Upper critical field in superconductors and the uncertainty principle: Upper limit to the maximum slope of H_{c2}

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Consideration of the formulas that give the upper critical field H_{c2} near the critical temperature T_c for superconductors in the dirty limit, together with the uncertainty principle, is shown to provide an upper limit to the maximum slope of H_{c2}. Analysis of reported values shows that amorphous superconductors are in this limit.

Amorphous superconductors all have a similar slope[1-8] dH_{c2}/dT of around 3 T/K for the upper critical field near the critical temperature T_c. Some crystalline materials show comparable values,[9] and this slope is in general among the highest found in superconductors, with the exception of the "heavy-fermion" superconductors.[10-12]

Using the theory developed by Maki[13] and relations first pointed out by Johnson and Poon[1] for superconductors in the "dirty limit," it is shown in the present Brief Report that this fact can be interpreted as a consequence of the uncertainty principle.

According to Refs. 1 and 2, one can obtain from the calculations of H_{c2} in the dirty limit by Maki[13] the following relation between the diffusivity D and the slope of the upper critical field near T_c

$$D=\frac{4kc}{\pi e}\left(\frac{dH_{c2}}{dt}\right)^{-1}, \tag{1}$$

where k is the Boltzmann constant, c the velocity of light, and e the electron charge. The diffusivity is defined as

$$D=\tfrac{1}{3}v_F l\,, \tag{2}$$

where l is the mean free path and v_F the Fermi velocity. Because of the uncertainty principle one has approximately

$$m^* v_F l \geq \hbar\,, \tag{3}$$

where m^* is the effective mass of the electron in the material. Combining Eqs. (1)–(3) one obtains

$$\frac{dH_{c2}}{dT}\leq\frac{6}{\pi}\frac{k}{\mu_B}\frac{m}{m^*}\,, \tag{4}$$

where we have multiplied and divided by the free electron mass m and introduced the Bohr magneton $\mu_B=e\hbar/2mc$.

Replacing the numerical constants yields the result (in T/K)

$$(dH_{c2}/dt)\leq 2.84(m^*/m) \tag{5}$$

and thus the maximum slope of H_{c2} is seen to be limited by a value close to 3 T/K multiplied by the ratio of the effective mass of the electron to the free-electron mass.

This means that amorphous superconductors, whose mean free path is very small and whose electrons have an effective mass close to that of the free electron, are near the maximum slope that the uncertainty principle allows.

There is a second way of obtaining an expression similar to Eq. (5) using better-known relationships derived from a combination of the macroscopic theory of Ginzburg-Landau (GL) and BCS theory [usually known as Ginzburg-Landau-Abrikosov-Gorkov (GLAG) theory] always keeping to the dirty limit.

One can use the general relation[14]

$$H_{c2}(T)=\frac{\Phi_0}{2\pi\xi(T)^2}\,, \tag{6}$$

where $\xi(T)$ is the GL coherence length, and Φ_0 the flux quantum. In the dirty limit and close to T_c one can further write[14]

$$\xi(T)^2=\xi(0)^2(1-t)\,, \tag{7}$$

with $\xi(0)=0.855\sqrt{\xi_0 l}$, where t is the reduced temperature and ξ_0 is the Pippard coherence length, which in the BCS weak-coupling limit can be written[14]

$$\xi_0=0.18\frac{\hbar v_F}{kT_c}\,. \tag{8}$$

Replacing (7) and (8) in (6) and multiplying and dividing by m^* results in

$$H_{c2}=\frac{\Phi_0 kT_c(1-t)m^*}{2\pi(0.855^2)0.18\hbar v_F l m^*}\,. \tag{9}$$

Again, by the uncertainty principle, $m^* v_F l$ must be greater than $\hbar$, so that

$$H_{c2}\leq\frac{\Phi_0 mk}{0.826\hbar^2}T_c(1-t)\frac{m^*}{m}=2.84(1-t)T_c\frac{m^*}{m} \tag{10}$$

in T/K, which reduces to Eq. (5) if the slope near T_c is calculated.

The limitations of this expression should be stressed. This is only an upper limit on H_{c2}, and other factors such as spin-orbit scattering[15] can lower H_{c2} further. Strictly, the expression is valid only near T_c and in the dirty limit.

Where applicable, Eqs. (5) and (10) mean that the higher the effective mass, the higher the possible slope of H_{c2}. If one were to naively apply GLAG theory, this seems to be the case in heavy fermions, although using the numbers available, they are well below the values allowed by Eq. (5). Taking UBe_{13}, for example, the slope is[11]

$dH_{c2}/dT = 42$ T/K, while the effective mass is quoted as 296 m which means a "possible" slope of 840 T/K, well above the measured value. Something similar[12] happens with UPt_3 and[13] $CeCu_2Si_2$, and it would be interesting to see if lowering the mean free path in these systems (for example, by rapid cooling or by neutron bombardment) could enhance the slope of H_{c2}.

Application of Eqs. (5) or (10) to high-T_c superconductors is problematic. On one hand they are not in the dirty limit because the mean free path is larger than the coherence length,[16] and on the other hand there is controversy[17] and considerable spread on the values reported for H_{c2}.

In conclusion, consideration of the uncertainty principle fixes an upper limit on the maximum slope attainable for H_{c2}. This result is not strictly new in the sense that it is implicit in Refs. 1 and 2 and some well-known formulas of superconductivity, but it may be useful to make it more explicit.

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