , \tag{24}$$

where we have replaced ω_+ with ω_c in the long-wavelength limit to obtain the approximate expression for ω_-. ω_+ is the principal 2D magnetoplasmon frequency modified by the modulation that is present in ϖ. ω_- is the mode due entirely to the modulation. Apart from the term depending on the sound velocity, the high-frequency mode ω_+ matches exactly our earlier quantum-mechanical result.[7] Close examination of the low-frequency mode reveals that its wave-number dependence and its oscillation period in $1/B$ both agree with those obtained in Ref. 7. While the wave-number dependence of ω_- is apparent, its periodicity can best be seen when the asymptotic expression of the Bessel function is employed. It turns out that $\omega_-^2\sim\cos^2(2\pi R_c/a-\pi/4)$ oscillates with precisely the same periodicity $\Delta(1/B)=ea/2k_F$ as predicted in Ref. 7. It is clear that such a periodicity follows from the commensurability condition $2R_c=(n+\frac{1}{4})a$, $n=1,2,3,\ldots$, with $R_c=k_F/eB$ being the cyclotron radius at the Fermi level. However the amplitude of the oscillation differs from that calculated in Ref. 7, especially in its dependence on the modulation strength V_0. Equation (24) shows a linear dependence of ω_- on V_0, which is to be compared with the square-root dependence on V_0 predicted for ω_- in Ref. 7. Such a discrepancy in the dependence on the modulation amplitude can be traced back to the starting points of the two theories. In the quantum-mechanical treatment the modulation is included only in the electron energy spectrum (to first order in V_0), while the electron wave functions are not affected by the modulation. Thus the electron density remains unmodulated. On the other hand, the present hydrodynamic theory puts the potential modulation and the electron-density modulation on an equal footing (albeit classically), including both in the starting equations. To fully remove the discrepancy thus requires a quantum-mechanical theory of complete self-consistency in dealing with the modulation, namely, solving the Schrödinger equation in conjunction with the Poisson equation, incorporating in full force the modulation in both equations, self-consistently.

The most important difference between the low-frequency mode of the present hydrodynamic theory [Eq. (24)] and that of the quantum-mechanical theory[7] is the absence of the Shubnikov–de Haas (SdH) oscillation in ω_- calculated here. This is not really surprising since the SdH oscillation is a quantum-mechanical phenomenon, originating from Landau quantization of the electron energy levels and the associated density of states. The classical nature of the present theory precludes any account of inherently quantum-mechanical effects (e.g., density-of-states effects associated with Landau quantization), as in the case of the SdH oscillations which become prominent when the magnetic field is large enough ($B>0.5$ T),

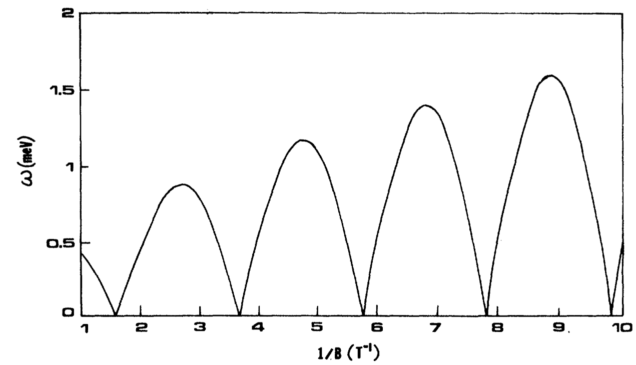

FIG. 1. ω_- vs reciprocal magnetic-field strength for a 1D superlattice modulation. $a = 382$ nm, $n_0 = 3.16\times 10^{11}$ cm^{-2}, $eV_0 = 1.5$ meV. Also $q_x = 0$, $q_y = 0.1k_F$.

such that Landau quantization can no longer be overlooked.

The hydrodynamic prediction of the low-frequency magnetoplasma mode as given by Eq. (24) is shown graphically in Fig. 1, where ω_- is plotted against the inverse magnetic field. The parameters used are average 2D electron density $n_0 = 3.16\times 10^{11}$ cm^{-2}, modulation period $a = 382$ nm, potential modulation amplitude $eV_0 = 1.5$ meV. The wave numbers used are $q_x = 0$, $q_y = 0.1k_F$, with the Fermi wave number given by $k_F = (2\pi n_0)^{1/2}$.

III. EFFECTS OF RETARDATION

Thus far we have implicitly assumed the speed of light to be infinite ($c\to\infty$) in our treatment of the response properties of the modulated 2DEG. Retardation effects associated with the finite speed with which electromagnetic signals propogate will be considered in this section following the procedure outlined by Orman and Horing.[25] In the present hydrodynamic theory, the modification brought about by the finite speed of light mainly affects the relations between the charge-current distributions and the fields generated by them. Thus, instead of the Poisson equation, these relations are furnished by the full set of Maxwell's equations. In what follows we consider a 2DEG of areal electron density n_s (or equivalently, a 2D current density $\mathbf{J}_s$, related to the charge density through the equation of continuity), with a known conductivity tensor $\overleftrightarrow{\sigma}$ as given in Sec. II with the effect of modulation already built into it as discussed in that section.

In the space above and below the 2D electron plane, the total electric field satisfies the homogeneous wave equation

$$\nabla^2\mathbf{E} - \frac{\kappa}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = \mathbf{0} , \tag{25}$$

along with

$$\nabla\cdot\mathbf{E} = 0 . \tag{26}$$

As before, the electric field is assumed to have the spatial-temporal dependence $\exp[i(\mathbf{Q}\cdot\mathbf{R}-\omega t)]$, with $\mathbf{Q} = (\mathbf{q}_\parallel, \mathbf{q}_\perp)$, decomposed into a vector parallel to the plane of the 2DEG, and one perpendicular to it. Equations (25) and (26) thus reduce to

$$Q^2 = q_\parallel^2 + q_\perp^2 = \frac{\kappa\omega^2}{c^2} \tag{27}$$

and

$$\mathbf{Q}\cdot\mathbf{E} = \mathbf{q}_\parallel\cdot\mathbf{E}_\parallel + \mathbf{q}_\perp\cdot\mathbf{E}_\perp = 0 . \tag{28}$$

Furthermore, the boundary condition on the electric field

$$\hat{\mathbf{z}}\cdot[\mathbf{E}_\perp(z=0^+) - \mathbf{E}_\perp(z=0^-)] = \frac{4\pi n_s}{\kappa} \tag{29}$$

requires that

$$E_\perp(z=0) = \frac{2\pi n_s}{\kappa} . \tag{30}$$

Employing another Maxwell equation

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t} , \tag{31}$$

or

$$\mathbf{Q}\times\mathbf{E} = \frac{\omega}{c}\mathbf{H} , \tag{32}$$

along with the boundary condition for the magnetic field

$$\hat{\mathbf{z}}\times[\mathbf{H}(z=0^+) - \mathbf{H}(z=0^-)] = \frac{4\pi}{c}\mathbf{J}_s , \tag{33}$$

one obtains the boundary value of the magnetic field

$$\hat{\mathbf{z}}\times\mathbf{H}(z=0) = \frac{2\pi}{c}\mathbf{J}_s , \tag{34}$$

where use has been made of the equation of continuity

$$\omega n_s = \mathbf{q}_\parallel\cdot\mathbf{J}_s . \tag{35}$$

Combining Eqs. (30) and (35) one writes $\mathbf{E}_\perp(z=0)$ in terms of the surface current density $\mathbf{J}_s$:

$$\mathbf{E}_\parallel(z=0) = -\frac{2\pi\omega}{c^2 q_\perp}\left[\overleftrightarrow{I} - \frac{\mathbf{q}_\parallel\mathbf{q}_\parallel}{Q^2}\right]\cdot\mathbf{J}_s , \tag{36}$$

which may be turned into a homogeneous equation for the total in-plane electric field $\mathbf{E}_\parallel(z=0)$, with the help of Ohm's law $\mathbf{J}_s = \overleftrightarrow{\sigma}\cdot\mathbf{E}_\parallel(z=0)$, as

$$\left[\overleftrightarrow{I} + \frac{2\pi\omega}{c^2 q_\perp}\left[\overleftrightarrow{I} - \frac{\mathbf{q}_\parallel\mathbf{q}_\parallel}{Q^2}\right]\cdot\overleftrightarrow{\sigma}\right]\cdot\mathbf{E}_\parallel(z=0) = 0 . \tag{37}$$

Obviously, self-sustaining oscillation frequencies are given by the roots of the determinantal equation

$$\left|\overleftrightarrow{I} + \frac{2\pi\omega}{c^2 q_\perp}\left[\overleftrightarrow{I} - \frac{\overleftrightarrow{q}_\parallel\overleftrightarrow{q}_\parallel}{Q^2}\right]\cdot\overleftrightarrow{\sigma}\right| = 0 . \tag{38}$$

In passing we note that to recover the nonretarded limit of the dispersion relation at this stage, as at any other stages in the rest of this section, one merely lets $c\to\infty$, which leads to $q_\perp = iq_\parallel \equiv iq$ according to Eq. (27), and Eq. (38) reproduces the results of Sec. II. The full dispersion relation given by Eq. (38) may be reduced to the form

$$\omega^4(1+\beta_c) - \omega^2\left[(1+\beta_c)\left[s^2q_\parallel^2+\tilde\omega^2+\omega_{p,2\mathrm{D}}^2\frac{q_\perp}{q_\parallel}\right]+\omega_c^2\right] + \tilde\omega^2 s^2 q_y^2(1+\beta_s)=0 , \quad (39)$$

where $\beta_c=\kappa\omega_{p,2\mathrm{D}}^2/c^2q_\perp q_\parallel$, and $\beta_s=\omega_{p,2\mathrm{D}}^2/s^2q_\perp q_\parallel$. This dispersion relation incorporates the full effect of retardation, as well as the effect of modulation. However, it is, in general, impossible to solve Eq. (39) analytically, whereas a numerical solution does not provide much insight into the nature of the various modes. [In this connection, it should be noted that $q_\perp$ is frequency dependent through Eq. (27)]. We shall not attempt to solve Eq. (39) here except to note that in the various limiting cases where analytic solutions do exist,[20–23,25,26] our calculations using Eq. (39) reproduce all these known results. Furthermore, corrections due to retardation for the two plasma modes discussed in Sec. II may be shown to have the form

$$\omega_\pm^2 \to \omega_\pm^2\left[1-\frac{\kappa\omega_{p,2\mathrm{D}}^2}{c^2q_\parallel^2}\right] \quad (40)$$

to lowest order in $\kappa\omega_{p,2\mathrm{D}}^2/c^2q_\parallel^2$. For nonvanishing wave vectors this factor is negligibly small. For example, corresponding to Fig. 1 this correction factor is $\sim 10^{-4}$.

IV. TWO-DIMENSIONAL MODULATION

It is straightforward to generalize the above formulation to treat a 2D superlattice of density-potential modulation. In doing so we shall ignore the effect of retardation, as it can be readily incorporated into our treatment as shown in Sec. III, when such a need arises. Moreover, incorporation of retardation will not alter the main conclusions of this section. Consider the case of a rectangular 2D superlattice of the form

$$V(x,y)=V_{x0}\cos\left[\frac{2\pi}{a}x\right]+V_{y0}\cos\left[\frac{2\pi}{b}y\right] , \quad (41)$$

with a and b as the lattice constants along the x and y directions, respectively, the notion of a fast variable $\alpha=\omega_c t$ superimposed on the slow drift of the cyclotron-orbit center (X,Y) is still valid. Corresponding to Eq. (14) we now have

$$V(X,Y)=\frac{1}{2\pi}\int_0^{2\pi}d\alpha\left[V_{x0}\cos\left[\frac{2\pi}{a}(X+R_c\cos\alpha)\right]+V_{y0}\cos\left[\frac{2\pi}{b}(Y+R_c\sin\alpha)\right]\right]$$
$$=V_{x0}J_0(2\pi R_c/a)\cos\left[\frac{2\pi}{a}X\right]+V_{y0}J_0(2\pi R_c/b)\cos\left[\frac{2\pi}{b}Y\right] . \quad (42)$$

This modulation potential along with its counterparts in electron density and static electric field can now be put into the linearized hydrodynamic equations (7) and (8) and the resultant dependence on X and Y integrated over the unit cell of the superlattice to single out the rms values of the modulation. Following essentially the same steps as outlined in Sec. II, one finds that the dielectric tensor in the present case of a rectangular modulation superlattice is given by

$$\overleftrightarrow{\epsilon}=\begin{bmatrix}1-\omega_{p,2\mathrm{D}}^2(\Omega^2-\tilde\omega_2^2-s^2q_y^2)/\mathcal{D}' & -\omega_{p,2\mathrm{D}}^2(s^2q_xq_y-i\omega\omega_c)/\mathcal{D}' \\ -\omega_{p,2\mathrm{D}}^2(s^2q_xq_y+i\omega\omega_c)/\mathcal{D}' & 1-\omega_{p,2\mathrm{D}}^2(\Omega^2-\tilde\omega_1^2-s^2q_x^2)/\mathcal{D}'\end{bmatrix} ,$$

where

$$\tilde\omega_1^2=\frac{1}{2n_0m}\frac{V_{x0}^2}{2\pi}\left[\frac{2\pi}{a}\right]^3[J_0(2\pi R_c/a)]^2 . \quad (44)$$

$$\tilde\omega_2^2=\frac{1}{2n_0m}\frac{V_{y0}^2}{2\pi}\left[\frac{2\pi}{b}\right]^3[J_0(2\pi R_c/b)]^2 , \quad (45)$$

and

$$\mathcal{D}'=\Omega^4-\Omega^2(s^2q^2+\tilde\omega_1^2+\tilde\omega_2^2) -\omega^2\omega_c^2+s^2q_y^2\tilde\omega_1^2+s^2q_x^2\tilde\omega_2^2 . \quad (46)$$

One can again set the longitudinal component of $\overleftrightarrow{\epsilon}$ to zero and solve for the normal modes. These turn out to be (setting $\tau=\infty$)

$$\omega_+^2=\omega_c^2+s^2q^2+\omega_{p,2\mathrm{D}}^2+\tilde\omega_1^2+\tilde\omega_2^2 \quad (47)$$

and

$$\omega_-^2=\frac{\tilde\omega_1^2q_y^2+\tilde\omega_2^2q_x^2}{q^2}\frac{\omega_{p,2\mathrm{D}}^2}{\omega_c^2} . \quad (48)$$

Thus the principal magnetoplasma mode is again modified by the modulation, this time by $\tilde\omega_1^2+\tilde\omega_2^2$, without interference between the two directions of modulation. However the low-frequency mode is an admixture of the two modulation directions which can be made more transparent if one defines the angle of propagation with respect to the x axis as $\tan\theta=q_y/q_x$, such that Eq. (48) can be rewritten as

$$\omega_-^2=\frac{\omega_{p,2\mathrm{D}}^2}{\omega_c^2}(\tilde\omega_1^2\sin^2\theta+\tilde\omega_2^2\cos^2\theta) . \quad (49)$$

For an arbitrary direction of propagation $(0<\theta<\pi/2)$

ω_- is a hybridization of $\tilde{\omega}_1$ and $\tilde{\omega}_2$, and two series of oscillations, with periods $\Delta(1/B)_x = ea/2k_F$ and $\Delta(1/B)_y = eb/2k_F$, respectively, should be observable in it. The expression for ω_- in the case of a unidirectional modulation can be readily recovered from Eq. (49) by setting $\theta=0$ or $\theta=\pi/2$. Another special limit is the square superlattice modulation potential with equal strengths in the x and y directions. Here $\tilde{\omega}_1=\tilde{\omega}_2\equiv\tilde{\omega}$, and it is seen that $\omega_-^2=\omega_{p,2\text{D}}^2\tilde{\omega}^2/\omega_c^2$, independent of the direction of propagation.

Another interesting case to which the present theory may be readily applied is the hexagonal superlattice modulation[9] as modeled by the potential

$$V(x,y)=V_0\left[\cos\left[\frac{2\pi}{a}x\right]+\cos\left[\frac{2\pi}{\sqrt{3}a}y\right]\right]\cos\left[\frac{2\pi}{\sqrt{3}a}y\right]. \quad (50)$$

Although the geometry of the unit cell is quite different, the magnetoplasma frequencies are similar to the square lattice case, with

$$\omega_+^2=\omega_c^2+s^2q^2+\omega_{p,2\text{D}}^2+2\tilde{\omega}^2\ , \quad (51)$$

and

$$\omega_-^2=\frac{\omega_{p,2\text{D}}^2}{\omega_c^2}\tilde{\omega}^2\ . \quad (52)$$

The only difference arises in the expression for $\tilde{\omega}$, which for the hexagonal superlattice modulation is given by

$$\tilde{\omega}^2=\frac{1}{\sqrt{3}}\frac{1}{2n_0m}\frac{V_0^2}{2\pi}\left[\frac{2\pi}{a}\right]^3[J_0(4\pi R_c/\sqrt{3}a)]^2\ . \quad (53)$$

This leads to a periodicity $\Delta(1/B)=\sqrt{3}ea/4k_F$ corresponding to the commensurability condition

$$2R_c=(n+\tfrac{1}{4})\frac{\sqrt{3}}{2}a,\quad n=1,2,3,\ldots\ . \quad (54)$$

Interestingly, the plasma frequency is again independent of the direction of propagation, just as in the square superlattice case. It seems that the directional dependence is a consequence of the different degrees of modulation along different directions, not directly related to the shape of the superlattice unit cell. On the other hand, the shape of the unit cell does determine the period of the commensurability oscillations. The latter correlates with the smallest dimension of the unit cell, such that the two opposite extreme points of the cyclotron orbit are both at potential maxima, hence inducing a maximum drift of the cyclotron-orbit center. This is exemplified in the present case of a hexagonal superlattice modulation, where instead of a, it is $\sqrt{3}a/2$, the height of the parallelogram-shaped unit cell, that enters the commensurability condition.

V. SUMMARY

We have applied the semiclassical hydrodynamic theory of linear response to the problem of collective-density fluctuations in a 2DEG subject to both a perpendicular magnetic field and a weak in-plane superlattice potential. Particular attention is focused on the long-wavelength limit of the magnetoplasma spectrum, which has two branches instead of only one principal mode for a homogeneous 2DEG. The potential modulation is responsible for the creation of a mode which exhibits commensurability oscillations periodic in $1/B$, in addition to its concurrent modification to the principal mode. The origin of the magnetoplasma oscillation is attributed to the resonant drift of the cyclotron-orbit center of the electron in the presence of both an external magnetic field and a modulation-induced spatially periodic electric field, as was pointed out by Beenakker[4] in an attempt to explain the corresponding magnetoresistance oscillations. Such an understanding of the commensurability oscillations seems reasonable, since it reaches basically the same conclusions as the corresponding quantum-mechanical theory,[7] as far as plasma excitations are concerned. However, true to its semiclassical nature, the hydrodynamic theory does not predict the SdH oscillations, which should be present at higher magnetic fields (upon which the commensurability oscillation is superposed, see Ref. 7).

Both a unidirectional modulation superlattice and 2D superlattices of various geometries are considered in this work, and their respective dielectric tensors and plasma modes are examined. A 2D rectangular superlattice with different degrees of modulation in the x and y directions is particularly interesting in that its plasma frequency depends on the direction of propagation. Such a directional dependence is absent for a square superlattice or a hexagonal superlattice, both with equal strengths of modulation in the two perpendicular directions.

The predicted plasma mode in these modulated 2D systems is yet to be observed experimentally, partly due to the low energy involved ($\sim$1 meV), which poses a stringent requirement of frequency resolution. Experimental techniques such as far-infrared transmission spectroscopy are in principle capable of detecting such resonances, and the theory presented here can hopefully stimulate further experimental interest in this problem.

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