PHYSICAL REVIEW B VOLUME 42, NUMBER 14 15 NOVEMBER 1990-I

Chaotic acoustoelectric oscillations in InSb in a magnetic field

C. W. Skorupka, L. M. Pecora, T. L. Carroll, and T. M. Tritt
Naval Research Laboratory (Code 6341), Washington, D.C. 20375-5000
(Received 5 June 1990; revised manuscript received 21 August 1990)

We report the first experimental time-series analysis of the noisy acoustoelectric voltage oscillations in dc-current-driven InSb in a transverse magnetic field. We show that these irregular oscillations, originally thought to be random noise, are actually deterministic in origin. The fractal dimensions of typical oscillations are presented. We have observed deterministic chaos from 20 to 230 K. We find that the chaotic regime coincides with the freezeout of the intrinsic carriers, and that the threshold for chaos as a function of temperature scales with the magnetoresistance.

Recently there has been much interest in chaos in semiconductors.[1] Several systems[2-4] (Si, Ge, GaAs, respectively) under a variety of external conditions (temperature, magnetic field, ac and dc bias) have been shown to exhibit deterministic chaos. Here we report on a novel mechanism for chaos in the narrow-band-gap semiconductor inductor indium antimonide (InSb).

In the 1960's a great deal of effort was devoted to the study of noisy radio-frequency oscillations in InSb. These oscillations (often associated with microwave emission) can be observed at liquid-nitrogen temperatures under a variety of conditions of electric (E) and magnetic (B) field magnitude and orientation.[5] There are two regimes where oscillations can be observed: at high E ($E > 200$ V/cm) and at low E ($E < 10$ V/cm). The high-field oscillations were attributed to a variety of underlying mechanisms, including plasma and avalanche instabilities. We have focused upon the low-E-field regime, where the oscillations are due to the acoustoelectric effect.[6] In the acoustoelectric effect, amplification of acoustic waves in a piezoelectric crystal occurs when the drift velocity of the carriers is greater than the velocity of sound. Thus there is a threshold for ac response in the applied dc current or electric field. The noisy oscillations in InSb have been attributed to the amplification of thermally generated acoustic waves, and thus were thought to be random in origin.

In this paper we present experimental evidence that the low-E-field acoustoelectric current oscillations in InSb are *not* random noise but deterministic chaos. The threshold for chaos as a function of magnetic field (B), current (I), and temperature (T) is reported, and is found to scale with the freezeout of intrinsic carriers and the magnetoresistance. Previous work on InSb by Seiler *et al.*[7] used power spectra to study chaotic oscillations arising from impact ionization of magnetically frozen-out donor states at below $T = 10$ K. To the best of our knowledge, however, ours is the first time series analysis of chaotic oscillations in InSb, and the first experimental report of deterministic chaos being associated with the acoustoelectric effect in *any* material.

Nominally n-type doped (carrier concentration $n \sim 3 \times 10^{14}$ cm^{-3}) InSb samples, with typical sample dimensions of $5.0 \times 1.0 \times 0.4$ mm^3 were mounted in a four-probe resistance configuration. A constant current source was used to supply current to the samples. Both tin and indium solders were used for contacts to copper leads. A magnetic field transverse to the current direction was applied to the samples, which were cooled in a Janis Varitemp Dewar. Time series of the voltage oscillations were recorded with an 8-bit LeCroy digitizer, sampling at a rate of 200 MHz. Time series as long as 128 000 points were recorded. We studied the threshold for chaos with respect to T, I, and B by holding two parameters constant and varying the third.

We have observed chaotic and periodic oscillations in a temperature range from ~ 20 to ~ 230 K. The frequencies of these oscillations were in the range of ~ 1 to ~ 10 MHz. The power spectra of typical chaotic and periodic wave forms are presented in Fig. 1.

Different routes to chaos can be observed under different conditions of T, I, and B. Most often the transi-

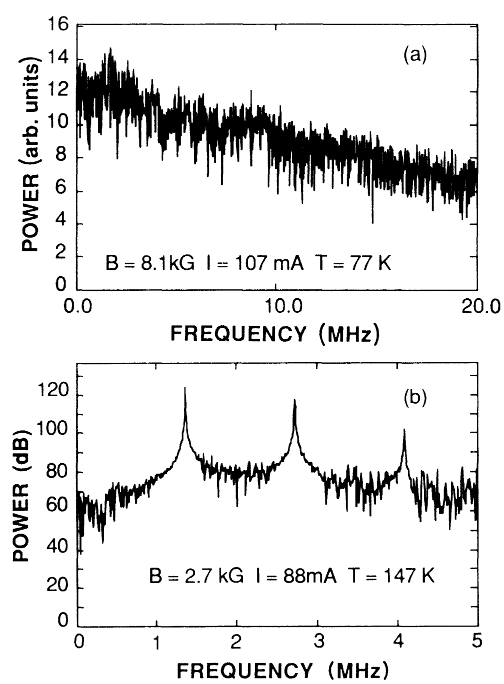

FIG. 1. Power spectra of (a) chaotic and (b) periodic time series.

tion was directly from the steady state to chaos, indicative of the crisis scenario, but at low-B fields it was often possible to observe periodic oscillations for values of current near the threshold of chaos. Occasionally a doubling of the period of such an oscillation could be induced by a small change of parameter, indicative of a period doubling route to chaos.[8] The amplitude of the ac response rises rapidly above the background noise, from a few multivolts to a few tens of multivolts as the applied current is increased a few multiamperes above threshold, eventually becoming as large as ~ 100 mV. The maximum amplitude depended on the temperature and the sample. For some samples it was possible to observe an upper threshold curve of I and B beyond which no oscillations, either chaotic or periodic, could be observed. Due to the presence of ambient rf background noise in our system, small amplitude acoustoelectric oscillations might be buried in the noise, and so a detailed study of the route to chaos was not possible. Measurements using a lower-noise apparatus are being performed and will be reported at a later date.

Along with period doubling and broad band power spectra, an estimate of a finite fractal dimension of the attractor can help distinguish deterministic chaos from random noise (although certain examples of colored noise can exhibit finite correlation dimension[9]). Here we calculate the dimension using the nearest-neighbor method of Termonia and Alexandrowicz.[10] We chose delay times in keeping with the criteria of,[11] i.e., the first minima in the correlation function, which corresponded typically to 25–50-ns delay times. We did not find that our dimension calculations were sensitive to the particular choice of delay time. The attractor dimensions of a typical chaotic time series and a periodic time series are plotted as a function of embedding dimension in Fig. 2. For comparison the dimension for a sample of the experimental background noise (ambient rf pickup) is plotted. This noise is ever present in the experiment, and thus constitutes a "noise floor" which limits the resolution of the measurements. The attractor dimension of a random signal is equal to its embedding dimension, whereas for a deterministic signal the attractor dimension will saturate. The calculated dimensions of the periodic oscillation and the chaotic oscillation begin to saturate to values of approximately 1 and 2.7–3.3, respectively, while for the background noise the calculated dimension is greater than 6. There have been few efforts to solve either analytically or numerically the nonlinear partial differential equations which govern the acoustoelectric effect.[12] The evidence that the broad band oscillations are deterministic in nature clearly emphasizes the need for such solutions.

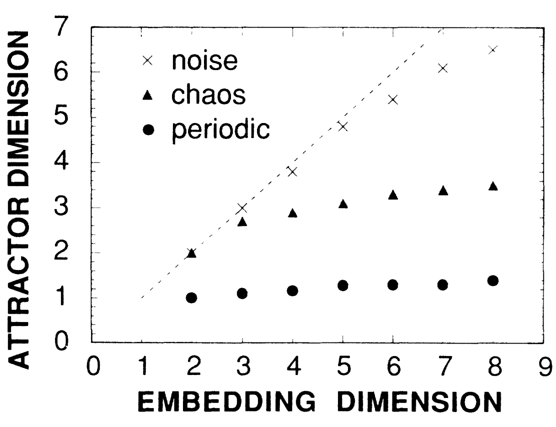

FIG. 2. Attractor dimension vs embedding dimension for typical chaotic and periodic time series, with a sample of the experimental background noise shown for comparison. Calculated dimensions of deterministic attractors saturate, while dimension of background noise does not. The dashed line is a guide to the eye for the expected ideal behavior of a high dimensional or random system.

In Fig. 3 the threshold current versus magnetic field for the transition from steady state directly to broad band chaos is plotted for a typical sample. No periodic oscillations were observed for that sample at that temperature. The threshold was estimated by choosing the values of I and B at which the acoustoelectric signal attained an arbitrary small amplitude, in this case twice the background noise level. We find that the product of I and B (Π) at threshold for a given temperature is approximately a constant. We plot Π as a function of temperature in Fig. 4. For comparison, the number of carriers and the magnetoresistance are presented. Above ~ 175 K the threshold for chaos coincides with the freeze-out of the intrinsic carriers, while below 175 K Π scales with the magnetoresistance. We account for this temperature dependence below.

The ac response due to acoustoelectric amplification[5,6] can occur when the drift velocity exceeds the velocity of wave propagation. The fact that the product of the mobility of our samples ($\mu > 10^5$ cm^2/V s) and threshold E field for chaos (~ 5 V/cm) yield drift velocities greater than the velocity of sound (2.35×10^5 cm/s) is consistent with such a description. Steele[13] developed a linear, small signal theory which demonstrated that a transverse magnetic field can lower the minimum drift velocity necessary for acoustic wave gain. The expression for the gain α as a function of current density and transverse magnetic field is

$$\alpha \approx \tfrac{1}{2} K^2 \left(\frac{j}{nev_s} - 1 \right) [1 + (\omega_c \tau)^2] \tau_D \omega ,$$

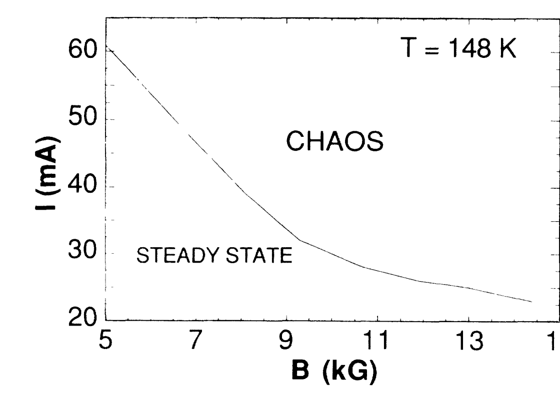

FIG. 3. Threshold for transition from steady state to chaos as a function of current and transverse magnetic field.

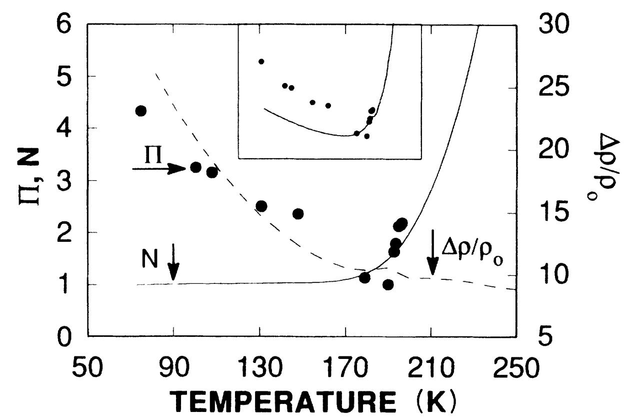

FIG. 4. Π, the product of current and transverse magnetic field at the threshold for chaos, plotted as a function of temperature (here Π is normalized to 1 at its minimum value). For comparison the carrier concentration N (normalized to 1 at $T=77$ K) and the magnetoresistance $\Delta\rho/\rho_0$ are shown. The inset is a comparison to a linear gain theory (solid line).

where K is the piezoelectric coupling constant, τ is the momentum relaxation time, ω_c is the cyclotron frequency, τ_D is the dielectric relaxation time, j is the current density, n is the number of carriers, v_s is the velocity of sound, and ω is the frequency of the acoustic wave. By inspection we see that the gain will increase as the carriers freeze out, and as the magnetic field is increased. In the inset of Fig. 4 we compare the temperature dependence of Π at the threshold for chaos with Π required for a constant gain from the above expression. We find that there is reasonable qualitative agreement between theory and experiment particularly above 175 K, suggesting that the gain at the threshold for chaos is roughly the same at all temperatures. The predicted threshold-temperature slope is markedly lower than the experimental measurement below 175 K. Thus, while the linear theory for constant gain can be used to estimate a lower bound for the threshold for chaos, such as analysis is inadequate for a full understanding of the threshold behavior.

In conclusion, this is the first time-series analysis of the noisy oscillations of the low-field instability in InSb. Our calculations of the attractor dimensions for these oscillations show that they are not random noise but low-dimensional, deterministic chaos. Our study of the temperature dependence of the threshold for chaos reveals that this threshold scales with the carrier concentration and the magnetoresistance. Comparison with a linear gain theory shows that a constant acoustoelectric gain sets a lower bound for the threshold for chaos. This suggests that the gain is an important parameter for determining the dynamics of this system. The behavior of the threshold for chaos and the fact that the acoustoelectric oscillations are deterministic rather than random in nature underscores the need for a reexamination of the acoustoelectric effect in terms of a full nonlinear analysis.

We wish to thank J. Davis and P. Thompson for supplying us with samples, D. King for aid in sample preparation, and R. Westervelt, S. Teitsworth, and X. N. Song for helpful discussions. We also wish to acknowledge the U.S. Office of Naval Technology for partial financial support.

[1]See, for example, R. P. Huebener, J. Peinke, and J. Parisi, Appl. Phys. A **48**, 107 (1989); Y. Abe, in *Proceedings of the Fifth International Conference on Hot Carriers in Semiconductors, Boston, Massachusetts, 1987,* edited by J. Shah and G. J. Iafrate [Solid State Electron. **31**, 795 (1988)].
[2]K. Yamada, N. Miura, and C. Hamaguchi, Appl. Phys. A **48**, 149 (1989).
[3]S. W. Teitsworth and R. M. Westervelt, Phys. Rev. Lett. **53**, 2587 (1984).
[4]A. Brandl, W. Kroninger, and W. Prettl, Phys. Rev. Lett. **64**, 212 (1990).
[5]See, for example, A. C. Baynham and A. D. Boardman, *Plasma Effects in Semiconductors* (Barnes and Noble, New York, 1971), or J. Pozhela, *Plasma and Current Instabilities in Semiconductors* (Pergamon, New York, 1981).
[6]D. L. White, J. Appl. Phys. **33**, 2547 (1962).
[7]D. G. Seiler, C. L. Littler, and R. J. Justice, Phys. Lett. **108A**, 462 (1985); X. N. Song, D. G. Seiler, and M. R. Loloee, Appl. Phys. A **48**, 137 (1989).
[8]P. Berge, Y. Pomeau, and C. Vidal, *Order within Chaos* (Wiley, New York, 1984).
[9]A. Osborne and A. Provenzale, Physica D **35**, 357 (1989).
[10]Y. Termonia and Z. Alexandrowicz, Phys. Rev. Lett. **51**, 1265 (1983).
[11]W. Liebert and H. Schuster, Phys. Lett. A **142**, 107 (1989).
[12]K. Nakamura, Phys. Lett. A **120**, 286 (1987).
[13]M. C. Steele, RCA Rev. **28**, 58 (1967).