Penetration depth in phenomenological marginal-Fermi-liquid model for CuO

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We have calculated the temperature-dependent London penetration depth in a recently proposed phenomenological, marginal-Fermi-liquid model that has been shown to be quite successful in explaining several of the observed anomalous properties of the normal state. Solutions with temperature dependence close to the two-fluid model and to experiment can be found. However, they do not correspond to a value of twice the gap to critical temperature as large as the recently suggested value of 8 for Y-Ba-Cu-O.

Recently, Varma[1] has reviewed work on phenomenological constraints that need to be placed on any theory of high-temperature superconductivity. It is evident that the single assumption about the existence of both charge- and spin-density excitations described by a polarizability $\tilde{P}(q,\omega)$ of the form[1-5]

$$\mathrm{Im}\tilde{P}(q,\omega)=-N(0)\tanh(\omega/2T), \quad |\omega|<\omega_c$$
$$=0, \quad |\omega|>\omega_c \tag{1}$$

(ω_c is a phenomenological cutoff) over the significant range of momentum transfer q, goes a long way in describing some of the anomalous normal-state properties associated with the copper oxides.

Kuroda and Varma[5] have calculated some superconducting properties associated with the model just described and find agreement with experiment. In this note, we present results of calculations for the penetration depth in the London limit. The basic equations given by Kuroda and Varma[5] involve coupling to charge (λ_ρ) and spin (λ_σ) fluctuations and have the form

$$\tilde{\Delta}(i\omega_n)=\pi T(\lambda_\rho-\lambda_\sigma) \times \sum_m F(\omega_n-\omega_m)\frac{\tilde{\Delta}(i\omega_m)}{[\tilde{\omega}^2(i\omega_m)+\tilde{\Delta}^2(i\omega_m)]^{1/2}} \tag{2}$$

and

$$\tilde{\omega}(i\omega_n)=\omega_n+\pi T(\lambda_\rho+\lambda_\sigma) \times \sum_m F(\omega_n-\omega_m)\frac{\tilde{\omega}(i\omega_m)}{[\tilde{\omega}^2(i\omega_m)+\tilde{\Delta}^2(i\omega_m)]^{1/2}} \tag{3}$$

with $i\omega_n \equiv i\pi T(2n-1)$, $n=0,\pm1,\pm2,\ldots$ and

$$F(\omega_n-\omega_m)=\int_0^{\omega_c}\frac{2\omega}{\pi}\frac{\tanh(\omega/2T)}{\omega^2+(\omega_n-\omega_m)^2}d\omega\ , \tag{4}$$

where the cutoff ω_c, λ_ρ, and λ_σ are the only parameters. We can introduce a constant g, defined as $(\lambda_\rho-\lambda_\sigma)/(\lambda_\rho+\lambda_\sigma)$, in terms of which the coupling to the spin fluctuations, λ_σ, is given as $(1-g)/(1+g)$ times the coupling to charge fluctuations. Therefore, for $g=1$ there is no coupling to spin fluctuations, only to charge.

Note, that in the gap channel [Eq. (2)], the charge fluctuations are pair creating while spin fluctuations are pair breaking, which is expected for an *s*-wave gap. Equations (2) and (3) with definition (4) are similar to the very well known Eliashberg gap equations[6-8] with phonons plus paramagnons. An important difference is that the spectral density is the same for spin and charge fluctuations in the present model, and is effectively temperature dependent through the $\tanh(\omega/2T)$ factor in Eq. (4). Also, as the gap $\Delta(T)$ in the superconducting state develops, the spectral density also becomes gapped and is to be cut off at $2\Delta(T)$. For a fixed choice of ω_c and g, the remaining parameter λ_ρ is chosen to get a critical temperature value of 100 K which we take to be representative of the oxides. The parameter λ_ρ, which is dimensionless, is plotted in the top frame of Fig. 1 for four values of g, namely, $g=0.4$, 0.5, 0.6, and 0.8 as a function of T_c/ω_c in the range 0–0.05. For low values of g it is seen that λ_ρ increases very rapidly with T_c/ω_c in the region around ~0.01 and ~0.03 for $g=0.4$ and 0.5, respectively. This indicates that the model rapidly becomes unphysical in this range.

The critical temperature T_c, and hence λ_ρ, follows on solution of the linearized form of Eqs. (2) and (3) which are valid near the critical temperature. Solutions at low temperature can also be obtained and can be analytically continued to the real frequency axis through Padé approximants[9-11] to obtain the gap edge Δ_0. Typically it is sufficient to solve (2) and (3) at the reduced temperature $t=T/T_c\cong0.1$ and Δ_0 follows from the real axis analytic continuation $\Delta(\omega)$ through solution of the equation $\Delta_0=\mathrm{Re}\Delta(\omega=\Delta_0)$, where only the real part of the complex gap $\Delta(\omega)$ enters. We have performed such calculations self-consistently to obtain the ratio $2\Delta_0/k_BT_c$ shown in the lower frame of Fig. 1. It is clear that for small g this dimensionless ratio increases very rapidly with increasing value of T_c/ω_c.

In obtaining our results for $2\Delta_0/k_BT_c$, we have fully recognized that the spin- and charge-fluctuation spectrum needs to be cutoff at $2\Delta(T)$ in the superconducting states as the gap develops. We have used a sharp cutoff in formula (4) with the lower limit on the integral placed at $2\Delta(T)$. We chose to use the temperature dependence of the BCS gap as we found that the analytic continuation at finite temperature gave a similar temperature dependence but with more numerical effort. This lower cutoff has the effect of suppressing both spin and charge fluctuations but the net result is that spin fluctuations are more effectively suppressed and so the gap is larger than it would be if no lower cutoff were applied. The gap in a fully oxygenated single crystal of YBa_2CuO_7 has been reported to be of the order of $2\Delta_0/k_BT_c \cong 8$ (Refs. 12–14) in far-infrared measurements. We see from our results that this value can be achieved for $g=0.4$ with a cutoff $\geq 10\,000$ K, while for $g=0.6$ it is ≥ 2500 K. It is worth noting that similar large values of $2\Delta/k_BT_c$ are also indicated in photoemission experiments[15–17] on cuprate superconductors.

The formula for the London penetration depth $\lambda_L(T)$ at temperature T is given by

$$\lambda_L(T)=\left[\frac{2\pi T\sigma_N}{\tau_N}\sum_{n=1}^{\infty}\frac{\Delta^2(i\omega_n)}{Z(i\omega_n)[\omega_n^2+\Delta^2(i\omega_n)]^{3/2}}\right]^{-1/2}, \tag{5}$$

where σ_N and τ_N are the normal-state conductivity and scattering time, respectively, and $\Delta(i\omega_n)$ and $Z(i\omega_n)$ are related to $\tilde{\Delta}(i\omega_n)$ and $\tilde{\omega}(i\omega_n)$ by

$$\begin{aligned}\tilde{\Delta}(i\omega_n)&=\Delta(i\omega_n)Z(i\omega_n)\ ,\\ \tilde{\omega}(i\omega_n)&=\omega_nZ(i\omega_n)\end{aligned} \tag{6}$$

with the Matsubara frequency $(i\omega_n)$.

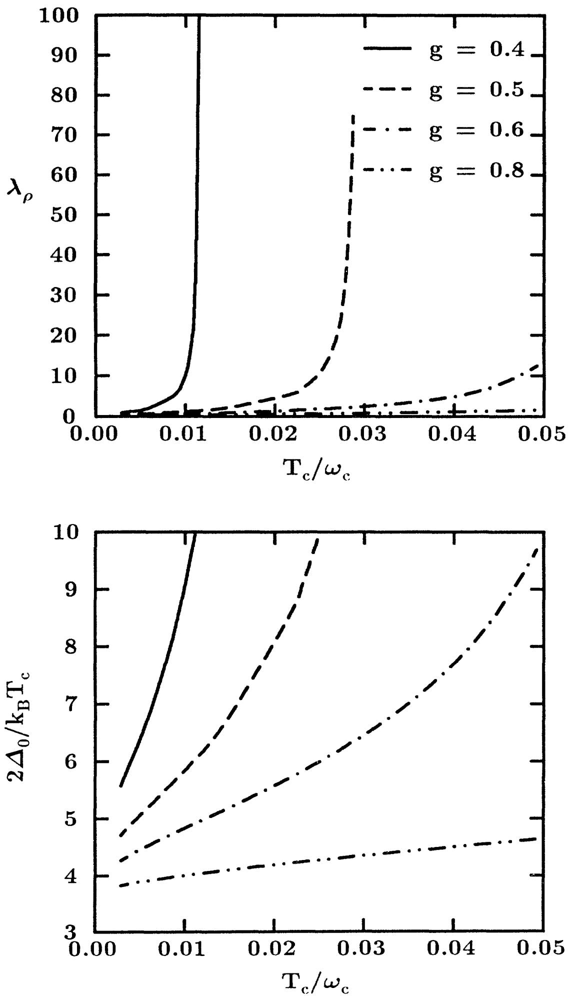

FIG. 1. Top frame gives the charge-fluctuation channel coupling λ_ρ as a function of T_c/ω_c for four values of $g=0.4$ (solid), 0.5 (dashed), 0.6 (dot-dashed), 0.8 (double dot–dashed). The bottom frame gives the gap to critical temperature ratio $2\Delta_0/k_BT_c$ for the same parameters.

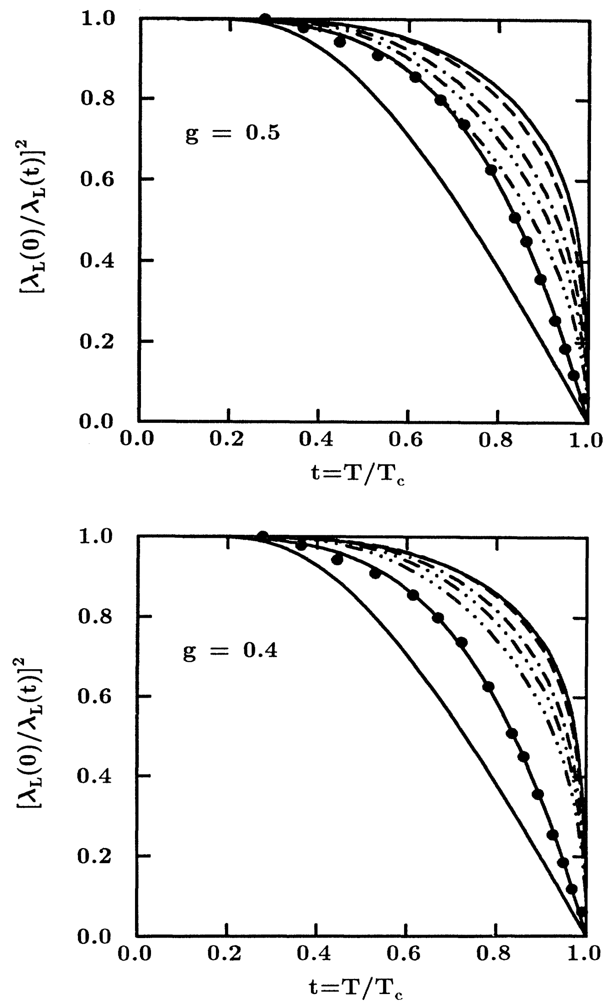

FIG. 2. The dependence of the square of the normalized penetration depth $\lambda_L(0)/\lambda_L(t)$ on reduced temperature t for several values of T_c/ω_c, namely, 0.0215 (solid line), 0.0144 (dashed line), 0.0086 (dash–dotted line), 0.0057 (dash–double dotted line) and 0.0029 (dash–triple dotted line). Also shown are data (solid dots) and the two-fluid model results (solid line) which represent the data very well. Finally, the lowest solid curve is the BCS prediction shown for comparison. The top frame is for $g=0.5$. The lower frame is for $g=0.4$ with the T_c/ω_c values of 0.0108, 0.0086, 0.0057, and 0.0043, and 0.0029 for the upper five curves in descending order.

We have solved the gap equations[2,3] at several temperatures T and applied the self-consistent cutoff to be the kernel of Eq. (4) at $2\Delta(T)$ for any temperature T. The results of such calculations are displayed in Figs. 2 and 3 for $g=0.4$ and 0.5, and $g=0.6$ and 0.8, respectively. The curves are labeled by values of T_c/ω_c. Also displayed are the two-fluid model results (second lowest solid curve), the results of a very recent careful experiment[18] (solid dots) and the BCS clean limit results (lowest solid curve). We see that, in many cases, the temperature variation is above the two-fluid model. This would also be true for a conventional strong-coupling system. As T_c/ω_c is lowered from higher values, the curves tend toward the weak-coupling curve. It is clear that results close to experiment can be found for specific values of the parameters considered although near $t=1$ the theoretical results will always fall above the experimental points. While the region of parameters, which are necessary to obtain overlap between experiment and theory for the temperature variation of the normalized London penetration depth, correspond to a value of $2\Delta_0/k_BT_c$ larger than the BCS value of 3.54, this ratio is not as large as the suggested experimental number of 8. To be specific, for $g=0.6$ and values of T_c/ω_c in the region of and up to ~ 0.01, overlap of the experimental and theoretical curves for the penetration depth can be achieved, but $2\Delta_0/k_BT_c \lesssim 5.0$. For $g=0.5$, T_c/ω_c would need to be around 0.003 with $2\Delta_0/k_BT_c \sim 4.7$. These numbers are not inconsistent with some tunneling[19,20] results. Also Timusk *et al.*[21] have pointed out that some problems remain in the interpretation of infrared data in terms of a large gap edge value. Also, in independent muon spin relaxation measurements in $R\mathrm{Ba_2Cu_3O}$ with $R=$Eu, Gd, and Er, Lichti *et al.*[22] find a temperature dependence for $[\lambda_L(0)/\lambda_L(T)]^2$ which falls considerably above the two-fluid model curves. This would be quite consistent with values of $2\Delta_0/k_BT_c$ as large as 8. While the situation remains somewhat ambiguous, it is clear that the present model is not inconsistent with the existing data.

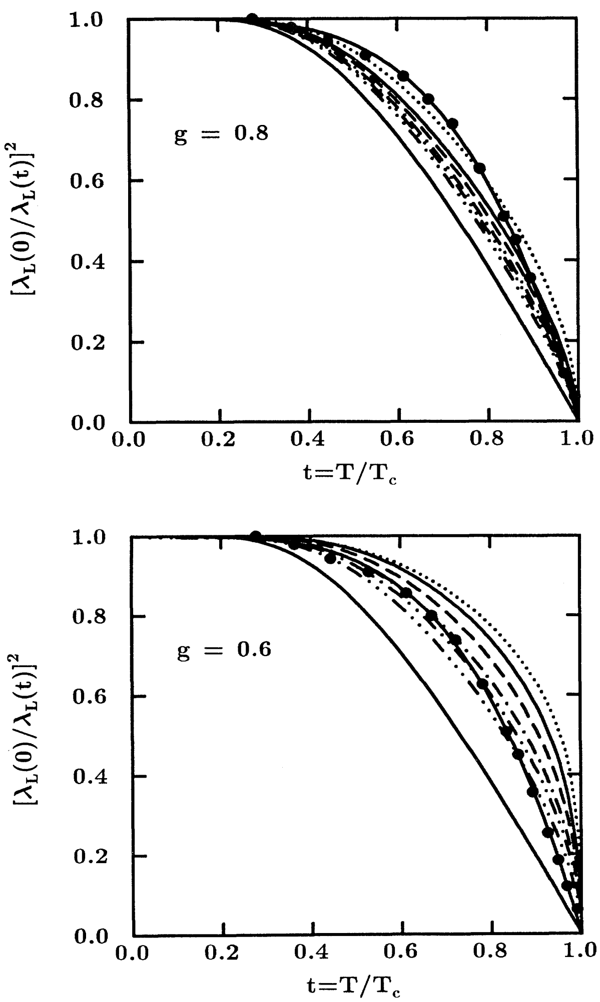

FIG. 3. Same as Fig. 2. The top frame is for $g=0.8$ with T_c/ω_c values (in descending order of the curves) of 0.0431 (dotted curve), 0.0215, 0.0144, 0.0086, 0.0057, and 0.0029. The lower frame is for $g=0.6$ with same parameters as in the top frame.

In Fig. 4, we show the deviation of the zero-temperature London penetration depth $\eta_{\lambda_L}(0)$ from BCS as a function of T_c/ω_c for the same values of g previously considered. By definition,

$$\lambda_L(0)\equiv\eta_{\lambda_L}(0)\lambda_L^{\mathrm{BCS}}(0) . \tag{7}$$

In Eq. (7), $\lambda_L^{\mathrm{BCS}}(0)$ is calculated for a superconductor with $T_c=11.6$ K and with a δ-function spectral density all placed at a frequency $\omega_E=200$ meV. We see from Fig. 4 that $\eta_{\lambda_L}(0)$ is always greater than 1 in the range considered and that it shows a very strong increase around $T_c/\omega_c=0.01$ for $g=0.4$ and around 0.03 for $g=0.5$. This reflects the unphysical increase in λ_ρ in this range of parameters as shown in the top from of Fig. 1. The ex-

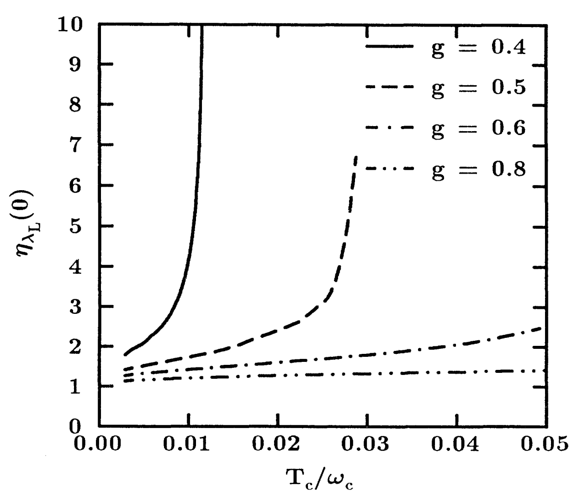

FIG. 4. The deviation of the zero-temperature London penetration depth $\lambda_L(0)$ from its BCS value as a function of T_c/ω_c for $g=0.4$ (solid), 0.5 (dashed), 0.6 (dot–dashed), 0.8 (double dot–dashed).

istence of this large correction to $\lambda_L(0)$ is hard to establish experimentally at this time because of uncertainties in the other parameters that enter the formula (5) for $\lambda_L(0)$.

In conclusion, we have calculated the absolute value of the zero-temperature London penetration depth as well as its temperature dependence on the basis of the phenomenological model [Eq. (1)] for the polarizability. This is important because this simple model has been found to explain well many of the anomalous normal-state properties observed in the oxides. We have compared the obtained temperature dependence with recent experimental results in Y-Ba-Cu-O which limits, to a certain restricted region, the parameters of the model. For this same region of parameters, the range of values for the ratio of the gap to T_c, $2\Delta_0/k_BT_c$, fall below a recent experimental suggestion of 8.

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