Macroscopic quantum tunneling in long Josephson junctions

Herbert Simanjuntak* and Leon Gunther
Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155
(Received 5 February 1990)

Macroscopic quantum tunneling in a long-overlap Josephson junction (JJ) is considered. Expressions for the nucleation rate of the phase of the JJ in the tunneling and thermal-activation regimes are presented.

In the last several years, there has been great interest in the effect of dissipation on macroscopic quantum tunneling (MQT).[1-8] Experimental tests of MQT, however, have been focused on small Josephson junctions[2] (JJ's) and superconducting quantum interference devices[3] (SQUID's). Giordano and Schuler[9] have reported experimental results on quantum nucleation in the current decay of long one-dimensional (1D) superconductors and theoretical studies on this effect have been reported by Shieh *et al.*[10]

In a JJ, the macroscopic variable is the phase of the junction and experiments on MQT are performed by switching the junction, using a current bias, from its metastable zero-voltage state to a nonzero-voltage state. Recently, we have related the expressions for the tunneling and thermal activation rates in a small JJ to the intrinsic I-V characteristic of the JJ to clarify the role of the dynamical resistance in MQT in JJ's.[7]

In calculations of the tunneling rate of the phase of a JJ one usually ignores, for a small junction, self-field effects. For a JJ with a size larger than or on the order of the Josephson penetration depth λ_J, however, self-field effects[11] cause the phase to be space dependent. Interestingly enough, this allows us to study nucleation problems in Josephson junctions which further provides another possible test of MQT in JJ's. Nucleation problems have, of course, been studied theoretically in other systems, such as, to mention a few, the vapor-liquid transition,[12] solids,[13] and ferromagnets.[14-17] See also Refs. 18–20 for a treatment of nucleation in the 1D sine-Gordon model with linear damping, which can thus be applied to shunted junctions. In these papers only the exponent of the quantum-nucleation rate was calculated. The thin-wall approximation was used in Refs. 18 and 19. (This approximation, however, is not very useful for comparison with experiment because it requires the bias current to be much less than the critical current, in which case the nucleation rate will be extremely small.) In Ref. 15 the exponent of the nucleation rate per unit area due to tunneling at temperature $T=0$ was calculated for ferromagnetic films while a detailed calculation of the prefactor was left incomplete. In Ref. 16 a numerical calculation was performed for the exponent of the rate of quantum nucleation in a ferromagnetic film and the exponent was estimated for a bulk ferromagnet. Estimates of the crossover temperature T_o between quantum tunneling and thermal activation were also given in Refs. 15 and 16.

Recently, Ivlev and Melnikov[21] studied nucleation along a string in the absence of dissipation. The mathematical model can be applied to the long JJ. Their focus was on the thin-wall approximation for which the exponent was calculated at $T=0$ and for T close to T_o. The exponent and prefactor were calculated for T above T_o. For nucleation at subcritical bias, which in our case is a subcritical bias current, they calculated the exponent and prefactor for T much greater than T_o, including quantum corrections.

Generally, the effect of dissipation due to external circuitry can complicate the analysis of experiments on MQT in JJ's. Often, JJ's are resistively shunted so as to avoid this problem and make the resistively shunted junction RSJ model applicable. (We look forward to studies on the intrinsic behavior of JJ's whose behavior has been discussed in detail for small junctions in a previous article.[7]) It happens that the main effect of dissipation is to "renormalize" the capacitance.[4,5,7] In this paper we consider an *intrinsic*, long-overlap JJ (Refs. 11 and 22) at subcritical current bias (see below) and present the thermal-nucleation rates of the phase of the junction at temperatures T much higher than the crossover temperature and close to the crossover temperature, as well as the quantum-nucleation rate at $T=0$. We use our model for the intrinsic junction.[7] The prefactor is calculated for all three cases.

We start with nucleation by thermal activation in the classical regime, but with quantum corrections included. The Euclidean action for the phase Φ of an overlap JJ (Refs. 11 and 22) is given by

$$B=W\int_{-L/2}^{L/2}dx\int_{-\beta\hbar/2}^{\beta\hbar/2}d\tau\left[\frac{C}{2}\left(\frac{\hbar}{2e}\frac{\partial\Phi}{\partial\tau}\right)^2+\frac{\lambda_J^2}{2}\frac{\hbar J_c}{2e}\left(\frac{\partial\Phi}{\partial x}\right)^2+V(\Phi)\right]$$
$$+2\frac{\hbar}{L}\int_{-L/2}^{L/2}dx\int_{-\beta\hbar/2}^{\beta\hbar/2}d\tau\int_{-\beta\hbar/2}^{\beta\hbar/2}d\tau'\alpha(\tau-\tau')\sin^2\{[\Phi(x,\tau)-\Phi(x,\tau')]/4\}\,, \quad (1)$$

where the potential $V(\Phi)$ is given by

$$V(\Phi)=\frac{\hbar J_c}{2e}\cos(\Phi)-\frac{\hbar J}{2e}\Phi . \quad (2)$$

Here, τ is the imaginary time variable, $\beta=1/kT$, where k is Boltzmann's constant, C is the capacitance *per unit area*, J_c is the critical current density, and J is the bias current density. λ_J is the Josephson penetration depth and W and L are the transverse dimensions of the junction. In the following, we will consider a large JJ with $L\gg\lambda_J$ and $W\ll\lambda_J$ for simplicity. The dissipation kernel $\alpha(\tau)$ is related to the I-V curve of the JJ (Refs. 4 and 7) (see below).

We next restrict our attention to the case when J is very close to J_c, so that the potential $V(\Phi)$ can be approximated by a quadratic plus a cubic term in Φ and the sine function in Eq. (1) can be approximated by its argument. We introduce the small parameter $\epsilon\equiv 1-(J/J_c)$, the reduced phase

$$\phi=\frac{J}{J_c}\frac{\Phi-\sin^{-1}(J/J_c)}{3(1-J^2/J_c^2)^{1/2}}\simeq\frac{\Phi-\pi/2}{3(2\epsilon)^{1/2}},$$

the effective Josephson penetration depth

$$\lambda_J^*\equiv\lambda_J/(1-J^2/J_c^2)^{1/4}\simeq\lambda_J/(2\epsilon)^{1/4},$$

and the frequency

$$\Omega_o\equiv(2eJ_c/\hbar C)^{1/2}(1-J^2/J_c^2)^{1/4}\simeq(2eJ_c/\hbar C)^{1/2}(2\epsilon)^{1/4}$$

of small oscillations around a local minimum of the potential $V(\Phi)$. Then, in terms of the reduced variables $t=\Omega_o\tau$, $y=x/\lambda_J^*$, $l=L/\lambda_J^*$ and the dimensionless functions

$$u(\phi)=\tfrac{1}{2}\phi^2(1-\phi) \quad (3)$$

and

$$\eta(t)=\frac{\hbar}{4}\frac{1}{CWL(\hbar/e)^2\Omega_o^3}\alpha(\tau=t/\Omega_o), \quad (4)$$

we can rewrite the action as

$$B=b_o\int_{-l/2}^{l/2}dy\int_{-\beta\hbar\Omega_o/2}^{\beta\hbar\Omega_o/2}dt\left[\frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2+\frac{1}{2}\left(\frac{\partial\phi}{\partial y}\right)^2+u(\phi)\right]$$
$$+\tfrac{1}{2}b_o\int_{-l/2}^{l/2}dy\int_{-\beta\hbar\Omega_o/2}^{\beta\hbar\Omega_o/2}dt\int_{-\beta\hbar\Omega_o/2}^{\beta\hbar\Omega_o/2}dt'\eta(t-t')[\phi(y,t)-\phi(y,t')]^2 . \quad (5)$$

where

$$b_o=9\epsilon\hbar W\lambda_J(\hbar CJ_c/2e^3)^{1/2} . \quad (6)$$

In Eqs. (3) and (5) we have shifted the potential so that the minimum is at $\phi=0$.

To find the *thermal-nucleation rate* of ϕ per unit length, we follow the path integral method,[23,24] wherein

$$\Gamma=\frac{2kT_o}{\hbar}\frac{1}{LZ_o}\mathrm{Im}\int D\phi(x,\tau)e^{-B(\phi(x,\tau))/\hbar} . \quad (7)$$

Here, Z_o is the partition function about the minimum of the potential $u(\phi)$.

For nucleation via thermal activation, a "bubble" ϕ has to reach a critical nucleus $\phi_c(y)$ before it can grow further. This critical nucleus is determined by minimizing the action[15,16,23,24] of Eq. (5), with $\phi(y,t)$ taken to be independent of time, which gives the equation

$$\frac{d^2\phi_c(y)}{dy^2}-\frac{du(\phi_c)}{d\phi_c}=0 . \quad (8)$$

As usual, we then consider fluctuations about the critical nucleus $\phi_c(y)$ given by

$$\phi(y,t)=\phi_c(y)+\sum_{n=-\infty;\alpha=0}c_{n,\alpha}Q_\alpha(y)\exp(i\nu_n t), \quad (9)$$

$$\nu_n=2\pi n/\beta\hbar\Omega_o,\ n=0,\pm1,\pm2,\ldots . \quad (10)$$

Then the action equation (5) becomes, up to Gaussian terms,

$$B=B_c+\tfrac{1}{2}b_o\beta\hbar\Omega_o\sum_{n,\alpha}\lambda_{n,\alpha}c_{n,\alpha}c_{-n,\alpha}, \quad (11)$$

where $B_c=B(\phi_c(y))$. The eigenvalues

$$\lambda_{n,\alpha}=\nu_n^2-2\eta_n+k_\alpha \quad (12)$$

are connected with the equation for the eigenvalues k_α and eigenfunctions $Q_\alpha(y)$,

$$\left[-\frac{d^2}{dy^2}+\frac{d^2u(\phi_c(y))}{d\phi_c^2}\right]Q_\alpha(y)=k_\alpha Q_\alpha(y), \quad (13)$$

with the normalization

$$\int_{-\infty}^{\infty}dy\,Q_\alpha(y)Q_\beta(y)=\delta_{\alpha,\beta} . \quad (14)$$

The η_n in Eq. (12) are the Fourier components of $\eta(t)$,

$$\eta(t)=\frac{1}{\beta\hbar\Omega_o}\sum_{n=-\infty}^{\infty}\eta_n\exp(i\nu_n t),\ \eta_o=0, \quad (15)$$

so that effectively[7]

$$\eta_n=-\frac{1}{\pi}\frac{\nu_n}{CWL\Omega_o}\left[\frac{1}{R_q}\cot^{-1}\left(\frac{\hbar\Omega_o\nu_n}{2\Delta}\right)+\frac{1}{R_N}\tan^{-1}\left(\frac{\hbar\Omega_o\nu_n}{2\Delta}\right)\right], \quad (16)$$

where R_q and R_N are respectively the quasiparticle and normal resistances of the JJ and Δ is the energy gap of the superconductors, being considered identical on both sides of the JJ.

The action equation (11) is now substituted into the path integral equation (7) so as to obtain the nucleation rate per unit length. The integral over the negative eigenvalue $\lambda_{0,0}$ as usual[23,24] must be continued into the complex plane, which results in a factor $i/2(\beta\hbar\Omega_o b_o|\lambda_{0,0}|)^{1/2}$, while the integration over the eigenvalue $\lambda_{0,1}=0$ produces a factor $(B_c/2\pi\hbar)^{1/2}l/[(\beta\hbar\Omega_o b_o)^{1/2}]$. The remaining integrals and Z_o are Gaussian integrals and can be performed directly so as to obtain the thermal-nucleation rate *per unit length*

$$\Gamma=\frac{kT_o}{\hbar}\frac{1}{\lambda_J^*}\left(\frac{A}{2\pi}\right)^{1/2}e^{-A}f_1, \quad (17)$$

where $A=B_c/\hbar$ and

$$f_1=\left(\frac{\prod_{\alpha=0}\lambda_{0,\alpha}^o}{\prod'_{\alpha=0}|\lambda_{0,\alpha}|}\right)^{1/2}\left(\prod_{n=1,\alpha=0}\frac{\lambda_{n,\alpha}^o}{\lambda_{n,\alpha}}\right) . \quad (18)$$

The prime on the product omits the zero eigenvalue $\lambda_{0,1}=0$. The eigenvalues $\lambda^o_{n,\alpha}$ are given by

$$\lambda^o_{n,\alpha}=\nu_n^2-2\eta_n+k^o_\alpha+1\ , \quad (19)$$

where the k^o_α satisfy the equation

$$\left[-\frac{d^2}{dy^2}\right]Q^o_\alpha(y)=k^o_\alpha Q^o_\alpha(y)\ . \quad (20)$$

The critical nucleus $\phi_c(y)$ of Eqs. (3) and (8) is $\phi_c(y)=\mathrm{sech}^2(y/2)$, which, along with its action $B_c(\phi(y))$, leads to

$$A=\frac{B_c}{\hbar}=\frac{12}{5}(2\epsilon)^{5/4}\frac{\hbar J_c W\lambda_J}{ekT}\ . \quad (21)$$

Note that $kT_o/\hbar\sim\Omega_o$ [see Eq. (24) below] and that the barrier energy $U_B=kTA$. The factor f_1 reflects quantum corrections in the classical regime and is given by Eq. (18). The calculation of f_1 can be done using the method of Ref. 23. The first factor (with $n=0$) in Eq. (18) gives $2\sqrt{15}$, while the second factor (over $n\neq0$ and α) diverges as a result of the breakdown of the model Lagrangian at small wavelengths.[23] We therefore need to introduce a small wavelength cutoff which results in

$$f_1=2\sqrt{15}\exp\left[6\left[\frac{\hbar\Omega_o}{2\pi kT}\right]\frac{\ln N}{(1+\hbar/\pi R_N CWL\Delta)^{1/2}}\right]\ . \quad (22)$$

The cutoff N is given by

$$N=\left[\frac{\hbar\Omega_o}{2\pi kT}\right]\frac{\lambda_J^*}{\xi_c}\ , \quad (23)$$

where ξ_c is on the order of the BCS coherence length of the superconductor. Equation (17) with f_1 in Eq. (22) is valid for $T_o\ll T\ll T_c$ (T_c is the transition temperature of the superconductor), so that we can ignore the $1/R_q$ term in Eq. (16), and for low damping ($\hbar/R_N CWL\Delta\ll1$).

To make a comparison with small junctions, the exponent A in Eq. (21) is $[\frac{36}{5}(\lambda_J^*/L)]$ times that for small junctions.[7] Furthermore, for small junctions the temperature dependence of the factor f_1 goes simply like[7] $\exp(\mathrm{const}/T^2)$.

Next we find the *crossover temperature* T_o between nucleation due to quantum tunneling and thermal activation, which is determined by the equation $\lambda_{1,0}=0$. With Eq. (16), as long as $(\hbar\Omega_o/2\pi)\ll\Delta$, the crossover temperature is approximately given by

$$kT_o=\frac{\sqrt{5}}{2}\frac{\hbar\Omega_o}{2\pi}\left[1+\frac{\hbar}{\pi R_N CWL\Delta}\right]^{-1/2}\ , \quad (24)$$

for all $\hbar/\pi R_N CWL\Delta$. This crossover temperature is $\sqrt{5}/2$ times that for small junctions.[5,7]

For T slightly greater than or equal to T_o ($0\le T-T_o\ll T_o$), the nucleation rate *per unit length* is given by

$$\Gamma=9(2\epsilon)^{3/2}\frac{J_c WT_o}{eT}\frac{\exp(x^2)}{\sqrt{b}}[1-\mathrm{erf}(x)]e^{-A}f_2\ , \quad (25)$$

where

$$x=3(2\epsilon)^{5/8}\left[\frac{\hbar J_c W\lambda_J}{2ekTb}\right]^{1/2}\left[\left[\frac{2\pi kT}{\hbar\Omega_o}\right]^2-2\eta_1-\frac{5}{4}\right] \quad (26)$$

and

$$b=\left[\frac{105\pi}{128}\right]^2\left[\frac{15}{32}\right]^3\left[\frac{8}{5}-\frac{1}{\lambda_{2,0}}\right]\ . \quad (27)$$

In Eq. (27) $\lambda_{2,0}=\nu_2^2-2\eta_2-\frac{5}{4}$, with η_n given by Eq. (16). The factor f_2 in Eq. (25) is given by

$$f_2=\left[\frac{\prod_{n=1;\alpha=0}\lambda^o_{n,\alpha}}{\prod'_{n=1;\alpha=0}\lambda_{n,\alpha}}\right]\ , \quad (28)$$

where the prime here now omits the eigenvalue $\lambda_{1,0}$. Equation (28) also diverges and therefore needs a cutoff $N'\sim N$, so that for low damping

$$f_2=\tfrac{1}{2}(r+\tfrac{3}{2})\frac{(2r+1)(r+1)(2r+3)}{(2r-1)(r-1)}\times\exp\left[6\left[\frac{\hbar\Omega_o}{2\pi kT}\right]\frac{\ln N'}{(1+\hbar/\pi R_N CWL\Delta)^{1/2}}\right]\ , \quad (29)$$

where $r=[\frac{5}{4}(T/T_o)^2+1]^{1/2}$.

We now turn to the problem of nucleation due to *quantum tunneling* in an overlap JJ. In the following discussion we will only consider $T=0$. At $T=0$, the main effect of the dissipation term (i.e., the last term) in Eq. (1) is to "renormalize" the capacitance, so that we can approximate the action equation (5) by[4,5,7]

$$B=b_o^*\int_{-l/2}^{l/2}dy\int_{-\infty}^{\infty}dt\left[\frac{1}{2}\left[\frac{\partial\phi}{\partial y}\right]^2+\frac{1}{2}\left[\frac{\partial\phi}{\partial y}\right]^2+u(\phi)\right]\ , \quad (30)$$

where b_o^* is given by Eq. (6) but with C now replaced by

$$C^*=C+\frac{\hbar}{\pi R_N WL\Delta}\ . \quad (31)$$

The usual path integral method[23,24] leads to the nucleation rate per unit length

$$\Gamma=\frac{\Omega_o^*}{\lambda_J^*}\frac{B_c}{2\pi\hbar}\exp(-B_c/\hbar)f_o\ , \quad (32)$$

where

$$\Omega_o^*\equiv(2eJ_c/\hbar C^*)^{1/2}(1-J^2/J_c^2)^{1/4}\simeq(2eJ_c/\hbar C^*)^{1/2}(2\epsilon)^{1/4}\ .$$

The action B_c is given by Eq. (30) with $\phi=\phi_c$ as the solution of the equation

$$\frac{d^2\phi_c}{d\rho^2}+\frac{1}{\rho}\frac{d\phi_c}{d\rho}-(\phi_c-\tfrac{3}{2}\phi_c^2)=0\ , \quad (33)$$

where $\rho=(y^2+t^2)^{1/2}$. The factor f_o in Eq. (32) is given by

$$f_o=\left[\frac{\prod_{n,\alpha}\Lambda^o_{n,\alpha}}{\prod'_{n,\alpha}|\Lambda_{n,\alpha}|}\right]^{1/2}\ , \quad (34)$$

where $\Lambda_{n,\alpha}$ and $\Lambda^o_{n,\alpha}$ are now the eigenvalues of their respective equations

$$\left[-\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial y^2}+\frac{\partial^2 u(\phi_c)}{\partial\phi_c^2}\right]\psi_{n,\alpha}(y,t)=\Lambda_{n,\alpha}\psi_{n,\alpha}(y,t) \quad (35)$$

and

$$\left[-\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial y^2}+1\right]\psi^o_{n,\alpha}(y,t)=\Lambda^o_{n,\alpha}\psi^o_{n,\alpha}(y,t)\ . \quad (36)$$

The prime on the product in the denominator of Eq. (34)

now omits the two zero eigenvalues of Eq. (35).

We have solved Eq. (33) numerically and used the result in Eq. (30) to give B_c in Eq. (32) as

$$B_c = 15.5\hbar\epsilon \left[\frac{2\hbar C^* J_c}{e^3} \right]^{1/2} W\lambda_J , \qquad (37)$$

which is $(6.5\lambda_J^*/L)$ times that for small JJ's.[7] The product f_o can be calculated by the method of Ref. 23. It also needs a small wavelength cutoff so that the leading order goes like

$$f_o = c_1 \exp\left[c_2 \ln\left[c_3 \frac{\lambda_J^*}{\xi_c} \right] \right] , \qquad (38)$$

where c_1, c_2, and c_3 are constants.[25]

To summarize, we have presented the calculation of the quantum-tunneling and thermal-nucleation rates of the phase ϕ of a long overlap JJ. It would be interesting, therefore, to see experimentally the variation of the nucleation rate with the size of the junction. Consequently, we are currently extending our calculations to the case of long but finite junctions, in which case nucleation at the ends of the junction can be dominant. It is very important to note that our results hold only for a perfectly homogeneous JJ. Modest inhomogeneities will lead to strong nucleation centers. The results for the thermal nucleation rates in Eqs. (17) and (25) can be used for a fit of the critical current density J_c, which is needed in order to check experimentally the validity of our theoretical prediction for quantum nucleation at $T=0$. In the RSJ model, η_n is replaced by $[-|\nu_n|/(2RCWL\Omega_o)]$, where R is the resistance in the model. T_o is given by $[\hbar\Omega_o/(2\pi k)]\{\frac{1}{2}[5+(RCWL\Omega_o)^{-2}]^{1/2}-(2RCWL\Omega_o)^{-1}\}$. The calculation of the nucleation rate in this model is complicated and is left for future study. Finally, the method presented in this paper can be applied straightforwardly to calculate the nucleation rate of other systems, such as ferromagnetic films and bulk ferromagnets.

We are grateful to Daniel Estève for his worthwhile criticism of our paper, and for pointing out a mathematical error.

*Present address: Jurusan Fisika, Fakultas Maternatika dan Ilmu Pasti Alam, Universitas Indonesia, Depok, Jawa Barat, Indonesia.

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[25] It is difficult to calculate f_o since we only have the numerical solution of ϕ_c. However, if we approximate the solution to $\phi_c = \mathrm{sech}^2(\rho/2)$ [the solution of Eq. (33) without the second term] that the constants are approximately given by $c_1 = 240\sqrt{3}\exp(-8.22)$ and $c_2 = 12/\sqrt{5}$.