Hydrodynamics of flux liquids

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A hydrodynamic theory of both isotropic and hexatic flux liquids in high-T_c superconductors is presented. Weak microscopic pinning centers are described within the flux-flow model of Bardeen and Stephen, while strong macroscopic pinning centers set the boundary conditions for the flow. This model is relevant for understanding recent transport measurements by Worthington *et al.* in bulk Y-Ba-Cu-O single crystals. Flux-line entanglement leads to a large intrinsic viscosity, which increases at the isotropic-to-hexatic transition.

I. INTRODUCTION

It is well known that the finite electrical resistance in type-II superconductors is associated with phase slip due to the motion of flux lines.[1] In very pure high-temperature (high-T_c) superconductors fluctuations can melt the Abrikosov flux-line lattice over a significant portion of the phase diagram.[2,3] The resulting flux liquid has no shear rigidity at long wavelengths, and it is hard to see how weak point pins such as oxygen vacancies could be very effective in suppressing flux-flow resistivity at these elevated temperatures.[4] It has, however, been argued that in bulk samples entanglement of the flux lines may yield very long viscous relaxation times, greatly affecting the response of flux liquid to a few, widely separated strong pins, such as twin boundaries.[2]

The motion of the flux lines will in general be affected *both* by intrinsic dynamical constraints associated with entanglement and extrinsic constraints associated with defects that can effectively pin the vortex lines. Defects will dominate at low temperatures and in very disordered samples, yielding the impurity-induced glassy state observed in recent transport measurements in epitaxial films of Y-Ba-Cu-O.[5] In contrast, vibrating-reed experiments[6] and recent transport and ac-susceptibility measurements by Worthington, Holtzberg, and Feild[7] in bulk Y-Ba-Cu-O may be better interpreted in terms of melting of the flux lattice along a line in the H-T phase diagram well below the mean-field transition line, $H_{c2}(T)$. The transport data of Ref. 7 indicate that a nonvanishing critical current develops suddenly at a line in the H-T phase diagram that coincides with the irreversibility line measured by ac susceptibility. This line is interpreted as the melting of a pinned vortex solid into a flux liquid. Above this line the I-V curves are linear for small currents and the data of resistance versus temperature at high fields display two distinct regimes separated by a characteristic shoulder that occurs at a temperature well above the melting of the flux solid (Fig. 2). Both the shoulder and the abrupt vanishing of the resistivity are smeared out in experiments at higher currents. At temperatures below the location of the shoulder and above the flux-lattice melting point the linear resistivity is considerably reduced as compared to the predictions of the simple free-flux-flow model of Bardeen and Stephen.[1] The crystals used by Worthington, Holtzberg, and Feild[7] contain many twins on the micrometer scale, and are otherwise much cleaner than the samples used in Ref. 5. This suggests that large-scale inhomogeneities must play an important role in reducing the flux-flow resistivity in clean bulk crystals.

A conventional approach would try to describe the transport data in terms of thermally activated flux flow. From this point of view one or more of the features of the data of Worthington, Holtzberg, and Feild would correspond to the increase in the vortex friction coefficient that occurs when the typical pinning barrier begins to exceed the thermal energy.[8,9] A model of thermally activated flow of independent vortex lines would, however, produce a small linear resistivity at all nonzero temperatures, contrary to the observation[7] that the linear resistivity vanishes below a well-defined temperature. Simple models which invoke a distribution of microscopic pinning centers[8,9] may be inadequate for the clean crystals used in Ref. 7 for another reason: Within the model of flux pinning and creep developed by Anderson and Kim,[1] the energy barrier associated with an oxygen vacancy can be estimated in terms of the superconductor condensation energy per unit volume, $H_c^2/8\pi$, where $H_c = H_{c2}/\sqrt{2}\kappa$ is the thermodynamic critical field.[4] Roughly speaking, pinning arises because this condensation energy can be avoided if the flux line runs through the normal region represented by an oxygen vacancy. Denoting by l the size of the pinning region, the characteristic pinning energy is $U_p \approx (H_c^2/8\pi)l^3$. Taking $l \approx 3$ Å for a single oxygen vacancy and using the parameters of Y-Ba-Cu-O with an applied field of 7 T, we estimate $H_c \approx 2\times 10^3$ G at 82 K and find a very small pinning energy, $U_p/k_B \approx 0.03$ K. Even if the actual pinning energy greatly exceeds this simple estimate, say $U_p/k_B = 5°$ K, it is hard to see how

independent weak pinning on these length scales can explain the broad resistive transition observed at liquid nitrogen temperatures in Y-Ba-Cu-O. Clearly collective effects must be included in some way. There is now considerable literature on collective flux pinning and creep in high-T_c superconductors, and a number of experimental results in thin films have been interpreted in this framework.[10,11] In this approach collective effects are described through the assumption that large rigid volumes of flux solid are coherently pinned by weak microscopic defects. Flux motion occurs by thermally activated jumps of these volumes of flux solid over the corresponding energy barriers. While collective pinning of the flux solid provides an interpretation for many of the experimental observations in thin superconductor films, it seems that a more careful treatment will be required for bulk samples where the pinning volumes are *physically connected* in the direction of the applied field [see, for example, Eq. (5) below]. It seems in any case unnatural to invoke large activated volumes of flux *solid* in a region where the solid may well be thermally melted on all scales larger than a few intervortex spacings.

Here we exploit an alternative point of view, where collective effects in the flux *liquid* are described through a flux liquid viscosity. Provided the barrier to flux-line crossing is sufficiently high, flux-line entanglement leads to a very large flux liquid viscosity.[2] The effect of a few large-scale inhomogeneities (e.g., twin boundaries) that act as "strong" pins can then be transmitted over large distances impeding the flow of the viscous flux liquid. In our model the hydrodynamic consequences of entanglement in the flux liquid can be quite dramatic. The resistivity data of Worthington, Holtzberg, and Feild can be interpreted in terms of a viscosity which increases abruptly at the shoulder and which diverges or jumps discontinuously to infinity when the linear resistivity goes to zero.

The difference between the I-V curves in films[5] and bulk crystals[7] may be due to very different twinning densities[12] and to the role of entanglement in increasing the flux liquid viscosity in thick samples. The epitaxially grown films generally have twins with spacing of order 100 Å, which would effectively act like microscopic pinning centers. The crystals on the other hand, have twins on the micrometer scale, and these twins can act as strong pins in three dimensions.[13] The enhanced pinning properties of twin boundaries for magnetic fields aligned with the twinning planes have been observed experimentally[14] and discussed theoretically.[15]

We discuss finally the existence of *two* distinct vortex liquid regimes, bounded by a shoulder in the resistivity (see Fig. 2).[7,14] Worthington, Holtzberg, and Feild[7] proposed that these regimes may correspond to isotropic and hexatic flux liquids, in agreement with our recent prediction of an entangled hexatic liquid of flux lines in *three dimensions*, interposed between a low-temperature crystalline phase and a high-temperature isotropic liquid.[16] The hexatic flux liquid is characterized by long-range bond-orientational order in planes normal to the applied field. In the hexatic the torsional rigidity of the vortices increases the effective viscosity of the flux lines,[17] which would lead to a shoulder in the resistivity at the isotropic-to-hexatic transition. The shoulder could also be explained, however, simply as the onset of significant barriers to flux cutting. Unambiguous evidence for a hexatic *glass* of flux lines in high-temperature superconductors has, in any case, recently been observed in flux decoration experiments on very pure single crystals of Bi-Sr-Ca-Cu-O at low temperature.[18] These experimental observations suggest the existence of an equilibrated hexatic flux liquid at higher temperatures, consistent with the conjecture of Worthington, Holtzberg, and Feild[7] for clean Y-Ba-Cu-O samples.

In this paper we present a theory of the hydrodynamics of both isotropic and hexatic melted flux liquids on scales large compared to the spacing between vortex lines. We apply this model to situations where defects occur on two well-separated energy and length scales. We assume macroscopic regions of flux liquid where the flux lines move in the presence of weak-pinning centers (e.g., oxygen vacancies) of microscopic size, described within the flux flow model of Bardeen and Stephen.[1] These large regions of flux liquid are interrupted by a few macroscopic defects (twin boundaries or other large-scale inhomogeneities) that are well separated relative to the mean flux-line spacing $d \approx (n_v^0)^{-1/2}$. Here $n_v^0 = B/\phi_0$ is the equilibrium two-dimensional density of flux lines with $\phi_0 = 2\pi\hbar c/2e$ the flux quantum. These strong-pinning centers act as macroscopic obstacles, pinning the flow via the boundary conditions in the hydrodynamic equations.

II. HYDRODYNAMIC MODES

Our starting point is a set of linearized hydrodynamic equations for a melted liquid of flexible flux lines in three dimensions. The external field $\mathbf{H}$ is aligned with the z direction. There are three hydrodynamic variables in an isotropic flux liquid, the areal vortex density $n_v(\mathbf{r},t)$ and the two components of a tangent field $\tau(\mathbf{r},t)$, describing the instantaneous tilt of the lines. They are given by

$$n_v(\mathbf{r},t) = \sum_i \delta^{(2)}[\mathbf{r}_\perp - \mathbf{r}_i(z,t)] , \tag{2.1}$$

$$\tau(\mathbf{r},t) = \sum_i \frac{\partial \mathbf{r}_i}{\partial z} \delta^{(2)}[\mathbf{r}_\perp - \mathbf{r}_i(z,t)], \tag{2.2}$$

with $\mathbf{r} = (\mathbf{r}_\perp, z)$ and $\mathbf{r}_i(z)$ a two-dimensional vector denoting the position of the ith vortex line in the xy plane. The fields $n_v(\mathbf{r},t)$ and $\tau(\mathbf{r},t)$ allow us to describe the local B fields parallel and perpendicular to $\hat{\mathbf{z}}$ in ter.ns of quantized vortex lines. Since vortices are only created or destroyed at the boundaries, the vortex density obeys a continuity equation

$$\partial_t n_v + \nabla_\perp \cdot \mathbf{j}_v = 0 , \tag{2.3}$$

where $\mathbf{j}_v = n_v^0 \mathbf{v}$ is the vortex current density and $\mathbf{v}$ the vortex flow velocity. The dynamics of the tangent field is governed by the equation[19]

$$\partial_t \tau_\alpha + \partial_\beta j^T_{\alpha\beta} = \partial_z j_{v\alpha} , \tag{2.4}$$

where $j^T_{\alpha\beta}$ is the antisymmetric tangent flux tensor. Fi-

nally, if flux lines cannot start or stop inside the sample, the flux density and the two components of the tangent field are not independent dynamical variables, but are related by a kind of continuity equation in the timelike variable z,

$$\partial_z n_v + \nabla_\perp \cdot \tau = 0 \ . \tag{2.5}$$

In the isotropic melted flux liquid we have therefore only two independent hydrodynamic variables. The flux-line velocity is overdamped, and relaxes rapidly due to interactions with the underlying lattice and via thermally activated jumps between weak-pinning centers.

In the flux-line hexatic discussed in Ref. 16 there is an additional hydrodynamic mode associated with the slowly decaying bond-orientational order parameter. Denoting by Θ a smoothed version of the microscopic bond-angle field, the corresponding hydrodynamic equation is readily constructed using standard methods,[17]

$$\partial_t \Theta = \tfrac{1}{2}\hat{\mathbf{z}} \cdot (\nabla_\perp \times \mathbf{v}) + \kappa_\perp \nabla_\perp^2 \Theta + \kappa_z \partial_z^2 \Theta \ , \tag{2.6}$$

where the irreversible couplings $\kappa_\perp$ and κ_z can be expressed as the products of a kinetic coefficient and the hexatic stiffness constants.[17]

Equations (2.3) and (2.4) must be supplemented with constitutive equations for the current density $\mathbf{j}_v$ and the tangent flux tensor $j^T_{\alpha\beta}$. The constitutive equation for $\mathbf{j}_v$ is presented here as an equation of motion for $\mathbf{v}$ which is obtained by balancing the force per unit volume acting on a small volume of the flux-line liquid,

$$-\gamma \mathbf{v} - \gamma' \rho_s \kappa n_v^0 \hat{\mathbf{z}} \times \mathbf{v} + \rho_s \kappa n_v^0 \hat{\mathbf{z}} \times \mathbf{v} + (\eta \nabla_\perp^2 + \eta_z \partial_z^2)\mathbf{v} + \eta_b \nabla_\perp (\nabla \cdot \mathbf{v}) - n_v^0 \left[\nabla_\perp \frac{\delta F_L}{\delta n_v(\mathbf{r})} - \partial_z \frac{\delta F_L}{\delta \tau(\mathbf{r})} \right] - \tfrac{1}{2}(\hat{\mathbf{z}} \times \nabla_\perp)(K_\perp \nabla_\perp^2 + K_z \partial_z^2)\Theta + \mathbf{f}_T = 0 \ , \tag{2.7}$$

where $\kappa = 2\pi\hbar/2m$ is the quantum of circulation and $\rho_s = m n_s$, with n_s the three-dimensional density of superconducting electrons. The first two terms in (2.7) represent the effective frictional forces per unit volume arising from the interaction of the normal core electrons in or near the vortex cores with the underlying crystal lattice. The effective drag has both a component along the flux-line velocity and one orthogonal to it.[1] The coefficient γ is a friction per unit volume. It is related to the flux-line mobility μ by $\gamma = n_v^0/\mu$. Weak point pins, with pinning energies $U_p \lesssim k_B T$, are assumed to be incorporated into γ. As an approximation, one can take

$$\gamma \approx \frac{n_v^0 \pi \hbar^2}{2e^2 \xi_c^2(T)} \sigma_n \ ,$$

where σ_n is the normal-state conductivity and ξ_c is the superconducting coherence length.[1] The coefficient γ' is dimensionless. The third term in Eq. (2.7) is the linearized Magnus force per unit volume on the flux-line continuum. The drag normal to $\mathbf{v}$ and the Magnus force determine the Hall angle θ_H, according to $\tan\theta_H = \Delta$, with

$$\Delta = (\gamma' - 1)\rho_s \kappa n_v^0 / \gamma \ .$$

In both the conventional and the high-T_c superconductors $\gamma' \approx 1$, i.e., the second and third terms in (2.7) cancel and the Hall angle is very small. Below we will for simplicity assume $\gamma' = 1$ and neglect the Hall effect. Although the viscous couplings η, η_z, and η_b would be small in a liquid of point vortices, flux-line entanglement will produce much larger values of η when there are significant barriers to line crossing.[2] The sixth term in (2.7) represents reversible forces (the first is just a pressure force) that are easily obtained from the free energy of an isotropic flux-line liquid,

$$F_L = \frac{1}{2(n_v^0)^2} \int d\mathbf{r} \int d\mathbf{r}' [c_l(\mathbf{r}-\mathbf{r}')\Delta n_v(\mathbf{r})\Delta n_v(\mathbf{r}') + c_{44}(\mathbf{r}-\mathbf{r}')\tau(\mathbf{r})\cdot\tau(\mathbf{r}')] \ , \tag{2.8}$$

where $\Delta n_v(\mathbf{r}) = n_v(\mathbf{r}) - n_\gamma^0$ and $c_l(\mathbf{r})$ and $c_{44}(\mathbf{r})$ are the local compressional and tilt moduli of the flux-line liquid (the in-plane shear modulus vanishes in the liquid[16]). Nonlocality affects the elastic properties of the flux-line lattice on scales less than the penetration length λ.[20] The seventh term in (2.7) is the force arising from inhomogeneities in the bond-angle field and it is present only in the hexatic. The Frank elastic constants $K_\perp$ and K_z were calculated in Ref. 16. Finally, in the presence of an externally imposed uniform transport current density $\mathbf{j}_T$, there will be a constant Lorentz force per unit volume on the flux liquid,

$$\mathbf{f}_T = -\frac{1}{c} n_v^0 \phi_0 \hat{\mathbf{z}} \times \mathbf{j}_T \ .$$

The antisymmetric tangent flux tensor can be written as[19]

$$j^T_{\alpha\beta} = \Gamma_T(\partial_\alpha \tau_\beta - \partial_\beta \tau_\alpha) \ , \tag{2.9}$$

where Γ_T is a kinetic coefficient. Because (2.9) describes diffusion of the component of the magnetic field in the xy plane, we can estimate Γ_T from the relaxation rate of an overdamped helicon,[1]

$$\Gamma_T \approx \frac{c^2}{4\pi\sigma_n} \ .$$

Equations (2.3)–(2.9) are a closed set of linearized hydrodynamic equations for the flux-line hexatic. The corresponding equations for the isotropic flux liquid are obtained by dropping (2.6) and the bond-angle field gradients in (2.7).

In the absence of applied transport current ($\mathbf{f}_T = 0$) the linearized hydrodynamic equations (2.3)–(2.9) can be

used to obtain the hydrodynamic modes of the flux liquid. For the isotropic liquid we find two diffusive modes of frequency ω_n and ω_τ, governing the decay of fluctuations in the flux-line density and the tangent field, respectively. In the flux-line hexatic there is an additional diffusive mode of frequency ω_Θ associated with fluctuations in the bond angle. The dispersion relations of the modes are

$$\omega_n = -\frac{i}{n_v^0\gamma}[q_\perp^2 c_l(q_\perp,q_z)+q_z^2 c_{44}(q_\perp,q_z)] ,$$
$$\omega_\tau = -i[\Gamma_T q_\perp^2 + q_z^2 c_{44}(q_\perp,q_z)/(n_v^0\gamma)] , \quad (2.10)$$
$$\omega_\Theta = -i(\kappa_\perp q_\perp^2 + \kappa_z q_z^2) .$$

In the limit of wavelengths much larger than the superconductor penetration length λ, the nonlocality of the elastic moduli can be neglected and one obtains conventional hydrodynamic diffusive modes. In the limit $\lambda\to\infty$, the density mode relaxes at a finite rate when $\mathbf{q}=0$, as in helium films.[21] If the Hall angle is finite, density and tangent field fluctuations are coupled and one finds[19] damped helicon modes analogous to those predicted for flux crystals.[1]

III. HYDRODYNAMIC TREATMENT OF THE RESISTIVITY

When a uniform current $\mathbf{j}_T$ is applied to the crystal, the flux liquid will flow according to the equations just discussed through the macroscopic obstacles represented by the few strong defects distributed throughout the fluid. Such defects anchor a portion of the flux liquid by forcing the flux-line flow velocity $\mathbf{v}$ to vanish at their vicinity. As an example, we consider an oversimplified, but illuminating model relevant for the understanding of recent transport measurements by Worthington, Holtzberg, and Feild.[7] Consider the flow of an isotropic flux liquid in a channel contained between two flat twin boundaries in the xz plane at $y=-W/2$ and $y=W/2$. The transport current is applied in the y direction, $\mathbf{j}_T=j_T\hat{\mathbf{y}}$, and yields a constant driving force

$$\mathbf{f}_T=\hat{\mathbf{x}}n_v^0\phi_0 j_T/c=\hat{\mathbf{x}}f_T$$

in the x direction. We search for a stationary flow profile with $\tau=0$ and $v_y=0$ everywhere in the channel. In this limit Eq. (2.7) (with $\gamma'\approx 1$) reduces to

$$-\gamma\mathbf{v}+\eta\nabla_\perp^2\mathbf{v}+\mathbf{f}_T=0 . \quad (3.1)$$

If drag is absent ($\gamma=0$), this is the equation for Poiseuille fluid flow with a pressure gradient $\nabla_\perp p=\mathbf{f}_T$. The combination of the drag and the viscous term introduces an important new length scale into the problem, $\delta=\sqrt{\eta/\gamma}$. Upon assuming for simplicity that $\mathbf{v}=0$ at the walls due to strong pinning by the twin boundaries, we find that the solution of (3.1) is

$$v_x(y)=v_\infty\left[1-\frac{\cosh(y/\delta)}{\cosh(W/2\delta)}\right] . \quad (3.2)$$

Here $v_\infty=f_T'/\gamma$ is the usual Bardeen-Stephen limiting flux-line velocity[1] and the constant force f_T' includes, in addition to the driving force f_T from the transport current, any constant pressure gradient in the y direction. The flow velocity drops to zero in a boundary layer of width $\delta=\sqrt{\eta/\gamma}$. It is clear from Fig. 1 that as δ increases the influence of the boundaries propagates into the interior of the channel and eventually chokes off the flow.

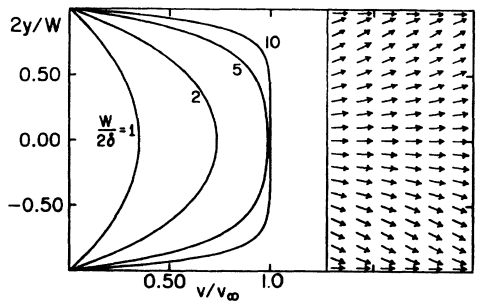

FIG. 1. Velocity profile for channel flow as given in Eq. (3.2) (left). Also shown on the right is a snapshot of the flux-line hexatic for $W/2\delta=5$. Each arrow indicates one of six equivalent bond directions.

To obtain an estimate of the screening length δ we need to estimate the viscosity η of the flux liquid. If the vortices in adjacent CuO_2 planes were decoupled, we could obtain the viscosity η_0 of a flux liquid from simple dimensional analysis,

$$\eta_0=k_B T\tau_0/d^3=k_B T/D_0 d ,$$

where τ_0 is a microscopic relaxation time and D_0 is the diffusion constant of an isolated point vortex in the xy plane. We take

$$D_0\approx\mu k_B T\approx 1\ \mathrm{cm}^2/\mathrm{sec} .$$

For $H=7$ T, we estimate the intervortex spacing $d\approx 100$ Å and at $T=80$ K, we obtain $\eta_0\approx 10^{-8}$ erg sec/cm^3. Using $\xi_c\approx 20$ Å for Y-Ba-Cu-O in the Bardeen-Stephen estimate of γ, we find $\delta_0\approx 5$ Å. Thus *if entanglement is neglected, the macroscopic obstacles of typical spacing $W\gg\delta$ will have a negligible effect on the flow.*

Flux-line entanglement, however, can greatly increase the viscosity and therefore the screening length δ. An estimate of the viscosity adapted from the theory of reptation dynamics in polymer melts[22] leads to $\eta\approx\mu_0\tau(L)$, where μ_0 is a short-time transient shear modulus, and

$$\tau(L)\approx\tau_0(L/\xi_z')^3$$

is a characteristic relaxation time. Here L is the length of the crystal in the z direction and ξ_z' is the spacing between entanglements along z. We expect that $\xi_z'\approx 4$–$5\,\xi_z$, where ξ_z is the entanglement correlation length defined in Ref. 2. The microscopic relaxation time τ_0 should be of order

$$\tau_0\approx(\xi_z'/c_0)D_0 n_v^0 ,$$

where $c_0\approx 10$ Å is the spacing between CuO_2 planes.

The parameter μ_0 is the transient shear modulus of a polymer gel of entanglement points separated by a distance ξ_z' along the flux lines. Upon adapting the method of de Gennes,[22] we find

$$\mu_0 \approx k_B T n_v^0 / \xi_z' .$$

For Y-Ba-Cu-O we estimate $\xi_z' \approx 4500$ Å and $\mu_0 \approx 240$ erg/cm^3. It is then apparent that in bulk crystals entanglement will greatly increase the viscosity. Using, for instance, the value $L = 0.025$ mm of the sample used by Worthington, Holtzberg, and Feild,[7] we find $\eta \approx 10^{-2}$ erg sec/cm^3 and $\delta \approx 10^4$ Å, a value comparable to the estimated mean separation of twin boundaries in the sample used in Ref. 7.

Recently, Obukhov and Rubinstein[23] have pointed out that relaxation times far longer than those predicted by reptation theory may be expected if the flux lines were truly impenetrable. These authors find a relaxation time requiring the collective motion of many flux lines which behaves like

$$\tau_{\rm coll}(L) \approx \tau_0 \exp[c(L/\xi_z)^3] , \tag{3.3}$$

where c is a dimensionless constant. The reptation theory underestimates this time because it allows disentanglements to proceed via the motion of flux ends in *three* dimensions. Much larger relaxation times result when the flux tips are constrained to move in two parallel planes at the top and the bottom of a superconducting sample. The true state of affairs is probably complicated by flux cutting. The possibility of flux cutting suggests that we replace L by a temperature-dependent length $L_{\rm eff}(T)$ which is the spacing between flux cutting events. We expect that $L_{\rm eff}(T) \approx \xi_z' e^{U_x/k_B T}$, where U_x is a typical flux cutting energy. If $L_{\rm eff} < L$, reptation theory with

$$\tau(L) = \tau_0 (L_{\rm eff}(T)/\xi_z')^3$$

may be a more appropriate description of the dynamics.

To find the voltage across the width W of the crystal associated with vortex motion, we use the Josephson relation for the voltage drop ΔV in terms of the phase difference $\Delta\theta$ between the two ends of the sample,

$$\Delta V = \frac{\hbar}{2e} \frac{d\Delta\theta}{dt} .$$

Since a phase slip of 2π occurs when a vortex crosses the length of the sample, we have

$$\frac{d\Delta\theta}{dt} = 2\pi n_v^0 \int_{-W/2}^{W/2} dy\, v_x(y) . \tag{3.4}$$

Defining the flux-flow contribution to the resistivity ρ_f by $|\Delta V|/W = j_T/\rho_f$ and using Eq. (3.2) in (3.4), we find

$$\rho_f = \rho_f^0 \left[1 - \frac{2\delta}{W} \tanh(W/2\delta) \right] , \tag{3.5}$$

where

$$\rho_f^0 = \pi^2 \hbar^2 n_v^0 \mu / e^2$$

is the Bardeen-Stephen flux-flow resistivity.[1] The same formula applies to a periodic array of twin boundaries with spacing W. Alternatively, one could average (3.4) over a distribution of channel spacings and orientations.

When considering the flow of an entangled flux-line hexatic in a channel, we have to solve the additional hydrodynamic equation for the bond-angle field. If we assume that the bond-angle field is pinned at the boundaries, we can then use (2.6) to eliminate the bond-angle field in Eq. (2.7). The resulting equation for the flow velocity $\mathbf{v}$ is formally identical to that obtained for the isotropic fluid with η replaced by the effective viscosity

$$\eta_{\rm eff} = \eta + K_\perp / 4\kappa_\perp .$$

The torsional rigidity of the hexatic increases the fluid viscosity[17] and therefore also increases the screening length δ. We expect that this viscosity enhancement starts at a second-order isotropic-to-hexatic phase transition in the universality class of the three-dimensional XY model. The viscosity should increase further as one approaches the (probably first-order) transition to an Abrikosov flux lattice.

The bond-angle field Θ is determined by the vorticity of the flow, via Eq. (2.6) specialized to a steady-state situation,

$$\nabla_\perp^2 \Theta = -\frac{1}{2\kappa_\perp} \hat{\mathbf{z}} \cdot (\nabla \times \mathbf{v}) . \tag{3.6}$$

Imposing $\Theta = 0$ at the walls, the solution of (3.6) is

$$\Theta(y) = \frac{v_\infty \delta_{\rm eff}}{2\kappa_\perp} \frac{\sinh(y/\delta_{\rm eff}) - 2y/w \sinh(w/2\delta_{\rm eff})}{\cos(w/2\delta_{\rm eff})} , \tag{3.7}$$

with $\delta_{\rm eff} = \eta_{\rm eff}/\gamma$. On the right-hand side of Fig. 1 we show the distortion induced in the flux-line hexatic in the channel flow problem for $W/2\delta = 5$ with $\Theta = 0$ at the walls. The viscosity is enhanced because the hexatic stiffness prevents the flux lines from rotating freely to accommodate spatial inhomogeneities in the vorticity.

We have attempted to extract the crucial hydrodynamic parameter $\eta(T)$ directly from the data of Worthington, Holtzberg, and Feild.[7] Following Bardeen-Stephen[1] we estimate the flux-flow resistivity ρ_f^0 as $\rho_f^0/\rho_n = B/H_{c2}(T)$, where $B = n_v^0 \phi_0$. We take $B \approx H$ and use the value

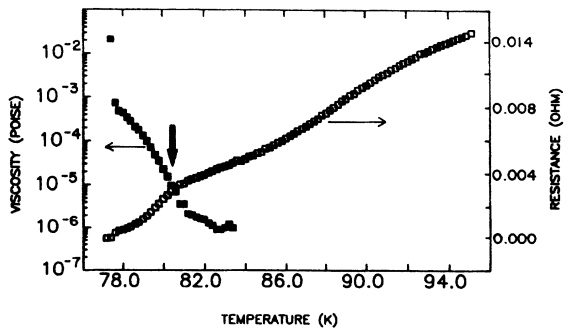

FIG. 2. Resistance data of Ref. 7 for $H = 9$ T and the corresponding viscosity $\eta(T)$ extracted from fits to Eq. (3.5). The vertical arrow indicates the shoulder in the resistivity, proposed by Worthington, Holtzberg, and Feild (Ref. 7) as the location of the isotropic-to-hexatic transition.

$$\frac{dH_{c2}(T)}{dT} = -2.4 \text{ T/K}$$

to estimate the mean-field $H_{c2}(T)$ curve.[7] For each value of the field we use this procedure to obtain a viscosity from Eq. (3.5) with $W = 4000$ Å using the resistance data only up to temperatures somewhat above the location of the experimentally observed shoulder in the resistivity. At higher temperatures amplitude fluctuations become important and our theory based on a London picture of well-separated flux lines is less reliable. As shown in Fig. 2, the viscosity of the flux liquid increases as the temperature decreases until it is many orders of magnitude larger than what one would estimate assuming independent point vortices in the CuO_2 planes. Most of this increase takes place in the region below the shoulder at 83 K identified by Worthington, Holtzberg, and Feild[7] as the hexatic phase where η begins to approach the estimate 10^{-2} erg sec/cm^3 discussed above.

IV. CONCLUSIONS

The hydrodynamic approach presented here makes no claim about the nature of the transition at which the linear resistivity vanishes. As Fig. 2 makes clear, the viscosity appears to diverge at this temperature. Such a divergence could result from the melting from a Abrikosov flux lattice.[6,7] Although an underlying first-order transition is expected in $d = 6 - \epsilon$ dimensions and also in $d = 3$,[24] this transition could be nearly continuous and characterized by a large viscosity for the experimentally relevant case of $d = 3$. An actual viscosity *divergence* associated with continuous melting appears in the theory of melting dynamics of classical particles in two dimensions.[17]

An apparent divergence of the viscosity would also result from the very long entanglement relaxation times estimated in Ref. 23. In this scenario, barriers to flux cutting would increase $L_{\text{eff}}(T)$ with decreasing temperature until $L_{\text{eff}}(T) \gtrsim L$, at which point the viscosity $\eta(L) \approx \mu_0 \tau_{\text{coll}}(L)$ is effectively infinite. An explanation of this type for the "irreversibility line"[25] was proposed in Ref. 2.

Our analysis strongly suggests that entanglement plays an important role in determining the transport properties of the flux-line liquid. More work is needed to determine how flux-line cutting reduces the various estimates of the viscosity $\eta(T)$, and to calculate more carefully the temperature dependence of the Bardeen-Stephen parameter $\gamma(T)$, which depends on the as yet poorly understood normal-state resistivity.

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