Temperature dependence of magnetoresistance oscillations in a two-dimensional electron gas subjected to a periodic potential

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We have measured the magnetoresistance at various temperatures of a two-dimensional electron gas subjected to a one-dimensional periodic potential. A series of oscillations periodic in inverse magnetic field is observed at low fields due to a resonant drift of electrons in the periodic potential, while at higher fields the Shubnikov–de Haas (SdH) oscillations are observed. The low-field oscillations persist to a much higher temperature than the SdH oscillations. We argue that the low-field oscillations are quenched when the thermal smearing of the cyclotron orbit diameter is equal to the period of the potential. Using this simple model, we show that the temperature at which the low-field oscillations are quenched is larger than that for the SdH oscillations by a factor $k_F a/2$, where a is the period and k_F the Fermi wave vector. In addition, an explicit expression for the temperature dependence of the low-field oscillations is calculated and compared with our experimental data. Excellent agreement is found between the predicted and observed temperature dependences.

Several groups have recently observed a series of low-field oscillations in the magnetoresistance of a two-dimensional electron gas (2D EG) subjected to a one[1,2] or two-dimensional[3] periodic potential. Although there have been a number of related interpretations of these oscillations,[1,2,4,5] it is clear that they are essentially classical in origin.[4] The important effect is that in perpendicular electric, E, and magnetic, B, fields, an electron experiences a drift in the direction $(\hat{\mathbf{E}}\times\hat{\mathbf{B}})$, where $\hat{\mathbf{E}}$ and $\hat{\mathbf{B}}$ are unit vectors in the direction of the electric and magnetic fields, respectively. There is a resonant enhancement of this drift whenever the classical cyclotron diameter at the Fermi energy, $2R_c$, is given by

$$2R_c=(n+\tfrac{1}{4})a \; , \tag{1}$$

where n is an integer and a is the period of the potential. Equation (1) defines the characteristic $1/B$ periodicity of the effect since $R_c=\hbar k_F/eB$, where $\hbar k_F$ is the Fermi momentum. In this Brief Report we present data on the magnetoresistance of a 2D EG in a one-dimensional periodic potential for temperatures ranging from 2 to 40 K.

Our samples are fabricated by depositing a "patterned gate" on the surface of a modulation-doped GaAs/(Al,Ga)As heterostructure in which a high-mobility two-dimensional electron gas is embedded. Before processing, the mobility of the 2D EG was in excess of 100 $m^2 V^{-1} s^{-1}$. The patterned gate comprises a layer of resist, patterned using electron beam lithography, over which a metal is evaporated. Where the metal adheres directly to the semiconductor surface its gating action is effective, however where the metal adheres to the resist the gating action is inhibited. The resist is patterned into lines, so that when the gate is reverse biased a one-dimensional periodic potential is generated in the plane of the 2D EG [see Fig. 1 (inset)]. The patterned gate covers a Hall bar of approximate dimensions 5×10 μm^2. Current flows perpendicular to the lines of constant potential. Due to screening effects, this potential is approximately sinusoidal, varying as $V(x)=V_0\cos(2\pi x/a)$, where x is the direction of current flow and V_0 is the amplitude of the potential. Figure 1 shows the magne-

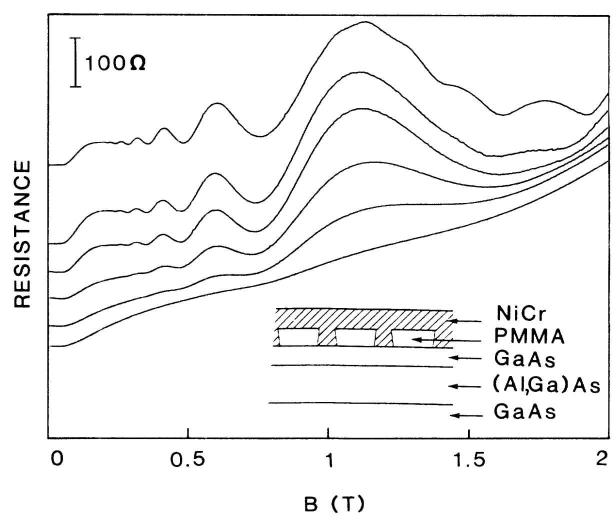

FIG. 1. Magnetoresistance at various temperatures of a 2D EG subjected to a periodic potential of period 0.15 μm. The curves are offset for clarity and correspond to temperatures 4.2 K (top), 6.9, 10, 15, 23, and 41 K (bottom). Inset: schematic representation of gate fabrication.

toresistance at various temperatures of a 2D EG subjected to a 1D periodic potential with period $a=0.15$ μm at a gate voltage $V_g=+0.25$ V, which corresponds to a weak potential modulation. At a temperature of 4.2 K a series of oscillations periodic in $1/B$ is clearly visible terminating at a field of ~ 1.2 T. At higher fields Shubnikov–de Haas (SdH) oscillations are observed. Also note the presence of the positive magnetoresistance at very low fields. By comparing the various traces in Fig. 1 it is clear that the SdH oscillations in the field range 1.5–2.0 T are quenched above a temperature $T\approx 10$ K, although the low-field oscillations are still clearly observed. Indeed the low-field oscillations persist to at least 41 K; at this temperature the final maximum of the series may still be discerned at a magnetic field of 1.2 T (see Fig. 1).

Generally the amplitude of the low-field oscillations decreases as the temperature increases in such a way that the temperature required to quench a particular maximum or minimum is an increasing function of magnetic field. This is shown in Fig. 2 in which the amplitude of a maximum or minimum at a given magnetic field is plotted against temperature. The amplitude of a given maximum was determined by drawing a straight line between the two neighboring minima and measuring the resistance difference between this line and the peak of resistance at the value of B where the peak occurs. A similar procedure was used to define the amplitudes of the minima.

These results may be understood within the semiclassical model introduced by Beenakker[4] for a one-dimensional periodic potential. Beenakker showed that the crossed magnetic and local electric field from the periodic potential caused electrons to drift in the $\pm y$ direction (the potential is chosen to vary in the x direction) generating an increase in the yy component of the diffusivity tensor. Inversion of the tensor leads to an increase in the xx component of the resistivity tensor, when the cyclotron orbit diameter $2R_c$ is given by Eq. (1). When $2R_c=(n-\frac{1}{4})a$, there is no drift and no enhancement of the magnetoresistance. Thus as the magnetic field is varied we observe a series of maxima in the magnetoresistance when $2R_c=(n+\frac{1}{4})a$ separated by minima when $2R_c=(n-\frac{1}{4})a$. At a finite temperature T, electrons within an energy $\sim k_B T$ of the Fermi energy ε_F contribute to the electrical transport. The thermal smearing of the Fermi circle leads to an equivalent smearing of the cyclotron radius δR_c. We anticipate that the low-field oscillations will be quenched if the range of cyclotron orbit diameters is approximately equal to the period of the potential. Thus the condition that must be satisfied to observe the low-field oscillations is

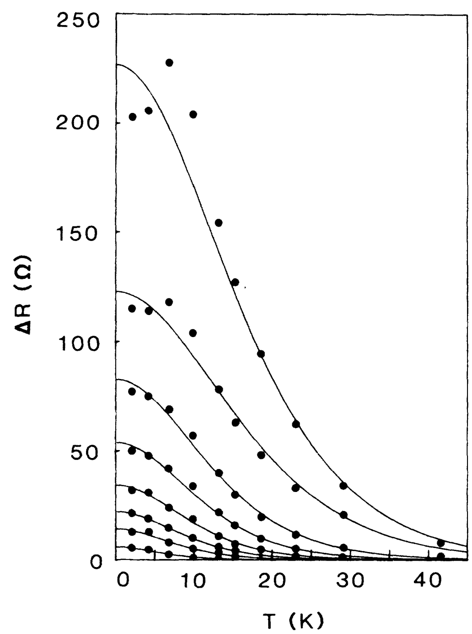

FIG. 2. Amplitude of maxima and minima of the low-field oscillatory structure shown in Fig. 1 plotted as a function of temperature. The curves are the best fits of Eq. (5).

$$k_B T \leq \hbar\omega_c k_F a/2 , \quad (2)$$

where the cyclotron frequency $\omega_c=(eB/m^*)$ and m^* is the electron effective mass. The equivalent condition for SdH oscillations is simply $kT\leq\hbar\omega_c$. Typically for our structures $k_F a \gg 1$; for the structure used to measure the data in Fig. 1, $k_F a\sim 12$.

Equation (2) predicts, therefore, that the quenching by temperature of the low-field oscillations shown in Fig. 1 should occur at a temperature ~ 6 times higher than the one required to quench the SdH oscillations. Also, since $\omega_c \propto B$, Eq. (2) indicates that the oscillations at low magnetic field disappear at a lower temperature than those at

TABLE I. Fitted parameters γ and $T_a(B)^{-1}$ for oscillation maxima and minima at various magnetic fields for a 1D potential with period $a=0.15$ μm. Also shown are the predicted values of $T_a(B)$.

B (T)	γ (Ω K^{-1})	$T_a(B)^{-1}$ (expt.) (K^{-1})	$T_a(B)^{-1}$ (predicted) (K^{-1})
0.26	2.1	0.34	0.31
0.32	3.9	0.27	0.25
0.35	5.3	0.24	0.23
0.41	7.5	0.22	0.20
0.48	10.5	0.20	0.17
0.60	14	0.17	0.13
0.74	16	0.13	0.11
1.1	30	0.13	0.07

TABLE II. Fitted values of the parameters γ and $T_a\ (B)^{-1}$ for samples with a 1D periodic potential with period $a=0.3\ \mu$m.

B (T)	γ ($\Omega\,\mathrm{K}^{-1}$)	$T_a(B)^{-1}$ (expt.) (K^{-1})	$T_a(B)^{-1}$ (predicted) (K^{-1})
0.18	8.8	0.28	0.24
0.21	12	0.24	0.21
0.26	17	0.23	0.17
0.32	22	0.19	0.14
0.47	34	0.17	0.09

higher magnetic field. Both predictions are in excellent qualitative agreement with experimental results.

We are able to calculate the temperature dependence of the low-field oscillations directly from the expression[2,4] for the relative change in the conductivity, $\delta\sigma_{yy}/\sigma_0$, in the direction perpendicular to the current and parallel to the lines of the constant potential:

$$\frac{\delta\sigma_{yy}}{\sigma_0}=\left[\frac{eV_0}{\varepsilon_F}\right]^2\left[\frac{l^2}{aR_c}\right]\frac{1}{\omega_c^2\tau^2}\cos^2\left[\frac{2\pi R_c}{a}-\frac{\pi}{4}\right], \qquad (3)$$

where V_0 is the amplitude of the potential, τ is the scattering time, and $l=v_F\tau$ is the electron mean free path. Using

$$\sigma(T)=-\int_0^\infty \sigma(\varepsilon)\frac{\partial f}{\partial\varepsilon}d\varepsilon\ , \qquad (4)$$

where f is the Fermi function and the assumption $kT\ll\varepsilon_F$, we obtain

$$\frac{\Delta\rho_{xx}}{\rho_0}=\left[\frac{eV_0}{\varepsilon_F}\right]^2\left[\frac{l^2}{aR_c}\right]\cos^2\left[\alpha\varepsilon_F^{1/2}-\frac{\pi}{4}\right]\times\left[\frac{T}{T_a(B)}\right]\mathrm{csch}\left[\frac{T}{T_a(B)}\right], \qquad (5)$$

where ρ_0 is the resistivity in zero magnetic field, $\alpha=2\pi(2m^*)^{1/2}/eBa$, and $k_BT_a(B)=\hbar\omega_c k_F a/4\pi^2$.

Equation (5) is not a complete expression for $\Delta\rho_{xx}/\rho_0$. It does not take into account two important effects. First, it ignores the effect of the periodic potential in distorting the electron cyclotron orbits; this is an important effect for the magnitude of the periodic potentials used in this and other experiments[1,2] and will be discussed in a later paper. Secondly, it assumes that electrons complete a cyclotron orbit without scattering. In practice scattering plays an important role and it is necessary to include an extra factor $\exp(-\pi/\omega_c\tau)$ in Eq. (5) to take into account the probability that an electron completes an orbit.

The temperature dependence enters explicitly in Eq. (5) and also via any temperature dependence of the scattering time τ in the extra term. We assume that the temperature dependence of τ is weak and we fit our data for ΔR, the amplitude of the magnetoresistance maxima and minima, to a function of the form $\gamma T\mathrm{csch}[T/T_a(B)]$. These fits are shown in Fig. 2 together with the data for ΔR. The fitted values for γ and $T_a(B)$ for various magnetic fields are shown in Table I together with the values for $T_a(B)$ predicted for the experimental sample. The predicted and fitted values are in excellent agreement.

The advantage of fitting individual maxima and minima, as in Fig. 2, is that for each fit the magnetic field is constant. Equation (5), in agreement with other theoretical predictions,[1,2,5] predicts that ΔR should grow linearly with B. This is neither seen in our data nor those of other authors.[1,2] The reasons for this discrepancy are mentioned above: the effect of the finite potential and the presence of scattering in the 2D EG. Inclusion of the extra term $\exp(-\pi/\omega_c\tau)$ into Eq. (5) allows a quantitative test of the latter effect. Using the fitted values of γ in Table I we can obtain a good fit yielding a scattering rate $\tau^{-1}=2.9\times10^{11}\ \mathrm{s}^{-1}$, a reasonable value for the *total* scattering rate (rather than the momentum relaxation rate) in this wafer. However, despite the good fit which indicates the scattering is important in these devices, we stress that for a full description of the magnetic-field dependence of the amplitude of the oscillations it is necessary to consider the effect of the finite amplitude of the periodic potential in distorting the cyclotron orbits.

We have repeated this experiment and data analysis for samples with a 1D periodic potential with $a=0.3\ \mu$m. In Table II the values of γ and $T_a(B)$ for this sample are tabulated for various values of magnetic field. Once again the predicted and fitted values of $T_a(B)$ are in good agreement. The agreement is better for oscillations at lower magnetic fields. At higher fields the shape is not really sinusoidal (see Fig. 1) and we should include higher harmonics in Eq. (3).

In conclusion, we have measured the temperature dependence of the low-field oscillations in the magnetoresistance of a 2D EG subjected to a 1D periodic potential. The quenching of these oscillations as the temperature is increased may be explained by k_BT smearing at the Fermi energy leading to smearing of the classical cyclotron radius. Excellent agreement is obtained between experiment and theory.

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