Brief Reports

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Linear polarization of radiation from planar channeled electrons and positrons

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The polarization of channeling radiation emitted by planar channeled electrons and positrons has been calculated using the many-beam method. The radiation was found to be almost completely polarized in the direction perpendicular to the channeling plane for emission in the forward direction. This is an exact, analytical result and holds for all instances in which the particles satisfy the conditions for planar channeling.

A charged particle directed into a crystal approximately parallel to one of the crystal planes will be planar channeled;[1,2] that is, it will experience a force which will "steer" the particle along the direction of the plane, provided that the trajectory is sufficiently different from an axis of the crystal to avoid being channeled by the axis. For negatively charged particles, such as electrons, the channel is provided by the crystal plane, while for positively charged particles, such as positrons, the channel is between the crystal planes (see Fig. 1).

In classical terms, the particle's momentum forms some small angle θ with respect to the crystal plane. This angle must be less than the critical channeling angle ψ_c for planar channeling to occur. The critical angle depends on the energy and type of the incident particle, but is on the order of a few mrad for electrons and positrons with energies on the order of 10 MeV.

From a quantum-mechanical viewpoint, the channel is the source of a potential well in the direction transverse to the particle's motion, which gives rise to transversely bound states for the particle. Transitions to lower-energy states lead to the phenomenon known as channeling radiation. The fact that the transverse potential is periodic allows the use of the Bloch-function or "many-beam" approach to this problem. This was first done by Andersen *et al.*[3]

The quantum-mechanical calculation of axial channeling radiation was first carried out by Kumakhov and Wedell,[4] and the following analysis utilizes their developments. One begins with the time-independent Dirac equation for an electron moving in a potential $V(x)$ periodic in the x direction (which is normal to the channeling planes)

$$[-i\underline{\alpha}\cdot\nabla-\underline{\beta}m+V(x)]\Phi=E\Phi \ , \tag{1}$$

where m and E are the electron's mass and energy, $\underline{\alpha}$ and $\underline{\beta}$ are the standard Dirac matrices, and $V(x)$ is the average value of $V(x,y,z)$ along the yz plane

$$V(x)=\frac{1}{d_y d_z}\int_0^{d_y}\int_0^{d_z} V(x,y,z)dy\,dz \ . \tag{2}$$

This approximation is valid if the electron is channeled and its energy satisfies the inequality[1]

$$E>\frac{3Ze^2}{2\pi(Ca)^3Nd_p}\sim Z^2 90 \text{ eV} \ , \tag{3}$$

where Z is the atomic number of the crystal atoms, a is the Thomas-Fermi screening length, Nd_p is the average number of atoms per unit area of the plane, d_p is the distance between planes, and C is a constant which Lindhard sets equal to $\sqrt{3}$. As will be shown below, the polarization is not dependent on the form of the atomic potentials used in Eq. (2).

Separating the wave function into large and small components,

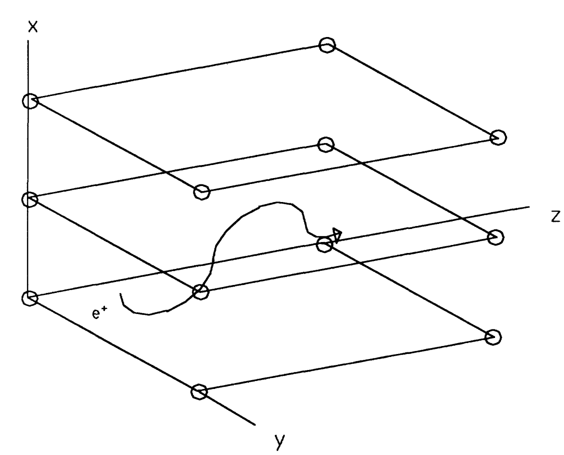

FIG. 1. Qualitative illustration of the classical motion of a channeled positron governed by a planar continuum potential.

$$\Phi=\begin{bmatrix}\phi_+\\ \phi_-\end{bmatrix} , \tag{4}$$

leads to a Pauli-type equation for the large components

$$\underline{\sigma}\cdot\nabla(E-V+m)^{-1}\underline{\sigma}\cdot\nabla\phi_+ + (E-V-m)\phi_+=0 . \tag{5}$$

Since the potential is inaependent of y and z, the solution of the wave equation can be written in the form

$$\phi_+ \propto \exp[i(p_z z + p_y y)]\psi(x)\chi . \tag{6}$$

We now take advantage of the fact that the particles of interest have kinetic energy on the order of 1 MeV, which is much larger than the planar potential energy, which is on the order of 10 eV. This allows us to transform Eq. (5) into a one-dimensional, relativistic Schrödinger equation

$$\frac{-1}{2\gamma m}\frac{\partial^2\psi}{\partial x^2}+V(x)\psi=E_\perp\psi , \tag{7}$$

where γ is the relativistic factor and $E_\perp$ is the transverse energy of the electron.

Now consider the emission of a photon by the channeled electron. Using quantum electrodynamics,[5] the single-photon spontaneous emission rate is proportional to the square of the radiative-transition matrix element, which is given by

$$\mathbf{j}(\mathbf{k})=\int \Phi_f^\dagger(\mathbf{r})\underline{\alpha}\Phi_i(\mathbf{r})\exp(-i\mathbf{k}\cdot\mathbf{r})d^3r , \tag{8}$$

where $\mathbf{k}$ is the photon momentum. In the many-beam formulation, the spatial part of the wave function is written as

$$\psi_{\mathbf{p}_{i(f)}}(\mathbf{r})=\exp(i\mathbf{p}_{i(f)}\cdot\mathbf{r})\sum_g c_g\exp(igx) , \tag{9}$$

where $\mathbf{p}_{i(f)}$ is the initial (final) momentum of the electron and the g are the reciprocal-lattice vectors. When this expression is inserted into the expression for the matrix element [Eq. (8)], the result, neglecting a spin-dependent term, is

$$j(\mathbf{k})=\frac{1}{\gamma mc}\sum_g c_g c^*_{g+K}(p_{xi}+g)\delta(\mathbf{p}_i-\mathbf{p}_f-\mathbf{k}-\mathbf{K}) , \tag{10}$$

where

$$\mathbf{K}=\mathbf{g}-\mathbf{g}' \tag{11}$$

is the momentum absorbed by the crystal lattice. (Although the above analysis was carried out for an electron, analogous equations may be derived for positrons. The only difference is that the potential in the Schrödinger equation [Eq. (7)] will have the opposite sign; the many-beam form of the wave function [Eq. (9)] can still be used. Therefore, the results found for electrons will be equally applicable for positrons.)

The differential intensity for a given polarization is, by Fermi's Golden Rule,

$$\frac{d^2I_i}{dk\,d\Omega}=\frac{e^2}{2\pi}\frac{k^2}{E^2}\delta(k-k\beta_\parallel-\omega)|\mathbf{j}\cdot\hat{\boldsymbol{\epsilon}}_i|^2 , \tag{12}$$

where ω is the energy difference between the two levels in the rest frame of the electron and $\hat{\epsilon}_i$ are the photon polarization vectors. The photon polarization is given by the difference in intensity for the two polarizations, dI_1 and dI_2, normalized by the total intensity, dI_1+dI_2

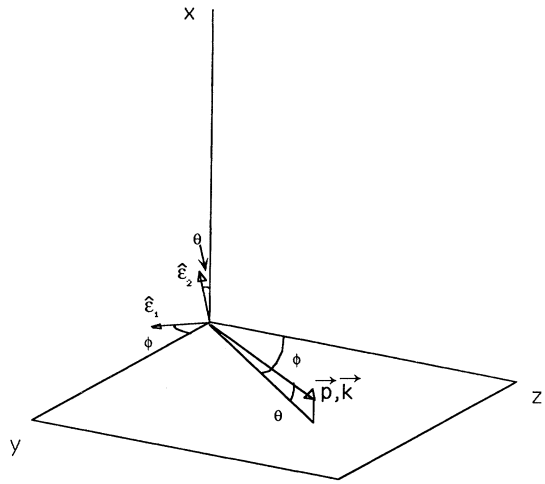

FIG. 2. Geometry of the channeling radiation problem. The set of vectors $(\mathbf{k},\hat{\epsilon}_1,\hat{\epsilon}_2)$ form an orthogonal basis.

$$P=\frac{dI_1-dI_2}{dI_1+dI_2} . \tag{13}$$

Following Kumakhov,[4,6] let us choose the photon polarization vectors as shown in Fig. 2. Let $\hat{\epsilon}_1$ be perpendicular to the plane formed by the photon momentum and $\hat{\epsilon}_2$, and let $\hat{\epsilon}_2$ be in the yz plane at 90° to the direction of the particle's motion. Here we consider only the case of forward emission, where the particle's momentum and the photon's momentum are parallel. The quantity dI_2 is seen to be zero, which tells us that the radiation is completely perpendicularly polarized with respect to the emission plane, which is the plane described by $\mathbf{k}$ and $\hat{\epsilon}_2$. Since the emission plane corresponds almost exactly to the channeling plane, the planar channeling radiation will be almost completely polarized in the direction normal to the channeling plane.

This result is in agreement with the results of Bloom *et al.*,[7] which were obtained from a classical, Monte Carlo calculation, as well as those of Sáenz *et al.*,[8] which were found using a single-plane type of approximation. The agreement with the work of Sáenz *et al.* is in contrast to the situation for axial channeling, where results for polarization calculated in the single-string approximation (analogous to the single plane in planar channeling) are significantly different from the results obtained in the many-beam approach.[9]

In summary, we have shown that the channeling radiation emitted in the forward direction by planar channeled electrons and positrons is completely linearly polarized, in the direction perpendicular to the emission plane. This result is true for any type of crystal and for any choice of crystal plane, provided only that the angle θ is less than the critical channeling angle.

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