Magnetic-field influence on polaronic electrons on liquid-helium films

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Using the harmonic-oscillator algebra, we calculate the ground-state energy of the system of an electron coupled to a ripplon on the outer surface of the liquid-helium film under the influence of a magnetic field of arbitrary strength. Analytical expressions obtained for limiting cases of strong field and weak field with the weak-coupling strength are essentially in agreement with those existing in the literature.

I. INTRODUCTION

There exists a considerable amount of work, both theoretical[1–9] and experimental,[10–13] on the system of electrons on the surface of liquid helium. Because of the combined effect of the attractive image potential and the surface barrier, electrons are confined to move on the outside surface of the liquid and form an ideal two-dimensional (2D) system. Aside from the fundamental proporties of the 2D electron gas, the polaronic states on the surface of liquid helium have attracted attention in recent years. A polaron is a system of an electron coupled to the ripplon field, an elementary excitation of the liquid-helium surface. The system is of particular interest, because the strength of the electron-ripplon coupling can be changed by adjusting the thickness of the film or by changing the substrate.

The problem was first formulated by Jackson and Platzman[2] and then treated by others[3,4] using different methods. We investigate in this paper the ground-state energy of a 2D polaron on the liquid-helium surface when a magnetic field of arbitrary strength is applied normal to the surface. Since we are only concerned with the weak coupling, no indication of phase-transition behavior[7,9,13] is expected. We use in our calculation the algebra of harmonic-oscillator operators introduced by Suzuki and Hensel.[14] The algebra was first employed by Larsen[15] to treat the 2D polaron in polar crystals. Its major advantage is that the complicated sum over the products of all the matrix elements and energy denominators can be replaced by a much simpler algebra. The method has been extended by two of the present authors to study the interface polaron in magnetic fields[16,17] with expected results.

II. THE HAMILTONIAN

Consider a film of liquid helium of thickness d. Its free surface is taken to be the xy plane, so that the space is vacuum when $z>0$ and the substrate with dielectric constant ϵ when $z<-d$. The magnetic field $\mathbf{B}_M$ is along the positive z direction. For an electron interacting with the ripplon on the free surface, the Hamiltonian of the coupled system is given by[7]

$$H=\frac{1}{2m}\left[\left(p_x-\frac{\beta^2}{4}y\right)^2+\left(p_y+\frac{\beta^2}{4}x\right)^2\right]+\sum_k \hbar\omega_{\mathbf{k}}a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + \sum_{\mathbf{k}}\left(V_k^* a_{\mathbf{k}}^\dagger e^{-i\mathbf{k}\cdot\mathbf{r}}+\text{H.c.}\right), \tag{1}$$

where we have defined

$$\beta^2=\frac{2e}{c}B_M\ , \tag{2a}$$

$$\omega_k=\left[\left({}'k+\frac{\sigma}{\rho}k^3\right)\tanh(kd)\right]^{1/2}, \tag{2b}$$

$$V_k=[2\pi\alpha\hbar^3 g'k\tanh(kd)/Sm\omega_k]^{1/2}\ . \tag{2c}$$

The notation is as follows. The electron mass is m, its position $\mathbf{r}$, and its momentum $\mathbf{p}$. The operator $a_{\mathbf{k}}^\dagger$ ($a_{\mathbf{k}}$) creates (annihilates) a ripplon of wave vector $\mathbf{k}$ and frequency ω_k. ρ and σ are the density and surface tension of the liquid helium, respectively, and g' is the acceleration caused by Van der Waals coupling between the fluid and the substrate. The surface area of the helium is denoted by S and the electron-ripplon coupling constant α is defined by

$$\alpha=\frac{(e\xi)^2}{8\pi\sigma}\Big/\frac{\hbar^2k_c^2}{2m}\ , \tag{2d}$$

$$e\xi=eE^{\text{ext}}+\frac{e^2}{4d^2}\frac{\epsilon-1}{\epsilon+1}\ , \tag{2e}$$

where the capillary constant $k_c=(\rho g'/\sigma)^{1/2}$. E^{ext} stands for the normal component of the external electric field. It is seen from the definition that α depends on the film thickness and the characteristics of the substrate. If we assume that the film thickness $d=100$ Å, then we have in cgs units[6,7] $g'\approx 10^8 g$, $k_c\approx 10^5\ \text{cm}^{-1}$, where g is the gravitational acceleration.

As proposed by Larsen[14], the Hamiltonian (1) becomes

$$H=H_0+H_{er}\ , \tag{3}$$

$$H_0=\frac{\hbar\beta^2}{2m}(A^\dagger A+\tfrac{1}{2})+\sum_k \hbar\omega_k a_k^\dagger a_k\ , \tag{4a}$$

$$H_{er}=\sum_k (V_k^* L_k M_k a_k^\dagger + V_k L_k^{-1} M_k^{-1} a_k)\ . \tag{4b}$$

The Suzuki-Hensel harmonic-oscillator operators in (4) are defined as

$$A=\frac{1}{\sqrt{\hbar}\beta}\left[\left[p_x-\frac{\beta^2}{4}y\right]-i\left[p_y+\frac{\beta^2}{4}x\right]\right]\ , \tag{5a}$$

$$B=A^\dagger-\frac{i\beta}{2\sqrt{\hbar}}(x+iy)\ , \tag{5b}$$

$$L_k=\exp\left[\frac{\sqrt{\hbar}}{\beta}(k_x+ik_y)A-\frac{\sqrt{\hbar}}{\beta}(k_x-ik_y)A^\dagger\right]\ , \tag{6a}$$

$$M_k=\exp\left[\frac{\sqrt{\hbar}}{\beta}(k_x-ik_y)B-\frac{\sqrt{\hbar}}{\beta}(k_x+ik_y)B^\dagger\right]\ . \tag{6b}$$

III. STRONG-FIELD LIMIT

Similar to the case of the electron-phonon system,[16,17] the unperturbed eigenstates can be written as

$$|l\rangle=(n!M!)^{-1/2}(A^\dagger)^n|0\rangle_A(B^\dagger)^M|0\rangle_B|n_{\mathbf{k}}\rangle\ , \tag{7}$$

where the vacuum states of A and B are defined by $A|0\rangle_A=B|0\rangle_B=0$ and the ripplon vacuum state is given by $a_k|0_k\rangle=0$. The strong-field limit is according to Ref. 14, $(\omega_c/\omega_0)=\lambda^2\to\infty$, where

$$\omega_c=\frac{\beta^2}{2m}=eB_M/mc\ , \tag{8a}$$

$$\omega_0=\hbar k_c^2/2m\ . \tag{8b}$$

In the strong-field limit, the electron can only be found in the lowest Landau level $n=0$. The effective Hamiltonian of the system is then

$$H=H_{\text{eff}}^0+H'\ , \tag{9}$$

$$H_{\text{eff}}^0=\tfrac{1}{2}\hbar\omega_c+\sum_{\mathbf{k}}\hbar\omega_k a_{\mathbf{k}}^\dagger a_{\mathbf{k}}\ , \tag{9a}$$

$$H'=\sum_{\mathbf{k}} V_{\mathbf{k}}^* e^{-\hbar k^2/2\beta^2}(M_k a_{\mathbf{k}}^\dagger+M_k^{-1}a_{\mathbf{k}})\ , \tag{9b}$$

where we have made use of the matrix element

$$_A\langle 0|L_k|0\rangle_A=e^{-\hbar k^2/2\beta^2}\ . \tag{10}$$

In the following, we treat H' as a small perturbation with the unperturbed ground-state energy

$$E^0=\langle j|H_{\text{eff}}^0|j\rangle=\hbar\omega_c/2\ , \tag{11}$$

where

$$|j\rangle=|M\rangle_B|0_{\mathbf{k}}\rangle=\frac{1}{\sqrt{M!}}(B^\dagger)^M|0\rangle_B|0_{\mathbf{k}}\rangle\ . \tag{12}$$

Because of the complicated dispersion relation (2b) and the interaction energy (2c), analytical expression for the ground-state energy can only be obtained in approximation. Following Ref. 6, we consider a linearized cutoff ripplon spectrum, that is, $\omega_k=sk$ with the cutoff wave number $k=k_c$, where $s=(g'd)^{1/2}$ for $k<k_0$. This has been shown[11] to be a good approximation for $d<100$ Å. Thus, the ground-state energy up to the second-order perturbation is found to be

$$\Delta E=-\frac{2\pi\alpha\hbar^2 g'd}{Sms^2}\frac{S}{(2\pi)^2}\int_0^{2\pi}d\phi\int_0^{k_c}k\,e^{-\hbar k^2/\beta^2}dk$$
$$=-\alpha\hbar\omega_c(1-e^{-\omega_0/\omega_c})\ . \tag{13}$$

Since $\omega_0/\omega_c\ll 1$, we find by expanding the exponential function and neglecting terms higher than second order

$$\Delta E=-\alpha\hbar\omega_0+\frac{\alpha}{2}\hbar\omega_c(\omega_0/\omega_c)^2\ . \tag{14}$$

The ground-state energy of a two-dimensional polaron on the liquid-helium film in strong magnetic field is therefore

$$E=E^0+\Delta E=\tfrac{1}{2}\hbar\omega_c\left[1+\alpha\left[\frac{\omega_0}{\omega_c}\right]^2\right]-\alpha\hbar\omega_0\ , \tag{15}$$

which is in agreement with the result of Ref. 7.

IV. ARBITRARY MAGNETIC FIELD

In an arbitrary magnetic field, the electron is no longer confined to the lowest Landau level. The Hamiltonian is still given by (3) but the unperturbed eigenstates are

$$|i\rangle=(n!M!)^{-1/2}(A^\dagger)^n|0\rangle_A(B^\dagger)^M|0\rangle_B|0\rangle \tag{16}$$

and the unperturbed eigenenergies

$$E^0=(n+\tfrac{1}{2})\hbar\omega_c\ . \tag{17}$$

The second-order perturbation energy from H' is

$$\Delta E = -\sum_{\mathbf{k}} V_k^2 \sum_n {}_A\langle 0|L_k^{-1}|n\rangle_A \frac{1}{n\hbar\omega_c + \hbar\omega_k} {}_A\langle n|L_k|0\rangle_A$$
$$= -\frac{\alpha\hbar^2 s}{m\omega_0}\int_0^{k_c} k^2 dk \int_0^\infty dt\, e^{-\omega_k t/\omega_0} \exp\left[-\frac{\hbar k^2}{\beta^2}(1-e^{-\lambda^2 t})\right], \quad (18)$$

where we have made use of the relation

$$(n\hbar\omega_c + \hbar\omega_k)^{-1} = \frac{1}{\hbar\omega_0}\left[n\lambda^2 + \frac{\omega_k}{\omega_0}\right]^{-1} = \frac{1}{\hbar\omega_0}\int_0^\infty e^{-[n\lambda^2 + (\omega_k/\omega_0)]t} dt$$

and the approximation of the linearized cutoff ripplon spectrum. It is noted that Eq. (18) is a limiting case of Eq. (18) of Ref. 7 when the two parameters V and W in that paper are taken to be equal to one another. The integral in (18) is still difficult to evaluate for arbitrary λ, for which numerical calculation is necessary. The result is plotted in Fig. 1 as a function of λ^2. The dashed line representing the strong-field limit (13) is also drawn for comparison. It is observed that Eq. (13) is practically accurate for $\lambda^2 \geq \frac{1}{2}$. The dotted line represents the weak-field limit, which is discussed below.

In the strong magnetic field limit, $\lambda^2 \to \infty$ and $e^{-\lambda^2 t} \to 0$. Equation (18) becomes identical to (13), as expected. In the weak-field limit, $\lambda^2 \to 0$ and $e^{-\lambda^2 t} \approx 1 - \lambda^2 t + \frac{1}{2}\lambda^4 t^2$. Thus Eq. (18) becomes

$$\Delta E = -2\alpha\eta\hbar\omega_0[1+\eta\ln\eta - \eta\ln(1+\eta)] - \frac{\alpha\hbar\omega_c}{(1+\eta)^2}, \quad (19)$$

where we have defined the parameter $\eta = sk_c/\omega_0$. The first term is the polaron self-energy, and the second term, which arises from the fourth-order term $\frac{1}{2}\lambda^4 t^2$, modifies the electron Landau levels in the form of mass renormalization, as we shall see. The ground-state energy of a two-dimensional polaron on the liquid-helium film is therefore given by

$$E = [n + \tfrac{1}{2} - \alpha/(1+\eta)^2]\hbar\omega_c - 2\alpha\eta\hbar\omega_0\{1 - \eta\ln[(1+\eta)/\eta]\}. \quad (20)$$

When the terms involving logarithms are neglected, as they are much smaller than unity, Eq. (20) reduces to the result of Ref. 7. When the fourth-order term is not included, it becomes the result of Ref. 9.

If we define the cyclotron frequency of the polaron in a magnetic field as $\omega_c^* = \beta^2/2m^*$, we find from (15) and (20) the effective mass of the polaron in weak and strong magnetic fields, respectively:

$$m^* = m\left[1 - \frac{2\alpha}{(1+\eta)^2}\right]^{-1} \approx m[1 + 2\alpha/(1+\eta)^2], \quad (21a)$$

$$m^* = m\left[1 + \alpha\left(\frac{\omega_0}{\omega_c}\right)^2\right]^{-1} \approx m[1 - \alpha(\omega_0/\omega_c)^2]. \quad (21b)$$

Therefore, the polaron effective mass is smaller than the bare electron mass in the strong field and is greater than the bare mass in the weak field. Our calculation shows that the polaron effective mass is a continuous function of the magnetic field as it goes from $m^*/m > 1$ in the low-field limit to $m^*/m < 1$ in the high-field limit. This is different from the variation of the effective cyclotron mass,[18] which is defined through the cyclotron resonance frequency.

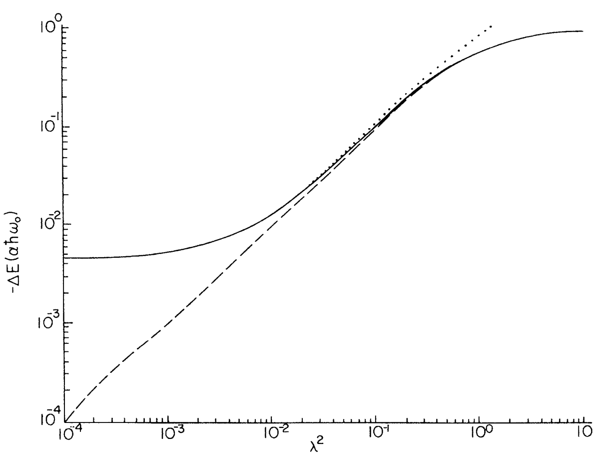

FIG. 1. Ground-state energy of a 2D polaron on the liquid-helium film in a magnetic field of arbitrary strength. The solid line represents results for an arbitrary field, the dashed line represents results in the strong-field limit, and the dotted line represents results in the weak-field limit.

V. DISCUSSION

We have calculated, in the presence of a magnetic field of arbitrary strength, the ground-state energy of a 2D polaron on the outer surface of the liquid-helium film in the weak-coupling limit. We find that the polaronic energy correction tends to increase monotonically in magnitude with increasing magnetic field as shown in Fig. 1. We also find that the interaction with ripplons results in the shift of the electron Landau levels. In the strong-field limit, the electron is most likely restricted to the ground level, which is shifted upward, while in the weak-field limit, the levels are shifted downward due to the ripplon effect. Consequently, the effective mass of the polaron is larger (smaller) than the electron mass in the weak (strong) magnetic field.

The three polaronic states discussed in Ref. 9 can be reproduced from our result (18) in appropriate limits. In the strong-field limit, or when $\lambda^2 \rightarrow \infty$, the polaron is magnetically trapped with $\alpha\hbar\omega_0$ as the limit of the trapping energy, and the polaron effective mass approaches the electron band mass from below. In the weak-field limit, or when $\lambda^2 \rightarrow 0$, the self-trapping energy follows from (19). It is $-2\alpha\eta\hbar\omega_0$ for strong coupling and approaches zero in the weak-coupling limit.

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