Soliton excitations in the alternating ferromagnetic Heisenberg chain

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Soliton excitations in the alternating ferromagnetic Heisenberg spin chain are investigated with the use of the coherent-state method combined with the Holstein-Primakoff bosonic representation of spin operators. When the Hamiltonian and the equations of motion are recast in dimensionless forms, we obtain two partial differential equations with nonlinear coupling. Their nonlinear modified terms are strongly restricted by two dimensionless small parameters that are used in the semiclassical approximation and in the long-wavelength approximation. With use of the method of multiple scales, these equations are reduced to the nonlinear Schrödinger equation, which describes the slow spatial components of the coherent-state amplitudes. The results show that solitonlike magnon localization and the two-magnon bound state are possible in the system. The possibility of observing a gap soliton in the alternating ferromagnetic Heisenberg chain is also discussed.

I. INTRODUCTION

Low-dimensional magnetism has attracted considerable attention in recent years. A great deal of experimental work has been devoted, in particular, to the study of linear and nonlinear excitations in one-dimensional and two-dimensional systems. Ferromagnetic and antiferromagnetic chain compounds, such as $CsNiF_3$ or TMMC $[(CH_3)_4NMnCl_3]$, have been shown to provide good systems exhibiting soliton-type nonlinear excitations. Such nonlinear excitations, which can be viewed as Bloch walls propagating along the chain, have been extensively studied with inelastic-neutron-scattering experiments.[1–5]

There are several theoretical methods to study the nonlinear excitations in quasi-one-dimensional magnets. In the classical methods,[6,7] the general soliton solutions are obtained for a continuum version of the classical Heisenberg chain. For the quantum spin system, a bosonic representation of the spin operators turns out to be a very convenient method for studying the soliton excitations, since the quantum corrections can be included in a systematic way. In the spin-coherent-state representation[8] one can work directly with spin operators, make no approximations to the Hamiltonian, and obtain an exact nonlinear equation of motion for the system.[9] The other coherent-state treatments[10–14] use a severely truncated Holstein-Primakoff (HP) expansion for $S_j^{\pm}$ and further approximate H by a Hamiltonian which is biquadratic in boson operators. Working in Glauber's coherent-state representation and making the semiclassical and long-wavelength approximations, one then finds solitary-wave profiles of the system.

The consistency and validity of the semiclassical treatment, which has been widely used in the study of nonlinear excitations in magnetic systems, have been reexamined in our previous papers.[15,16] We found that the nonlinear modified terms of the equation of motion of the coherent amplitude are strongly constrained by the relation between the semiclassical and long-wavelength approximations that are represented by two small dimensionless parameters, ε and η, respectively. Arguments concerning how to consistently treat the problems of nonlinear excitations in the ferromagnetic chain with use of the coherent-state method have been settled. In this paper, we extend our method to study nonlinear excitations in the alternating ferromagnetic Heisenberg chain. Alternating interactions can occur in layered materials that can exhibit quasi-one-dimensional character[17] and other materials for such studies may well be grown synthetically in a layered manner by molecular-beam epitaxy. Another reason for considering the alternating-bond model lies in the fact that it can be considered as a first step towards studying nonlinear excitations in the alternating antiferromagnetic chain and in the square-lattice antiferromagnet. It should be noted that the spin soliton has been observed in a one-dimensional antiferromagnetic system of Galvinoxyl single crystals.[18]

The organization of this paper is as follows. In Sec. II we rewrite the model Hamiltonian in a dimensionless form, introduce the HP transformation for spin operators, and make the semiclassical approximation. In Sec. III we use Glauber's coherent-state representation and make the long-wavelength approximation to the equations of motion. In Sec. IV the method of multiple scales is introduced to reduce these equations with nonlinear coupling into an envelope-function equation. In Sec. V, one-soliton and two-soliton solutions are given. The possibility of observing a gap soliton in the alternating ferromagnetic Heisenberg chain is predicted. The last section is the summary and discussion.

II. THE MODEL HAMILTONIAN, THE HP TRANSFORMATION, AND THE SEMICLASSICAL APPROXIMATION

The Hamiltonian for the alternating ferromagnetic Heisenberg spin chain is given by

$$\begin{aligned} H-H_0 = &-J_1\sum_j^{N/2}(\mathbf{S}_{2j}\cdot\mathbf{S}_{2j+1}-S^2\hbar^2)-J_2\sum_j^{N/2}(\mathbf{S}_{2j+1}\cdot\mathbf{S}_{2j+2}-S^2\hbar^2)-D_1\sum_j^{N/2}(S_{2j}^zS_{2j}^z-S^2\hbar^2)\\ &-D_2\sum_j^{N/2}(S_{2j+1}^zS_{2j+1}^z-S^2\hbar^2)-g\mu_Bf\sum_j^{N/2}(S_{2j}^z-S\hbar)-g\mu_Bf\sum_j^{N/2}(S_{2j+1}^z-S\hbar)\ , \end{aligned} \tag{1}$$

where $\mathbf{S}_n$ is the spin on site n and J_1,J_2 are the bond strengths, both of which will be taken to be positive. D_1,D_2 are the uniaxial crystal-field anisotropy parameters and f is the intensity of an external magnetic field in the z direction. The ground-state ($T=0$) configuration of this system corresponds to all the spins aligned in the z-axis direction. $H_0=-NS^2\hbar^2(J_1+J_2+D_1+D_2)/2-Ng\mu_BfS\hbar$ is the ground-state energy of the system. Hence $H-H_0$ denotes the energy of excitations.

Introducing the dimensionless spin $\tilde{\mathbf{S}}_n=\mathbf{S}_n/\hbar$ and defining $\tilde{S}_n^{\pm}=\tilde{S}_n^x\pm i\tilde{S}_n^y$, we can recast the Hamiltonian (1) into the dimensionless form,

$$\begin{aligned} \tilde{H}=(H-H_0)/(JS_c^2)= &-\sum_j(\tilde{\mathbf{S}}_{2j}\cdot\tilde{\mathbf{S}}_{2j+1}-S^2)/S^2-\tilde{J}\sum_j(\tilde{\mathbf{S}}_{2j+1}\cdot\tilde{\mathbf{S}}_{2j+2}-S^2)/S^2\\ &-\tilde{D}_1\sum_j(\tilde{S}_{2j}^z\tilde{S}_{2j}^z-S^2)/S^2-\tilde{D}_2\sum_j(\tilde{S}_{2j+1}^z\tilde{S}_{2j+1}^z-S^2)/S^2\\ &-\tilde{f}\sum_j(\tilde{S}_{2j}^z-S)/S-\tilde{f}\sum_j(\tilde{S}_{2j+1}^z-S)/S\ , \end{aligned} \tag{2}$$

where $S_c=\lim_{\hbar\to 0,S\to\infty}(S\hbar)$ and

$$\begin{aligned} &\tilde{J}=J_2/J_1,\quad \tilde{f}=g\mu_B/(J_1S_c)\ ,\\ &\tilde{D}_1=D_1/J_1,\quad \tilde{D}_2=D_2/J_1 \end{aligned} \tag{3}$$

are dimensionless parameters of the system. $\tilde{S}_n^+$, $\tilde{S}_n^-$, and $\tilde{S}_n^z$ satisfy the commutation relations

$$[\tilde{S}_n^{\pm},\tilde{S}_{n'}^z]=\mp\tilde{S}_n^{\pm}\delta_{nn'}\ , \tag{4}$$

$$[\tilde{S}_n^+,\tilde{S}_{n'}^-]=2\tilde{S}_n^z\delta_{nn'}\ , \tag{5}$$

with $\tilde{\mathbf{S}}_n\cdot\tilde{\mathbf{S}}_n=S(S+1)$. After this, we can introduce the HP transformation for the spin operators[19]

$$\tilde{S}_n^+=[2S-a_n^\dagger a_n]^{1/2}a_n\ , \tag{6}$$

$$\tilde{S}_n^-=a_n^\dagger[2S-a_n^\dagger a_n]^{1/2}\ , \tag{7}$$

$$\tilde{S}_n^z=S-a_n^\dagger a_n\ . \tag{8}$$

a_n and $a_n^\dagger$ satisfy Bose commutation relations

$$[a_n,a_{n'}^\dagger]=\delta_{nn'}\ , \tag{9}$$

$$[a_n,a_{n'}]=[a_n^\dagger,a_{n'}^\dagger]=0\ . \tag{10}$$

In low-temperature $a_n^\dagger a_n \ll 2S$, we can use the semiclassical expansions

$$\tilde{S}_n^+/S=\sqrt{2}[\varepsilon a_n-\varepsilon^3a_n^\dagger a_na_n/4 -\varepsilon^5a_n^\dagger a_na_n^\dagger a_na_n/32+O(\varepsilon^7)]\ , \tag{11}$$

$$\tilde{S}_n^-/S=\sqrt{2}[\varepsilon a_n^\dagger-\varepsilon^3a_n^\dagger a_n^\dagger a_n/4 -\varepsilon^5a_n^\dagger a_n^\dagger a_na_n^\dagger a_n/32+O(\varepsilon^7)]\ , \tag{12}$$

where $\varepsilon=1/\sqrt{S}$ is a small dimensionless parameter used in this approximation. Then the dimensionless Hamiltonian (2) can be written as a power series in ε,

$$\tilde{H}=\tilde{H}_1+\tilde{H}_2\ , \tag{13}$$

$$\begin{aligned} \tilde{H}_1=&\varepsilon^2\sum_j[(\tilde{f}+2\tilde{D}_1+1)a_{2j}^\dagger a_{2j}+a_{2j+1}^\dagger a_{2j+1}-(a_{2j}a_{2j+1}^\dagger+a_{2j+1}a_{2j}^\dagger)]\\ &+\varepsilon^4\sum_j[-\tilde{D}_1a_{2j}^\dagger a_{2j}a_{2j}^\dagger a_{2j}-a_{2j}^\dagger a_{2j}a_{2j+1}^\dagger a_{2j+1}+(a_{2j}a_{2j+1}^\dagger a_{2j+1}^\dagger a_{2j+1}+a_{2j}^\dagger a_{2j}a_{2j}a_{2j+1}^\dagger+\text{H.c.})/4]\\ &+\varepsilon^6\sum_j(a_{2j}a_{2j+1}^\dagger a_{2j+1}^\dagger a_{2j+1}a_{2j+1}^\dagger a_{2j+1}+a_{2j}^\dagger a_{2j}a_{2j}^\dagger a_{2j}a_{2j}a_{2j+1}^\dagger-2a_{2j}^\dagger a_{2j}a_{2j}a_{2j+1}^\dagger a_{2j+1}^\dagger a_{2j+1}+\text{H.c.})/32+O(\varepsilon^8), \end{aligned} \tag{14}$$

$$\tilde{H}_2=\varepsilon^2\sum_j[(\tilde{f}+2\tilde{D}_2+1)a^\dagger_{2j+1}a_{2j+1}+\tilde{J}(a^\dagger_{2j+2}a_{2j+2})-\tilde{J}(a_{2j+1}a^\dagger_{2j+2}+a_{2j+2}a^\dagger_{2j+1})]$$
$$+\varepsilon^4\sum_j[-\tilde{D}_2a^\dagger_{2j+1}a_{2j+1}a^\dagger_{2j+1}a_{2j+1}-\tilde{J}a^\dagger_{2j+1}a_{2j+1}a^\dagger_{2j+2}a_{2j+2}$$
$$+\tilde{J}(a_{2j+1}a^\dagger_{2j+2}a^\dagger_{2j+2}a_{2j+2}+a^\dagger_{2j+1}a_{2j+1}a_{2j+1}a^\dagger_{2j+2}+\text{H.c.})/4]$$
$$+\varepsilon^6\sum_j(a_{2j+1}a^\dagger_{2j+2}a^\dagger_{2j+2}a_{2j+2}a^\dagger_{2j+2}a_{2j+2}+a^\dagger_{2j+1}a_{2j+1}a^\dagger_{2j+1}a_{2j+1}a_{2j+1}a^\dagger_{2j+2}$$
$$-2a^\dagger_{2j+1}a_{2j+1}a_{2j+1}a^\dagger_{2j+2}a^\dagger_{2j+2}a_{2j+2}+\text{H.c.})/32+O(\varepsilon^8)\ , \tag{15}$$

where H.c. represents the corresponding Hermitian-conjugate terms. The Heisenberg equation of motion for operator a_n

$$i\hbar\,\partial a_n/\partial t=[a_n,H] \tag{16}$$

can be written in the dimensionless form

$$i\tilde{\omega}_0\,\partial a_n/\partial\tilde{t}=[a_n,\tilde{H}]\ , \tag{17}$$

where $\tilde{\omega}_0=\hbar\omega_0/(J_1S_c^2)$ (ω_0 is a typical frequency of the excitations) and $\tilde{t}=\omega_0 t$ is the dimensionless time. Because of the alternating bond strengths of the system, the operators a_{2j} and a_{2j+1} will satisfy different equations of motion. This is similar to the diatomic chain in which there are two different atoms in each elementary cell.[20] As a result, we have

$$i\tilde{\omega}_0\,\partial a_{2j}/\partial\tilde{t}=\varepsilon^2[(\tilde{f}+2\tilde{D}_1+\tilde{J}+1)a_{2j}-a_{2j+1}-\tilde{J}a_{2j-1}]$$
$$+\varepsilon^4[a^\dagger_{2j}a_{2j}a_{2j+1}/2+\tilde{J}a^\dagger_{2j}a_{2j-1}a_{2j}/2+a^\dagger_{2j+1}a_{2j}a_{2j}/4$$
$$+\tilde{J}a^\dagger_{2j-1}a_{2j}a_{2j}/4+a^\dagger_{2j+1}a_{2j+1}a_{2j+1}/4+\tilde{J}a^\dagger_{2j-1}a_{2j-1}a_{2j-1}/4$$
$$-a^\dagger_{2j+1}a_{2j+1}a_{2j}-\tilde{J}a^\dagger_{2j-1}a_{2j-1}a_{2j}-\tilde{D}_1(1+2a^\dagger_{2j}a_{2j})a_{2j}]+O(\varepsilon^6) \tag{18}$$

and

$$i\tilde{\omega}_0\,\partial a_{2j+1}/\partial\tilde{t}=\varepsilon^2[(\tilde{f}+2\tilde{D}_2+1+\tilde{J})a_{2j+1}-\tilde{J}a_{2j+2}-a_{2j}]$$
$$+\varepsilon^4[\tilde{J}a^\dagger_{2j+1}a_{2j+1}a_{2j+2}/2+a^\dagger_{2j+1}a_{2j}a_{2j+1}/2+\tilde{J}a^\dagger_{2j+2}a_{2j+1}a_{2j+1}/4$$
$$+a^\dagger_{2j}a_{2j+1}a_{2j+1}/4+\tilde{J}a^\dagger_{2j+2}a_{2j+2}a_{2j+2}/4+a^\dagger_{2j}a_{2j}a_{2j}/4$$
$$-\tilde{J}a^\dagger_{2j+2}a_{2j+2}a_{2j+1}-a^\dagger_{2j}a_{2j}a_{2j+1}-\tilde{D}_2(1+2a^\dagger_{2j+1}a_{2j+1})a_{2j+1}]+O(\varepsilon^6)\ . \tag{19}$$

Equations (18) and (19) are Heisenberg's equations of motion for operators a_{2j} and a_{2j+1} in the semiclassical approximation.

III. GLAUBER'S COHERENT-STATE REPRESENTATION AND THE LONG-WAVELENGTH APPROXIMATION

In this section we introduce Glauber's coherent-state representation for Bose operators[21]

$$|\alpha\rangle=\prod_n|\alpha_n\rangle\ , \tag{20}$$

$$|\alpha_n\rangle=\exp(-\tfrac{1}{2}|\alpha_n|^2)\sum_{m=0}^{\infty}[(\alpha_n)^m/\sqrt{m}\,!]|m\rangle\ , \tag{21}$$

with $\langle\alpha|\alpha\rangle=1$. The semiclassical approach allows us to consider the projections of spins which can be continuously distributed along the z axis. The states (20) are the eigenstates of the operators a_n with eigenvalue α_n:

$$a_n|\alpha\rangle=\alpha_n|\alpha\rangle\ . \tag{22}$$

For the system in the state $|\alpha\rangle$, we can find the equations for the averages $\langle\alpha|a_{2j}|\alpha\rangle$ and $\langle\alpha|a_{2j+1}|\alpha\rangle$ using Eqs. (18) and (19). Using this, we obtain

$$i\tilde{\omega}_0\,\partial\alpha_{2j}/\partial\tilde{t}=\varepsilon^2[(\tilde{f}+2\tilde{D}_1+\tilde{J}+1)\alpha_{2j}-\alpha_{2j+1}-\tilde{J}\alpha_{2j-1}]$$
$$+\varepsilon^4[\alpha^*_{2j}\alpha_{2j}\alpha_{2j+1}/2+\tilde{J}\alpha^*_{2j}\alpha_{2j-1}\alpha_{2j}/2+\alpha^*_{2j+1}\alpha_{2j}\alpha_{2j}/4$$
$$+\tilde{J}\alpha^*_{2j-1}\alpha_{2j}\alpha_{2j}/4+\alpha^*_{2j+1}\alpha_{2j+1}\alpha_{2j+1}/4+\tilde{J}\alpha^*_{2j-1}\alpha_{2j-1}\alpha_{2j-1}/4$$
$$-\alpha^*_{2j+1}\alpha_{2j+1}\alpha_{2j}-\tilde{J}\alpha^*_{2j-1}\alpha_{2j-1}\alpha_{2j}-\tilde{D}_1(1+2\alpha^*_{2j}\alpha_{2j})\alpha_{2j}]+O(\varepsilon^6) \tag{23}$$

and

$$i\tilde{\omega}_0\,\partial\alpha_{2j+1}/\partial\tilde{t}=\varepsilon^2[(\tilde{f}+2\tilde{D}_2+1+J)\alpha_{2j+1}-\tilde{J}\alpha_{2j+2}-\alpha_{2j}] +\varepsilon^4[\tilde{J}\alpha^*_{2j+1}\alpha_{2j+1}\alpha_{2j+2}/2+\alpha^*_{2j+1}\alpha_{2j}\alpha_{2j+1}/2+\tilde{J}\alpha^*_{2j+2}\alpha_{2j+1}\alpha_{2j+1}/4 +\alpha^*_{2j}\alpha_{2j+1}\alpha_{2j+1}/4+\tilde{J}\alpha^*_{2j+2}\alpha_{2j+2}\alpha_{2j+2}/4+\alpha^*_{2j}\alpha_{2j}\alpha_{2j}/4 -\tilde{J}\alpha^*_{2j+2}\alpha_{2j+2}\alpha_{2j+1}-\alpha^*_{2j}\alpha_{2j}\alpha_{2j+1}-\tilde{D}_2(1+2\alpha^*_{2j+1}\alpha_{2j+1})\alpha_{2j+1}]+O(\varepsilon^6)\ , \tag{24}$$

where asterisks denote the complex conjugate. When the $O(\varepsilon^4)$ terms are neglected, Eqs. (23) and (24) are linear. It is easy to get the linear dispersion relation

$$\omega=(\omega_1+\omega_2)/2+\delta_1\{(\omega_1-\omega_2)^2+4S_c^2[J_1^2+J_2^2+2J_1J_2\cos(2kd_0)]\}^{1/2}/2\ , \tag{25}$$

with

$$\omega_m=g\mu_B f+S_c(2D_m+J_1+J_2-D_m/S),\quad m=1,2 \tag{26}$$

when returning to the dimensional quantities. Here d_0 is the lattice constant. $\delta_1=\pm 1$ labels the two branches: the upper branch ω_+ of this relation ($\delta_1=+1$) is designated "optic" and the lower branch ω_- ($\delta_1=-1$) "acoustic" by analogy with the phonon case. The dispersion curve is shown in Fig. 1. There is a frequency gap at the border of Brillouin zone $k=k_B=\pm\pi/2d_0$.

Although (18) and (19) have been transformed to the c-number equations in Glauber's coherent-state representation, to solve them is very difficult because of their nonlinearity and discreteness. If we assume that a typical wavelength of the excitation $\lambda_0 >> 2d_0$ (in case of soliton excitation, λ_0 will correspond to the soliton width), then we may take the continuum approximation

$$\alpha_{2j}(t)\rightarrow\psi_1(x,t)\ , \tag{27}$$

$$\alpha_{2j\pm2}(t)\rightarrow\psi_1(x,t)\pm d\psi_{1x}+d^2\psi_{1xx}/2!+d^3\psi_{1xxx}/3!+\cdots =\psi_1\pm\eta\psi_{1\tilde{x}}+\eta^2\psi_{1\tilde{x}\tilde{x}}/2!\pm\eta^3\psi_{1\tilde{x}\tilde{x}\tilde{x}}/3!+O(\eta^4)\ , \tag{28}$$

$$\alpha_{2j+1}(t)\rightarrow\psi_2(x,t)\ , \tag{29}$$

$$\alpha_{2j-1}(t)\rightarrow\psi_2-\eta\psi_{2\tilde{x}}+\eta^2\psi_{2\tilde{x}\tilde{x}}/2!-\eta^3\psi_{2\tilde{x}\tilde{x}\tilde{x}}/3!+O(\eta^4)\ , \tag{30}$$

$$\alpha_{2j+3}(t)\rightarrow\psi_2+\eta\psi_{2\tilde{x}}+\eta^2\psi_{2\tilde{x}\tilde{x}}/2!+\eta^3\psi_{2\tilde{x}\tilde{x}\tilde{x}}/3!+O(\eta^4)\ , \tag{31}$$

$$\sum\rightarrow(1/d)\int dx=(1/\eta)\int d\tilde{x}\ , \tag{32}$$

where $d=2d_0$, $\tilde{x}=x/\lambda_0$. $\eta=d/\lambda_0$ is a small dimensionless parameter used in the long-wavelength approximation. Here we must point out that α_j is not a continuous variable as j changes from point to point ($j=1,2,3\ldots$), but α_{2j} and α_{2j+1} are. The reason for this is that the bond strengths in the Hamiltonian (1) are alternating. The system is divided into two sublattices with lattice constant $d=2d_0$.[22] Equations (23) and (24) in the continuum approximation become

$$i\tilde{\omega}_0\psi_{1\tilde{t}}=\varepsilon^2[(\tilde{f}+2\tilde{D}_1+1+\tilde{J})\psi_1-(1+\tilde{J})\psi_2+\eta\tilde{J}\psi_{2\tilde{x}}-\eta^2\tilde{J}\psi_{2\tilde{x}\tilde{x}}/2!+\eta^3\psi_{2\tilde{x}\tilde{x}\tilde{x}}/3!+O(\eta^4)] +\varepsilon^4\{-\tilde{D}_1\psi_1-2\tilde{D}_1|\psi_1|^2\psi_1+(\tilde{J}+1)(2|\psi_1|^2\psi_2+\psi_1^2\psi_2^*+|\psi_2|^2\psi_2-4|\psi_2|^2\psi_1)/4 +\eta\tilde{J}[-2|\psi_1|^2\psi_{2x}-\psi_1^2\psi_{2x}^*-(|\psi_2|^2\psi_2)_x+4\psi_1(|\psi_2|^2)_x]/4+O(\eta^2)\}+O(\varepsilon^6)\ , \tag{33}$$

$$i\tilde{\omega}_0\psi_{2\tilde{t}}=\varepsilon^2[(\tilde{f}+2\tilde{D}_2+1+\tilde{J})\psi_2-(1+\tilde{J})\psi_1-\eta\tilde{J}\psi_{1\tilde{x}}-\eta^2\tilde{J}\psi_{1\tilde{x}\tilde{x}}/2!-\eta^3\psi_{1\tilde{x}\tilde{x}\tilde{x}}/3!+O(\eta^4)] +\varepsilon^4\{-\tilde{D}_2\psi_2-2\tilde{D}_2|\psi_2|^2\psi_2+(\tilde{J}+1)(2|\psi_2|^2\psi_1+\psi_2^2\psi_1^*+|\psi_1|^2\psi_1-4|\psi_1|^2\psi_2)/4 +\eta\tilde{J}[2|\psi_2|^2\psi_{1\tilde{x}}+\psi_2^2\psi_{1\tilde{x}}^*+(|\psi_1|^2\psi_1)_{\tilde{x}}-4\psi_2(|\psi_1|^2)_{\tilde{x}}]/4+O(\eta^2)\}+O(\varepsilon^6)\ . \tag{34}$$

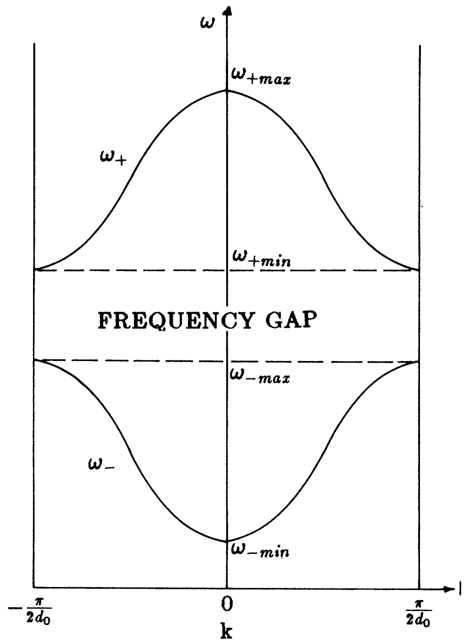

FIG. 1. The two branches ($\delta_1=\pm1$) of the linear dispersion curve for alternating-bond ferromagnetic chain. ω_+ ($\delta_1=+1$) is the optic branch and ω_- ($\delta_1=-1$) is the acoustic branch.

All quantities in Eqs. (33) and (34) are dimensionless. ε and η, which are two small expansion parameters used in the semiclassical approximation and in the long-wavelength approximation, respectively, are explicitly written.

All works made so far on the soliton excitations in the Heisenberg ferromagnet in the HP representation are based on the semiclassical approximation and the long-wavelength approximation.[10–14] The two approximations had been thought to be independent of each other. In our previous papers,[15,16] we have shown that the relative ratio of ε to η is very important for determining the nonlinear modified terms of equations of motion. For a given physical system, ε and η are related to the characteristic quantities of the system. That is to say, $\eta=g(\varepsilon)$ (g is a function of ε). Theoretically, we cannot determine which case is the most important because different cases correspond to different physical pictures. Only from the experimental conditions and initial excitation conditions can we estimate which case is more appropriate. The reason is the same as in the nonlinear theory of long waves in shallow water.[23,24] In this paper, we only consider the case $\eta=O(\varepsilon)$. Retaining terms in Eqs. (33) and (34) to $O(\varepsilon^4)$, we have

$$i\psi_{1t}=\omega_1\psi_1-S_c(J_1+J_2)\psi_2+J_2S_cd\psi_{2x}-J_2S_cd^2\psi_{2xx}/2$$
$$+(S_c/S)[-2D_1|\psi_1|^2\psi_1+(J_1+J_2)(2|\psi_1|^2\psi_2+\psi_1^2\psi_2^*+|\psi_2|^2\psi_2-4|\psi_2|^2\psi_1)/4]\ , \tag{35}$$

and

$$i\psi_{2t}=\omega_2\psi_2-S_c(J_1+J_2)\psi_1-J_2S_cd\psi_{1x}-J_2S_cd^2\psi_{1xx}/2$$
$$+(S_c/S)[-2D_2|\psi_2|^2\psi_2+(J_1+J_2)(2|\psi_2|^2\psi_1+\psi_2^2\psi_1^*+|\psi_1|^2\psi_1-4|\psi_1|^2\psi_2)/4]\ . \tag{36}$$

IV. REDUCTION OF THE EQUATIONS OF MOTION

To solve Eqs. (35) and (36) exactly is very difficult because they are nonlinear and coupled. In spite of this, we can use the method of multiple scales to reduce them to another nonlinear equation which can be solved exactly. This method has been used recently by de Sterke and Sipe to study gap solitons in nonlinear periodic structures.[25,26] We introduce the multiple-scale variables $x_j=\mu^jx$, $t_j=\mu_jt$ ($\mu\ll1$, $j=0,1,2,\ldots$). These variables are considered to be independent.[23,24] Then the first spatial and temporal derivatives can be written as

$$\partial/\partial x=\partial/\partial x_0+\mu\partial/\partial x_1+\mu^2\partial/\partial x_2+\cdots\ , \tag{37}$$

$$\partial/\partial t=\partial/\partial t_0+\mu\partial/\partial t_1+\mu^2\partial/\partial t_2+\cdots\ , \tag{38}$$

from which expressions for higher derivatives follow straightforwardly. Similarly, the quantities ψ_1 and ψ_2 are written as expansions

$$\psi_1=\mu\psi_1^{(1)}+\mu^2\psi_1^{(2)}+\mu^3\psi_1^{(3)}+\cdots\ , \tag{39}$$

$$\psi_2=\mu\psi_2^{(1)}+\mu^2\psi_2^{(2)}+\mu^3\psi_2^{(3)}+\cdots\ . \tag{40}$$

$\psi_s^{(j)}$ ($s=1,2$ and $j=1,2,3,\ldots$) are functions of all x_j and t_j, but these arguments will not be written explicitly. Equations (37)–(40) are now substituted into (35) and (36) and terms with equal powers of μ are collected. This substitution results in equations for $\psi_s^{(j)}$ as follows:

$$(i\,\partial/\partial t_0-\omega_1)\psi_1^{(j)}+L_1\psi_2^{(j)}=\beta_1^{(j)}\ , \tag{37$'$}$$

$$(i\,\partial/\partial t_0-\omega_2)\psi_2^{(j)}+L_2\psi_1^{(j)}=\beta_2^{(j)}\ , \tag{38$'$}$$

$j=1,2,3,\ldots$ with

$$L_1=S_c(J_1+J_2)-J_2S_cd\,\partial/\partial x_0+(J_2S_cd^2/2)\partial^2/\partial x_0^2\ , \tag{39'}$$

$$L_2=S_c(J_1+J_2)+J_2S_cd\,\partial/\partial x_0+(J_2S_cd^2/2)\partial_2/\partial x_0^2\ , \tag{40'}$$

$$\beta_1^{(1)}=0\ , \tag{41}$$

$$\beta_1^{(2)}=-i\,\partial\psi_1^{(1)}/\partial t_1+J_2S_cd\,\partial\psi_2^{(1)}/\partial x_1-J_2S_cd^2\,\partial^2\psi_2^{(1)}/\partial x_0\partial x_1\ , \tag{42}$$

$$\begin{aligned}\beta_1^{(3)}=&-i[\partial\psi_1^{(2)}/\partial t_1+\partial\psi_1^{(1)}/\partial t_2]+J_2S_cd\,(\partial\psi_2^{(2)}/\partial x_1+\partial\psi_2^{(1)}/\partial x_2)\\&-J_2S_cd^2[2\,\partial^2\psi_2^{(2)}/\partial x_0\partial x_1+(2\partial^2/\partial x_0\partial x_2+\partial^2/\partial x_1^2)\psi_2^{(1)}]/2\\&+S_c(J_1+J_2)[2|\psi_1^{(1)}|^2\psi_2^{(1)}+\psi_1^{(1)}(\psi_2^{(1)})^*+|\psi_2^{(1)}|^2\psi_2^{(1)}-4|\psi_2^{(1)}|^2\psi_1^{(1)}]/4S-2D_1(S_c/S)|\psi_1^{(1)}|^2\psi_1^{(1)}\ ,\end{aligned} \tag{43}$$

$\dots,$

$$\beta_2^{(1)}=0\ , \tag{44}$$

$$\beta_2^{(2)}=-i\partial\psi_2^{(1)}/\partial t_1-J_2S_cd\,\partial\psi_1^{(1)}/\partial x_1-J_2S_cd^2\partial^2\psi_1^{(1)}/\partial x_0\partial x_1\ , \tag{45}$$

$$\begin{aligned}\beta_2^{(3)}=&-i(\partial\psi_2^{(2)}/\partial t_1+\partial\psi_2^{(1)}/\partial t_2)-J_2S_cd(\partial\psi_1^{(2)}/\partial x_1+\partial\psi_1^{(1)}/\partial x_2)\\&-J_2S_cd^2[2\,\partial^2\psi_1^{(2)}/\partial x_0\partial x_1+(2\partial^2/\partial x_0\partial x_1+\partial^2/\partial x_1^2)\psi_1^{(1)}]/2\\&+S_c(J_1+J_2)[2|\psi_2^{(1)}|^2\psi_1^{(1)}+\psi_2^{(1)}\psi_2^{(1)}(\psi_1^{(1)})^*+|\psi_1^{(1)}|^2\psi_1^{(1)}-4|\psi_1^{(1)}|^2\psi_2^{(1)}]/4S-2D_2(S_c/S)|\psi_2^{(1)}|^2\psi_2^{(1)}\ .\end{aligned} \tag{46}$$

Equations (37′) and (38′) can be rewritten as

$$(i\,\partial/\partial t_0-\omega_1)(i\partial/\partial t_0-\omega_2)\psi_1^{(j)}-L_1L_2\psi_1^{(j)}=(i\,\partial/\partial t_0-\omega_2)\beta_1^{(j)}-L_1\beta_2^{(j)}\ , \tag{47}$$

$$(i\,\partial/\partial t_0-\omega_2)\psi_2^{(j)}=\beta_2^{(j)}-L_2\psi_1^{(j)}\ , \tag{48}$$

$j=1,2,3,\dots$. We shall solve $\psi_1^{(j)}$ from Eq. (47) and get $\psi_2^{(j)}$ from Eq. (48).

(i) Let $j=1$; then we obtain the equations

$$(i\,\partial/\partial t_0-\omega_1)(i\,\partial/\partial t_0-\omega_2)\psi_1^{(1)}-L_1L_2\psi_1^{(1)}=0\ , \tag{49}$$

$$(i\,\partial/\partial t_0-\omega_2)\psi_2^{(1)}=-L_2\psi_1^{(1)}\ . \tag{50}$$

These are just the linear wave equations. We can obtain a harmonic solution for them, but, since the differential operators are only in the fast scales x_0 and t_0, we may write

$$\psi_1^{(1)}=A(x_1,x_2,\dots;t_1,t_2,\dots)\exp(i\vartheta)\ , \tag{51}$$

$$\psi_2^{(1)}=-[\chi/(\omega-\omega_2)]A(x_1,x_2,\dots,t_1,t_2,\dots)\exp(i\vartheta)\ , \tag{52}$$

$$\vartheta=kx_0-\omega t_0\ , \tag{53}$$

$$\omega=(\omega_1+\omega_2)/2+\delta_1[(\omega_1-\omega_2)^2+4\chi\chi^*]^{1/2}/2\ , \tag{54}$$

$$\chi=S_c(J_1+J_2)-J_2S_ck^2d^2/2+iJ_2S_ckd\ . \tag{55}$$

Equation (54) is the linear dispersion relation of the excitations $\delta_1=+1$ labels the "optic branch" and $\delta_1=-1$ the "acoustic branch." They are the long-wavelength approximation of Eq. (25). The function A in Eqs. (51) and (52) are the undetermined envelope function of the slow scales x_j and $t_j\,(j=1,2,\dots)$.

(ii) When $j=2$ we have

$$(i\,\partial/\partial t_0-\omega_1)(i\partial/\partial t_0-\omega_2)\psi_1^{(2)}-L_1L_2\psi_1^{(2)}=(i\,\partial/\partial t_0-\omega_2)\beta_1^{(2)}-L_1\beta_2^{(2)}\ , \tag{56}$$

$$(i\,\partial/\partial t_0-\omega_2)\psi_2^{(2)}=\beta_2^{(2)}-L_2\psi_1^{(2)}\ . \tag{57}$$

Using (51) and (52) we obtain

$$(i\,\partial/\partial t_0-\omega_1)(i\partial/\partial t_0-\omega_2)\psi_1^{(2)}-L_1L_2\psi_1^{(2)}=-i(\partial A/\partial t_1+C_g\partial A/\partial x_1)\exp(i\vartheta)\ , \tag{58}$$

where $C_g=d\omega/dk$ is the group velocity of linear waves. The terms proportional to $\exp(i\vartheta)$ on the right-hand side of Eq. (58) are secular terms that must be eliminated in order for the theory to be valid.[23,24]. Hence the function A must evolve according to the equation

$$\partial A/\partial t_1+C_g\,\partial A/\partial x_1=0\ . \tag{59}$$

Consequently we have

$$A=A(\xi;x_2,\dots;t_2,\dots)\ , \tag{60}$$

$$\xi=x_1-C_gt_1\ . \tag{61}$$

From here we conclude, that on the first slow space and time scales the waves travel with the group velocity. The particular integrals of $\psi_1^{(2)}$ and $\psi_2^{(2)}$ are

$$\psi_1^{(2)}=D(\xi,x_2,\dots;t_2,\dots)\exp(i\vartheta)\ , \tag{62}$$

$$\begin{aligned}\psi_2^{(2)}=&-[i\chi/(\omega-\omega_2)^2+J_2S_cd(1+ikd)/(\omega-\omega_2)]\\&\times\partial A/\partial\xi\exp(i\vartheta)-[\chi/(\omega-\omega_2)]D\exp(i\vartheta)\ ,\end{aligned} \tag{63}$$

where D is another undetermined function.

(iii) Let $j=3$; then we obtain the equations for $\psi_1^{(3)}$ and $\psi_2^{(3)}$:

$$(i\,\partial/\partial t_0-\omega_1)(i\partial/\partial t_0-\omega_2)\psi_1^{(3)}-L_1L_2\psi_1^{(3)}=(i\,\partial/\partial t_0-\omega_2)\beta_1^{(3)}-L_1\beta_2^{(3)}\ , \quad (64)$$

$$(i\,\partial/\partial t_0-\omega_2)\psi_2^{(3)}=\beta_2^{(3)}-L_2\psi_1^{(3)}\ . \quad (65)$$

Substituting Eqs. (51), (52), (62), and (63) into Eq. (64), we obtain

$$(i\,\partial/\partial t_0-\omega_1)(i\,\partial/\partial t_0-\omega_2)\psi_1^{(3)}-L_1L_2\psi_1^{(3)}=-2[\omega-(\omega_1+\omega_2)/2][i\,\partial A/\partial t_2+\Gamma_1\partial^2A/\partial\xi^2+(\Gamma_2/S)|A|^2A]\exp(i\vartheta)\ , \quad (66)$$

with

$$\Gamma_1=\tfrac{1}{2}d^2\omega/dk^2\ , \quad (67)$$

$$\Gamma_2=(S_c(J_1+J_2)/\{4[\omega-(\omega_1+\omega_2)]\})\times\{4(\omega-\omega_1)+[1+(\omega-\omega_1)/(\omega-\omega_2)](\chi+\chi^*)+4(\omega-\omega_1)[D_1+D_2(\omega-\omega_1)^2/(\omega-\omega_2)^2]/(J_1+J_2)\}\ . \quad (68)$$

Again in order to apply perturbation theory, we must demand the coefficient of $\exp(i\vartheta)$ (the secular term) be zero. This then forces function A to evolve according to the equation

$$i\,\partial A/\partial t_2+\Gamma_1\partial^2A/\partial\xi^2+(\Gamma_2/S)|A|^2A=0\ . \quad (69)$$

V. ONE-SOLITON AND TWO-SOLITON SOLUTIONS

From Eq. (69) we can see that the envelope function A satisfies the nonlinear Schrödinger (NLS) equation, which belongs to completely integrable systems and can be solved exactly by the inverse-scattering transform.[23,24] In the case of the phonon localization and two-phonon bound states of multivibrational excitations in anharmonic molecular crystals, the evolution of coherent amplitude of lattice vibrations is also reduced to the NLS equation.[27]

By making the transformation

$$A=(1/\mu)u(X,t) \quad (70)$$

and noting that $\xi=\mu(x-C_gt)$ and $t_2=\mu^2t$, Eq. (69) can be rewritten as

$$i\,\partial u/\partial t+\Gamma_1\partial^2u/\partial X^2+(\Gamma_2/S)|u|^2u=0\ , \quad (71)$$

where $X=x-C_gt$. The single-soliton solution is

$$u=(2\Gamma_1\kappa_0^2S/\Gamma_2)^{1/2}\text{sech}\{\kappa_0[x-(C_g+2\Gamma_1\kappa)t-x_0]\}\times\exp(iKx-i\Omega t-\varphi_0)\ , \quad (72)$$

with

$$K=k+\kappa\ , \quad (73)$$

$$\Omega=\omega+[C_g\kappa+(\kappa^2-\kappa_0^2)\Gamma_1]\ , \quad (74)$$

where κ_0, κ, x_0, and φ_0 are integration constants. Equation (72) is a wave packet traveling to the right with velocity $C_g+2\Gamma_1\kappa$. If κ is set to be zero, it becomes

$$u=(2\Gamma_1\kappa_0^2S/\Gamma_2)^{1/2}\text{sech}[\kappa_0(x-x_0)]\exp(ikx-i\Omega t-\varphi_0)\ , \quad (75)$$

$$\Omega=\omega-(1/2)\kappa_0^2\omega''(k) \quad (76a)$$

at $k=0$ or $\pi/2d_0$ ($C_g=0$). Here $\omega''(k)=d^2\omega/dk^2$. For the acoustic branch we see that

$$\Omega|_{k=0}=\Omega_{-\min}=\omega_{-\min}-\tfrac{1}{2}\kappa_0^2\omega''_{-\min}<\omega_{-\min}\ , \quad (76b)$$

since

$$\omega''_{-\min}=(d^2\omega^-/dk^2)_{k=0}>0\ , \quad (77)$$

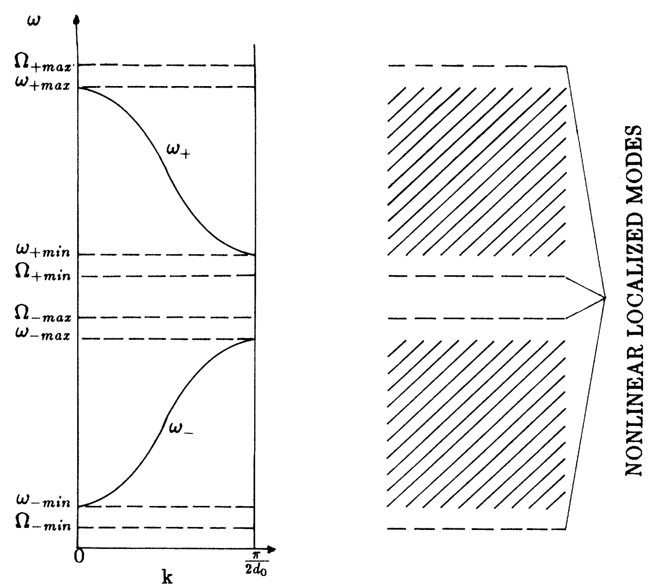

FIG. 2. Four frequency values $\Omega_{-\min}$, $\Omega_{-\max}$, $\Omega_{+\min}$, and $\Omega_{+\max}$ enter into the frequency gap of the linear dispersion curve of the system. They denote the nonlinear localized vibrating modes of the chain.

and

$$\Omega|_{k=\pi/2d}=\Omega_{-\max}=\omega_{-\max}-\tfrac{1}{2}\kappa_0^2\omega''_{-\max}>\omega_{-\max} . \quad (78)$$

For the optical branch, we have

$$\Omega|_{k=\pi/2d}=\Omega_{+\min}=\omega_{+\min}-\tfrac{1}{2}\kappa_0^2\omega''_{+\min}<\omega_{+\min} . \quad (79)$$

$$\Omega|_{k=0}=\Omega_{+\max}=\omega_{+\max}-\tfrac{1}{2}\kappa_0^2\omega''_{+\max}>\omega_{+\max} . \quad (80)$$

So the soliton frequencies have four values ($\Omega_{-\min}$, $\Omega_{-\max}$, $\Omega_{+\min}$, and $\Omega_{+\max}$), which enter into the frequency gap of the linear dispersion curve as shown in Fig. 2. This shows the possibility of observing a gap soliton in the alternating ferromagnetic Heisenberg chain. The concept of the gap soliton was first introduced by Chen and Mills when they studied the nonlinear optical response of superlattices.[28] Although the above treatments involve the semiclassical and the long-wavelength approximations, we believe that gap solitons are possible in an alternating ferromagnetic chain.

With the use of the inverse-scattering transform, we can obtain the two-soliton bound-state solution[27]

$$u=(2\Gamma_1 S/\Gamma_2)^{1/2}Q\{\kappa_1\text{sech}[\kappa_1(x-C_g t+x_0)]\exp(-i\Omega_1 t)+\kappa_2\text{sech}[\kappa_2(x-C_g t-x_0)]\exp(-i\Omega_2 t)\} , \quad (81)$$

$$Q=(\kappa_2^2-\kappa_1^2)/((\kappa_1^2+\kappa_2^2)-2\kappa_1\kappa_2\{\tanh[\kappa_1(x-C_g t+x_0)]\tanh\kappa_2(x-C_g t-x_0) -\text{sech}[\kappa_1(x-C_g t+x_0)]\text{sech}[\kappa_2(x-C_g t-x_0)]\cos\Omega t\}) \quad (82)$$

with

$$\Omega_1=\omega-\tfrac{1}{2}\kappa_1^2\omega''(k) , \quad (83)$$

$$\Omega_2=\omega-\tfrac{1}{2}\kappa_2^2\omega''(k) , \quad (84)$$

$$\Omega=\tfrac{1}{2}(\kappa_1^2-\kappa_2^2)\omega''(k)=\Omega_2-\Omega_1 . \quad (85)$$

Equation (81) represents two bound solitons, which move to the right with velocity C_g. When $C_g=0$ (i.e., $k=0$ or $\pi/2d_0$), they are localized two-soliton bound states in which one soliton vibrates around equilibrium position $x=-x_0$ with frequency Ω_1 and the other around $x=x_0$ with frequency Ω_2. The mutual interaction between them is described by the function Q in equation (82). They may be called two-magnon bound states of the alternating ferromagnetic Heisenberg chain.

From Eqs. (83) and (84) we can see that when $k=0$ or $\pi/2d_0$, (81) represents two-gap-soliton bound states of the chain.

Recently, Bell *et al.* investigated the two-magnon states of the alternating ferromagnetic chain with spin $S=\frac{1}{2}$ by using the Bethe ansatz and scaling approach.[29] They found that a bound state exists in one of the gaps at $k=0$, which is possibly a good candidate for detection by a light-scattering experiment. Our approach, developed above, provides the same conclusion.

In the coherent-state representation, the energy of the system (1) is

$$E=\langle\alpha|H|\alpha\rangle/\langle\alpha|\alpha\rangle=(1/d)\int dx\mathcal{H}(x,t) , \quad (86)$$

where $\mathcal{H}(x,t)$ is the energy density. For the single-soliton bound state (72), it is easy to get

$$E=[4\Gamma_1\kappa_0 S_c^2/\Gamma_2 d][g\mu_B f/S_c+2D_1+J_1+J_2+(g\mu_B f/S_c+2D_2+J_1+J_2)\chi\chi^*/(\omega-\omega_2)^2 +(J_1+J_2)(\chi+\chi^*)/(\omega-\omega_2)] . \quad (87)$$

The spatial configuration of the spin is given by

$$\langle S_{2j}^z\rangle=\langle\alpha|(S-a_{2j}^\dagger a_{2j})|\alpha\rangle/\langle\alpha|\alpha\rangle , \quad (88)$$

$$\langle S_{2j+1}^z\rangle=\langle\alpha|(S-a_{2j+1}^\dagger a_{2j+1})|\alpha\rangle/\langle\alpha|\alpha\rangle . \quad (89)$$

For the single-soliton case, we have

$$\langle S_{2j}^z\rangle\to S-\psi_1^*\psi_1=S(1-(2\Gamma_1\kappa_0^2/\Gamma_2)\text{sech}^2\{\kappa_0[x-(C_g+2\kappa\Gamma_1)t-x_0]\}) , \quad (90)$$

$$\langle S_{2j+1}^z\rangle\to S-\psi_2^*\psi_2=S(1-\{2\chi\chi^*\Gamma_1\kappa_0^2/[\Gamma_2(\omega-\omega_2)^2]\}\text{sech}^2\{\kappa_0[x-(C_g+2\kappa\Gamma_1)t-x_0]\}) , \quad (91)$$

and, for the two-soliton bound state, we obtain

$$\langle S^z_{2j}\rangle \to S - S(2\Gamma_1/\Gamma_2)Q^2\{\kappa_1^2\mathrm{sech}^2[\kappa_1(x-C_g t+x_0)]+\kappa_2^2\mathrm{sech}^2(x-C_g t-x_0) + 2\kappa_1\kappa_2\mathrm{sech}[\kappa_1(x-C_g t+x_0)]\mathrm{sech}[\kappa_2(x-C_g t-x_0)]\cos\Omega t\}\ , \tag{92}$$

$$\langle S^z_{2j+1}\rangle \to S - S\{2\chi\chi^*\Gamma_1/[(\omega-\omega_2)^2\Gamma_2]\}Q^2\{\kappa_1^2\mathrm{sech}^2[\kappa_1(x-C_g t+x_0)]+\kappa_2^2\mathrm{sech}^2[\kappa_2(x-C_g t-x_0)] + 2\kappa_1\kappa_2\mathrm{sech}[\kappa_1(x-C_g t+x_0)]\mathrm{sech}[\kappa_2(x-C_g t-x_0)]\cos\Omega t\}\ , \tag{93}$$

where Q has been given in Eq. (82). Equations (90)–(93) show analytically the magnon localization in the alternating ferromagnetic Heisenberg chain. They are typical nonlinear excitations of the system.

VI. DISCUSSION AND SUMMARY

We have investigated soliton excitations in the alternating ferromagnetic Heisenberg chain. For studying the nonlinear excitations in ferromagnets and antiferromagnets, the approach developed above is consistent and systematic. We have emphasized the importance of the relative ratio of ε to η for determining the modified terms of the equations of motion. This is an example of Kruskal's "principle of maximal balance," which states that in a perturbation expansion involving two or more small parameters a scaling that reduces the problem as little as possible is of interest.[30]

For the alternating chain, the number of equations of motion is more than one, and, with nonlinear coupling, it is very difficult to solve them. The method of multiple scales used here can reduce these equations to a single equation, for example, the NLS equation in this paper. This equation plays an important role in many nonlinear phenomena,[24] and its properties have been widely studied.

In summary, soliton excitations in the alternating ferromagnetic Heisenberg spin chains with uniaxial crystal-field anisotropy have been investigated with help of the coherent-state method combined with the *HP* transformation. After recasting the Hamiltonian and the equations of motion in the dimensionless forms, we obtained two coupled partial differential equations. Their nonlinear terms are strongly restricted by two small dimensionless expansion parameters, ε and η, which have been used in the semiclassical approximation and in the long-wavelength approximation, respectively. By using the method of multiple scales, the system of coupled equations has been reduced to the NLS equation. The single-soliton- and two-soliton bound-state solutions are obtained by the inverse-scattering transform. These results show that solitonlike magnon localization and two-magnon bound state in the alternating ferromagnetic Heisenberg chain are possible. The possibility of observing a gap soliton in this system is also discussed. Our next aim is to compare our theory with experiment and to investigate the nonlinear excitations in the alternating antiferromagnetic Heisenberg chain, which will be given in a future publication.

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