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Nuclear-spin relaxation and spin-wave collective modes in a disordered two-dimensional electron gas

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The spin-polarization of nuclei near a two-dimensional electron gas (2D EG) may be relaxed by spin-flip excitations of the electron system. The spectrum of low-energy electronic spin-flip excitations depends on the disorder broadening of Landau levels and on the interaction enhancement of the Landau-level spin splitting. Disorder vertex corrections to the spin-flip response function capture the fact that the nuclear relaxation rate depends on local rather than thermodynamic Landau-level broadening, while interaction vertex corrections can strongly enhance the relaxation rate. We illustrate these effects by summing the disorder and interaction ladder diagrams for the spin-flip response function in the strong-magnetic-field limit. Our approach is also able to describe the effect of disorder on the spin-wave collective modes of a spin-polarized 2D EG.

I. INTRODUCTION

In metals the relaxation of polarized nuclear spins[1] is usually dominated by processes involving the hyperfine contact interaction between nuclear and electronic magnetic moments. Studies of nuclear relaxation rates can often provide unique information about the local electronic structure near a given nuclear site.[2] Recently[3] Berg *et al.* have demonstrated the possibility of observing nuclear-spin relaxation which is due to hyperfine interactions with electrons in a two-dimensional electron gas (2D EG) in a strong perpendicular magnetic field. This development provides a potentially important probe of the 2D EG at strong magnetic fields, which can provide information complementary to that available from the conventional transport and optical studies. Previous theoretical studies of nuclear-spin relaxation near 2D EG systems[3-5] have not taken account of the vertex corrections, which are important because of the local nature of the contact interaction. In this article, we demonstrate that the vertex corrections can have substantial quantitative and qualitative consequences and that their importance complicates the interpretation of the spin-relaxation experiments.

For nuclei near a 2D EG in GaAs, the contribution to the "spin-lattice" relaxation rate due to hyperfine interactions is given by[1,3,6]

$$T_1^{-1} = \hbar^{-1} A^2 \Omega^2 |\phi(z)|^4 k_B T \lim_{\omega \to 0} \left. \frac{\mathrm{Im}\chi_{+-}(\mathbf{x}_\perp, \mathbf{x}_\perp; \omega)}{\hbar\omega} \right|_{T=0}, \tag{1}$$

where $\phi(z)$ is the envelope function for the quasi-2D EG evaluated at the site of the relaxing nuclei, $\Omega = 45$ Å^3 is the unit cell volume, $A \equiv 40$ μeV, and $\chi_{+-}(\mathbf{x}_\perp, \mathbf{x}_\perp; \omega)$ is the local spin-flip response function for the quasi-2D electrons.[7] For an ideal 2D EG in a strong magnetic field the single particle spectrum is discrete because of the quantization of kinetic energy into macroscopically degenerate Landau levels. As emphasized by Vagner and coworkers[4] the low-energy electronic spin-flip excitations which relax the nuclear spins are present only if the Landau levels are sufficiently broadened that Landau levels with different spin indices overlap. (Without disorder the lowest electronic spin-flip excitations occur at the Zeeman energy and T_1^{-1} is zero.) Thus the inclusion of disorder is essential to any discussion of the nuclear-spin relaxation rate in a 2D EG at strong magnetic fields. It is well established[8] that electron-electron interactions can dramatically enhance the g factor which parametrizes the energy separation between up-spin and down-spin Landau levels. It is therefore equally essential to include electron-electron interactions in any discussion of the spin-flip response in the present system. In the next section we discuss the spin-flip response function of a disordered strong-field 2D EG within the self-consistent Born approximation. The following section discusses the effects of adding electron-electron interactions in the time-dependent Hartree-Fock approximation. Both of these approximations have deficiencies but are adequate to illustrate the important physical features which are misrepresented when vertex corrections are neglected. In Sec. IV we use these results to calculate the nuclear-spin relaxation rate. We conclude in Sec. V with a discussion of the implications of our results for the interpretation of experimental nuclear-spin relaxation data.

II. SPIN-FLIP RESPONSE IN THE SCBA

In this section we neglect electron-electron interactions and include only the effect of disorder within the self-consistent Born approximation (SCBA) which provides a qualitative and semiquantitative description[9] of disorder effects of a 2D EG in a strong magnetic field. The SCBA is expressed diagrammatically in Fig. 1. We will assume that the field is sufficiently strong that the low-energy spin-flip transitions in the disordered 2D EG do not involve a change in orbital Landau-level index. (The spin-splitting is about 65 times smaller than the Landau-level splitting in GaAs 2D EG systems when electron-electron interactions are neglected.) Assuming that Landau-level mixing by disorder can be neglected in a strong field, we follow Ando *et al.* in summing[10] the disorder ladders and evaluate[11] the remaining frequency sum[12] by performing a contour integration. The result for the low-frequency (i.e., intra-Landau-level) spin-flip response function is

$$\chi_{+-}(\mathbf{x},\mathbf{x}';\omega)=\int\frac{d\mathbf{q}}{(2\pi)^2}\chi_{+-}(\mathbf{q};\omega)e^{i\mathbf{q}\cdot(\mathbf{x}-\mathbf{x}')}\ , \tag{2}$$

where

$$\begin{aligned}\chi_{+-}(q,\omega)=\frac{e^{-q^2l^2/2}[L_n(q^2\ell^2/2)]^2}{2\pi l^2}\int d\varepsilon\, n_F(\varepsilon)\Bigg[&\frac{G^-(\varepsilon-\varepsilon_\downarrow)G^-(\varepsilon-\varepsilon_\uparrow+\omega)}{1-B(q)G^-(\varepsilon-\varepsilon_\downarrow)G^-(\varepsilon-\varepsilon_\uparrow+\omega)}\\&-\frac{G^+(\varepsilon-\varepsilon_\downarrow)G^-(\varepsilon-\varepsilon_\uparrow+\omega)}{1-B(q)G^+(\varepsilon-\varepsilon_\downarrow)G^-(\varepsilon-\varepsilon_\uparrow+\omega)}\\&+\frac{G^-(\varepsilon-\varepsilon_\downarrow)G^+(\varepsilon-\varepsilon_\uparrow+\omega)}{1-B(q)G^-(\varepsilon-\varepsilon_\downarrow)G^+(\varepsilon-\varepsilon_\uparrow+\omega)}\\&-\frac{G^+(\varepsilon-\varepsilon_\downarrow)G^+(\varepsilon-\varepsilon_\uparrow+\omega)}{1-B(q)G^+(\varepsilon-\varepsilon_\downarrow)G^+(\varepsilon-\varepsilon_\uparrow+\omega)}\Bigg]\ ,\end{aligned} \tag{3}$$

where

$$B(q)=\int\frac{d\mathbf{p}}{(2\pi)^2}e^{i(\mathbf{q}\times\mathbf{p})\cdot\hat{z}l^2}e^{ip^2l^2/2}|V_0(\mathbf{p})|^2L_n(p^2l^2/2) \tag{4}$$

and $l^2=\hbar/eB$. Here n is the index of the Landau level at the Fermi energy and $V_0(\mathbf{p})$ is the Fourier transform of the disorder potential from one of the impurities which are taken to be randomly distributed. In Eq. (3) $\varepsilon_\uparrow$ and $\varepsilon_\downarrow$ are the energies for the unbroadened up and down-spin levels and

$$G(z)=\frac{2}{\pi}\int_{-\Gamma}^{+\Gamma}\frac{d\omega'}{\Gamma}\frac{\sqrt{1-\omega'^2/\Gamma^2}}{z-\omega'} \tag{5}$$

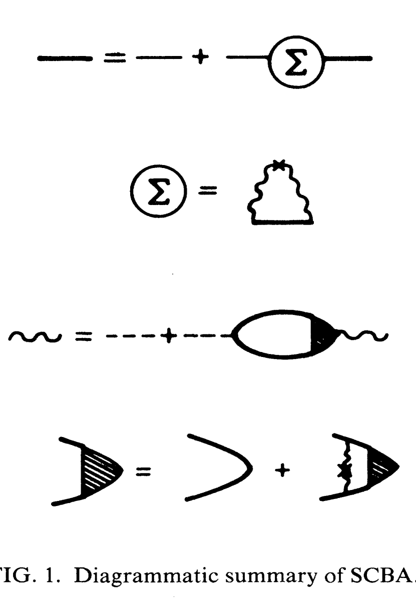

FIG. 1. Diagrammatic summary of SCBA.

is the SCBA Green's function $[G^{\pm}(\varepsilon)=G(\varepsilon\pm i0^+)]$ where 2Γ is the Landau-level width $[\Gamma^2/4=B(q=0)]$. Note that the Landau-level density-of-states in the SCBA goes strictly to zero for energies farther than Γ from the Landau level center. This feature of the SCBA is an artifact[13] but it does not influence the qualitative conclusions we wish to draw concerning χ_{+-}. The factor outside the integral in Eq. (3) is the form factor for the cyclotron orbit of an electron in the nth orbital Landau level. All the results presented in this paper and discussed below will be for the extreme quantum limit where $n=0$. (Only trivial changes are necessary to consider other value of n as long as the magnetic field is strong enough that Landau level mixing can be neglected.)

We plot $\mathrm{Im}\chi_{+-}(q,\omega)/\omega$ versus ω for $ql=0.6$, in Figs. 2 and in 3. The results shown in Fig. 2 are for spin splitting, $\Omega_{ss}\equiv\varepsilon_\uparrow-\varepsilon_\downarrow=0.3\Gamma$ and those in Fig. 3 are for $\Omega_{ss}=1.7\Gamma$. As discussed in more detail below $\mathrm{Im}\chi_{+-}$ shows the oscillator strength weighted spectrum of the spin-flip excitations in the system. Note that in the low-frequency limit $\mathrm{Im}\chi_{+-}(q,\omega)/\omega$ approaches a constant. According to Eq. (1) the nuclear relaxation rate is proportional to the integral over wave vector of these low-frequency limits. Comparing Figs. 2(a) and 2(b) we see that the vertex corrections change the results qualitatively. When vertex corrections are not included $[B(q)\equiv 0]$, $e^{q^2l^2/2}\mathrm{Im}\chi_{+-}(q,\omega)$ is independent of q and is proportional to the convolution of the density states of occupied majority-spin electrons with the density of states of unoccupied minority-spin electrons at an energy larger by ω as illustrated schematically in Fig. 4. In the limit where ω is small compared to Γ it follows that $\mathrm{Im}\chi_{+-}(q,\omega)$ is proportional to ω times the product of the majority and minority-spin densities of states at the Fermi level. For the case illustrated in Fig. 3 where Γ is small compared to Ω_{ss}, $\mathrm{Im}\chi_{+-}(q,\omega)$ would be sharply peaked for ω near Ω_{ss}. On the other hand for Γ large compared to Ω_{ss}, as illustrated in Fig. 2, $\mathrm{Im}\chi_{+-}(q,\omega)$ should fall smoothly from a weak maximum near $\omega = \Omega_{ss}$ and reach zero when $\omega = 2\Gamma + \Omega_{ss}$. As seen in Figs. 2 and 3 the effect of the vertex corrections may be described as leading to a reduction of the apparent Landau-level width, which is especially strong at small q. In fact as we show below using an *exact* sum rule argument, the width of the peak in $\mathrm{Im}\chi_{+-}(q,\omega)$ approaches zero and its position is centered around $\omega = \Omega_{ss}$ as q goes to zero. Despite its deficiencies, the SCBA treatment of disorder is able to capture this qualitative property of χ_{+-} because it conserves total particle number.[14] The importance of this property in interpreting nuclear-spin relaxation experiments will be discussed in Sec. IV.

In order to derive a sum rule for the spin-flip response function we first construct a Lehmann representation that is appropriate for the case when an orbital Landau level is spin-split. If the energies are measured with respect to the centers of the Landau levels (which are at $\pm\Omega_{ss}/2$) we can write the response function as

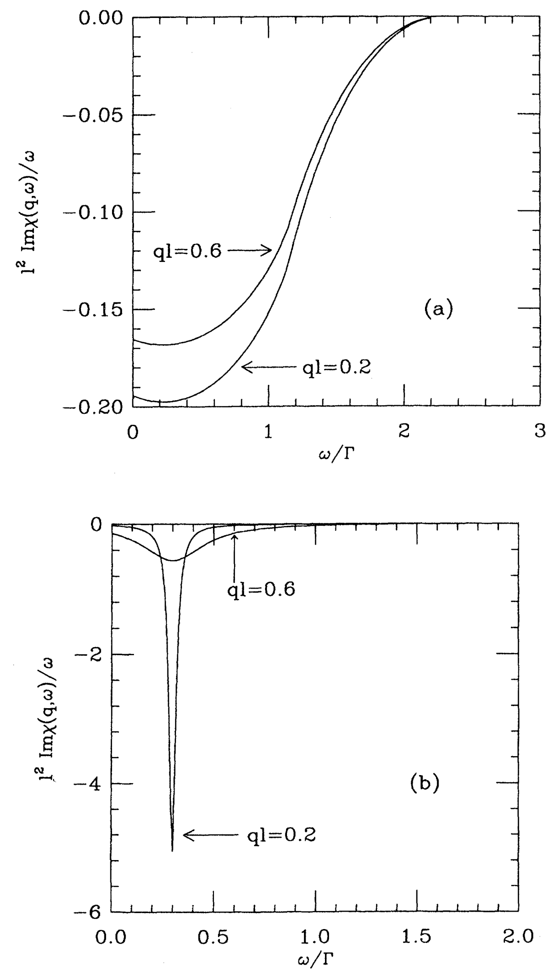

FIG. 2. Plot of $\mathrm{Im}\chi_{+-}(q,\omega)/\omega$ vs ω for the case $\Omega_{ss}/\Gamma = 0.3$ for two values of ql, (a) when no vertex corrections are included, (b) when disorder vertex corrections are included.

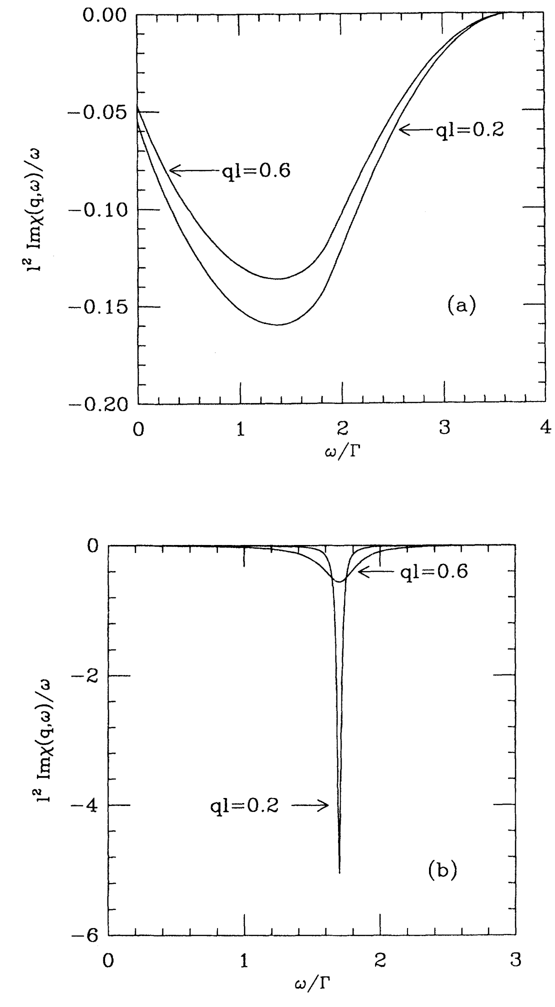

FIG. 3. Same as Fig. 2 but with $\Omega_{ss}/\Gamma = 1.7$.

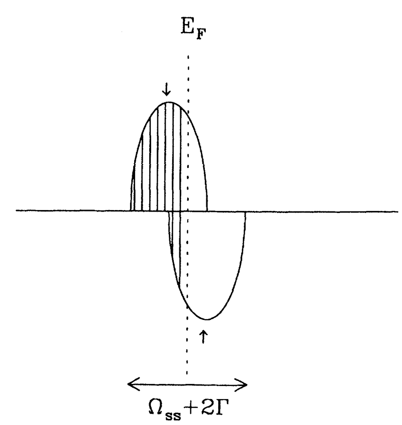

FIG. 4. A schematic plot of the density of states vs energy for the disordered broadened, overlapping spin bands. The shaded part denotes occupied states

$$\chi_{+-}(q,\omega)=\frac{1}{A}\sum_{i,f}\frac{f(\varepsilon_i-\Omega_{ss}/2)-f(\varepsilon_f+\Omega_{ss}/2)}{\omega+\varepsilon_i-\varepsilon_f-\Omega_{ss}+i\eta}|\langle\phi_f|e^{i\mathbf{q}\cdot\mathbf{r}}|\phi_i\rangle|^2 , \tag{6}$$

where A is the area of the system, and f and i range over the labels of the exact eigenstates ($|\phi\rangle$) of the orbital Hamiltonian,

$$\mathcal{H}=\frac{N\hbar\omega_c}{2}+\frac{1}{A}\sum_{\mathbf{k}}V_{\rm dis}(\mathbf{k})\bar{\rho}(\mathbf{k}) . \tag{7}$$

$V_{\rm dis}$ is the impurity potential and $\bar{\rho}$ is the projection of the density operator on the lowest Landau level. We define the quantities $\Lambda^{\pm}(q,\omega)$ to be

$$\Lambda^{\pm}(\mathbf{q},\omega)=\frac{1}{A}\sum_{i,f}f(\varepsilon_i\mp\Omega_{ss}/2)|\langle\phi_f|e^{i\mathbf{q}\cdot\mathbf{r}}|\phi_i\rangle|^2\delta(\omega+\varepsilon_i-\varepsilon_f) . \tag{8}$$

In terms of $\Lambda^{\pm}$ the spin-flip response function is given by

$$\chi_{+-}(q,\omega)=\int d\omega'\left[\frac{\Lambda^{+}(q,\omega')}{\omega-\omega'-\Omega_{ss}+i\eta}-\frac{\Lambda^{-}(q,\omega')}{\omega+\omega'-\Omega_{ss}+i\eta}\right] . \tag{9}$$

In particular,

$$\mathrm{Im}\chi_{+-}(q,\omega)=-\pi[\Lambda^{+}(q,\omega-\Omega_{ss})-\Lambda^{-}(q,-\omega+\Omega_{ss})] . \tag{10}$$

Using the Hamiltonian, Eq. (7), we can evaluate the first two moments of $\Lambda^{\pm}$:

$$\int d\omega\,\Lambda^{\pm}(q,\omega)=(2\pi l^2)^{-1}\nu^{\pm}e^{-q^2l^2/2} , \tag{11}$$

$$\int d\omega\,\omega\Lambda^{\pm}(q,\omega)=e^{-q^2l^2/2}\int\frac{d^2\mathbf{k}}{(2\pi)^2}V_{\rm dis}(\mathbf{k})n^{\pm}(\mathbf{k})(e^{il^2(\mathbf{k}\times\mathbf{q})\cdot\hat{z}}-1) , \tag{12}$$

where $\nu^{\pm}$ is the filling factor and $n^{\pm}$ is the density for majority (minority) spins. For isotropic disorder the right-hand side of Eq. (12) vanishes as q^2 in the long-wavelength limit. Note that the first moment of $\Lambda^{\pm}(q,\omega)$ will remain small at all q if the disorder potential is smooth. It follows from these sum rules[15] that the typical value of ω' contributing to the right-hand side of Eq. (9) vanishes as q goes to zero and hence that

$$\lim_{q\to 0}\chi_{+-}(q,\omega)=\frac{\nu^{+}-\nu^{-}}{2\pi\ell^2}\frac{1}{\omega-\Omega_{ss}+i\eta} . \tag{13}$$

$\mathrm{Im}\chi_{+-}$ approaches a δ function centered at Ω_{ss}. This is an *exact* result which is badly violated when vertex corrections are neglected that the SCBA is able to recapture when impurity ladders are included.

III. EXCHANGE-ENHANCED SPIN SPLITTING AND SPIN-WAVE MODES

We turn our attention now to study the effect of the electron-electron interactions on the spin-flip response function. As we have discussed in the previous section χ_{+-} depends qualitatively on the ratio of the spin-splitting to the Landau-level width. It is well known[8,16] that in a 2D EG system the spin-splitting is often enhanced by a factor of 10 or more because of electron-electron interactions. In this paper we use a Hartree-Fock approximation to the electronic self-energy to describe this spin-splitting. We expect that this approximation will overestimate the spin-splitting,[17] at least at intermediate magnetic fields, but a quantitative estimate of the exchange enhancement is not important to our present purposes. The Hartree-Fock approximation for the self-energy has the advantage that the consistent vertex correction (the "conserving" approximation in the sense of Kadanoff and Baym[14]) entering the spin-flip response function is a simple ladder sum which can easily be evaluated as we describe below. This consistency is essential.

The Hartree-Fock approximation in the treatment of electron-electron interactions is the precise analog of the SCBA for the perturbative treatment of disorder and it may be summarized by the same set of diagrams as in Fig. 1 if disorder lines are replaced by Coulomb interaction lines. In the Hartree-Fock approximation[16] the spin-splitting between the centers of the disorder broadened Landau levels is given by

$$\Omega_{ss}=g_0\mu_B H-\Sigma_{\downarrow}+\Sigma_{\uparrow} , \tag{14}$$

Σ_σ is the self-energy of electrons with spin σ. The Hartree-Fock result for the self-energy of electrons in the lowest Landau level is $\Sigma_\sigma=-\nu_\sigma\sqrt{\pi/2}(e^2/\epsilon\ell)$. The filling factors ν_σ must be determined self-consistently[16] by minimizing the total energy including that from the occupied states within the disorder-broadened Landau levels. We observe[18] that the sum of the Coulomb interaction ladders is equivalent to a random-phase-approximationlike sum of bubbles with the Coulomb interaction replaced by an exchange effective interaction $-I(q)$. When only the lowest Landau level is occupied (the filling factor ν is smaller than 2) this effective interaction is equal to

$$I(q)=2\pi l^2\left[\frac{\pi}{2}\right]^{1/2}\frac{e^2}{\epsilon l}e^{q^2l^2/4}I_0(q^2l^2/2) , \tag{15}$$

where I_0 is a modified Bessel function. The result for the spin-flip response function is

$$\chi_{+-}(q,\omega)=\frac{\chi^0_{+-}(q,\omega)}{1+I(q)\chi^0_{+-}(q,\omega)} , \tag{16}$$

where $\chi^0_{+-}(q,\omega)$ is the result obtained neglecting electron-electron vertex corrections, i.e., when the exchange enhancement of the spin splitting is included in the single-particle energy levels but interactions are otherwise neglected.

In trying to understand the physical content of these electron-electron vertex corrections it is useful to start from the weak disorder limit where the exchange enhanced spin splitting, Ω_{ss}, is much larger than Γ. In Fig. 5 we plot $\mathrm{Im}\chi_{+-}(q,\omega)$ against ω for several values of q and $\Omega_{ss}=3\Gamma$. (In the Hartree-Fock approximation this occurs when $e^2/l\Gamma=\sqrt{18/\pi}$). Without the electron-electron vertices this quantity would be zero for $\omega<\Omega_{ss}-2\Gamma=\Gamma$. In Fig. 5 we see that sharp peaks occur in $\mathrm{Im}\chi_{+-}$ at q-dependent resonance frequencies below $\omega=\Gamma$. From Eq. (16) we see that

$$\mathrm{Im}\chi_{+-}(q,\omega)=\frac{\mathrm{Im}\chi^0_{+-}(q,\omega)}{[1+I(q)\mathrm{Re}\chi^0_{+-}(q,\omega)]^2+[I(q)\mathrm{Im}\chi^0_{+-}(q,\omega)]^2} . \tag{17}$$

It follows that the positions of the resonances shown in Fig. 5, which are associated with spin-wave excitations of the spin-polarized electron gas, are given by the equation

$$I(q)\mathrm{Re}\chi^0_{+-}(q,\omega)=-1 . \tag{18}$$

(In Fig. 5 the resonances are artificially broadened by adding a constant to $\mathrm{Im}\chi^0_{+-}$.) For the case where the resonance frequency is separated from Ω_{ss} by more that $\sim\Gamma$ it follows from Eq. (9) and the sum rules discussed in the previous section that Eq. (13) may be used for $\mathrm{Re}\chi^0_{+-}$ at all wave vectors. When this is done an explicit expression for the resonance frequency follows from Eq. (18):

$$\omega=\Omega_{ss}+\frac{\nu^+-\nu^-}{2\pi\ell^2}I(q) . \tag{19}$$

Note that the second term on the right-hand side of Eq. (19) exactly cancels the self-energy contribution to the spin-splitting. Equation (19) agrees with the result obtained by Kallin and Halperin[19] for $\nu^+=1$ and $\nu^-=0$ in the absence of disorder. As they discuss, the cancellation of the many-body corrections to the resonance frequency in the $q\rightarrow 0$ limit is an exact result which the time-dependent Hartree-Fock approximation captures because it conserves total particle number. In the next section we discuss the consequences of this cancellation for the interpretation of nuclear-spin-relaxation experiments.

In Figs. 6(a) and 6(b) we plot $\mathrm{Im}\chi_{+-}(q,\omega)/\omega$ versus ω for several values of q with the electron-electron vertex

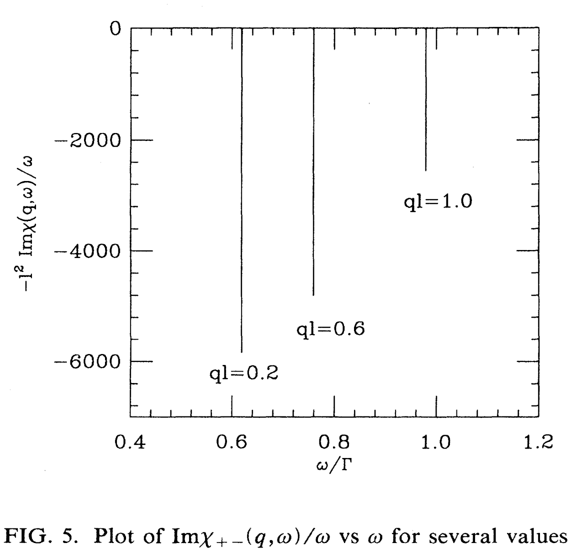

FIG. 5. Plot of $\mathrm{Im}\chi_{+-}(q,\omega)/\omega$ vs ω for several values of ql for the case $\Omega_{ss}/\Gamma=3.0$. Both disorder and electron-electron vertex corrections are included (a small constant has been added to $\mathrm{Im}\chi^0_{+-}$ in order to make the width of the peaks visible).

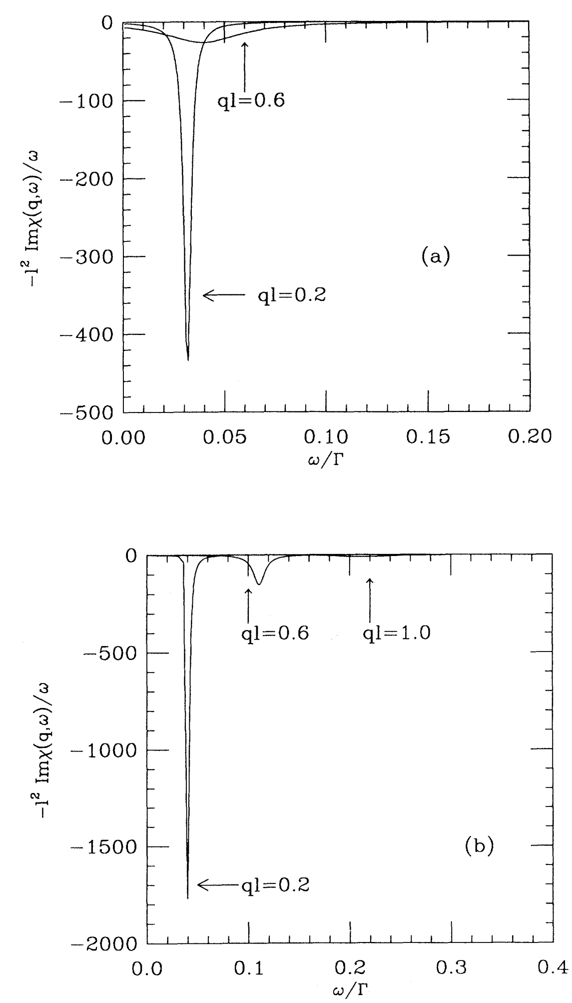

FIG. 6. Same as Fig. 5 but for different values of spin-splitting: (a) $\Omega_{ss}/\Gamma=0.3$, (b) $\Omega_{ss}/\Gamma=1.7$.

corrections included at $\Omega_{ss}=0.3\Gamma$ and $\Omega_{ss}=1.7\Gamma$. Using the bare g factor of GaAs ($g_0=0.5$) these values of Ω_{ss}, calculated within the Hartree-Fock approximation, correspond to enhancements of approximately 10 and 40 times, respectively, when the magnetic field is $B=16$ T. Figures 6 may be compared with Figs. 2(b) and 3(b) to see the effect of adding the electron-electron ladders. For $\Omega_{ss}=0.3\Gamma$ the collective resonance discussed above lies within the continuum of particle-hole excitations described by $\mathrm{Im}\chi^0_{+-}$ and is "Landau" damped. (Damping occurs when the resonance frequency exceeds $\Omega_{ss}-2\Gamma$.) For the $\Omega_{ss}=1.7\Gamma$ however the damping is weak and, in fact, the resonance peaks shown in Fig. 6(b) are still artificially broadened. Even for $\Omega_{ss}=0.3\Gamma$ the shape of $\mathrm{Im}\chi^0_{+-}$ is strongly altered by the electron-electron ladders although the spin-wave resonances are no longer clearly identifiable.

IV. NUCLEAR-SPIN RELAXATION RATE

We are now in a position to discuss the effect of vertex corrections on the relaxation rate T_1^{-1} from Eq. (1). Results for the relaxation rate as a function of the filling factor are shown in Figs. 7(a), 7(b), and 7(c) with no vertex corrections, disorder vertex corrections, and disorder and electron-electron vertex corrections, respectively, for $\Omega_{ss}=0.3\Gamma$ and $\Omega_{ss}=1.7\Gamma$. Note that the vertex corrections do not change the range of the filling factor over which the relaxation rate is nonzero but do change the shape of the curve describing its dependence on the filling factor. (We emphasize that the fact that the relaxation rate drops strictly to zero at some filling factors is an artifact of the SCBA.)

We start by discussing the role played by disorder vertex corrections. (We expect that interactions will play a minor role in relatively low-mobility systems where the exchange enhancement of the Landau-level spin-splitting is small.) Comparing Figs. 7(a) and 7(b) we see that the disorder vertex corrections cause a slight enhancement of the relaxation rate for the smaller spin-splitting and a more substantial decrease in the relaxation rate for the larger value of the spin-splitting. (This conclusion disagrees with the claims of Vagner *et al.*[4] that vertex corrections are negligible.) This behavior can be understood in terms of the results presented in Sec. II where it was pointed out that the effect of the vertex corrections is to produce an effective narrowing of the Landau level. When the spin-splitting is small the narrowing increases the effective density-of-states for both up spins and down spins at the Fermi level and increases the number of allowed low-energy spin-flip transitions. When the spin-splitting is larger, however, the decrease of the effective Landau-level width causes the Fermi level to lie more in the tails of the effective majority and minority-spin densities of states and the relaxation rate is lowered. This tendency is illustrated in Fig. 8 where we plot the relaxation rate with and without disorder vertex corrections as a function of Ω_{ss} for $\nu=1$.

Comparing Figs. 7(b) and 7(c) we see that the electron-electron vertex corrections can increase the relaxation rate by more than an order of magnitude when the spin-

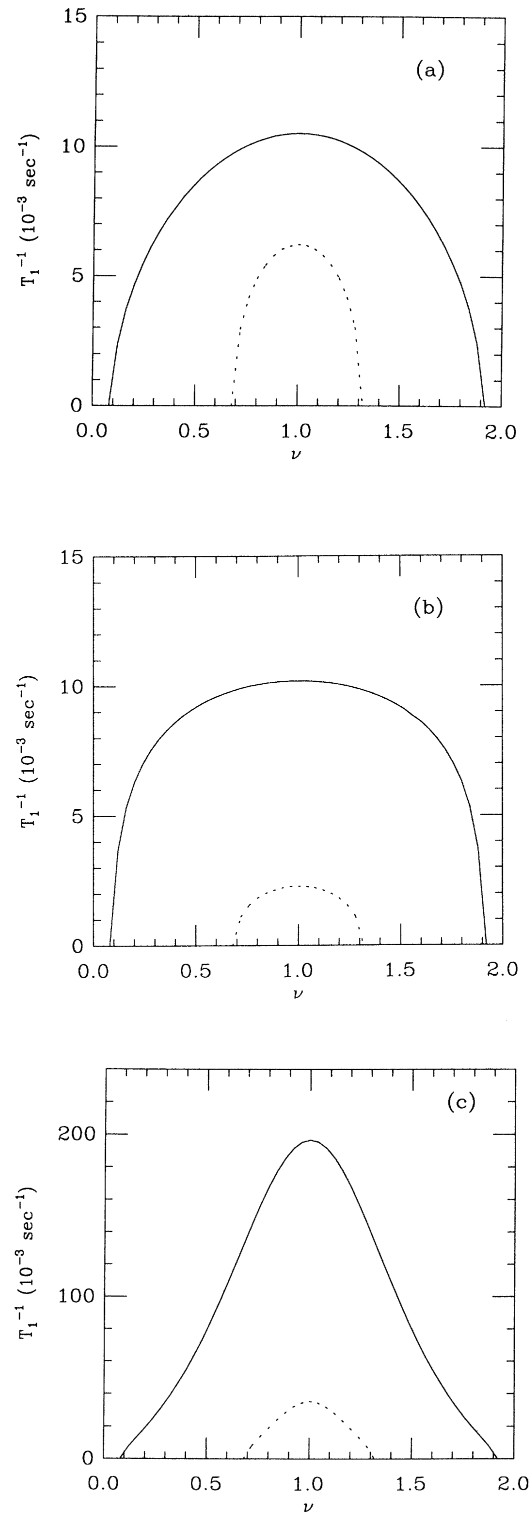

FIG. 7. Plots of the relaxation rate vs filling factor for $\Omega_{ss}/\Gamma=0.3$ (solid line) and $\Omega_{ss}/\Gamma=1.7$ (dashed line) (a) when no vertex corrections are included, (b) when disorder vertex corrections, and (c) when both disorder and electron-electron vertex corrections are included.

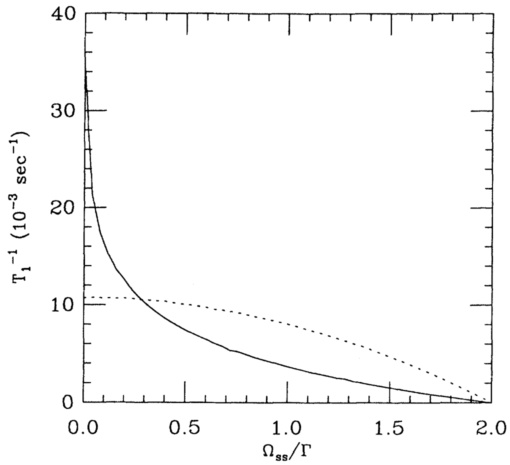

FIG. 8. Plot of the relaxation rate vs spin-splitting when the filling factor $\nu=1$. The solid line is for the case when disorder ladders are included and the dashed line for the case when no vertex corrections are included.

splitting is exchange enhanced. This is easily understood using Eq. (17) from which it follows that

$$\lim_{\omega\to 0} \mathrm{Im}\chi_{+-}(q,\omega)=\frac{\lim_{\omega\to 0}\mathrm{Im}\chi^0_{+-}(q,\omega)}{[1+I(q)\mathrm{Re}\chi^0_{+-}(q,\omega=0)]^2} . \quad (20)$$

In Fig. 9 we plot $1+I(q)\mathrm{Re}\chi^0_{+-}(q,\omega=0)$ versus q for the two values of Ω_{ss} illustrated in Fig. 7(c). The inverse of this quantity is the exchange-enhancement of the static long-wavelength limit of the spin-flip response function. We can evaluate this quantity exactly in the long-wavelength limit $q\to 0$ using Eq. (13) for χ_{+-} and noting that $(\nu^+-\nu^-)I(q=0)/(2\pi l^2)$ is the exchange contribution to Ω_{ss} [see Eq. (19)]. The result is $1+I(q=0)\chi_{+-}(q=0,\ \omega=0)=\Omega^{\mathrm{bare}}_{ss}/\Omega_{ss}$, i.e., the inverse of the enhancement of the spin-splitting. For the range of q which contributes importantly we see that enhancements of the contribution to the relaxation rate of ~ 20

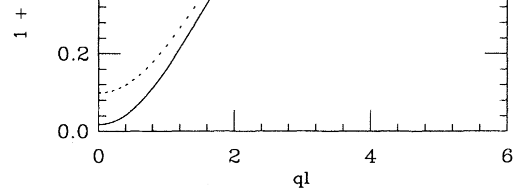

FIG. 9. Plot of $1+I(q)\mathrm{Re}\chi^0_{+-}(q,\omega=0)$ vs q for $\Omega_{ss}/\Gamma=0.3$ (dashed line) and $\Omega_{ss}/\Gamma=1.7$.

can be expected in agreement with the differences between Figs. 7(b) and 7(c). As in the case of disorder scattering electron-electron vertex corrections partially offset the effects of electron-electron self-energy corrections on the nuclear-spin relaxation rate which reduce the relaxation rate by enhancing the spin-splitting. Since the Hartree-Fock approximation, which we employ, is expected to overestimate the enhanced spin-splitting, the above results may overestimate the importance of electron-electron vertex corrections to the relaxation rate in typical situations. As in the disorder case the motivation for using the Hartree-Fock approximation is that the consistent vertex correction is a simple ladder sum which can be evaluated without difficulty. We do not expect that improvements on the Hartree-Fock approximation will alter any of the qualitative conclusions which we draw below.

V. SUMMARY AND DISCUSSION

The nuclear-spin relaxation rate for electrons near a two-dimensional electron gas is proportional to the integral over wave vectors of $\lim_{\omega\to 0}\mathrm{Im}\chi_{+-}(q,\omega)/\omega$, i.e., to an oscillator strength weighted sum over the low-energy spin-flip excitations in the system. In the case of electrons in a strong magnetic field, there are no low-energy spin-flip excitations unless the majority-spin and minority-spin Landau levels are sufficiently broadened that they overlap. Thus disorder is essential in determining the nuclear-spin relaxation rate. We find that when the vertex corrections consistent with the SCBA for the disorder averaged Green's function are included, the effective Landau-level broadening seen in nuclear-spin relaxation is reduced. The results presented here are for the case of a disorder potential from randomly located short-range scatterers. If follows from the exact sum rules discussed in Sec. II that the effective Landau-level broadening seen in the relaxation will be reduced compared to the thermodynamic Landau-level broadening which appears in the one-particle Green's function's spectral weight to an even greater degree if the impurity potential is smooth, as it is in typical high-mobility two-dimensional electron systems. The reduction in the effective Landau-level broadening occurs because of the local nature of the spin-relaxation probe of the electron system; the effective density of states seen is the local density of states at the position of an individual nucleus. Simultaneous measurement of the nuclear-spin relaxation rate and of thermodynamic properties such as the magnetization[20] or the specific heat[21] can potentially be a useful probe for distinguishing inhomogenous broadening of the Landau level by long-distance fluctuations in the disorder potential from the local broadening produced by short-range scatterers.

In high-mobility 2D EG systems the spin splitting may be strongly exchange enhanced. When vertex corrections are neglected the larger value of the spin splitting will lead to a large reduction of the relaxation rate. However, when vertex corrections are included this effect can be substantially reduced or, in some circumstances, even overcome. The inclusion of electron-electron vertex corrections increases the relaxation rate by the square of the exchange-enhancement of the static spin-flip response function averaged appropriately over the wave vectors which contribute importantly to the relaxation.

Our results suggest that the quantitative interpretation of nuclear-spin relaxation data is quite complicated. This is especially true in light of the screening dependence[22] and hence magnetic-field dependence of the Landau-level width, which is treated as a parameter here. However our results also show that the nuclear-spin relaxation data provide information about electronic structure which is qualitatively different from that available from transport, optical, or thermodynamic experiments in that the relaxation rates depend on the local Landau-level broadening and are insensitive to large length scale inhomogeneities. When combined with separate measurements of the enhanced Landau-level splitting[23] and possibly also thermodynamic properties this probe has the potential to provide unique information on the nature of the disorder potential and also to provide new information on exchange-enhanced response functions.

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