Collective flux creep in high-T_c superconductors

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(Received 29 June 1990; revised manuscript received 6 November 1990)

We develop a scaling approach to flux-line pinning in high-T_c superconductors. Our main result is a nonlinear relation $V(j_{\text{ex}}) \sim \exp(-Cj_{\text{ex}}^{-\mu})$ between the voltage V and the external current density j_{ex}, where the exponent μ is related to the roughness exponent and has been estimated in an earlier paper [T. Nattermann, Phys. Rev. Lett. **64**, 2454 (1990)]. For the low-frequency ac resistivity we find two contributions $\rho_1(\omega) \sim \omega^2[1-(T/T^*)\ln\omega\tau_0]^{4/\psi}$ and $\rho_2(\omega) \sim \omega[1-(T/T^*)\ln\omega\tau_0]^{2/\psi}$, where $\psi \approx 1$ in three dimensions, and where T^* is a characteristic temperature. Similar power laws are obtained for the dynamic susceptibility $\chi''(\omega)$, whereas the magnetization due to the change of the applied field varies in time as $(\ln t)^{-1/\mu}$.

I. INTRODUCTION

The conventional theory of flux creep in superconductors[1] is based on various concepts. Apart from thermal fluctuations, the flux line motion is due to the Lorentz force density $\mathbf{p}_{\text{ex}} = \mathbf{j}_{\text{ex}} \times \mathbf{B}/c$, which acts perpendicular to the flux lines. Here, $\mathbf{j}_{\text{ex}}$ is the current density of an external source or transport current density and $\mathbf{B}$ the spatially averaged magnetic induction. The flux lines are pinned by defects such as vacancies, interstitials, impurities, voids, twin boundaries, etc., which even for $\mathbf{j}_{\text{ex}}=0$ lead to a distortion of the flux-line lattice (FLL). The distortion of the FLL due to the Lorentz force density and due to pinning is described by elastic continuum theory. At zero temperature flux-line motion is only possible if p_{ex} exceeds the average pinning force density, and the current density $j_{\text{ex}} < j_c(0,B)$ is dissipationless, where $\mathbf{j}_c(T,B)$ is the critical current density. For $j_{\text{ex}} > j_c$ flux-line motion with velocity $\mathbf{v}$ leads to an electric field $\mathbf{E} = \mathbf{B} \times \mathbf{v}/c$ and hence to a finite voltage V. At finite temperatures there is a finite probability that the flux lines overcome the pinning energy barriers: One has a strong crossover from thermally activated *flux creep* for $j_{\text{ex}} \ll j_c(T,B)$ to *flux flow* for $j_{\text{ex}} \gg j_c(T,B)$.

Flux creep depends on various parameters. At low fields of the order of H_{c1} one has essentially pinning by a single flux line. At higher fields the interactions between the flux lines become sufficiently strong, leading to a thermally activated motion of bundles of flux lines.[2] The volume V_c of such a bundle which moves in a thermally activated jump depends on T and B and can be estimated only very crudely. There is a variety of possible pinning centers which might pin more or less strongly and in general one considers two limiting cases: Either one has strongly pinning defects which all pin independently *(strong-pinning limit)* or weak pinning centers which pin collectively and where a single pin does not disturb the flux lines significantly *(weak-pinning limit)*. There is also collective pinning of a single flux line. Here we consider only weak collective pinning as it is most likely the case in high-T_c superconductors[3] [see, however, Ref. 3(a)].

In the simplest case the activation (or barrier) free energy $U_0(T,B)$ which is overcome by a jump of a flux line or flux-line bundle is determined by the condensation energy density $H_c^2/8\pi$ gained by the flux line by transversing a nonsuperconducting impurity multiplied by a suitable volume V_c. Here, H_c is the thermodynamic critical field. For a *single* flux line this volume is of the order of $V_c \approx \xi_\perp^2 L_c$ where $\xi_\perp$ is the Ginzburg-Landau coherence length perpendicular to the line (or the radius of the normal conducting core of the flux line) and L_c the activated length. In thin films L_c is determined by the film thickness d and for $d \ll L_c$ one has two-dimensional (2D) collective pinning.[3] The crossover from 2D to 3D pinning indeed has been observed.[3] For 3D systems the relevant scales are either the coherence lengths $\xi_\parallel$ and $\xi_\perp$, the flux-line lattice constant $a_0 = 1.075\ (\Phi_0/B)^{1/2}$, or the mean distance between the impurities, where Φ_0 is the flux quantum. Yeshurun and Malozemoff[4] and Tinkham[5] estimate for $H \gg H_{c1}$ the activation energy $U_0(T,B) = \beta H_c^2 \xi_\parallel a_0^2$ where the constant β is proportional to the number of flux lines in the bundle and to the free energy difference between flux lines in a square and a triangular lattice in units of $H_c^2/8\pi$. The square lattice corresponds to the saddle point and the triangular lattice to the stable flux-line configuration. Various expressions for V_c and L_c have been discussed by Kes *et al.*[6] For strong pinning, L_c is the mean distance between two pinning centers.

The concept of a "flux-line bundle" and hence the meaning of V_c has been considerably clarified by Larkin and Ovchinnikov (LO)[7] who showed that translational long-range order (TLRO) of the FLL is unstable against weak pinning. Flux lines are correlated only over a finite volume which in the conventional theory is identified with the volume V_c which is activated during a jump.

The FLL is strongly distorted and is treated in the theory of LO essentially as an uncorrelated fluid on scales larger than the correlation length for TLRO. The volume V_c is determined by the elastic energy of the FLL (which is a function of T, H_c, and B) the density and strength of the pinning centers, and the characteristic lengths $\zeta_\parallel$, $\zeta_\perp$, and a_0.

At zero temperature the barrier energy $U_0(T=0,B)$ is connected to the critical current density $j_c\ (0,B)$ since for $j_{ex}=j_c$ the Lorentz force density has to be equal to the pinning force density in the volume V_c

$$U_0(0,B)=j_c(0,B)BV_c d_p/c \ . \qquad (1.1)$$

Here, d_p is the distance over which the flux-line (bundle) moves with $d_p \approx \xi$ for an isolated vortex line or $b \equiv B/B_{c2} < 0.2$ and $d_p \approx a_0/2$ for a flux-line bundle or $b > 0.2$.[8]

In *conventional* superconductors one has $U_0(T,B)/T \gg 1$ (with $k_B=1$) for all temperatures $T \le T_c$. The flux-line velocity v is given by[1]

$$v=2v_0\exp(-U_0/T)\sinh(Bj_{ex}V_c d_p/cT)$$

$$\approx v_0\exp[-(cU_0-Bj_{ex}V_c d_p)/cT] \ , \qquad (1.2)$$

where the second part of (1.2) holds since a voltage is measurable only for $Bj_{ex}V_c d_p/cT \gg 1$. Here, v_0 is a microscopic velocity proportional to an attempt frequency (see Ref. 9 for an explicit calculation of v_0). The flux creep with velocity v, the generated field $E=Bv/c$, and the resistivity $\rho(B,T)$ can be observed only if $Bj_{ex}V_c d_p/c$ is of the order of U_0 or larger since otherwise the voltage is unmeasurably small.[10–12]

In *high-T_c superconductors* such as Y-Ba-Cu-O, La-Ba-Cu-O, Bi-Sr-Ca-Cu-O, or Tl-Ca-Ba-Cu-O the ratio U_0/T is considerably smaller since the coherence lengths ξ_c and ξ_{ab} in the c and a,b directions are extremely small and since T_c is large. A small coherence length or size of the vortex cores also explains why these systems pin weakly. One observes in the magnetization a well-defined irreversibility line in the B-T plane which can be interpreted as a strong crossover from flux creep to flux flow or a depinning transition. Below this line one has a strong difference between the field-cooled and zero-field-cooled magnetizations. This difference represents an irreversible magnetization which scales as a function of temperature and external field.[13] In the resistivity one observes in a broad range of fields or temperatures either flux creep or flux flow.[14–19] In part of these experiments one observes deviations from the Arrhenius law (1.2) which is found in conventional superconductors. These deviations might be due either to a strong temperature dependence of the activation from energy U_0, to a strong crossover from flux creep to flux flow or due to a true phase transition. A "shoulder" in the resistivity versus temperature which appears in fairly high external fields $\mathbf{H} \perp \mathbf{j}_{ex}$ (Refs. 14, 18–20) has been interpreted as a depinning transition.[21] Evidence for a sharp equilibrium phase transition at considerably lower temperatures for a given field comes from the current-voltage (I-V) curve[22,23] for very low currents. These data indicate also a vanishing linear resistivity in the flux-creep region, i.e., below the transition line. This transition would become "soft" if one measures the resistivity with too high currents. The vanishing linear resistivity is in contrast to the conventional theory discussed so far and will be discussed in Sec. III.

The experiments mentioned so far are connected to the motion of flux lines which are created at sufficiently high fields. In addition one observes in zero or small fields some kind of phase transition which possibly can be identified with a Kosterlitz-Thouless transition.[24] The corresponding data on $Bi_2Sr_2CaCu_2O_x$ (Bi 2:2:1:2),[25,26] $ErBa_2Cu_3O_{7-y}$,[27] and $YBa_2Cu_3O_{7-y}$ (Y 1:2:3)[28–30] all indicate that these systems in a certain field-temperature region behave as two-dimensional superconductors in which vortex pairs can be created spontaneously and dissociate at the Kosterlitz-Thouless temperature T_{KT}. The temperature T_{KT} turns out to be only a few degrees below the transition temperature T_c and depends on the applied field or current since both stimulate pair dissociation.

The melting of a three-dimensional FLL without pinning has been considered by various authors.[31–36] The corresponding melting temperature T_m, as obtained from the Lindemann criterion, turns out to be well below T_c. However, in a system with collective pinning one would rather expect a transition from a vortex glass into a vortex liquid[37,38] or a depinning transition.[39] The experimental evidence for such a transition seems to be inconclusive. The interpretation of vibrating reed data as evidence for FLL melting[40] (see also Ref. 41) has been objected by several authors[42–45] and it is not obvious that the proposed vortex glass-liquid transition[22,37,38] is identical with a depinning transition.[42,46] If the latter is a sharp phase transition[22] it should be connected with infinite energy barriers which by some mechanism disappear at the transition temperature.

In two dimensions one has melting by dissociation of dislocation pairs in the FLL (Refs. 24, 31, and 47) and possibly this mechanism can be observed also in layered high-T_c superconductors.[48]

So far we considered the case of an external field $\mathbf{H}$ in direction of the c axis and the current density j_{ex} in the ab plane. The situation still became more puzzling when a series of experiments on the resistivity transition[17,18,49–51] indicated also a broadening for $\mathbf{H} \parallel \mathbf{j}_{ex}$. In these data the "shoulder" is missing.[18] They were interpreted as a breakdown of conventional theory since the Lorentz force density due to the external field $\tilde{\mathbf{p}}_{ex}=\mathbf{j}_{ex}\times\mathbf{H}/c$ should be zero. However, as mentioned before, the flux lines in a vortex glass and in particular in a vortex liquid are not parallel to H, and there are still local Lorentz force densities $\mathbf{p}_{ex}=\mathbf{j}_{ex}\times\mathbf{B}_{loc}/c$ which can lead to flux creep or flux flow and hence to a finite resistivity. In a recent paper[52] it has been shown that the "shoulder" in the resistivity of Y 1:2:3 or more exactly the difference between the resistivities for $\mathbf{H} \perp \mathbf{j}_{ex}$ and $\mathbf{H} \parallel \mathbf{j}_{ex}$ are due to the Lorentz force density $\tilde{p}_{ex}$. The absence of a similar anomaly in the resistivity for $\mathbf{H} \parallel \mathbf{j}_{ex}$

then suggests that in the range of fields and temperatures investigated one has thermally assisted flux flow (TAFF) without any phase transition. In Bi 2:2:1:2 the "shoulder" is missing for all angles between **H** and $\mathbf{j}_{ex}$ *within* the *ab* planes[50,51] indicating a TAFF or vortex liquid state for all fields between 0.5 and 3 T and for all measuring temperatures. This can be explained by the fact that Bi 2:2:1:2 has a considerably larger anisotropy than Y 1:2:3 and hence behaves more as a 2D system with very small energy barriers U_0, at least for flux lines in the *ab* planes.

In the classical theory of flux creep[1,2,6,9,11,21] the barrier energy U_0 is assumed to be the same for all flux-line bundles. However, a better fit to some of the experimental data is obtained if one assumes a broad distribution of barriers.[53,54] Actually, the pinning of a flux line or of a flux-line bundle of linear size L is a complicated stochastic problem. The system has many metastable states which correspond to different flux-line configurations. In the case of weak pinning (which we consider in this paper) the barrier energy U_0 depends on the scale L on which we consider the flux-line bundle and is the free energy difference between two neighboring locally stable configurations of the flux-line bundle and the saddle point. There is pinning on all length scales, as in the case of a ferromagnetic domain wall or interface,[55-60] of a charge density wave,[61,62] or a dilute Ising antiferromagnet in a uniform magnetic field (the random field Ising model).[63] In these systems a random distribution of point defects leads to *roughening* of the domain walls or charge density waves.[55,61,62,64] Hence one expects a similar roughening of a single flux line,[55,57,65-68] a FLL,[37-39,69] or also of a single dislocation in a random field or random potential.[70]

In this paper we apply the concept of scaling to the roughening of a FLL and similar systems with random point defects. In Sec. II we present a general theory which holds for flux-line lattices, interfaces, and charge density waves. We derive expressions for the time-dependent response to a small external force density p_{ex} and in particular for the creep velocity, based on a scaling assumption. In Sec. III we apply this theory to the FLL in superconductors and calculate the dc and ac resistivities and the dissipative part of the dynamic susceptibility due to thermally activated flux creep. Our main result will be a nonanalytic relation $V(j_{ex}) \sim \exp(-Cj_{ex}^{-\mu})$ between the voltage V and the current density j_{ex} where C is a T- and H-dependent constant. The exponent μ is related to the roughness exponent ζ and the barrier energy exponent Ψ by $\mu = \Psi/(d-\zeta-\Psi)$ and has been estimated by one of us[39] to be $\mu = 0.5$ for $d=3$. A similar expression for $V(j_{ex})$ has been derived in a different way[37,38,69] though with different values for μ. The experimental data of Ref. 22 suggest $\mu = 0.4$. This result leads to the linear resistivity $\rho \sim (dV/dj_{ex})j_{ex} \to 0 = 0$, but for all finite current densities j_{ex} to a finite resistivity V/j_{ex}. It holds in the flux creep or vortex-glass region. For the transition observed in Ref. 22 other mechanisms (such as the formation of dislocations in the FLL) should be responsible which are not considered in this paper.

II. GENERAL THEORY

In this section we describe a general scaling approach to the pinning of flux lines and similar objects such as domain walls in Ising systems, dislocations, or charge density waves in a d-dimensional space. These objects have different dimensions D: $D = d-1$ for domain walls, $D=1$ for flux lines and dislocations, and $D=d$ for charge density waves or flux-line lattices. All these objects are pinned by inhomogeneities of the solid. Here, we consider *weak pinning* by a random distribution of point defects. In order to simplify reading we will mostly refer to flux lines as the objects which are pinned.

All these objects are distorted by the point defects. Our basic assumption is that *density fluctuations* of these point defects lead on scale L to a typical distortion

$$w \approx L^{\zeta} \quad (1 \geq \zeta \geq 0, L \geq L^*) \ . \tag{2.1}$$

Here the length scale L^* is assumed to be large compared to the characteristic lengths ξ and a_0, and also compared to the distance between the point defects. For a FLL, L^* will be identified with the characteristic length in the LO theory [see Eq. (3.5) below] which has been estimated in Ref. 6 and depends strongly on the number and strength of the pinning centers. As explained in the Introduction, the point defects lead to many metastable states or many different flux-line configurations. A flux line can be pinned on any length scale $L \geq L^*$; there is a cut-off L^* since a single point defect does not disturb a flux-line significantly, and since for $L < L^*$ there are too few point defects in the volume V^D in order to obtain a significant fluctuation of the number of impurities which may pin a flux line collectively.

To give the "typical" distortion w a more precise meaning, we connect w with the correlation function $\langle \mathbf{u}(\mathbf{x})\mathbf{u}(\mathbf{y})\rangle$ of the displacement field $\mathbf{u}(\mathbf{x})$. Here, $\mathbf{u}(\mathbf{x})$ is the local displacement of the object from its equilibrium configuration if the disorder is absent and $\mathbf{x}$ is a D-dimensional position vector. We then define as our central quantity the *roughness* in arbitrary direction ($\mathbf{n}$ is a unit vector)

$$w(L\mathbf{n}) \equiv \left[V^{-1} \int d^D x \langle [\mathbf{u}(\mathbf{x}+L\mathbf{n}) - \mathbf{u}(\mathbf{x})]^2 \rangle_s \right]^{1/2} , \tag{2.2}$$

where V is the volume of the system. Here, $\langle \ \rangle_s$ denotes the average over thermal fluctuations and the randomness. More precisely, we assume that at low temperatures the average over neighboring metastable configurations $\mathbf{u}_s(\mathbf{x})$, $\mathbf{u}_{s'}(\mathbf{x})$ with energies close to the ground-state energy

$$\left[V^{-1} \int d^D x \langle [\mathbf{u}_s(\mathbf{x}+L\mathbf{n}) - \mathbf{u}_{s'}(\mathbf{x}+L\mathbf{n}) - \mathbf{u}_s(\mathbf{x}) + \mathbf{u}_{s'}(\mathbf{x})]^2 \rangle_{ss'} \right]^{1/2}$$

agrees roughly with definition (2.2)

The low-frequency dynamics of the system is governed by the free energy barriers U_0 between neighboring metastable states which also depend on the length scale L, and in addition on temperature and on the field $\mathbf{B}$. We assume that typical barriers will scale with the exponent Ψ and that there is a smallest free energy $T^*=U_0(L^*)$ below which there is no pinning. [As long as the distribution of $U_0(L)$ has no long power-law tail we can obtain the correct long-time behavior of our system by considering only typical barriers.]

We have

$$U_0(L)=T^*\left[\frac{L}{L^*}\right]^{\Psi} \quad (D\geq\Psi\geq 0)\ , \tag{2.3}$$

$$w(L\mathbf{n})=w^*\left[\frac{L(\mathbf{n})}{L^*}\right]^{\zeta} \quad (1\geq\zeta\geq 0)\ , \tag{2.4}$$

where (2.4) makes (2.1) more precise with $w=w^*$ for $L=L^*$. The characteristic quantities T^* and w^* will be estimated below.

We can estimate the pinning force from $U_0(L)/w(L)$ and therefore define a pinning force density in direction of the gradient $\nabla U_0(L)$ by

$$F_p(L)=\frac{U_0(L)}{w(L)}L^{-D}=F_p^*\left[\frac{L}{L^*}\right]^{\psi-D-\zeta} \tag{2.5}$$

with the exponent $\Psi-D-\zeta\leq 0$ from (2.3) and (2.4). Excluding for the moment the case $\Psi=D$ and $\zeta=0$, $p(L)$ decreases with increasing length scale L or barrier free energy $U_0(L)$ and one has the strongest pinning density $F_p^*=F_p(L^*)$ (the depinning threshold) for the smallest length $L=L^*$ or the smallest barrier free energy T^*. For sufficiently large scales L the force density $F_p(L)$ will be small and thermal fluctuations or a small driving force density p_{ex} will lead to flux flow. Between these limits one has $F_p(L)>p_{ex}$ and at $T=0$ pinning will prevent the motion of the object.

At finite temperatures part of the free energy barriers $U_0(L)$ will be overcome by thermal activation. According to the Arrhenius law, on the time scale $t(L)$ barriers $U_0(L)$ are jumped over. We have with (2.3)

$$t(L)=\tau_0\exp\left[\frac{U_0(L)}{T}\right]=\tau_0\exp\left[\frac{T^*}{T}\left[\frac{L}{L^*}\right]^{\Psi}\right]\ , \tag{2.6}$$

where τ_0 is a microscopic time. Hence, after the time $t(L)$ barriers of size

$$L<L_t'=L^*\left[\frac{T}{T^*}\ln\frac{t}{\tau_0}\right]^{1/\Psi} \quad (L_t'\geq L^*) \tag{2.7}$$

are ineffective for pinning. For $L_t'<L^*$ Eqs. (2.6) and (2.7) are meaningless since there is no pinning on these scales. In order to get rid of the condition $L_t'\geq L^*$ we replace L_t' by $L_t=L_t'+L^*$, i.e.,

$$L_t=L^*\left[1+\frac{T}{T^*}\ln\frac{t}{\tau_0}\right]^{1/\Psi} \tag{2.8}$$

which is correct $T\to 0$ or $t\to\tau_0$ and for $(T/T^*)(\ln t/\tau_0)\gg 1$ and interpolates between both limits. Introducing $L=L_t$ into (2.5) we obtain the pinning force density which is effective at the time t

$$F_p(t)=F_p^*\left[1+\frac{T}{T^*}\ln\frac{t}{\tau_0}\right]^{-1/\mu}\ ,\quad \mu=\frac{\psi}{D+\zeta-\psi}\ . \tag{2.9}$$

Let us consider the pinning on each length scale L separately. A given p_{ex} will compensate the pinning force density $F_p(L_p)$ on the length scale L_p. One has from (2.5) $F_p(L_p)=F_p^*(L_p/L^*)^{\psi-D-\zeta}=p_{ex}$ or

$$L_p=L^*\left[\frac{F_p^*}{p_{ex}}\right]^{1/(D+\zeta-\psi)}\ . \tag{2.10}$$

On scales $L<L_p$ the system can overcome the relevant barriers only by thermally activated jumps or flux creep. For $L>L_p$ the external force density overcompensates the pinning force density and one has flux flow. The force density p_{ex} induces the flux-line velocity v (domain wall velocity, etc.) which is dominated by the creep motion on the largest scale which is still the scale L_p. Hence

$$v\approx\frac{w(L_p)}{t(L_p)}\approx\frac{w^*}{\tau_0}\exp\left[-\frac{T^*}{T}\left[\frac{F_p^*}{p_{ex}}\right]^{\mu}\right] \tag{2.11}$$

from (2.6) and (2.10) with $w\approx w^*$.

We defined the roughness (2.2) by a spatial and thermal average over the distortions $\mathbf{u}(\mathbf{x})$ which can be described by elastic continuum theory. Here we consider the simplest model for a D-dimensional object (flux line, domain wall, FLL, etc.) with the free energy

$$F=\int d^Dx\left[\tfrac{1}{2}\Gamma(\nabla\mathbf{u})^2+V(\mathbf{x},\mathbf{u})-\mathbf{p}_{ex}\mathbf{u}\right]\ , \tag{2.12}$$

where Γ is an elastic constant, $V(\mathbf{x},\mathbf{u})$ includes the interaction with the point defects, and $\mathbf{p}_{ex}$ is an external force density. For vortex lines in a superconductor one has $\mathbf{p}_{ex}=\mathbf{j}_{ex}\times\mathbf{B}/c$.

A weak external force density $\mathbf{p}_{ex}$ leads to a distortion $\delta\mathbf{u}=\hat{\chi}\mathbf{p}_{ex}$ where the susceptibility $\hat{\chi}(L)$ in general is a tensor. On length scales where the interactions with the defects can be neglected we obtain for (2.12) for a displacement δu due to an infinitesimal external force p_{ex} acting on scale L of the object

$$\delta u\ p_{ex}=\Gamma(\delta u)^2L^{-2} \tag{2.13}$$

or

$$\hat{\chi}(L)=\Gamma^{-1}L^2\ . \tag{2.14}$$

For processes on the time scale $t(L)$ this leads with (2.8) to the time dependent susceptibility

$$\hat{\chi}(L_t)=\Gamma^{-1}L_t^2=\hat{\chi}^*\left[1+\frac{T}{t^*}\ln\frac{T}{\tau_0}\right]^{2/\Psi} \tag{2.15}$$

with $\chi^*=\Gamma^{-1}(L^*)^2$.

We close this section by estimating the smallest pinning length L^* and free energy barrier T^*. Following LO we estimate L^* by extremizing the total free energy in a system of the volume $(L^*)^D$ with the density n_p of pinning centers. In this volume one has

$$N = n_p(L^*)^D(w^*)^{d-D} \tag{2.16}$$

pinning centers (point defects) with the pinning energy v_p for a single defect. Here, the roughness w^* on the smallest length scale is determined by the characteristic lengths of the flux lines such as the coherence length ξ or the lattice constant a_0. For weak (collective) pinning the pinning free energy is determined by the *fluctuations* of the number of pinning centers. We have to extremize the sum of the elastic and the pinning free energies

$$F \approx \Gamma(w^*)^2(L^*)^{D-2} - v_p[n_p(L^*)^D(w^*)^{d-D}]^{1/2} , \tag{2.17}$$

which leads to

$$L^* = w^*[(w^*)^D\Gamma/\Delta]^{2/(4-D)} \quad (D<4) \tag{2.18a}$$

with

$$\Delta = v_p[n_p(w^*)^d]^{1/2} , \tag{2.18b}$$

apart from a numerical constant. This result holds for $N \gg 1$ or for many pinning centers in the volume L^d. The existence of a finite correlation length L^* above which distortions are larger than w^* means that translational long-range order of the object (flux line, FLL , etc.) is destroyed by a random distribution of point defects, and that L^* is the smallest size of the object which moves in a depinning process. From (2.18) one has $L^* \to \infty$ either for $n_p \to 0$ or for $v_p \to 0$ and $D<4$, i.e., in the absence of point defects.

For arbitrary length scales and $T=0$ the elastic free energy scales as

$$F_{\rm el}(L) = \Gamma(\nabla \mathbf{u}_g)^2 L^D \approx \Gamma w^2(L) L^{D-2} \approx \Gamma(w^*)^2(L^*)^{D-2}\left[\frac{L}{L^*}\right]^{\chi} , \tag{2.19}$$

where $\mathbf{u}_g$ denotes the ground-state configuration and $\chi = 2\zeta + D - 2$.

Since $F_{\rm el}$ has to (over-) compensate the free energy gain $F_p(L)$ due to the interaction with the randomly distributed impurities, we conclude that both scale in the same way. It is then tempting to assume that the free energy barriers $U_0(L)$ Eq. (2.3) also scale with the same power χ of L, i.e., $\chi = \Psi$ (for additional arguments see Ref. 39).

This admittedly crude estimate (which ignores that $F_{\rm el}$ and U_0 might scale with different exponents χ and Ψ) leads to the smallest barrier free energy

$$T^* = \Gamma(w^*)^2(L^*)^{D-2} \tag{2.20}$$

and to $\Psi = 2\zeta + D - 2$. For domain walls with $D = d-1$ we have $\chi = 2\zeta + d - 3$ in agreement with Ref. 55. In particular the relation $\chi = 2\zeta - 1$ for $d=2$ has been derived by various authors.[56–58,60] For a FLL with $D=d$ this leads together with the estimate[39] $\zeta = 0(\log)$ for $d<4$ to $\Psi = d-2$ and $\mu = (d-2)/2$. With (2.20) the depinning threshold F_p^* in (2.5) can be written as

$$F_p^* = (T^*/w^*)(L^*)^{-D} = \Gamma w^*(L^*)^{-2} \tag{2.21}$$

which leads to the smallest possible response

$$\chi^* = w^*/F_p^* . \tag{2.22}$$

III. APPLICATION TO FLUX-LINE LATTICE

A. Static properties

In order to apply the theory presented in Sec. II to the flux-line lattice of a superconductor we have to generalize the free energy (2.12) slightly in order to take into account anisotropy. We have for $D=d$ (Refs. 33 and 69)

$$F = \int d^dx \left[\frac{1}{2}(c_{11}-c_{66})(\mathrm{div}\mathbf{u})^2 + \frac{1}{2}c_{66}(\nabla_\perp \mathbf{u})^2 + \frac{1}{2}c_{44}\left[\frac{\partial \mathbf{u}}{\partial z}\right]^2 + V(\mathbf{x},\mathbf{u}) - \mathbf{p}_{\rm ex}\mathbf{u}\right] , \tag{3.1}$$

where c_{11}, c_{44}, and c_{66} are, respectively, bulk, tilt, and shear elastic modules. In general the modules $c_{11}(\mathbf{k})$ and $c_{44}(\mathbf{k})$ depend[34,35] on the wave vector $\mathbf{k} = (\mathbf{k}_\perp, \mathbf{k}_z)$ which is important if one considers the possibility of melting[35] of the FLL. Here we shall ignore the compressibility $\sim(\mathrm{div}\mathbf{u})^2$ assuming sufficiently strong fields B. With this restriction we can transform the free energy (3.1) into (2.12) and hence apply all results of the preceding section for $D=d$.[71]

For this purpose we consider the free energy of a block of size $L_z L^{d-1}$ and write $z = \gamma z'$ with $\gamma = (c_{44}/c_{66})^{1/2}$. If we now chose $L_z = \gamma L$, the system is elastically isotropic in the new coordinates z', and the transverse coordinates $R_\perp$ with the elastic modulus

$$\Gamma = c_{66}\gamma = (c_{44}c_{66})^{1/2} \tag{3.2}$$

and the block size L^d. For the rescaling of the random part we write the pinning free energy

$$F_{\rm pin} = \sum_n \int_0^{L_z} dz \; V_0(z, \mathring{\mathbf{R}}_n + \mathbf{u}_n(z)) = \sum_n \int_0^{L} dz' \gamma V_0(\gamma z', \mathring{\mathbf{R}}_n + \mathbf{u}_n(\gamma z')) , \tag{3.3}$$

where the summation runs over all flux lines on the unperturbed sites. Here, V_0 is a random variable in both, z' and u_n.

The scaling factor γ in (3.3) has to be taken into account if we estimate the correlated FLL volume[7] by extremizing the total free energy $F = F_{\rm el} + F_{\rm pin}$. We have instead of the second term in (2.17)

$$F_{\rm pin} \approx -v_p(\gamma L^d n_p)^{1/2}$$

with the density $n_p = N/L_z L^{d-1}$ of pinning centers. Extremizing the total free energy $\Gamma(w^*)^2 L^{d-2} + F_p$ with respect to L then leads to the distance L^* over which the flux lines are correlated (apart from a constant)

$$L^* = w^* \left[(w^*)^d \frac{\Gamma}{\Delta \gamma^{1/2}} \right]^{2/(4-d)} ,$$
$$L_z^* = \left[\frac{C_{44}}{C_{66}} \right]^{1/2} L^* \quad (d<4) , \tag{3.4}$$

where the energy Δ is defined in (2.18b). For $d=3$ the result (3.4) reduces with (2.18b) and (3.2) to

$$L^* = \frac{c_{44}{}^{1/2} c_{66}{}^{3/2}}{v_p{}^2 n_p} (w^*)^4 , \quad L_z^* = \frac{c_{44} c_{66}}{v_p{}^2 n_p} (w^*)^4 , \tag{3.5}$$

and agrees for $w^* = a_0$ and $v_p = a_0 f$ with the result of LO. Here, f is the pinning force of a single point defect. Both the activated volume

$$V_c^* = (L^*)^2 L_z^* = \frac{c_{44}{}^2 c_{66}{}^4}{v_p{}^6 n_p{}^3} (w^*)^6 \tag{3.6}$$

and the roughness w^* depend on the field B and on temperature. Here, V_c^* is the *smallest* activated volume since for smaller volumes the point defects (pinning centers) are inefficient. It connects the smallest pinning free energy barrier (2.20)

$$T^* = \Gamma (w^*)^2 L^* = \frac{c_{44} c_{66}{}^2}{n_p v_p{}^2} (w^*)^6 \quad (d=3) \tag{3.7}$$

to the critical current density at $T=0$

$$j^* = F_p^* c/B = \frac{n_p{}^2 v_p{}^4 c}{c_{44} c_{66}{}^2 (w^*)^7 B} , \tag{3.8}$$

where $F_p^* = T^*/w^* V_c^* \equiv U_0(L^*)/V_c^* w^*$ is the *largest* possible pinning force density [see (2.21)]. At $T=0$ we have $j^*(0,B) = j_c(0,B)$ since the critical current density j_c per definition is the largest possible current density. By comparing with (1.1) we can then identify w^* with the distance d_p over which the (smallest) flux bundle moves. A relation between j^* and j_c at finite temperatures is given below.

The field and temperature dependence of the elastic constants has been investigated by various authors,[35,72,73] whereas little is known about v_p. For thin films LO estimate $F_p^* = n_p \xi_\perp c_{66}$ where $\xi_\perp$ is the coherence length perpendicular to the z axis or within the plane of the film. This expression has been used by Yeh[73] in order to estimate the free energy barriers T^* for various fields and temperatures in high-T_c superconductors.

So far we considered a FLL for $p_{ex}=0$. A small external current density j_{ex} leads to the Lorentz force density $p_{ex} = B j_{ex}/c$ perpendicular to the flux lines and to an enhancement of the jump rate over the barriers $U_0(L)$ in the direction of p_{ex} and to a reduction in the opposite direction. A given p_{ex} will compensate the pinning force density $F_p(L_p)$ [see (2.10)] and leads to the flux-line velocity (2.11) which generates the electric field $\mathbf{E} = \mathbf{B} \times \mathbf{v}/c$ and hence the voltage

$$V(j_{ex}) = V_0 \exp \left[-\frac{T^*}{T} \left[\frac{j^*}{j_{ex}} \right]^\mu \right] , \quad \mu = \frac{\Psi}{d-\zeta-\Psi} , \tag{3.9}$$

where V_0 is proportional to the microscopic velocity $v_0 = w^*/\tau_0$. Since $\mu > 0$ for $d=3$ (see Ref. 39) we have the remarkable result that the linear resistivity $\rho \sim (dV/dj_{ex})j_{ex} \to 0$ vanishes and one has a true superconducting state without energy dissipation in the limit $j_{ex} \to 0$. This is in contrast to the conventional theory of flux creep which predicts a finite resistivity or ohmic behavior for all fields $B > H_{c1}$.

However, a finite current density j_{ex} leads to a finite voltage, as observed in many experiments. With the exponents[39] $\zeta = 0(\log)$ and $\Psi = d-2$ we have $\mu = 0.5$ for $d=3$, in fair agreement with the exponent $\mu = 0.4 \pm 0.2$ found in Ref. 22. However, the data of Ref. 24 seem to indicate a phase transition in the field-temperature plane which is not predicted by our theory. Equation (3.9) describes only the flux creep (or "vortex glass") region. Actually, a theory which considers the spontaneous generation of dislocations in the FLL predicts a phase transition into a liquid-like state in which all energy barriers have finite heights. Above this "depinning" or glass transition the conventional theory of pinning (see Introduction) should apply.[38]

For $j_{ex} = j^* - \delta j$, $\delta j \gg j^*$ or for j_{ex} near to the critical current density j_c, our result (3.9) agrees with that of the conventional theory. One has

$$V(j_{ex}) \sim V_0 \exp \left[-\frac{T^*}{T} \right] \left[1 - \frac{T^*}{T} \frac{\delta j}{j^*} \mu \right] , \tag{3.10}$$

where the first term is due to thermal fluctuations for $j_{ex}=0$ and where one has the linear resistivity

$$\rho \approx V_0 \frac{T^* \mu}{T j^*} \exp \left[-\frac{T^*}{T} \right] . \tag{3.11}$$

This result could have been expected: At the critical state and for $T=0$ one has $j_{ex} = j^* = j_c$ and the Lorentz force density p_{ex} is equal to the largest pinning force density F_p^*. Smaller force densities $p < F_p^*$ become unimportant and one has essentially a single free energy barrier as in the conventional theory.

Finally, we derive a relation between $j^*(T,B)$ and the critical current density $j_c(T,B)$ at *finite* temperatures. In the *conventional theory* one defines j_c by $j_c = j_{ex}(V_{min})$ where V_{min} is the smallest measurable voltage. We have from (1.2) for $v \sim V = V_{min}$

$$j_c(T,B) = \frac{cT}{B V_c d_p} \sinh^{-1} \{ \exp[U_0/T - \ln(2V_0/V_{min})] \} , \tag{3.12}$$

which reduces for $T \to 0$ to definition (1.1), becoming independent of V_{min}. In the present *scaling approach* which takes into account energy barriers on all scales L the voltage is determined by (3.9) and the same criterion leads to

$$j_c(T,B)=j^*\left[1+\frac{T}{T^*}\ln\frac{V_0}{V_{\min}}\right]^{-1/\mu} . \quad (3.13)$$

Here again we took into account that there is no pinning for $p_{ex}>F_p^*$ or $j_{ex}>j^*$ and interpolated between $j_c=j^*$ for $T=0$ and the result for $(T/T^*)\ln(V_0/V_{\min})\gg 1$ [see the discussion below (2.7)].

B. Dynamic properties

1. ac resistivity

In the preceding section we have shown that the motion of a flux line or flux-line bundle as induced by an external current leads to energy dissipation and hence to a finite resistivity. As shown by Bardeen and Stephen,[74] one obtains also a finite resistivity if a single flux line moves in a viscous medium without pinning. This motion leads to scattering of normal electrons near or in the flux-line core and hence to energy dissipation as described by the viscosity coefficient[1,75]

$$\eta=\Phi_0 H_{c2}/c^2\rho_n \quad (H\ll H_{c2}) , \quad (3.14)$$

where ρ_n is the resistivity in the normal state at the same temperature. In the case of pinning and if one applies an ac current density $j_{ex}(\omega)$ one has two types of flux line motion: A pinned flux line or flux-line bundle might perform small oscillations or might move by depinning. We will consider both processes, restricting ourselves to the discussion of a single flux line. The theory could be formulated as well for a flux-line bundle. In all cases the restoring force is determined by the elastic energy density $\Gamma u^2 L^{-2}$ for a distortion u and the driving force by the Lorentz force density $p_{ex}=j_{ex}B/c\approx j_{ex}\Phi_0/a_0{}^2c$ [see Eq. (2.13)].

In the case of small oscillations perpendicular to the z direction we have the equation of motion of a single flux line of length L [with Γ replaced by c_{66} from Eq. (3.1)]

$$\eta\dot{u}(t)+c_{66}(a_0/L)^2u(t)=j_{ex}(t)\Phi_0/c \quad (3.15)$$

with the viscosity coefficient (3.14) and where $j_{ex}(t)=j_0+j(t)$ might also contain a static contribution j_0. The Fourier transformation of (3.15) leads to

$$u(\omega)=\frac{\Phi_0L^2}{cc_{66}a_0{}^2}\frac{j(\omega)}{1+i\omega\tau(L)} \quad (\omega\neq 0) \quad (3.16)$$

with the relaxation time

$$\tau(L)=\frac{\eta L^2}{a_0{}^2c_{66}}=\frac{\Phi_0H_{c2}L^2}{c^2a_0{}^2c_{66}\rho_n} \quad (3.17)$$

and η from Eq. (3.14). The resistivity $\rho_0(\omega)$ due to oscillations can be obtained either from the dissipated energy density[10]

$$P(\omega)=\tfrac{1}{2}\mathrm{Re}[Bj(t)\dot{u}(t)]_\omega=\tfrac{1}{2}[j(\omega)]^2\rho(\omega) \quad (3.18)$$

($[\ ]_\omega$ means the Fourier transform), or from the electric field

$$\mathbf{E}(t)=\mathbf{B}\times\mathbf{v}/c , \quad E(\omega)=i\omega Bu(\omega)/c . \quad (3.19)$$

One has with (3.16) and (3.17)

$$\rho_1(\omega,L)=\mathrm{Re}\frac{E(\omega)}{j(\omega)}=\frac{B\Phi_0\eta L^4}{c^2c_{66}^2a_0{}^4}\frac{\omega^2}{1+[\omega\tau(L)]^2}$$
$$=\rho_n\frac{B}{H_{c2}}\frac{[\omega\tau(L)]^2}{1+[\omega\tau(L)]^2} . \quad (3.20)$$

The resistivity (3.20) still depends on the length L of the oscillating flux line. For a given force density p_{ex} the system will adjust itself in such a way that the resistivity (or response $V=j\rho$) as a function of L becomes a maximum.

It turns out that the function $L^4\{1+[\omega\tau(L)]^2\}^{-1}$ is extremal for $L\to\infty$ and ρ_1 is determined by the length scale L_{depin} due to depinning processes (see below). The latter turns out to be identical with the maximal length scale (2.8) on which the FLL is not pinned, if t is replaced by $1/\omega$, i.e.,

$$L_{\text{depin}}=L_{t=1/\omega}$$
$$=L^*\left[1+\frac{T}{T^*}\ln\frac{1}{\omega\tau_0}\right]^{1/\Psi} \quad (\omega>0) . \quad (3.21)$$

This leads to the resistivity due to small flux-line oscillations

$$\rho_1(\omega)=\rho_n\frac{B}{H_{c2}}\frac{[\omega\tau(L_{t=1/\omega})]^2}{1+[\omega\tau(L_{t=1/\omega})]^2} \quad (\omega>0) \quad (3.22)$$

with $\tau(L_{t=1/\omega})$ from (3.17) and (3.21).

In the case of depinning we have the same solution (3.16) of the distortion $u(\omega)$. However, now the relaxation time $\tau(L)$ is determined by the time scale $t(L)$ (2.6) in which barriers with the free energy $U_0(L)$ are jumped over. This leads to

$$\rho_2(\omega,L)=\frac{B\Phi_0L^2}{c^2c_{66}a_0{}^2}\frac{\omega^2t(L)}{1+[\omega t(L)]^2}$$
$$=\rho_n\frac{B}{H_{c2}}\frac{\omega^2\tau(L)t(L)}{1+[\omega t(L)]^2} . \quad (3.23)$$

The relevant length scale $L_{t=1/\omega}$ for a given frequency ω again is determined from $\partial\rho_2/\partial L=0$ which leads with $\omega t(L_{t=1/\omega})\approx 1$ to (3.21) and with (3.23) to[39]

$$\rho_2(\omega)=\rho_n\frac{B}{H_{c2}}\omega\tau(L^*)\left[1+\frac{T}{T^*}\ln\frac{1}{\omega\tau_0}\right]^{2/\Psi} \quad (\omega>0) . \quad (3.24)$$

In deriving (3.24) we assumed $L_{t=1/\omega}\gg L^*$ which should hold for a sufficiently small current density j_{ex} or for $\omega\tau_0\ll 1$. In addition we took into account that $L_{t=1/\omega}$ cannot be smaller than L^* which is the smallest flux-line length which pins [see Eq. (2.8)].

The resistivities (3.22) and (3.24) due to small oscillations and due to depinning have different frequency and temperature dependences compared to the resistivity obtained in the conventional theory.[10] The latter agrees with Eq. (3.20) if $c_{66}(a_0/L)^2$ is replaced by a single elastic constant k and if the relaxation time (3.17) is replaced by η/k. For $\omega\tau(L_{t=1/\omega})\gg 1$ the resistivity $\rho_1(\omega)$ Eq. (3.22) and the resistivity in Ref. 10 both extrapolate to $\rho_1=\rho_n B/H_{c2}$.

The resistivity $\rho_2(\omega)$ due to depinning has some similarity with a recent result of Ref. 38. However, Eq. (3.24) contains the free-energy barrier exponent defined in (2.3) whereas in the result (8.14) of Ref. 38 enters the ratio of different exponents. With $\Psi=1$ from Ref. 39 the resistivity (3.24) vanishes for $\omega\to 0$ as $\omega(\ln\omega)^2$ and remains at all frequencies and temperatures larger than $\rho_1(\omega)$. Near $\omega\tau(L_{t=1/\omega})=1$ both resistivities are of the same order of magnitude.

2. *Dynamic susceptibility*

There is a close relation between the ac resistivity $\rho(\omega)$ and the dissipative part $\chi''(\omega)$ of the complex susceptibility as the response to an external field $\delta H(\omega)$. The energy density dissipation (3.18) can be written as

$$P(\omega)=\tfrac{1}{2}\rho(\omega)[j(\omega)]^2$$
$$=\frac{\omega}{2}\chi''(\omega)[\delta H(\omega)]^2 \quad (\omega>0) \tag{3.25}$$

with the total external fields and current densities $H=H_0+\delta H(t)$ and $j_{\text{tot}}(t)=j_0+j(t)$ and their static components H_0 and j_0. For $j(t)$ and $\delta H(t)$ holds the Maxwell equation

$$\text{rot}\delta\mathbf{H}=(4\pi/c)\mathbf{j}\ . \tag{3.26}$$

Here we calculate $\delta H(\omega)$ and $j(x,\omega)$ for a slab of thickness l with the external field $\mathbf{H}(t)$ parallel to its surface (in z direction) and the current density $j_{\text{tot}}(t)$ in y direction. As a simplest approximation we use the Bean model[76] in which one assumes the current density $\mathbf{j}(t)$ which is an average over many flux lines to be constant within the sample. We have for $l\ll\lambda_L$ where λ_L is the London penetration depth

$$\delta H(x,\omega)=\delta H(0,\omega)x/l,$$
$$j(x,\omega)=-(c/4\pi)\left[\frac{\partial H}{\partial x}\right] \tag{3.27}$$
$$=(c/4\pi)\delta H(0,l)/l\ .$$

This leads to the spacial average $\delta H(\omega)=\frac{1}{2}\delta H(0,\omega)$ and with (3.23) and (3.25) to (for $l\gg\lambda_L, l^2$ has to be replaced by $l\lambda_L$)

$$\chi''(\omega,L)=\left[\frac{c}{2\pi l}\right]^2\frac{\rho(\omega)}{\omega}$$
$$=\frac{B^2L^2}{(2\pi l)^2c_{66}}\frac{\omega t(L)}{1+[\omega t(L)]^2}\ , \tag{3.28}$$

where $\rho(\omega)$ is the real part of the complex resistivity. Equation (3.28) represents the energy dissipation for depinning processes on the length scale L. The dynamic susceptibility due to oscillations of the flux lines can be calculated in the same way from the resistivity $\rho_1(\omega)$ Eq. (3.20). The relevant length scale $L_{t=1/\omega}$ (3.21) again follows from the maximum energy dissipation which leads to

$$\chi''(\omega)=\frac{B^2(L^*)^2}{2(2\pi l)^2c_{66}}\left[1+\frac{T}{T^*}\ln\frac{1}{\omega\tau_0}\right]^{2/\Psi}\ . \tag{3.29}$$

Note that Eq. (3.29) holds only in a restricted frequency range in which there are depinning processes.

3. *Relaxation of the magnetization*

In this section we calculate the relaxation of the magnetization, considering again a thin slab of thickness l with the boundary conditions $B(x=0)=B_0=H$ for $t\geq 0$ and $B(x=l)=B_0+\Delta B(t)$ for $t\geq 0$. We follow closely the standard theory[1,11] which is based on the condition of flux conservation for flux-line motion in x direction

$$\frac{\partial B}{\partial t}=\frac{\partial j_f}{\partial x}=-\frac{\partial}{\partial x}(Bv)\ , \tag{3.30}$$

where j_f is the flux-line current density and v the flux-line velocity which enters into (3.9). Here, the flux lines are driven either by the current density $j_{\text{tot}}=-c(\partial H/\partial x)/4\pi$ or by the field gradient $\partial B/\partial x$.

Following Ref. 1 we introduce the parameter

$$\alpha=-\frac{\partial}{\partial x}\left[\frac{B^2}{8\pi}\right] \tag{3.31}$$

and take into account the x dependence of $B(x,t)$ only in the exponent of (3.9). We have for $B\approx H$ from (3.30) and (3.9) with V and V_0 replaced by v and v_0

$$\frac{\partial\alpha}{\partial t}=v_0\frac{B^2}{4\pi}\frac{\partial^2}{\partial x^2}\exp\left[-\frac{T^*}{T}\left[\frac{\alpha_c}{\alpha}\right]^\mu\right]\ , \tag{3.32}$$

where

$$\alpha_c=-F_p^*=-Bj^*/c\ . \tag{3.33}$$

The ansatz

$$\exp\left[-\frac{T^*}{T}\left[\frac{\alpha_c}{\alpha}\right]^\mu\right]=f(x)g(t) \tag{3.34}$$

or

$$\frac{\alpha}{\alpha_c}=\left[-\frac{T}{T^*}\ln(fg)\right]^{-1/\mu} \tag{3.35}$$

then leads to

$$\frac{\partial\alpha}{\partial t}=v_0\frac{B^2}{4\pi}g\frac{\partial^2 f}{\partial x^2}\approx\frac{\alpha_c T}{\mu T^* g}\frac{\partial g}{\partial t}\left[-\frac{T}{T^*}\ln(f_0g_0)\right]^{-(1/\mu)-1}\ , \tag{3.36}$$

where we replaced $\ln(fg)$ by $\ln(f_0g_0)$ with the initial values $f_0=f(x=0)$ and $g_0=g(t=0)$. In general, the field $B(x)$ varies smoothly within the sample. We write[1]

$$f(x)=1+a_1x-a_2x^2\ . \tag{3.37}$$

Depending on the initial condition, the sign of the coefficient a_2 will determine whether the field $B(x,t)$ will increase or decrease as a function of time. The time integration (3.36) then leads with $\ln f_0=0$ to

$$\ln g(t)=-\ln\left[g_0^{-1}+\frac{t}{\tau_1}\right] \tag{3.38}$$

with the relaxation time

$$\tau_1=\frac{4\pi\alpha_c[(T/T^*)\ln g_0^{-1}]^{-(1/\mu)-1}}{T^*\mu v_0 B^2 a_2} . \tag{3.39}$$

We have in linear approximation[76] of $B(x)$ from (3.35)

$$\frac{\alpha}{\alpha_c}=\left[\frac{T}{T^*}\ln\left[g_0^{-1}+\frac{t}{\tau_1}\right]\right]^{-1/\mu} \tag{3.40}$$

and by integration of (3.31) with the initial conditions $B(x=0,t\geq 0)=B_0$ and $\alpha_c=-B_0 j_c/c$

$$B^2(x,t)=B_0^2-8\pi x\alpha_c\left[\frac{T}{T^*}\ln\left[g_0^{-1}+\frac{t}{\tau_1}\right]\right]^{-1/\mu} , \tag{3.41}$$

$$B(x,t)=B_0+\frac{4\pi}{c}j_c x\left[\frac{T}{T^*}\ln\left[g_0^{-1}+\frac{t}{\tau_1}\right]\right]^{-1/\mu}$$
$$\equiv B_0+\Delta B(t)x/l . \tag{3.42}$$

In particular for a thin slab of thickness l we have the average magnetic induction $\bar{B}(t)=B_0+\Delta B(t)/2$ and the magnetization $M=(B-H)/4\pi$. This leads to our final result

$$M(t)=\frac{lj_c}{2c}\left[\frac{T}{T^*}\ln\left[g_0^{-1}+\frac{t}{\tau_0}\right]\right]^{-1/\mu} \tag{3.43}$$

with the boundary condition $B_0\equiv B(x=0)=H$. Equation (3.43) describes the decay (or increase) of the magnetization for a fixed field at $x=0$ after a sudden change of the field at $x=l$. This change in field is determined by the initial slope

$$\frac{\partial B}{\partial x}\Big/_{t=0}=\frac{4\pi}{c}j_c\left[\frac{T}{T^*}\ln g_0^{-1}\right]^{-1/\mu} \tag{3.44}$$

which defines g_0. With[39] $\Psi=1$, $\mu=\frac{1}{2}$ the dynamic susceptibility (3.29) and the inverse magnetization $M^{-1}(t)$ from (3.43) decay with the same power of the logarithm.

IV. CONCLUSIONS

We developed a theory for the pinning of flux lines and similar objects such as domain walls in Ising systems, dislocations, or charge density waves in a d-dimensional space, based on a scaling approach. The underlying idea is to take into account that flux lines or flux-line bundles are pinned by density fluctuations of impurities, which exist on all length scales L. This results in pinning barriers which increase in height as L^{ψ}, $\psi>0$, if L goes to infinity. Here, the impurities act as a random distribution of pinning centers which leads to many flux-line configurations with roughly the same energy. There is also a certain analogy between such a "vortex glass" and a spin glass to which similar methods have been applied.[77]

An application of this approach to the FLL of a superconductor leads to the current-voltage relation (3.9) which, for small currents, gives a vanishing linear resistivity and which is one of our main result. Equation (3.9) agrees for three dimensions and the critical exponent $\mu=0.5$ (Ref. 39) fairly well with the experimental data of Koch *et al.*[22] who find $\mu=0.4\pm0.2$. A qualitatively similar relation has been found in Refs. 37, 38, and 69. However, these authors obtain different exponents μ and do not derive the prefactors T^* and j^*, Eqs. (3.7) and (3.8). A nonlinear I-V curve similar to that of Ref. 22 was also observed in thin films of $YBa_2Cu_3O_x$,[23] $ErBa_2Cu_3O_x$,[78] and amorphous Nb_3Ge,[79] and also in a $YBa_2Cu_3O_x$ single crystal.[80] No attempt has been made by these authors to fit their results to an equation of the form (3.9) and to extract the critical exponent μ. In addition, in thin films, a nonlinear I-V relation for small fields might also be due to a Kosterlitz-Thouless transition which is not considered in this paper.

The depinning transition observed[22] in Y-Ba-Cu-O is possibly due to the creation of dislocation pairs and their subsequent unbinding in the vortex glass state which are not included in our theory. Hence our results hold only for the flux-creep region or for sufficiently low temperatures and fields. For very small fields ($H\approx H_{c1}$) one has essentially independent vortex lines which yields a different critical exponent μ.[68]

Our scaling approach leads also to dynamical properties of a superconductor which differ from those obtained from the conventional theory which assumes a single characteristic pinning length L. In the ac resistivity, one has two contributions due to flux-line motion both of which depend on the frequency-dependent depinning length $L_{\text{depin}}(\omega)$, (3.21). The resistivity $\rho_1(\omega)$, Eq. (3.22), due to oscillations varies for small frequencies as $\rho_1(\omega)\sim\omega^2[1-(T/T^*)\ln\omega\tau_0]^{4/\psi}$ (with $\psi=1$ for three dimensions) which replaces the ω^2 law of the conventional theory.[10] In addition, there is the low-frequency contribution $\rho_2(\omega)$, Eq. (3.24), due to transitions over energy barriers which varies as $\rho_2\sim\omega[1-(T/T^*)\ln\omega\tau_0]^{2/\psi}$. Here T^* is the temperature corresponding to the smallest pinning free energy barrier. Unfortunately, there do not seem to be experimental data available in order to test this result. A similar ln behavior has been found for the conductivity [Eq. (8.14) of Ref. 38]. In contrast to Ref. 38 we calculated $\rho_2(\omega)$ explicitly, including all prefactors and give an estimate for the exponent ψ.

The dissipative part $\chi''(\omega)$ of the dynamic susceptibility is closely related to $\rho_2(\omega)$ since both responses are proportional to the energy dissipation due to flux-line motion in a depinning process. For the frequency range in which depinning is important, the relation (3.25) leads to $\chi''(\omega)\sim[1-(T/T^*)\ln\omega\tau_0]^{2/\psi}$. According to the Kramers-Kronig relations, the real part $\chi'(\omega)$ then should vary as $(\ln\omega\tau_0)^{2/\psi}$. A constant susceptibility

$\chi''(\omega)$ due to viscous motion of flux lines indeed has been observed.[81] However, due to different initial conditions, one can also obtain other frequency dependences of $\chi''(\omega)$ or time dependences of the magnetization $M(t)$.[6] In deriving (3.25) we assumed a homogeneous distribution of vortices over the whole sample. Since the susceptibility or magnetization measures the change of the number of vortices in the sample, there is no contribution to $\chi(\omega)$ which corresponds to the resistivity $\rho_1(\omega)$.

The frequency dependence of the susceptibility $\chi'(\omega)$ is closely related to the time dependence of the magnetization $M(t)$. The derivation of $M(t)$ (Sec. III B) follows closely the standard theory[1,11] and leads to $M(t) \sim [\ln(\text{const}+t)]^{-1/\mu}$ with $\mu=\frac{1}{2}$ in three dimensions. Here, the constant depends on the initial condition. This result contrasts the conventional $\ln t$ flux-creep behavior[1,11] which is often observed.[82-85] (Unfortunately, often the $\ln t$ behavior is assumed to be trivial and is not plotted.) However, sometimes, clear deviations from this $\ln t$ law are observed.[43,83,86,87] At present, it is not clear whether these deviations exist also in the homogeneous flux line state (i.e., the fully penetrated sample) or are due to a macroscopic variation of the flux-line density. For the latter case, deviations from the $\ln t$ law have been demonstrated in Refs. 6, 88, and 89.

ACKNOWLEDGMENTS

This research was supported by a Grant from the German Israeli Foundation for Scientific Research and Development.

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