Kosterlitz-Thouless transition in high-T_c superconductor films

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Dynamical theory for the polarization of bound vortex-antivortex pairs near the Kosterlitz-Thouless transition ($T_{KT}=88.4$ K) has been applied to thin films of $YBa_2Cu_3O_7$. Calculations show that the correct order of magnitude is predicted for the loss function $\omega G/c^2$ at T_{KT}, but the temperature dependence below the transition is wrong. The theoretical value drops much more rapidly with decreasing temperature than observed experimentally. Similar disagreement is found for the penetration depth $\lambda(T)$. Estimates of the loss function at microwave frequencies show rather large effects near the critical temperature, but these become negligible by 80 K. The performance of microwave devices operating at liquid-N_2 temperature should not be degraded by vortex-antivortex pairs.

I. INTRODUCTION

Recently, Fiory *et al.*[1] observed renormalization of the penetration depth $\lambda(T)$ in thin films of $YBa_2Cu_3O_7$ near the critical temperature. They attributed the substantial decrease of λ^{-1} relative to λ_{BCS}^{-1} to the impending unbinding of vortex-antivortex pairs due to a Kosterlitz-Thouless (KT) transition.[2] The transition itself was observed to satisfy the universal relationship

$$\lambda_\perp^{-1}(T_{KT})=32\pi^2 kT_{KT}/\phi_0^2$$

predicted by theory. Here $\lambda_\perp=\lambda^2/d$ is the transverse penetration depth and d is the film thickness. In one 500-Å film, this occurred at $T_{KT}=88.4$ K. They also measured the dissipation $c^{-2}\omega G$ [$G=$ Re (Z^{-1}) where Z is the sheet impedance of the film at $\omega=8.4\times10^4$ s^{-1}], which was sharply peaked just above T_{KT}. This was the first observation of the KT transition in an intrinsic type-II superconductor. Previously, the KT transition had been identified in dirty superconductors such as granular Al (Ref. 3) and Hg-Xe (Ref. 4) films.

While the KT transition has been accepted as pertinent for neutral superfluids such as liquid He films (see review by Minnhagen[5]), its significance for superconducting films was first pointed out by Beasley *et al.*[6] They noted that for all practical purposes, vortices in very thin superconducting films interact with a logarithmic potential which typifies the topological defects of KT theory. Only for separations $r>\lambda_\perp$ does the potential change from ln r to $1/r$. Typically, near T_{KT}, $\lambda_\perp$ is comparable to the extent of the film.

The theory of the sheet impedance has been developed and discussed by Ambegaokar *et al.*,[7] Halperin and Nelson,[8] and Hebard and Fiory,[3] which we follow closely in this paper. We will restrict our attention here to $T<T_{KT}$ where the emphasis is on the dielectric constant of the bound vortex-antivortex pairs. Both the renormalization of the penetration depth and the dissipation will be discussed in terms of the polarization of the pairs.

The purpose of the present work is (1) to analyze the Fiory *et al.* experiment more completely (i.e., can theory quantitatively predict the temperature dependence of λ and the foot on the dissipation peak below T_{KT}); (2) to determine the feasibility of studying the KT transition by microwave measurements; and (3) to explore the practical implications of these effects in microwave devices.

The paper is organized as follows. Section II is a summary of the KT theory and the calculation of the r dependence of the dielectric constant. Section III describes the sheet impedance theory and the ω dependence of the polarization. Section IV gives a comparison of theory to the experimental results of Fiory *et al.*[1] Section V extends previous analyses to microwave measurements of the sheet impedance of a film. Section VI contains our conclusions.

II. KOSTERLITZ-THOULESS THEORY AND $\tilde{\epsilon}(r)$

KT theory[2,7,8] for the scale-dependent dielectric constant $\tilde{\epsilon}(r)$ involves two functions $K(l)$ and $y(l)$, where $l=\ln(r/\xi)$ and ξ is the coherence length. These functions obey the Kosterlitz recursion relations

$$\frac{d}{dl}[K(l)]^{-1}=4\pi^3 y^2(l)\ , \tag{2.1}$$

$$\frac{dy}{dl}(l)=[2-\pi K(l)]y(l)\ . \tag{2.2}$$

The dielectric constant is given by

$$\hat{\epsilon}(r)=K(0)/K(l)\ , \qquad (2.3)$$

where the unrenormalized coupling constant (essentially the reduced superfluid density) is

$$K(0)=\phi_0^2 d/16\pi^3\lambda_{\rm BCS}^2(T)k_BT\ , \qquad (2.4)$$

$$\phi_0=hc/2e\ . \qquad (2.5)$$

The activity $y(0)$ can be expressed as

$$y(0)=\exp[-CK(0)/4]\ , \qquad (2.6)$$

where the nonuniversal constant C is related to the energy of the vortex core.

The renormalized penetration depth can be found from

$$\lambda^{-2}=\lambda_{\rm BCS}^{-2}/\epsilon_\infty\ , \qquad (2.7)$$

where

$$\epsilon_\infty=\hat{\epsilon}(r=\infty)\ . \qquad (2.8)$$

At the KT transition ($T=T_{\rm KT}$), the renormalized coupling constant $K(l=\infty)$ approaches a universal value[9]

$$K(\infty)=2/\pi\ . \qquad (2.9)$$

The dielectric constant ϵ_∞, on the other hand, approaches a nonuniversal value ϵ_c at the transition. The constant C and ϵ_c are related such that the recursion relations (2.1) and (2.2) reproduce $\epsilon_\infty=\epsilon_c$ when $K(0)=2\epsilon_c/\pi$. For the low-temperature superconductors, ϵ_c is typically 1.1 to 1.8, whereas Fiory *et al.*[1] found $\epsilon_c=4.6$ for their film of $YBa_2Cu_3\,O_7$, which corresponds to $C=2.23946$. A calculation of $\hat{\epsilon}(r)$ at $T_{\rm TK}$ obtained by solving (2.1) and (2.2) numerically is shown in Fig. 1 (solid curve). As $r\to\infty$, $\epsilon(r)\to 4.6$ as required by the experimental data of Fiory *et al.*[1]

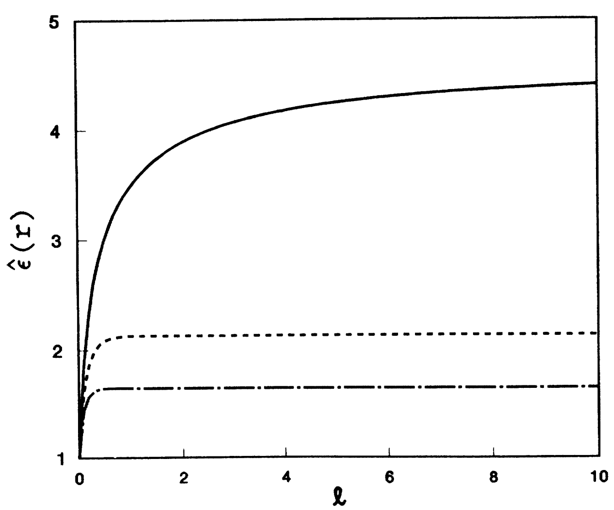

FIG. 1. Scale-dependent dielectric constant $\hat{\epsilon}(r)$ vs $l=\ln(r/\xi)$ for $T=88.4$ K $=T_{\rm KT}$ (Kosterlitz-Thouless transition temperature, solid curve); 88.3 K, $\tau=(T_{\rm KT}-T)/T_{\rm KT}=1.13\times10^{-3}$ (dashed); and 88.2 K, $\tau=2.26\times10^{-3}$ (chained). The mean-field critical temperature $T_{c0}=89.4$ K.

Temperature is introduced into the calculation via Eq. (2.4) where we take for simplicity

$$\lambda_{\rm BCS}^2(T)=\lambda_{\rm BCS}^2(0)T_{c0}/(T_{c0}-T)\ , \qquad (2.10)$$

where T_{c0} is the mean-field critical temperature. Substituting into (2.4) and making use of the universal jump condition[9] (2.9), we find

$$K(0)=(2\epsilon_c/\pi)(T_{c0}-T)T_{\rm KT}/(T_{c0}-T_{\rm KT})T\ . \qquad (2.11)$$

Using the values suggested by Fiory *et al.*,[1] $T_{\rm KT}=88.4$ K and $T_{c0}=89.4$ K, we calculated $\hat{\epsilon}(r)$ for several temperatures in Fig. 1. The inputs to the calculation were $T_{\rm KT}$, T_{c0}, ϵ_c, and C. The first three quantities were experimentally determined[1] and the fourth was chosen so that the recursion relations give the proper value of ϵ_∞ at $T=T_{\rm KT}$, namely ϵ_c. It can be seen that only very near $T_{\rm KT}$ does $\hat{\epsilon}(r)$ show any significant r dependence. Likewise, as one sees in Fig. 2, ϵ_∞ differs from unity only very close to the transition (within a few tenths of a K). The strong temperature dependence is in large part due to the rapid change of the penetration depth near the critical temperature as given by Eq. (2.10) and appears to be characteristic of a realistic range of input parameters. The assumed linear temperature dependence of $\lambda_{\rm BCS}^{-2}(T)$ near T_{c0}, which is the essential feature, agrees with experiment on thicker films where KT renormalization is not expected. Furthermore, it is quite general and applies to any Ginzburg-Landau superconductor, even in the presence of pair breaking.

III. SHEET IMPEDANCE

In this section, we summarize the theory of Ambegaokar *et al.*,[7] Halperin and Nelson,[8] and Hebard and Fiory[3] for the calculation of the sheet impedance of the superconducting film. We restrict our attention to $T<T_{\rm KT}$ where both the resistance and inductance are influenced by the polarization of the bound vortex-antivortex pairs.

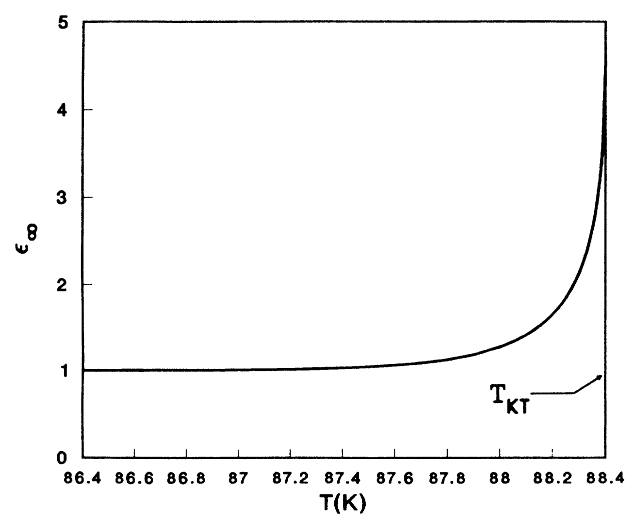

FIG. 2. $\epsilon_\infty=\hat{\epsilon}(r=\infty)$ vs T. At $T_{\rm KT}$, $\epsilon_\infty=4.6$.

This is characterized by a frequency-dependent dielectric constant $\epsilon(\omega)$ obtained from the scale-dependent dielectric constant $\hat{\epsilon}(r)$ and the vortex diffusion constant D according to

$$\epsilon(\omega)=1+\int_{\xi}^{\infty}dr\,\frac{d\hat{\epsilon}}{dr}(r)\frac{14D/r^2}{-i\omega+14D/r^2}\ , \tag{3.1}$$

where

$$D=\frac{2e^2\xi^2k_BT}{\hbar^2\pi\sigma_n d}\ , \tag{3.2}$$

$$\sigma_n^{-1}=\frac{10^9}{c^2}\rho_n\ (\Omega\,\mathrm{cm})\ , \tag{3.3}$$

and σ_n^{-1} is in sec. Equation (3.3) is the Bardeen-Stephen formula for a dirty superconductor and may not apply to $YBa_2Cu_3O_7$. However, if we substitute $\xi=15$ Å, $T=89.4$ K, ρ_n (Ω cm) = 100 $\mu\Omega$ cm, and $d=500$ Å, we find $D=8.2\times10^{-4}$ cm^2/sec, which is in fair agreement with the result 2×10^{-4} cm^2/sec determined from flux-flow resistance measurements by Fiory *et al.*[1]

The real and imaginary parts of $\epsilon(\omega)$ are shown in Figs. 3(a) and 3(b), respectively, for several temperatures near T_{KT}. They are plotted against a frequency-dependent scale l_ω which is defined by

$$l_\omega=\ln(r_\omega/\xi)\ , \tag{3.4}$$

where

$$r_\omega=(14D/\omega)^{1/2}\ . \tag{3.5}$$

We note that there is no strong temperature dependence of l_ω since ξ in the denominator of the argument of ln in (3.4) cancels the ξ^2 term in D.

The sheet impedance is expressed as

$$Z(\omega)=-i\omega m\epsilon(\omega)/n_se^2d\ . \tag{3.6}$$

This can be written to a good approximation as

$$Z(\omega)=-i4\pi\lambda_{\mathrm{BCS}}^2\omega\epsilon(\omega)/c^2d\ . \tag{3.7}$$

Equation (3.6) neglects the normal channel contribution, which must be added in parallel so that the total impedance is given by

$$Z_{\mathrm{tot}}^{-1}=Z(\omega)^{-1}+\sigma_{1\mathrm{qp}}d\ , \tag{3.8}$$

where the quasiparticle conductivity is (in the two-fluid model)

$$\sigma_{1\mathrm{qp}}=\sigma_n(T/T_{c0})^4\simeq\sigma_n\ . \tag{3.9}$$

For low frequencies (as in Ref. 1), the magnitude of the first term in (3.8) is of order $10^5\ \Omega^{-1}$, whereas the normal channel is only $\frac{1}{20}\ \Omega^{-1}$ and can be neglected. For microwave frequencies, the complete expression must be retained.

IV. COMPARISON TO EXPERIMENT

In this section we compare the theory summarized in the previous sections to the experimental results of Fiory *et al.*[1] In Fig. 4(a) we show the calculated results for the loss function $\omega G/c^2$ where $G=\mathrm{Re}[Z(\omega)^{-1}]$ and the sheet impedance is given by Eq. (3.7). [We choose $l_\omega=7.1$, which corresponds to $D=2\times10^{-4}$ cm^2/sec, $\xi=15$ Å, and $\omega=8.4\times10^4$ s^{-1}. Using typical values in (3.2) would have given $l_\omega=7.8$, not significantly different.] We see that the absolute value at T_{KT} compares well with experiment, but that the temperature dependence is incorrect. The theoretical results decrease much too rapidly with decreasing temperature. At 88.0 K, experiment is four orders of magnitude larger than theory. As we remarked above, the strong temperature dependence in the theory is due mostly to the singularity in $\lambda_{\mathrm{BCS}}(T)$ at T_{c0}.

The lack of agreement between theory and experiment is also evident in the renormalization of the penetration depth $\lambda(T)$. In Fig. 4(b), we compare the experimental results to that calculated from KT theory. Dynamical

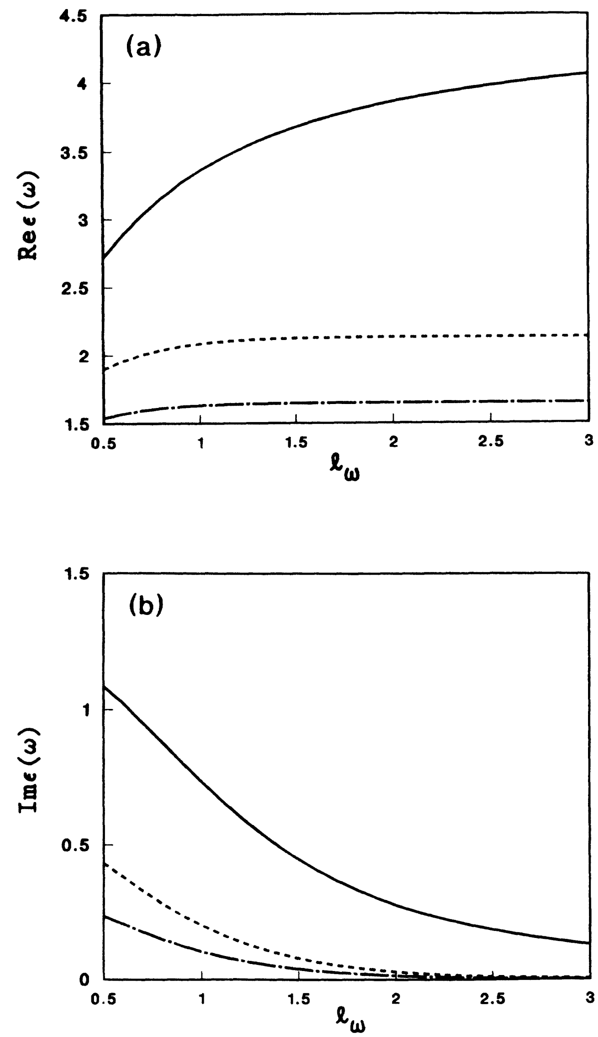

FIG. 3. (a) Real part of frequency dependent dielectric constant $\epsilon(\omega)$ vs $l_\omega=\ln(r_\omega/\xi)$ where $r_\omega=(14D/\omega)^{1/2}$ for various temperatures (88.4 K, solid; 88.3 K, dashed; 88.2 K, dash dotted). (b) Imaginary part of frequency-dependent dielectric constant $\epsilon(\omega)$ vs l_ω.

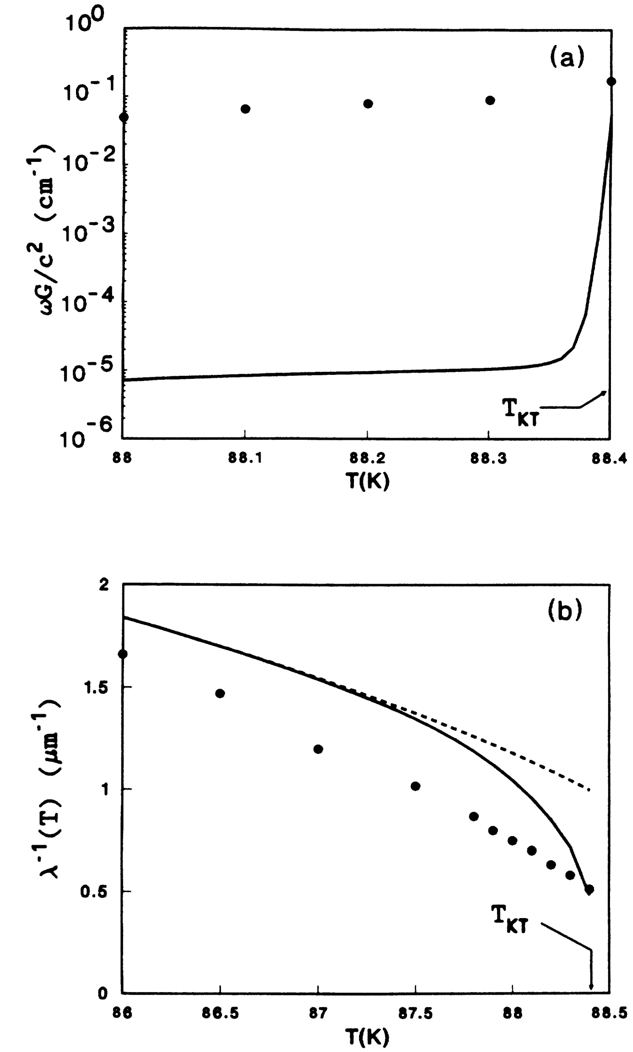

FIG. 4. (a) Loss function vs temperature for Kosterlitz-Thouless theory (solid curve) and experimental results (solid points) of Ref. 1. $\omega=8.4\times10^4$ s^{-1} and $\lambda_{BCS}(0)=0.106\mu m$ [see Eq. (2.10)]. (b) Penetration depth at $\omega=8.4\times10^4$ s^{-1} vs temperature for KT theory (solid curve), BCS theory [as simplified in Eq. (2.10), dashed curve], and experimental results of Ref. 1 (solid points).

corrections to account for the effects of measuring at $\omega>0$ are included by using $\mathrm{Re}\epsilon(\omega)$ instead of ϵ_∞. The dashed curve, denoted BCS, is a plot of Eq. (2.10). Although the disagreement is not as severe as for the loss function, nonetheless one sees substantial disagreement. The theoretical value of λ at T_{KT} agrees with experiment as it must due to the fitted value of C.

Both measurements indicate that the effects of the bound vortex-antivortex pairs persist below T_{KT} to a greater extent than KT theory predicts. Several possible explanations need to be considered. First, one can conjecture that decoupling of the CuO_2 planes occurs, although this seems unlikely given the relatively modest Ginzburg-Landau mass ratio of 25 for $YBa_2Cu_3O_7$. If d were 11.7 Å (the interplane spacing) instead of 500 Å (the film thickness), Fiory *et al.*[1] noted that T_{KT} should be about 82 K.

A more likely explanation is inhomogeneities in the film that lead to a progressive dilution of the current paths as $T\rightarrow T_{c0}$. If these inhomogeneities exist on a length scale small compared with $\lambda_\perp$, their effect would be to increase $\lambda_\perp$ (i.e., renormalize it) but not to undermine existence of a KT transition itself. In particular, the universal relation between T_{KT} and $\lambda_\perp$ predicted by theory (and seen experimentally) would remain valid. Other possibilities include vortex trapping and effects due to Josephson coupling of grains, the latter of which would have effects similar to inhomogeneities.

The temperature at which vortex trapping becomes important can be estimated by equating the pinning energy, $U_p(t)(t=T/T_{c0})$, to the thermal energy kT. The pinning energy is[10]

$$U_p(t)=[H_c^2(t)/8\pi]\pi\xi^2(t)d \ , \tag{4.1}$$

where the critical field $H_c(t)=H_c(0)(1-t^2)$ and $H_c(0)=1.8$ kOe. If we let $\xi^2(t)=\xi^2(0)/(1-t)$ with $\xi(0)=15$Å, then for $d=500$Å (which yields a maximal estimate of the pinning energy) and $t\simeq1$, we find

$$U_p(t)=U_p(0)(1-t) \ , \tag{4.2}$$

where $U_p(0)=0.11$ eV. Setting $U_p(t)=$ kT gives $1-t=0.07$ or $T=83$ K. This temperature is below the range ($1-t=0.01$ to 0.02) where there are significant effects from vortex-antivortex polarization. However, if our estimate of $1-t$ were too large by a factor of 4 or more, then vortex trapping would become important. This seems unlikely, but it cannot be ruled out entirely.

V. CALCULATION OF MICROWAVE PROPERTIES

To explore the potential of microwave measurements to observe KT effects and to evaluate the significance of these effects on devices, we extend the calculations of Sec. III to higher frequencies. Here we take the same parameters as before, except that we use (3.2) and (3.3) to determine D and l_ω since we consider a wider temperature range. Note that two typical frequencies, $\omega=2\pi\times10^9$ s^{-1} and $2\pi\times10^{10}$ s^{-1}, correspond to $l_\omega=2.193$ and 1.042, respectively.

In Fig. 5(a) $\mathrm{Im}\epsilon(\omega)$ is plotted against l_ω for $87.4<T<88.4$ K. The frequency range is $1.86\times10^{11}>\omega>1.25\times10^9$ s^{-1} ($3\times10^{10}>f>2\times10^9$ Hz). The loss function $\omega G/c^2$ is plotted for the same ranges of frequency and temperature in Fig. 5(b). One can see immediately that going to microwaves substantially increases the magnitude of the effects of the vortex-antivortex pairs. The loss function varies from roughly 0.01 to 5 cm^{-1}, the latter being three times greater than the peak in $\omega G/c^2$ observed by Fiory *et al.*[1] *above* T_{KT}, where the pairs are not all bound and the superconductor is better described as a plasma of free vortices.

In addition to the loss due to vortices (which is plotted in Figs. 5 and 6), the total loss function has a contribution from the normal channel, Eqs. (3.8)–(3.9), given by

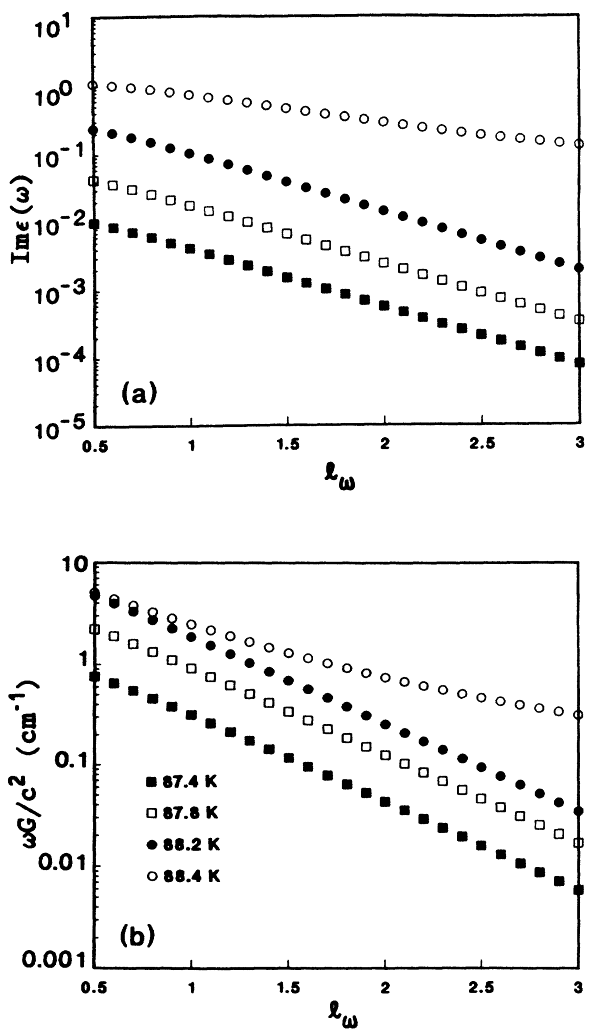

FIG. 5. (a) Imaginary part of $\epsilon(\omega)$ vs l_ω for various temperatures at microwave frequencies. $\tau=(T_{\rm KT}-T)/T_{\rm KT}=1.13\times10^{-2}$ (solid square), 6.79×10^{-3} (open square), 2.26×10^{-3} (solid circle), 0 (open circle). $\omega=2\pi\times10^9$ s^{-1} at $l_\omega=2.193$ and $2\pi\times10^{10}$ s^{-1} at 1.042. $d=500$Å and $\lambda_{\rm BCS}(0)=0.106\mu$m. (b) Loss function vs l_ω. The normal channel contribution is omitted.

$$\omega\sigma_n d/c^2=2\pi f d/10^9\rho_n(\Omega-cm) \tag{5.1}$$

$$=10^{-10}\pi f\ , \tag{5.2}$$

for $\rho_n(\Omega-\text{cm})=100$ $\mu\Omega$-cm and $d=500$Å.

A plot of the vortex loss function for $f=10^9$ Hz and 10^{10} Hz over the temperature range 86 to 88.4 K is shown in Fig. 6. The loss is clearly observable over most of the range. However, if we go to 85 K, the loss becomes rather small (1.4×10^{-5} and 1.4×10^{-4} cm^{-1}, respectively) and at 80 K vanishes to our numerical accuracy. Consequently, very thin high-T_c superconductive devices operated at liquid N_2 temperature should not experience loss due to the nearby KT transition.

For $f=10^{10}$ Hz the quasi-particle loss (5.2) is π and we can see from Fig. 6 that it is comparable to the vortex

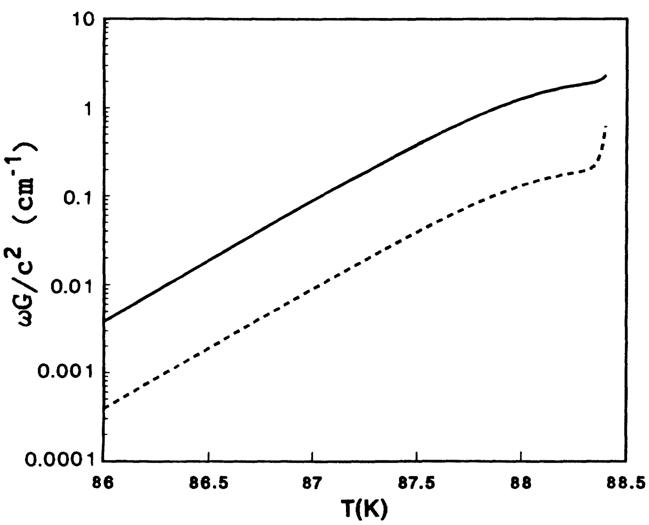

FIG. 6. Loss function vs temperature for $f=10^9$ Hz (dashed line) and 10^{10} Hz (solid line). The normal channel contribution is omitted.

loss near $T_{\rm KT}$. In this case, the total loss should drop by an amount of order 50% as we cool several degrees below 88.4 K. For $f=10^9$ Hz, the quasiparticle loss is smaller than the vortex loss near $T_{\rm KT}$ and at still lower frequencies it becomes negligible compared to the vortex part at $T_{\rm KT}$.

VI. CONCLUSIONS

From our analysis of the experimental results of Fiory *et al.*,[1] we conclude that the dynamical KT theory developed by Ambegaokar *et al.*[7] and by Halperin and Nelson[8] and applied by Hebard and Fiory[3] to low-temperature superconductors does not fully explain the effects observed in the $YBa_2Cu_3O_7$ films of Ref. 1. Fitting the theory at the observed transition temperature $T_{\rm KT}$, we find the temperature dependence of the data to be incorrect. The loss function $\omega G/c^2$ drops abruptly (almost four orders of magnitude in less than 0.1 K decrease in T) compared to the slower decrease observed in the experiment (vanishes about 0.7 K below $T_{\rm KT}$). Likewise, the penetration depth $\lambda(T)$ is renormalized (increased) over a much larger temperature range than predicted by theory. Decoupling of the CuO_2 planes might be responsible for the disagreement between theory and experiment—an interesting possibility. But more mundane effects such as inhomogeneities, vortex pinning, and, perhaps, Josephson coupling of grains have not been ruled out. Further work is required to understand fully the nature of the observed Kosterlitz-Thouless transition in high-T_c films.

We have extended the calculations to higher frequencies (in the microwave range) to see if observable effects are predicted for a temperature range of a few K below $T_{\rm KT}$. Indeed, we find within existing KT theory predictions of substantial loss above 86 K for 500-Å films where $T_{\rm KT}=88.4$ K. However, below 80 K, the loss is negligible and the operation of microwave devices at liquid N_2 temperature should not be degraded by the losses due to the polarization of bound vortex-antivortex pairs.

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[1]A. T. Firoy, A. F. Hebard, P. M. Mankiewich, and R. E. Howard, Phys. Rev. Lett. **61**, 1419 (1988).

[2]J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973).

[3]A. F. Hebard and A. T. Fiory, Physica **109 & 110B**, 1637 (1982).

[4]A. M. Kadian, K. Epstein, and A. M. Goldman, Phys. Rev. B **27**, 6691 (1983).

[5]P. Minnhagen, Rev. Mod. Phys. **59**, 1001 (1987).

[6]M. R. Beasley, J. E. Mooij, and T. P. Orlando, Phys. Rev. Lett. **42**, 1165 (1979).

[7]V. Ambegaokar, B. I. Halperin, D. R. Nelson, and E. D. Siggia, Phys. Rev. Lett. **40**, 783 (1978); Phys. Rev. B **21**, 1806 (1980); V. Ambegaokar and S. Teitel, Phys. Rev. B **19**, 1667 (1979).

[8]B. I. Halperin and D. R. Nelson, J. Low Temp. Phys. **36**, 599 (1979).

[9]D. R. Nelson and J. M. Kosterlitz, Phys. Rev. Lett. **39**, 1201 (1977).

[10]A. F. Hebard, P. L. Gammel, C. E. Rice, and A. F. J. Levi, Phys. Rev. B **40**, 5243 (1989).