

Current-carrying states in Josephson junctions

Akira Furusaki and Masaru Tsukada

Department of Physics, University of Tokyo, Tokyo 113, Japan

(Received 24 October 1990; revised manuscript received 23 January 1991)

A unified treatment of clean Josephson junctions—both tunnel junctions and weak links—is developed for the static case. The quasiparticle local density of states in Josephson junctions is calculated, and it is found that the energy spectrum depends strongly on the phase difference between the two superconductors. In the case of tunnel junctions, it is predicted that bound states which are localized around the tunnel barrier appear in the energy gap. The Josephson current flows via the bound states. The possibility to observe such current-carrying states by scanning tunneling spectroscopy is discussed.

I. INTRODUCTION

The Josephson effect is a phenomenon observed both in tunnel junctions¹ and in weak links,² but it has been studied by different theoretical approaches for each case. The properties of tunnel junctions have been analyzed within perturbation theory based on models which are composed of two independent superconductors connected by a tunneling Hamiltonian.³ This approach separating the system into two noninteracting subsystems as an unperturbed state is justified by the fact that the tunneling probability is extremely small. On the other hand, weak links represent a strongly coupled system. That is, electrons can travel through a metallic link almost freely and the transmission probability is nearly unity. Thus higher-order processes, in which several Cooper pairs move from one superconductor to the other, become important and cause deviations of the current-phase relation from a simple sinusoidal function.⁴ Therefore weak links must be treated not as a composite system but as a single system.

Recently Arnold extended the tunneling Hamiltonian method to include higher-order processes and discussed the proximity effect in the tunneling spectroscopy of tunnel junctions.⁵ He obtained an expression of the dc Josephson current, reproducing the Ambegaokar-Baratoff result⁶ for tunnel junctions and the Kulik-Omel'yanchuk result⁷ for superconductor-orifice-superconductor junctions, which can be regarded as extremely thin tunnel junctions. Since his main interest lay in the current-voltage characteristics of tunnel junctions rather than the dc Josephson effect, he did not fully discuss the dc Josephson effect in tunnel junctions, nor that in weak links.

One of the purposes of this paper is to give an expression of the dc Josephson current which can be used both for weak links and for tunnel junctions. In this sense, our theory may be considered to be an extension of that of Arnold, but our formulation⁸ is completely different

from, and simpler than, his. In his theory, the Green's function of the total system is obtained as properly composing of those of the subsystems, while we directly obtain it by solving a scattering problem. The other main purpose of our work is to clarify the effect of the phase difference between two superconductors on the local density of states (LDOS) of weak links and, especially, of tunnel junctions. While the LDOS of weak links was evaluated by Ishii⁴ in the simplest case, that of tunnel junctions has not been discussed. This is probably due to the fact that no drastic change can be expected for ordinary tunnel junctions where the lowest order process dominates tunneling. However, there exist some junctions such as point contacts where the higher-order processes become important. The higher-order processes give rise to excess current in the case of metal-superconductor point contacts.^{9,10} In such junctions, it is likely that some changes occur in the LDOS when the Josephson current flows. Thus it is clearly of interest to elucidate the change of the LDOS also in tunnel junctions.

The organization of this paper is as follows. The dc Josephson current in weak links as well as in tunnel junctions is calculated in a unified way by using a simple model in Sec. II. The phase dependence of the LDOS is studied in Sec. III and its relevance to current-carrying states is discussed. Finally all the results are summarized in the last section.

II. S-N-S JUNCTION

In this section we describe the superconductor-normal-material-superconductor (S-N-S) junction as a model for both types of Josephson junctions. Two superconducting electrodes ($x < 0$ and $x > D$) are linked by a normal material ($0 < x < D$), in which impurity scattering is assumed to be negligibly small, and the system is translationally invariant along the y and the z axes. The motion of quasiparticles is governed by the Bogoliubov-de Gennes equation,¹¹

$$\begin{pmatrix} -\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{r}) - \mu & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -\left[-\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{r}) - \mu\right] \end{pmatrix} \begin{pmatrix} u(\mathbf{r}) \\ v(\mathbf{r}) \end{pmatrix} = E \begin{pmatrix} u(\mathbf{r}) \\ v(\mathbf{r}) \end{pmatrix}, \quad (1)$$

where the diagonal potential is given by

$$U(\mathbf{r}) = \begin{cases} 0, & x \leq 0 \\ U, & 0 \leq x \leq D \\ 0, & x \geq D, \end{cases} \quad (2)$$

$\Delta(\mathbf{r})$ is the pair potential, or the order parameter, m is the effective mass of electrons, and μ is the chemical potential. The model can be regarded as a weak link or a tunnel junction, depending on the value of U . That is to say, if U is smaller than μ , the model corresponds to a weak link where the normal material is metallic; on the other hand, if U is larger than μ , the model corresponds to a tunnel junction. So far most works in the literature have been concerned with the special case of $U=0$. Kuli¹² and Ishii⁴ studied this case, using the same model as that of the present paper. The case of a periodic S - N arrangement was also investigated by van Gelder,¹³ and Büttiker and Klapwijk¹⁴ discussed the effect of the Aharonov-Bohm flux on the energy spectrum of an S - N loop.

In principle, the pair potential must be determined from the gap equation self-consistently, but here we adopt the simplest form,

$$\Delta(x) = \Delta [e^{i\phi_L} \Theta(-x) + e^{i\phi_R} \Theta(x-D)]. \quad (3)$$

It is assumed that the phonon-mediated attractive interaction between electrons exists only in superconductors, and spatial variations of the pair potential near the interfaces, i.e., the proximity effect, are neglected. The steplike form of the pair potential is a reasonable assumption for tunnel junctions. For weak links, however, its validity depends on the degree of the proximity effect near the S - N interfaces. If the Fermi velocities are different for S and N , and if the temperature is low, the pair potential is not much reduced in superconductors, and a proximity effect is not seen. Moreover, when the length of the normal metal, D , is much longer than the coherence length, variations of the pair potential can be neglected. Thus the above form of the pair potential is physically reasonable in these cases. Otherwise, especially near the critical temperature, the steplike form is not a good approximation¹⁵ and a self-consistent treatment is required.

The dc Josephson current can be obtained from the probability amplitudes of the Andreev reflection^{8,16} which are obtained by solving the Bogoliubov-de Gennes equation. By applying the method to our model, the supercurrent may be calculated. We do not reproduce details of the calculation but instead write only its result.

The dc Josephson current, j , per unit area is obtained¹⁷ as

$$j = -\frac{2e\Delta^2}{\hbar} \int \frac{d\mathbf{k}_{\parallel}}{(2\pi)^2} \frac{1}{\beta} \sum_{\omega_n} \frac{\sin\varphi}{\Gamma_n}, \quad (4)$$

where

$$\Gamma_n = (K^2 \Omega_n^2 + \omega_n^2) \cosh \left[\frac{2\omega_n D}{\hbar v_N} \right] + 2K \omega_n \Omega_n \sinh \left[\frac{2\omega_n D}{\hbar v_N} \right] - (K^2 - 1) \Omega_n^2 \cos(2k_N D) + \Delta^2 \cos\varphi,$$

$$\varphi = \phi_R - \phi_L, \quad \Omega_n = \sqrt{\omega_n^2 + \Delta^2},$$

$$\omega_n = \pi(2n+1)/\beta, \quad \beta = 1/k_B T,$$

$$k_N = \left[\frac{2m}{\hbar^2} (\mu - U) - \mathbf{k}_{\parallel}^2 \right]^{1/2},$$

$$k_S = \left[\frac{2m\mu}{\hbar^2} - \mathbf{k}_{\parallel}^2 \right]^{1/2},$$

$$K = \frac{k_N^2 + k_S^2}{2k_N k_S}, \quad v_{N(S)} = \frac{\hbar k_{N(S)}}{m}.$$

This expression of the dc Josephson current covers tunnel junctions and weak links. All the important informations are included in Γ_n which is the determinant of the matching equations of wave functions. As we shall see, Γ_n determines not only the temperature and phase dependence of the Josephson current but also the energy spectrum of the quasiparticles.

When the normal material is metallic ($U < \mu$), k_N is a real number for $\mathbf{k}_{\parallel} \approx 0$. Thus behavior of Γ_n is mainly determined by the first two terms on the right-hand side of Eq. (6). The characteristic length scale is $\xi_N = \hbar v_N / \pi k_B T$. If the normal region is longer than this characteristic length, Eq. (6) reduces to

$$\begin{aligned} j &\approx -\frac{e\Delta^2}{\hbar} \int \frac{d\mathbf{k}_{\parallel}}{(2\pi)^2} \frac{1}{\beta} \frac{8 \sin\varphi \exp(-2\omega_0 D / \hbar v_N)}{K^2 \Delta^2 + (1+K^2)\omega_0^2 + 2K\omega_0 \Omega_0} \\ &\sim -\frac{e v_{N0}}{D} \frac{\Delta^2 k_{N0}^2 \sin\varphi}{[K \sqrt{\Delta^2 + (\pi k_B T)^2} + \pi k_B T]^2} \\ &\quad \times \exp(-2\pi k_B T D / \hbar v_{N0}), \end{aligned} \quad (6)$$

where

$$k_{N0} = \left[\frac{2m}{\hbar^2} (\mu - U) \right]^{1/2}, \quad v_{N0} = \frac{\hbar}{m} k_{N0}. \quad (7)$$

The dc Josephson current depends exponentially on T and D .

On the other hand, for tunnel junctions ($U > \mu$), k_N and K are imaginary, and the third term dominates Γ_n . The characteristic length in this case is $1/|k_{N0}|$. Since the length of the normal region must be much shorter than ξ_N , Eq. (6) can be approximately written as

$$\begin{aligned}
j &\approx -\frac{e\Delta^2}{\hbar} \int \frac{d\mathbf{k}_{\parallel}}{(2\pi)^2} \frac{1}{\beta} \sum_{\omega_n} \frac{2 \sin\varphi}{-\bar{K}^2 \Omega_n^2 + \omega_n^2 + (1 + \bar{K}^2) \Omega_n^2 \cosh(2\kappa D) + \Delta^2 \cos\varphi} \\
&= -\frac{e\Delta^2}{2\hbar} \sin\varphi \int \frac{d\mathbf{k}_{\parallel}}{(2\pi)^2} \frac{1}{1+Z} \left[\frac{1+Z}{\cos^2(\varphi/2) + Z} \right]^{1/2} \tanh \left[\frac{\beta\Delta}{2} \left[\frac{\cos^2(\varphi/2) + Z}{1+Z} \right]^{1/2} \right].
\end{aligned} \tag{8}$$

In the above, we define κ , \bar{K} , and Z by

$$\begin{aligned}
\kappa &= \left[\frac{2m}{\hbar} (U - \mu) + \mathbf{k}_{\parallel}^2 \right]^{1/2}, \\
\bar{K} &= \frac{\kappa^2 - k_S^2}{2\kappa k_S}, \\
Z &= (1 + \bar{K}^2) \sinh^2(\kappa D).
\end{aligned} \tag{9}$$

Equation (8) is essentially the same as the result obtained by Arnold.⁵ Since Z depends on D exponentially, the dc Josephson current decreases rapidly as the insulating barrier becomes thicker. It should be noted that the parameter Z is just what was used in the theory of Blonder *et al.*,⁹ who considered superconductor-insulator-normal metal (S - I - N) point contacts. According to the experimental work by Blonder and Tinkham,¹⁰ there exist some point contacts for which Z is of the order of unity. Thus it is natural to expect that a small Z value is realized as well in some Josephson tunnel junctions.

We show the temperature dependence of the critical current I_C in Fig. 1 for various lengths of the normal region. Every curve is normalized by the value at $T=0$ K. In the numerical calculation we set $U=0.5$ eV in Fig. 1(a) and $U=1.1$ eV in Fig. 1(b). Since we set $\mu=1$ eV, the former figure corresponds to the case where the normal material is metallic (weak link) and the latter is the case of an insulating barrier (tunnel junction).

We see that the I_C - T curves for weak links gradually change according to the length of the normal region D . When D is longer than ξ_N , I_C is an exponentially decreasing function, while it changes similarly as Kulik and Omel'yanchuk's result for D being shorter than ξ_N . Since the profile of the pair potential is assumed to be steplike regardless of the temperature, the temperature dependence $I_C(T) \propto (T_C - T)^2$ argued by de Gennes¹⁸ cannot be reproduced in our theory. It is necessary to determine the pair potential self-consistently in order to get the correct temperature dependence near T_C .

As for the tunnel case, the I_C - T curves approach the universal result of the Ambegaokar-Baratoff's (AB) formula, which almost coincides with the curve of 4 Å, as the length of the barrier increases. This is the reason why AB's formula successfully explains many experimental data of tunnel junctions.

III. LOCAL DENSITY OF STATES

Here we consider the phase dependence of the LDOS on the phase difference, φ . As one can see from Eq. (4), the main contribution of the ω_n summation arises from the poles of $\Gamma(z)$, which is analytic continuation of Γ_n ,

on the real axis. Thus the Josephson effect is closely related with energy spectrum, especially, of bound states.

By definition, the LDOS, $N(x, E)$, is obtained from the imaginary part of the retarded Green's function. The retarded Green's function can be evaluated by combining the out-going solutions of the Bogoliubov-de Gennes equation.⁸ Neglecting the rapidly oscillating terms like $\sin(2k_S x)$ and $\sin(2k_N x)$, we obtain the LDOS:

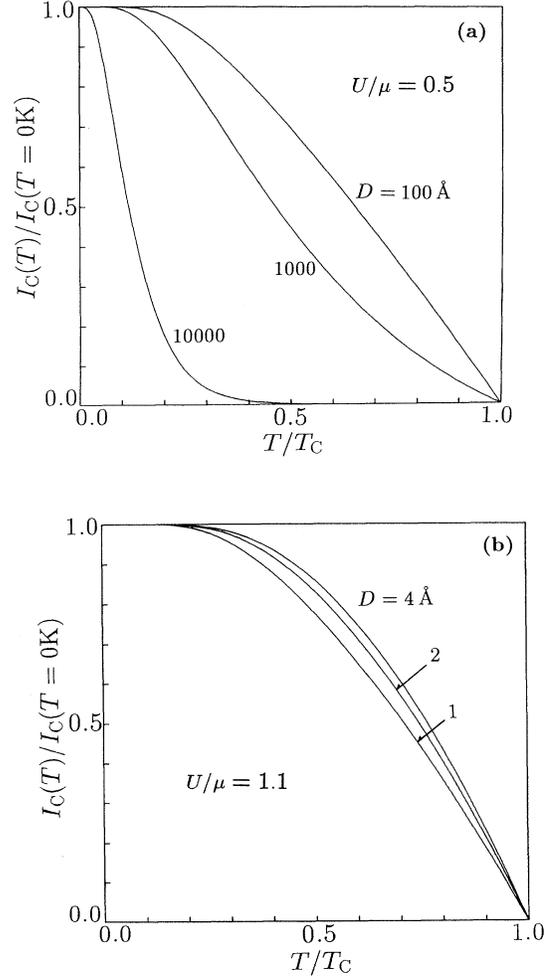


FIG. 1. Temperature dependence of the critical current of the S - N - S junction: (a) weak link ($D=100, 1000, 10000$ Å) and (b) tunnel junction ($D=1, 2, \text{ and } 4$ Å). We set $\Delta(T=0)=1$ meV, $\mu=1$ eV, and $U=0.5$ eV for the weak link, and $U=1.1$ eV for the tunnel junction.

$$x \geq D$$

$$\frac{N(x, E)}{N(0)} = \int_0^{\pi/2} d\theta \sin\theta \operatorname{Re} \left\{ \frac{E}{\Omega} + \frac{\Delta^2}{\Gamma(E)} \left[\frac{E}{\Omega} \left[\cos \left[\frac{2ED}{\hbar v_N} \right] - \cos\varphi \right] - iK \sin \left[\frac{2ED}{\hbar v_N} \right] \right] \exp \left[\frac{2i\Omega}{\hbar v_S} (x - D) \right] \right\}, \quad (10)$$

$$0 \leq x \leq D$$

$$\frac{N(x, E)}{N(0)} = \int_0^{\pi/2} d\theta \sin\theta \cos\theta \operatorname{Re} \left\{ \frac{k_{S0}}{k_N} \frac{1}{\Gamma(E)} \left[-2KE\Omega \cos \left[\frac{2ED}{\hbar v_N} \right] + i(E^2 + K^2\Omega^2) \sin \left[\frac{2ED}{\hbar v_N} \right] - i(K^2 - 1)\Omega^2 \sin(2k_N D) \right] \right\}.$$

In the above, we define Ω and $N(0)$, which is the density of states per volume at the Fermi surface, by

$$\Omega = \sqrt{E^2 - \Delta^2}, \quad (11)$$

$$N(0) = \frac{mk_{S0}}{2\pi^2\hbar^2} = \frac{m}{2\pi^2\hbar^2} \left[\frac{2m\mu}{\hbar^2} \right]^{1/2},$$

and θ is related with k_N , v_N , and v_S of Eq. (6) by

$$|\mathbf{k}_{\parallel}| = k_{S0} \sin\theta. \quad (12)$$

The energy of bound states is determined by the condition

$$\begin{aligned} \Gamma(E) &= [K^2\Delta^2 - E^2(1 + K^2)] \cos \left[\frac{2ED}{\hbar v_N} \right] \\ &\quad - 2KE\sqrt{\Delta^2 - E^2} \sin \left[\frac{2ED}{\hbar v_N} \right] \\ &\quad - (K^2 - 1)(\Delta^2 - E^2) \cos(2k_N D) + \Delta^2 \cos\varphi = 0. \end{aligned} \quad (13)$$

In the case $U=0$, this equation reduces to¹²

$$\left[\tan \left[\frac{ED}{\hbar v_N} + \frac{\varphi}{2} \right] - \frac{\sqrt{\Delta^2 - E^2}}{E} \right] \left[\tan \left[\frac{ED}{\hbar v_N} - \frac{\varphi}{2} \right] - \frac{\sqrt{\Delta^2 - E^2}}{E} \right] = 0. \quad (14)$$

Discrete energy levels (Andreev levels)^{4,19} are formed for fixed v_N in the normal region surrounded by the off-diagonal potential barrier, and the degeneracy of the discrete levels is removed when the phases of the superconductors are different from each other. In the case of S - N superlattices, the discrete levels have dispersion and the energy spectrum shows a band structure.¹³ It should be noted that the factorization of Eq. (14) does not hold for the general cases of $U \neq 0$, for which the energy spectrum becomes more complicated.

In Fig. 2 we show a numerically calculated LDOS of a weak link both at the midpoint of the normal region and in the superconductor for $\varphi=0$ and $\pi/2$. The parameters chosen are $\mu=1$ eV, $\Delta(T=0)=1$ meV, and $U=0.5$ eV. Variation of \mathbf{k}_{\parallel} gives rise to a dispersion of the bound state's energy, and peaks in the figures correspond to the discrete levels at $\mathbf{k}_{\parallel}=0$. The figures are slightly different from those of $U=0$ which was calculated by Ishii,⁴ since

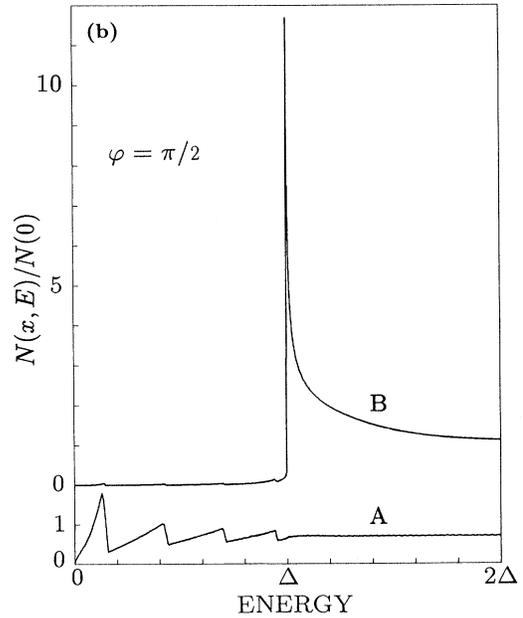
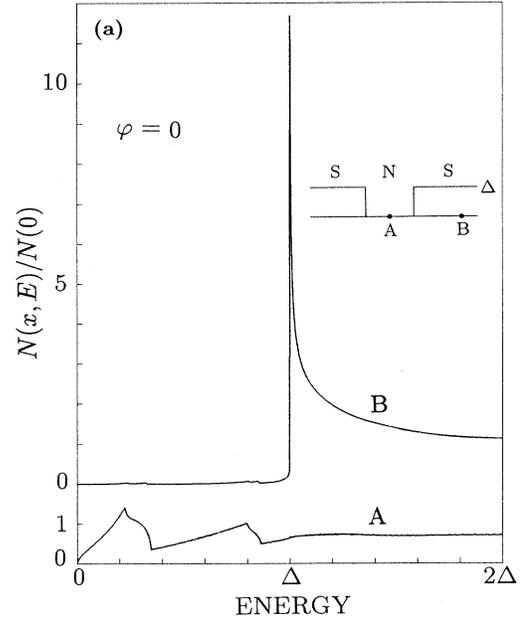


FIG. 2. Local density of states in a weak link at $x = D/2$ and $x = D + \xi$ with $U=0.5$ eV for (a) $\varphi=0$ and (b) $\varphi=\pi/2$. $\Delta(T=0)=1$ meV, $\mu=1$ eV, and $D = \xi = 12400$ Å.

the normal reflection, such as electron→electron and hole→hole, at the S - N interfaces was not taken into account in his calculation.

In actual samples, the electrons are scattered by impurities and the energy spectrum is somewhat changed. However, as long as the phase of electron wave functions is coherent in the normal region, these bound states still exist. The effect of impurity scattering in mesoscopic S - N - S junctions has been investigated by Al'tshuler *et al.*²⁰

As for tunnel junctions, the energy of bound states is, within the approximation used in Eq. (8), is given by

$$\tilde{\Delta} = \Delta \left[\frac{\cos^2 \varphi + Z}{1 + Z} \right]^{1/2}. \quad (15)$$

It is seen from the definition of Z that the bound state energy gets closer to Δ as $|\mathbf{k}_{\parallel}|$ increases. As compared with the bound states in weak links, the appearance of bound states in tunnel junctions is mysterious at first glance; there are no energy levels in the insulator and the energy gap opens in the superconductors. However, this can be understood by the following consideration. Although supercurrent is carried by Cooper pairs deep in superconductors, it flows via quasiparticles near the tunnel barrier. That is, Cooper pairs approaching the barrier gradually change into quasiparticles, going through the barrier, and then they recondense into Cooper pairs; bound states represent this process. This conversion of a Cooper pair into quasiparticles and vice versa was demonstrated in weak links,¹⁴ but it has not been recognized in tunnel junctions. When the phases of two superconductors are different, the supercurrent flows and the current-carrying bound states appear in the energy gap; when the supercurrent does not flow, these states are all degenerate at $E = \Delta$. The result for the case of $Z \sim 1$ is shown in Fig. 3. A peak due to the bound states is clearly seen at the point B when $\varphi = \pi/2$, while the LDOS vanishes in the energy-gap region when $\varphi = 0$.

The formation of bound states in our non-self-consistent calculations suggests that in a self-consistent treatment the pair potential decreases near the tunnel barrier when the supercurrent flows. Thus some quasiparticles may be trapped in the quantum well, which is surrounded by the tunnel barrier and the reduced pair potential, and finally self-consistent calculations may give results similar to those which we obtain from the steplike model.

Since there has been no direct method which is appropriate to detect bound states, they have never been observed experimentally in tunnel junctions, nor in weak links. However, it may become possible to observe them in the near future. Scanning tunneling spectroscopy (STS) may be a promising method for this application. In fact, bound states in vortices of a type-II superconductor have been successfully observed by STS.²¹ In our case, we locate a tip above a Josephson junction and measure the tunnel conductance dI/dV , which is proportional to the LDOS,²² between the junction and the tip, while changing the phase difference controlled by the magnitude of the dc Josephson current.

Unfortunately, since Z increases rapidly as a function of D , the bound states have an energy very close to Δ in

ordinary tunnel junctions, and their contribution is hardly observed separately from that of the continuum of scattering states. Therefore one must prepare a junction, probably a point contact, with $Z \sim 1$, and the experiment requires high energy resolution and very low temperature. It is crucial to prepare a clean sample also for weak links. An extension of our calculation was recently used to analyze the STS data of a weak link,²³ but the sawtoothlike form of the LDOS has not been observed, probably because of the dirtiness of the sample surface and because of thermal smearing.

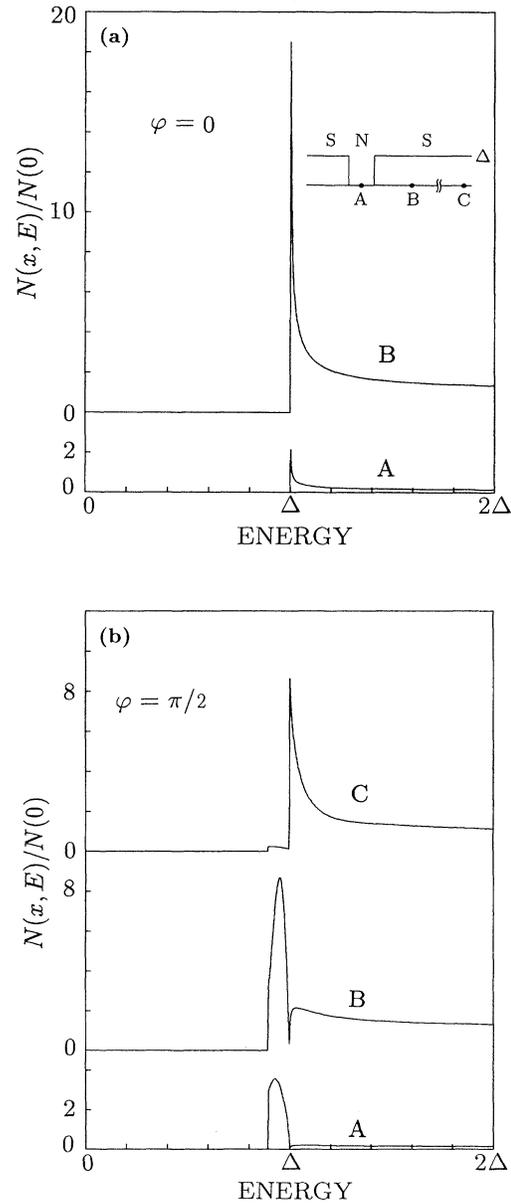


FIG. 3. Local density of states in a tunnel junction at $x = 0.5D$, $2.5D$, and $D + \xi$ for (a) $\varphi = 0$ and (b) $\varphi = \pi/2$. $\Delta(T=0) = 1$ meV, $\mu = 1$ eV, $D = 2$ Å, $U = 1.1$ eV, and $\xi = 12400$ Å.

IV. CONCLUSION

We have obtained an expression of the dc Josephson current which holds for weak links as well as tunnel junctions. It contains well-known results on the dc Josephson effect as limiting cases. We have also shown that the energy spectrum is definitely affected by the phase difference between the two superconductors. In other words, the dc Josephson effect is correlated with a change of the energy spectrum of the system. Especially, the most interesting finding is that current-carrying bound states appear even in tunnel junctions and not only in weak links. This change of the energy spectrum may be detected by STS. In the case of tunnel junctions, one must use a junction in which the parameter Z is of the or-

der of unity so that the LDOS of the bound states can be detected separately from that of the scattering states. In the case of weak links, it is necessary to perform experiments with very clean junctions in order not to smear the structure in the LDOS.

ACKNOWLEDGMENTS

We would like to thank Dr. Y. Tanaka for valuable discussions. The present work was partly supported by a Grant-in-Aid from the Ministry of Education, Science and Culture in Japan. The numerical calculation was performed on a Hitachi HITAC S-820 computer at the Computer Center of the University of Tokyo.

-
- ¹B. D. Josephson, *Adv. Phys.* **14**, 419 (1965).
²K. K. Likharev, *Rev. Mod. Phys.* **51**, 101 (1979).
³M. H. Cohen, L. M. Falicov, and J. C. Phillips, *Phys. Rev. Lett.* **8**, 316 (1962).
⁴C. Ishii, *Prog. Theor. Phys.* **44**, 1525 (1970); **47**, 1464 (1972).
⁵G. B. Arnold, *J. Low Temp. Phys.* **59**, 143 (1985).
⁶V. Ambegaokar and A. Baratoff, *Phys. Rev. Lett.* **10**, 486 (1963); **11**, 104 (1964).
⁷I. O. Kulik and A. N. Omel'yanchuk, *Fiz. Nizk. Temp.* **4**, 296 (1978) [*Sov. J. Low Temp. Phys.* **4**, 142 (1978)].
⁸A. Furusaki and M. Tsukada, *Solid State Commun.* (to be published).
⁹G. E. Blonder, M. Tinkham, and T. M. Klapwijk, *Phys. Rev. B* **25**, 4515 (1982).
¹⁰G. E. Blonder and M. Tinkham, *Phys. Rev. B* **27**, 112 (1983).
¹¹P. G. de Gennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966), Chap. 5.
¹²I. O. Kulik, *Zh. Eksp. Teor. Fiz.* **57**, 1745 (1969) [*Sov. Phys. JETP* **30**, 944 (1970)].
¹³A. P. van Gelder, *Phys. Rev.* **181**, 787 (1969).
¹⁴M. Büttiker and T. M. Klapwijk, *Phys. Rev. B* **33**, 5114 (1986).
¹⁵Y. Tanaka and M. Tsukada, *Phys. Rev. B* **37**, 5087 (1988).
¹⁶A. F. Andreev, *Zh. Eksp. Teor. Fiz.* **46**, 1823 (1964) [*Sov. Phys. JETP* **19**, 1228 (1964)].
¹⁷A. Furusaki and M. Tsukada, in *Proceedings of the 19th International Conference on Low Temperature Physics, Brighton, 1990*, edited by D. S. Betts (North-Holland, Amsterdam, 1990), p. 967.
¹⁸P. G. de Gennes, *Rev. Mod. Phys.* **36**, 225 (1964).
¹⁹A. F. Andreev, *Zh. Eksp. Teor. Fiz.* **49**, 655 (1965) [*Sov. Phys. JETP* **22**, 455 (1966)].
²⁰B. L. Al'tshuler, D. E. Khmel'nitskii, and B. Z. Spivak, *Solid State Commun.* **48**, 841 (1983); B. L. Al'tshuler and B. Z. Spivak, *Zh. Eksp. Teor. Fiz.* **92**, 609 (1987) [*Sov. Phys. JETP* **65**, 343 (1987)].
²¹H. F. Hess, R. B. Robinson, R. C. Dynes, J. M. Valles, Jr., and J. V. Waszczak, *Phys. Rev. Lett.* **62**, 214 (1989).
²²J. Tersoff and D. R. Hamann, *Phys. Rev. B* **31**, 805 (1985).
²³H. Takayanagi, K. Inoue, and Y. Tanaka, in *Proceedings of the 19th International Conference on Low Temperature Physics, Brighton, 1990*, edited by D. S. Betts (North-Holland, Amsterdam, 1991) (to be published); K. Inoue and H. Takayanagi, *Phys. Rev. B* **43**, 6214 (1991).