Pairing in two dimensions: A systematic approach

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A systematic study of pairing fluctuations in two dimensions about a BCS-like mean-field theory is performed. It is based upon the large-N limit of models with symplectic [Sp(N)] symmetry. The leading corrections to the critical temperature and the chemical potential in two dimensions are found to be of order $(\ln N)/N$ and do not significantly alter the physical scenario of the mean-field theory. The results are applicable to models with short-range attractive interactions between opposite-spin electrons, e.g., the negative U Hubbard model or the t-J model.

I. INTRODUCTION

It has by now become quite clear that the electronic properties of the high-temperature superconductors in cuprates like $La_{2-x}Sr_xCuO_4$ (Ref. 1) and $YBa_2Cu_3O_{6+x}$ (Ref. 2) are dominated by the two-dimensional CuO_2 layers[3] found in all of these compounds. Despite the intense theoretical activity generated by these observations, there remains a paucity of systematic results on superconductivity in two dimensions. In the case where the electrons are tightly bound into nonoverlapping Cooper pairs and the problem can be mapped onto an interacting Bose gas, systematic results are available. As shown by Popov[4] and Fisher and Hohenberg[5] an expansion can be made for the thermodynamic properties of a dilute Bose gas which satisfies the condition

$$\ln\ln(1/na^2)\gg 1 , \tag{1.1}$$

where n is the density and a is the range of the interaction between the bosons.

In this paper we shall provide a systematic treatment of fluctuations about the opposite limit of overlapping Cooper pairs where the BCS theory[6] is a good first approximation. Nonsystematic treatments of this limit have recently been carried out[7] but found that strong infrared fluctuations lead to large shifts in the chemical potential μ and suppression of the critical temperature for the onset of superconductivity, T_c, to 0; these unphysical effects persist even after interactions between the Cooper pairs have been included.[8]

Our calculation is a large-N expansion based upon generalizing the SU(2) spin symmetry of the electrons to the symplectic groups Sp(N) [note Sp(1)$\cong$SU(2)]. Such expansions were recently used to study frustrated quantum antiferromagnets and superconductivity in the t-J model at zero temperature.[9] We shall present our calculation in the context of a negative-U Hubbard model, although the results are rather more general and can be applied to other models with a short-range attraction between the opposite-spin electrons, e.g., the t-J model.[10,11] At temperatures well away from T_c the validity of the expansion requires that

$$N\gg 1 . \tag{1.2}$$

Near T_c, long-wavelength fluctuations must be treated with some care. By a careful consideration of the large-N expansion we will show that infrared divergences are cut off by a small "mass" in a manner that is reminiscent of Popov's calculation.[4] We find that in this critical region validity of the large-N expansion imposes the more stringent requirement

$$\ln N\gg 1 . \tag{1.3}$$

We shall study the model

$$H=-t\sum_{\langle ij\rangle}c_{i\alpha}^{\dagger}c_j^{\alpha}-\frac{U}{4N}\sum_i(\mathcal{J}^{\alpha\beta}c_{i\alpha}^{\dagger}c_{i\beta}^{\dagger})(\mathcal{J}_{\gamma\delta}c_i^{\delta}c_i^{\gamma})$$
$$-\mu\sum_i(c_{i\alpha}^{\dagger}c_i^{\alpha}-2N\nu) , \tag{1.4}$$

where $c_{i\alpha}^{\dagger}$ creates "electrons" at the site i of a d-dimensional cubic lattice. The index $\alpha=1,\ldots,2N$ and identifies the electrons as Sp(N) spinors. The tensor $\mathcal{J}$ is the $2N\times 2N$ matrix

$$\mathcal{J}_{\alpha\beta}=\mathcal{J}^{\alpha\beta}=\begin{bmatrix} & 1 & & & & \\ -1 & & & & & \\ & & & 1 & & \\ & & -1 & & & \\ & & & & \ddots & \\ & & & & & \ddots \end{bmatrix} \tag{1.5}$$

which generalizes the familiar ϵ tensor of SU(2) to Sp(N). The interaction term in H favors on-site pairing of electrons into Sp(N) singlets. For $N=1$ H reduces to the Hamiltonian of the negative-U Hubbard model. Finally, the chemical potential μ must be adjusted to maintain an average on-site density of $2N\nu$ electrons per site (ν is the filling factor).

At $N=\infty$, the BCS decoupling of the interaction term[6]

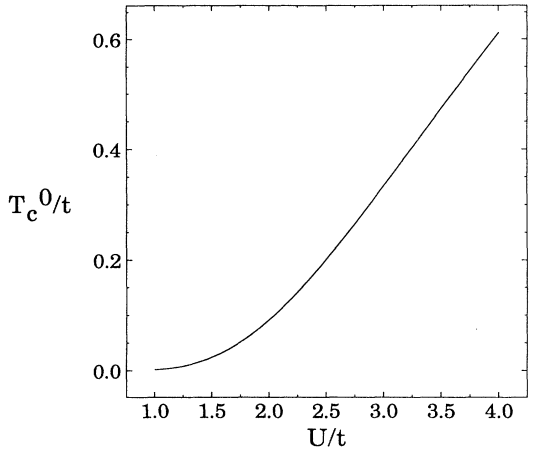

FIG. 1. Mean-field value of the critical temperature T_c^0/t of a negative-U Hubbard model on a square lattice with hopping parameter t and a quarter-filled band ($\nu=\frac{1}{4}$).

becomes exact. The critical temperature T_c^0 and the chemical potential μ^0 can be determined by solving the BCS (Ref. 6) equations. In Figs. 1 and 2 we show the values of T_c^0 and $\mu^0(T_c^0)$ as a function of U/t for the quarter-filled band $\nu=\frac{1}{4}$ on a square lattice. These results have the following asymptotic limits:

$$T_c^0 \sim \begin{cases} te^{-at/U} \\ U \end{cases}, \quad \mu^0 \sim \begin{cases} -t & \text{for } U/t \ll 1 \\ -U & \text{for } U/t \gg 1 \end{cases}, \tag{1.6}$$

where a is a constant of order unity.

Finite N fluctuations lead to shifts in these quantities which have the following form in $d=2$:

$$\begin{aligned} T_c &= T_c^0 + T_c^0 \left[\frac{\ln N}{N}\right] A(U/t) + O\left[\frac{1}{N}\right] \\ &\quad + O\left[\frac{1}{N \ln N}\right] + \cdots, \\ \mu(T_c) &= \mu^0(T_c^0) + T_c^0 \left[\frac{\ln N}{N}\right] B(U/t) \\ &\quad + O\left[\frac{1}{N}\right] + O\left[\frac{1}{N \ln N}\right] + \cdots. \end{aligned} \tag{1.7}$$

This expansion indicates that the mean-field results for μ and T_c are quite stable and, unlike Ref. 7, a new physical regime does not emerge when the one-loop fluctuations have been included. For small N there can be large renormalizations of various parameters, but we expect the mean-field picture to be essentially correct. The functions $A(U/t)$ and $B(U/t)$ are determined in this paper. The results are summarized in Figs. 3 and 4 which show A and B for $\nu=\frac{1}{4}$ on the square lattice. We have

$$\begin{aligned} A &\sim \begin{cases} -T_c^0/t \\ -(U/t)^2 \end{cases}, \\ B &\sim \begin{cases} -(t/U)(T_c^0/t)^2 & \text{for } U/t \ll 1 \\ (U/t)^2 & \text{for } U/t \gg 1 \end{cases}. \end{aligned} \tag{1.8}$$

It is important to note that the above results are valid for any finite value of U/t. However, it is clear from the asymptotic expressions quoted above that the limits $U/t \to \infty$ and $N \to \infty$ do not commute. In this paper we have taken the $N \to \infty$ limit first. In the case with $U/t \to \infty$ first, the system is expected to behave as a gas of bosons with hopping amplitude t^2/U; such a regime is not observed in the present calculation.

We also note that in the presence of a coupling to electromagnetic fluctuations, a two-dimensional superconductor is not expected to have a true phase transition.

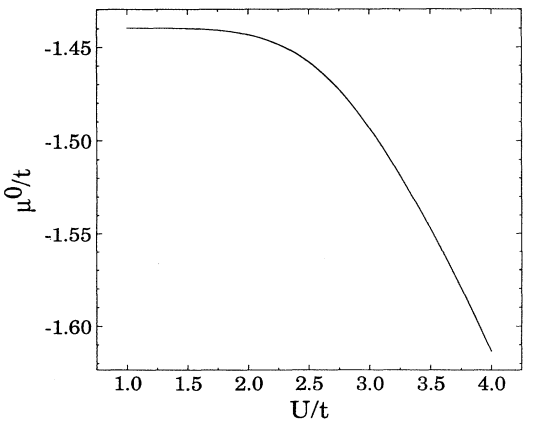

FIG. 2. Mean-field chemical potential μ^0/t for the model of Fig. 1. The bottom of the band is at $-4t$.

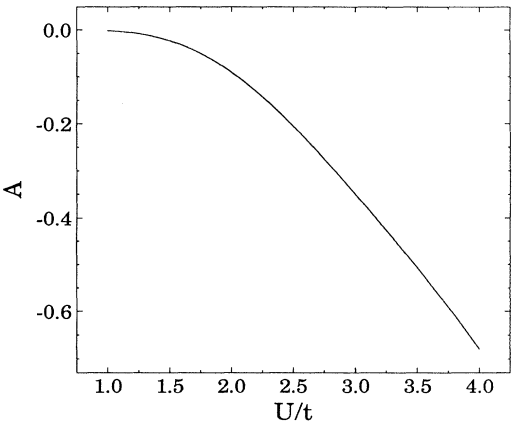

FIG. 3. Coefficient $A(U/t)$ of the $(\ln N)/N$ correction to T_c [Eq. (1.7)] for the model of Fig. 1.

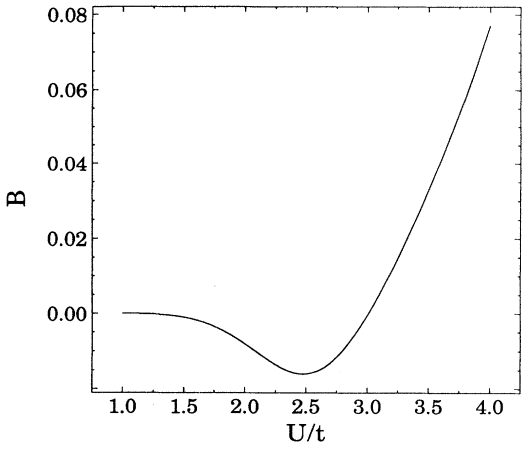

FIG. 4. Coefficient $B(U/t)$ of the $(\ln N)/N$ correction to μ [Eq. (1.7)] for the model of Fig. 1.

However, for a sufficiently large London penetration depth there will be a very rapid crossover at the critical temperatures calculated in this paper. See Ref. 12 for further discussion on this point.

The outline of the rest of this paper is as follows. In Sec. II we will set up the general framework for the large-N expansion on H for all d away from T_c. In this regime there are no subtleties and a simple order-by-order series in $1/N$ exists. We will discuss the modification of this expansion as T approaches T_c in Sec. III and also obtain expressions for the functions A and B. The extension of these results to the t-J model is briefly discussed in Sec. IV. The basic points are recapitulated in Sec. V. Finally in an Appendix we will present a renormalization-group-based analysis which shows that higher loops do not affect the results of Sec. III.

II. GENERAL FRAMEWORK

In this section we will set up the framework for the large-N expansion for the thermodynamic properties of the Hamiltonian H. We will begin by examining temperatures well away from T_c in $d \geq 2$; under these conditions standard methods can be used to perform a simple power-series expansion in $1/N$ for all the physically measurable properties of H in both the low- and high-temperature phases. The modification of the analysis when the temperature approaches T_c in $d=2$ will be discussed in Sec. III.

We begin by expressing the partition function associated with H as follows:

$$\begin{aligned} Z &= \int \prod_i \mathcal{D}c_i^{\alpha}(\tau)\mathcal{D}c_{i\alpha}^{\dagger}(\tau)\mathcal{D}\Psi_i(\tau)\mathcal{D}\Psi_i^*(\tau) \\ &\quad \times \exp\left[-\int_0^{\beta} d\tau \mathcal{L}\right], \\ \mathcal{L} &= \sum_i \frac{N|\Psi_i|^2}{U} - t\sum_{\langle ij\rangle} c_{i\alpha}^{\dagger}c_j^{\alpha} - \frac{1}{2}\sum_i \Psi_i^* \mathcal{J}^{\alpha\beta} c_{i\alpha}^{\dagger}c_{i\beta}^{\dagger} \\ &\quad + \text{H.c.} - \mu\sum_i (c_{i\alpha}^{\dagger}c_i^{\alpha} - 2N\nu) . \end{aligned} \tag{2.1}$$

We have decoupled the quartic term in (1.4) by the Hubbard-Stratanovich field Ψ_i. The fermions can now be integrated out leading to the formal expression

$$Z = \int \prod_i \mathcal{D}\Psi_i(\tau)\mathcal{D}\Psi_i^*(\tau)\exp\{-N\tilde{S}[\Psi_i(\tau)]\} , \tag{2.2}$$

where $\tilde{S}$ is a calculable functional of $\Psi_i(\tau)$. The prefactor N now guarantees the existence of a $1/N$ expansion as long as all the parameters in $\tilde{S}$ are of order 1: this will be the case well away from T_c. Instead of displaying $\tilde{S}$ in its full complexity we anticipate some of our subsequent results by retaining only a few terms. We will ultimately only be interested in determining the coefficient of the $(\ln N)/N$ corrections to T_c and μ; the logarithmic dependence on N will arise from singular infrared fluctuations which appear near T_c. It is not difficult to verify *a posteriori* that none of the following approximations will modify this coefficient. Also, away from T_c the following approximations will lead to numerical changes but not modify the structure of the perturbation theory. The gain will be a considerable simplification of the discussion.

1. We will retain only the zero frequency component of $\Psi_i(\tau)$. The logarithmic terms arise from the purely classical thermal fluctuations of the $d=2XY$ model. The finite frequency fluctuations of Ψ are dissipative in nature near T_c and well described by a time-dependent Ginzburg-Landau model.[13] The fluctuation-dissipation theorem guarantees that all of the thermodynamic properties can be obtained from a purely static thermal ensemble.[14] The action will therefore be expressed solely in terms of the field

$$\psi_i = \frac{1}{\beta}\int_0^{\beta} d\tau \Psi_i(\tau) . \tag{2.3}$$

2. The expansion of $\tilde{S}$ in powers of ψ will be truncated at order $|\psi|^4$. Furthermore the coefficients of the $|\psi|^2$ and $|\psi|^4$ terms will be expanded in momentum and only certain low-order terms retained. All of the omitted terms are subdominant near T_c.

These steps lead to the following effective action:

$$\begin{aligned} Z &= \exp\left[-\frac{NVF_0}{T}\right]\int \mathcal{D}\psi\mathcal{D}\psi^* \exp\left[-\frac{N}{T}S\right], \\ F_0 &= -2T\int \frac{d^dk}{(2\pi)^d}\ln(1+e^{-(\varepsilon_{\mathbf{k}}-\mu)/T}) , \\ S &= \int d^dx \left[K|\nabla\psi|^2 + r|\psi|^2 + \frac{v}{2}|\psi|^4 + 2\mu\nu\right], \end{aligned} \tag{2.4}$$

where we have set the lattice spacing to unity and V is the volume of the system. The parameters K, r, and v are functions of μ and T and are given by

$$r + K\mathbf{q}^2 + O(\mathbf{q}^4) \equiv \frac{1}{U} - \int \frac{d^dk}{(2\pi)^d}\frac{1-f(\varepsilon_{\mathbf{k}+\mathbf{q}})-f(\varepsilon_{\mathbf{k}})}{\varepsilon_{\mathbf{k}+\mathbf{q}}+\varepsilon_{\mathbf{k}}-2\mu} \tag{2.5}$$

and

$$v=\int \frac{d^d k}{(2\pi)^d}\frac{1}{4(\varepsilon_{\mathbf{k}}-\mu)^3}[1-2f(\varepsilon_{\mathbf{k}})+2(\varepsilon_{\mathbf{k}}-\mu)f'(\varepsilon_{\mathbf{k}})] . \tag{2.6}$$

Here $\varepsilon_{\mathbf{k}}=-2t\sum_\alpha \cos k_\alpha$ is the fermion dispersion relation and f is the Fermi function

$$f(x)=\frac{1}{e^{(x-\mu)/T}+1} . \tag{2.7}$$

The large-N expansion proceeds by expanding about a saddle point value of $\psi=\psi_0$, chosen to be real. The value of ψ_0 will be one that minimizes the free energy density $\mathcal{F}=-(T/NV)\ln Z$. Infrared divergences will appear in $d=2$ in the equations determining ψ_0 but these will cancel order by order in $1/N$ in any physically measurable quantity.[4] The fluctuations about ψ_0 will be parametrized as

$$\psi=\psi_0+\psi_1 . \tag{2.8}$$

The ψ_1 fluctuations will be controlled by a normal (Σ) and anomalous (D) self-energies leading to the following normal (G) and anomalous Green's functions (F):[4]

$$G(\mathbf{k})=\langle \psi_1(\mathbf{k})\psi_1^*(\mathbf{k})\rangle=\frac{T}{N}\frac{K\mathbf{k}^2+r-\Sigma(\mathbf{k})}{[K\mathbf{k}^2+r-\Sigma(\mathbf{k})]^2-D^2(\mathbf{k})} , \tag{2.9}$$

$$F(\mathbf{k})=\langle \psi_1(\mathbf{k})\psi_1(-\mathbf{k})\rangle=\frac{T}{N}\frac{D(\mathbf{k})}{[K\mathbf{k}^2+r-\Sigma(\mathbf{k})]^2-D^2(\mathbf{k})} .$$

The stationarity condition $\partial\mathcal{F}/\partial\psi_0$ can be shown[4] to be equivalent to either $\psi_0=0$ or

$$r=\Sigma(0)-D(0) . \tag{2.10}$$

To leading order in $1/N$ the self-energies have the values

$$\Sigma=-2\psi_0^2 v, \quad D=-\psi_0^2 v . \tag{2.11}$$

For the rest of this section we will assume that we are in the low-temperature phase which has $\psi_0\neq 0$ and $r<0$. Equation (2.10) now leads to the result

$$\psi_0^2=-r/v . \tag{2.12}$$

The next order requires evaluation of the diagrams shown in Fig. 5 while using the Green's functions from Eq. (2.9). Evaluating the diagrams and solving Eq. (2.10) for ψ_0^2 we obtain to order $1/N$

$$\psi_0^2=-\frac{r}{v}-\frac{T}{N}\int \frac{d^d k}{(2\pi)^d}\frac{2K\mathbf{k}^2-r}{K\mathbf{k}^2(K\mathbf{k}^2-2r)} . \tag{2.13}$$

Notice that fluctuation correction to ψ_0 diverges in $d=2$ at all temperatures. If we cut off the log divergence by a small momentum k_0 we find

$$\psi_0^2=\frac{|r|}{v}\left[1-\frac{Tv}{4\pi NK|r|}\ln\left[\frac{\Lambda}{k_0}\right]+\cdots\right] . \tag{2.14}$$

We expect this series to exponentiate[4] to $\psi_0^2\sim k_0^\alpha$ with $\alpha=(Tv)/(4\pi NK|r|)$. However, in the physically measurable $\langle|\psi|^2\rangle=\psi_0^2+\langle|\psi_1|^2\rangle$ we expect all the infrared divergences to cancel. We evaluate $\langle|\psi_1|^2\rangle$ from Eq. (2.9) and obtain

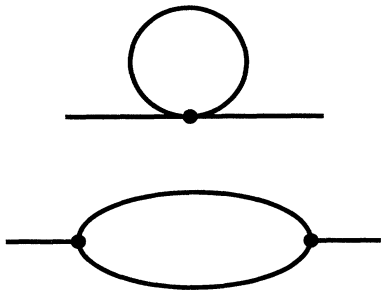

FIG. 5. Graphs nominally of order $1/N$ for the self-energies Σ and D. Arrows must be placed on all the lines and all combinations summed over.

$$\langle|\psi|^2\rangle=-\frac{r}{v}-\frac{T}{N}\int\frac{d^d k}{(2\pi)^d}\frac{1}{(K\mathbf{k}^2-2r)} , \tag{2.15}$$

which is well defined for all d and $r<0$. The chemical potential is determined by the condition $\partial\mathcal{F}/\partial\mu=0$ which gives us

$$\begin{aligned}2v&\approx n_0-\left[\frac{\partial r}{\partial\mu}\right]\langle|\psi|^2\rangle\\&=n_0+\left[\frac{\partial r}{\partial\mu}\right]\left[\frac{r}{v}+\frac{T}{N}\int\frac{d^d k}{(2\pi)^d}\frac{1}{K\mathbf{k}^2-2r}\right] ,\end{aligned} \tag{2.16}$$

where

$$n_0=-\frac{\partial F_0}{\partial\mu}=2\int\frac{d^d k}{(2\pi)^d}f(\varepsilon_{\mathbf{k}}) . \tag{2.17}$$

In Eq. (2.16) we have dropped terms proportional to $\partial K/\partial\mu$ and $\partial v/\partial\mu$; these will turn out to be subdominant in the critical region considered in Sec. III.

The superfluid density ρ_s is an important physical quantity and will be used in determining the critical temperature. It can be obtained by coupling ψ to an external transverse vector potential and evaluating the current response. A standard calculation gives

$$\rho_s=\frac{2NK}{T}\left[-\frac{r}{v}-\frac{2T}{N}\int\frac{d^d k}{(2\pi)^d}\frac{1}{K\mathbf{k}^2-2r}+O\left[\frac{1}{N^2}\right]\right] . \tag{2.18}$$

The leading term in the superfluid density can be checked to be consistent with the value of the exponent α quoted above [the relationship $\alpha=1/(2\pi\rho_s)$ is expected to be exact to all orders in $1/N$ (Ref. 4)]. Provided $|r|$ is of order 1, all of the above expressions define a consistent $1/N$ expansion. Other physical quantities can be calculated in a similar manner.

III. CRITICAL REGION

In this section we will extend the previous analysis into the critical region in two dimensions. The phase transi-

tion is of the Kosterlitz-Thouless[15] type driven by the unbinding of vortex pairs, and occurs when the superfluid density reaches the universal value[16,12]

$$\rho_s = 2/\pi \ . \tag{3.1}$$

We note in passing that none of the results of this section would be modified if we had instead used the condition $\rho_s = 0$ to determine T_c (as was done by Popov[4]). The shift in T_c between these two criteria is of order $1/N$ while the fluctuation effects considered below lead to a shift of order $(\ln N)/N$.

At $N = \infty$, the critical temperature and the chemical potential are determined by solving the equations

$$r = 0, \quad n_0 = 2\nu \ , \tag{3.2}$$

which determine the values of T_c^0 and μ^0. The solutions of these equations are shown in Figs. 1 and 2 as a function of U/t for a quarter-filled band ($\nu = \frac{1}{4}$). For small U/t, $T_c^0 \sim te^{-\alpha t/U}$ (α is a constant of order 1), and $\mu^0 \sim$ const. For large U/t, $T_c^0 \sim -\mu \sim U$.

Upon examining the fluctuation corrections we see that the terms which are nominally of order $1/N$ are singular. We now assert that the leading correction to T_c can be determined simply by solving Eqs. (2.16), (2.18), and (3.1) self-consistently. This is easily done and we find that, at T_c,

$$r \sim -\frac{\ln N}{N} \ . \tag{3.3}$$

The corrections to T_c and μ have the form shown in Eqs. (1.7) in the Introduction with the coefficients A and B given by

$$A = -\frac{1}{2\pi K}\left[\frac{1}{2}\left[\frac{\partial r}{\partial \mu}\right]^2 + \upsilon\frac{\partial n_0}{\partial \mu}\right] \Big/ \left[\frac{\partial n_0}{\partial \mu}\frac{\partial r}{\partial T} - \frac{\partial r}{\partial \mu}\frac{\partial n_0}{\partial T}\right] ,$$
$$B = \frac{1}{2\pi K}\left[\frac{1}{2}\frac{\partial r}{\partial \mu}\frac{\partial r}{\partial T} + \upsilon\frac{\partial n_0}{\partial T}\right] \Big/ \left[\frac{\partial n_0}{\partial \mu}\frac{\partial r}{\partial T} - \frac{\partial r}{\partial \mu}\frac{\partial n_0}{\partial T}\right] , \tag{3.4}$$

evaluated at $\mu = \mu^0$ and $T = T_c^0$. The numerical values of A and B are shown in Figs. 3 and 4 for $\nu = \frac{1}{4}$. For small U/t, $A \sim -T_c^0/t$ and $B \sim -(t/U)(T_c^0/t)^2$. For large U/t, $A \sim -B \sim -(U/t)^2$.

It now remains to verify that terms that were nominally of higher order in $1/N$ do not modify the above result when r takes the anomalously small value of Eq. (3.3). Individual graphs which are nominally of order $1/N^2$ will be infrared divergent in the low-temperature phase. However, these divergences must cancel when all the terms have been collected. The resulting expression will, however, be singular as $|r| \to 0$. In particular we expect it to be quadratically divergent in $d = 2$; the final result will be of order

$$\frac{1}{N^2}\frac{1}{|r|} \ .$$

The leading dependence on $|r|$ should have no $\ln|r|$ factors. Using (3.3) we see that this correction is of order $1/(N \ln N)$ which is subdominant to the corrections already considered. In the Appendix we use the renormalization group to explicitly demonstrate the absence of any additional corrections to (1.7) and (3.4), as suggested by the above scenario.

IV. EXTENSION TO THE t-J MODEL

The Sp(N) based large-N approach to the t-J model was studied in Ref. 9 at $N = \infty$ and $T = 0$. Superconducting ground states of various symmetries were found. Here we show how the results of Secs. II and III can be easily extended to this model.

First we recall the model and the notations of Ref. 9. The constraints on electron occupation present in the t-J model are implemented by the following representation for the "electrons" c_i^α:

$$c_i^\alpha = f_i^\alpha b_i^\dagger \ . \tag{4.1}$$

Here the f_i^α are fermions which carry spin while the spinless bosons b_i keep track of the holes. The local constraint of the t-J model is

$$f_{i\alpha}^\dagger f_i^\alpha + b_i^\dagger b_i = N \tag{4.2}$$

for every site i. We will consider the following Hamiltonian:

$$H_{tJ} = -\frac{t}{N}\sum_{\langle ij\rangle} b_i f_{i\alpha}^\dagger f_j^\alpha b_j^\dagger - \frac{J}{N}\sum_{\langle ij\rangle}(\mathcal{J}^{\alpha\beta} f_{i\alpha}^\dagger f_{j\beta}^\dagger)(\mathcal{J}_{\gamma\delta} f_j^\delta f_i^\gamma)$$
$$+ \sum_i \lambda_i (f_{i\alpha}^\dagger f_i^\alpha + b_i^\dagger b_i - N) + \mu \sum_i (b_i^\dagger b_i - N\delta) \ . \tag{4.3}$$

The last two terms enforce the local constraint (4.2) and fix the average hole density at $N\delta$ (δ is the doping fraction). In the large-N limit the b bosons condense completely at $N = \infty$ and $T = 0$; thus $\langle |b_i| \rangle = \sqrt{N\delta}$. The critical temperature for superconductivity is of order unity: at these temperatures $\langle |b_i| \rangle$ will have corrections only at order $1/N$. These fluctuations can therefore be neglected and for all of the subsequent analysis we may replace b_i by $\sqrt{N\delta}$. The exchange term in H_{tJ} may be decoupled by the link field

$$\Delta_{ij} = \frac{1}{N}\langle \mathcal{J}^{\alpha\beta} f_{i\alpha}^\dagger f_{j\beta}^\dagger \rangle \ . \tag{4.4}$$

The subsequent analysis is now very similar to that performed on H [Eq. (1.4)] with the exchange constant J playing the role of the negative U in H. The main difference is that while the pairing field in (2.1) was com-

pletely on site, the field Δ_{ij} takes values on the links. Depending upon values of t/J and δ different superconducting states will be obtained characterized by different spatial distributions of the Δ_{ij}. Near T_c we only need to focus on the appropriate linear combination of the Δ_{ij} which is becoming massless: the effective action describing these critical fluctuations will have a form identical to S [Eq. (2.4)]. The subsequent analysis is very similar to that of Secs. II and III and $(\ln N)/N$ corrections to T_c can be obtained.

V. CONCLUSION

The symplectic groups Sp(N) [Sp(1)$\cong$SU(2)] were used recently to develop systematic theories of frustrated quantum antiferromagnets and a mean-field theory of superconductivity in the t-J model.[9] In this paper we have shown how this large-N expansion is also a useful tool in studying finite temperature pairing fluctuations in two dimensions. A separate development which was useful in the present analysis was the theory of the dilute Bose gas in two dimensions of Refs. 4 and 5. As was the case in this theory, our analysis also found that a small "mass" regulated the infrared singular pairing fluctuations near T_c. Shifts of order $(\ln N)/N$ in T_c and μ were found. However, the basic physical picture of the BCS-like mean-field theory—of a Fermi liquid condensing due to an attractive pairing interaction of quasiparticles—remained intact.

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APPENDIX

In this appendix we will use the renormalization group to demonstrate that the $(\ln N)/N$ corrections in Eqs. (1.7) and (3.4) are not affected by higher-order terms. We will examine the following partition function:

$$Z=\int \mathcal{D}\psi\mathcal{D}\psi^*\exp(-L)\ ,$$
$$L=\int d^2x\left[|\nabla\psi|^2+r|\psi|^2+\sum_{n=1}^{\infty}\frac{v_{2n}}{N^n}|\psi|^{2n}\right]\ , \tag{A1}$$

obtained from expanding the action $\tilde{S}$ in Eq. (2.2) to all orders in ψ, rescaling ψ by $1/\sqrt{N}$, and setting $K/T=1$. All the coupling constants v_{2n} are of order unity. We are interested in the critical value of $r=r_c$ at which a phase transition occurs in this model for large N.

The renormalization-group transformation proceeds by integrating out fields in a momentum shell of width e^l, followed by an appropriate rescaling of fields and coordinates (see Ref. 5). This results in scale-dependent coupling constants $r_l, v_{4l}, \ldots$ which can be obtained by solving the flow equations. We begin by presenting these flow equations for small r_l and v_{4l}, and dropping all other couplings. To lowest nontrivial order we obtain

$$\frac{dr_l}{dl}=2r_l+\frac{1}{N\pi}v_{4l}\ , \tag{A2}$$

$$\frac{dv_{4l}}{dl}=2v_{4l}\ . \tag{A3}$$

These equations can be easily integrated to yield

$$v_{4l}=v_4e^{2l}\ , \tag{A4}$$

$$r_l=\left[r+\frac{v_4}{N\pi}l\right]e^{2l}\ , \tag{A5}$$

where $r_{l=0}\equiv r$, etc. The crucial term here is the one linear in l in (A5). It arises from "resonance" due to the identical initial growth rates ($\sim e^{2l}$) of r_l and v_{4l}.[5] In the following we assume $v_4>0$. We see from (A5) that for $r>0$, r_l flows monotonically to $+\infty$ as $l\to\infty$ indicating that the system is in a disordered phase. For $r<0$, however, r_l decreases initially before turning around. The minimum value of r_l occurs at

$$l=l_m=-\frac{N\pi r}{v_4}-\frac{1}{2}\ , \tag{A6}$$

and the minimum value of r_l is

$$r_{l_m}\approx-\frac{v_4}{2N\pi}\exp\left[-\frac{2N\pi r}{v_4}\right]\ , \tag{A7}$$

where we have assumed that $r\gg 1/N$. When $r_{l_m}\sim -1$, then the increase in r_l for $l>l_m$ can no longer be trusted and the system can be considered to have reached the ordered phase. Solving for this condition we find the critical value of $r=r_c$ at which the phase transition occurs,

$$r_c\approx-\frac{v_4}{2N\pi}(\ln N)\ . \tag{A8}$$

For $(\ln N)\gg 1$ the previous approximations are justified. It is now easy to see that this critical value of r is precisely the one that would be obtained from the calculation in the main part of the paper—compare Eq. (2.18). We also note from Eq. (A6) that the value of $l_m=l^*$ associated with this value of r is

$$l^*=\tfrac{1}{2}(\ln N)\ . \tag{A9}$$

We now examine the effects of higher-order terms. We will consider the modification of the flow equation for v_{4l}, Eq. (A3), by a second-order term

$$\frac{dv_{4l}}{dl}=2v_{4l}-\frac{2}{N\pi}v_{4l}^2\ . \tag{A10}$$

Upon integrating this we find that Eq. (A4) is modified to

$$v_{4l}=\frac{v_4e^{2l}}{1+v_4(e^{2l}-1)/(N\pi)}\approx\frac{v_4e^{2l}}{1+v_4e^{2l}/(N\pi)} \tag{A11}$$

for large N. Using this result, we solve for r_l from Eq.

(A2) and obtain

$$r_l = \left\{ r + \frac{v_4}{N\pi} \left[l - \frac{1}{2} \ln \left[1 + \frac{v_4}{N\pi} e^{2l} \right] \right] \right\} e^{2l} . \quad \text{(A12)}$$

For $l = l^*$ and $(\ln N) \gg 1$ we now observe that the difference between the expressions (A5) and (A12) can be neglected. The subsequent analysis therefore remains unchanged.

A similar analysis can be carried out to show that none of the v_{2n} couplings for $n > 2$ affect the leading $(\ln N)/N$ term in r_c.

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