dc Josephson current for very strong coupling

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We have calculated the dc Josephson critical current $J_c(T)$ for various coupling strengths covering weak, intermediate, strong, and very strong coupling regimes. We consider the temperature T variation of $J_c(T)$ and its deviation from BCS values near T_c and at zero temperature. The deviations can be quite significant but are too small to explain a recent experiment.

I. INTRODUCTION

Recently, Blezius *et al.*[1] have given results for the London penetration depth of a superconductor as a function of strong-coupling index T_c/ω_{ln} spanning the weak, intermediate, strong, and very strong coupling regime. The fundamental parameter T_c/ω_{ln} is the ratio of the critical temperature T_c to the characteristic boson energy ω_{ln} associated with the electron-boson spectral density. This energy was first introduced by Allen and Dynes[2] in the case of phonon superconductors. Similar, but much less extensive results were also given by Rammer.[3]

In this paper, we wish to consider the dc Josephson[4] critical current $J_c(T)$ which is also closely related to the local-limit penetration depth $\lambda_l(T)$.[5-9] In Sec. II, we give the necessary formulas for J_c and λ_l and the relationship between them. Numerical results are given in Sec. III, and a short conclusion can be found in Sec. IV.

II. FORMALISM

The formula for the local-limit penetration depth $\lambda_l(T)$ is given by[5-9]

$$\lambda_l(T)=\left[4\pi\sigma_N T\sum_{n=1}^{\infty}\frac{\Delta^2(i\omega_n)}{\omega_n^2+\Delta^2(i\omega_n)}\right]^{-1/2}, \tag{1}$$

where $\Delta(i\omega_n)$ is the nth Matsubara gap associated with the Matsubara frequency $i\omega_n\equiv i(\pi T)(2n-1)$, $n=0, \pm1, \pm2, \ldots,$ and T is the temperature. In Eq. (1), σ_N is the normal-state conductivity. The dc Josephson critical current is proportional to $\lambda_l^{-2}(T)$ and so[5-9]

$$\frac{J_c(T)}{J_c(0)}=\left[\frac{\lambda_l(0)}{\lambda_l(T)}\right]^2. \tag{2}$$

To evaluate Eqs. (1) or (2), it is necessary to have numerical solutions of the Eliashberg equations which are a set of two coupled equations for $\Delta(i\omega_n)$ and the renormalization factors $Z(i\omega_n)$. They are[10,11]

$$\Delta(i\omega_n)Z(i\omega_n)=\pi T\sum_{m=-\infty}^{\infty}[\lambda(i\omega_n-i\omega_m)-\mu^*(\omega_c)\theta_c(\omega_c-|\omega_m|)]\frac{\Delta(i\omega_m)}{[\omega_m^2+\Delta^2(i\omega_m)]^{1/2}} \tag{3}$$

and

$$Z(i\omega_n)=1+\frac{\pi T}{\omega_n}\sum_{m=-\infty}^{\infty}\lambda(i\omega_n-i\omega_m)\frac{\omega_m}{[\omega_m^2+\Delta^2(i\omega_m)]^{1/2}}. \tag{4}$$

The electron-boson spectral density $\alpha^2F(\nu)$ appears through the relation

$$\lambda(Z)=\int_0^{\infty}\frac{2\nu d\nu\alpha^2F(\nu)}{\nu^2-Z^2}, \tag{5}$$

and $\mu^*(\omega_c)$ is the Coulomb pseudopotential with cutoff ω_c. In much of what follows we will use a δ function for the spectral density centered at the Einstein frequency ω_E with weight A

$$\alpha^2F(\omega)=A\delta(\omega-\omega_E). \tag{6}$$

Results will be quoted as a function of T_c/ω_{ln} with $\omega_{ln}\equiv\omega_E$ for T_c/ω_{ln} up to 4. Such curves are completely independent of A. Also, for convenience, we will take $\mu^*=0$, a restriction which is of no importance qualitatively. In the conventional strong-coupling region with $0.0\le T_c/\omega_{ln}\lesssim 0.2$, we will also quote some results for real materials and give a semiphenomenological form that fits the data reasonably well.

III. NUMERICAL RESULTS

The temperature dependence of $J_c(T)/J_c(0)$ is exhibited in Fig. 1 for BCS (solid line) and five finite values of T_c/ω_{ln} namely 0.115 (intermediate coupling) 0.333 (strong coupling), and 1.092 (very strong coupling) as well as 4.083 and 7.307. This last case corresponds to a rather small value of ω_{ln} even for a T_c of 96 K. We note from Fig. 1 that the temperature dependence of this reduced quantity is not so different from BCS even in extreme strong coupling, except perhaps at low T/T_c where it is seen that the two last curves ($T_c/\omega_{ln}=4.083$ and 7.307) are flat only for a very small temperature interval. This is unexpected since we are dealing with s-state superconductivity. The argument for a large flat region in this case is that the deviations vary exponentially with the ex-

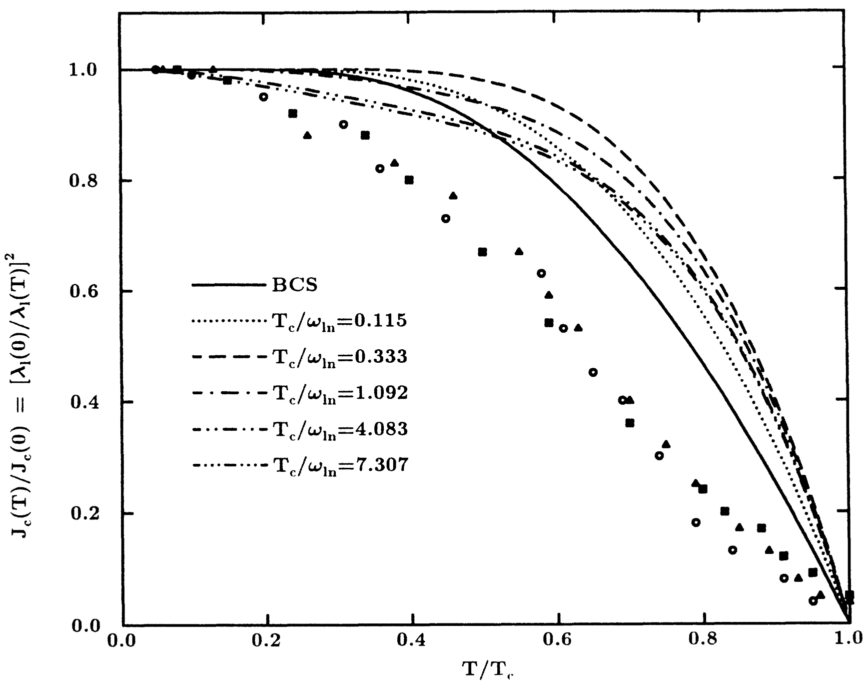

FIG. 1. Temperature variation of the reduced Josephson critical current $J_c(T)/J_c(0) \equiv [\lambda_l(0)/\lambda_l(T)]^2$ for several values of coupling strength T_c/ω_{ln}, namely 0.0 (BCS), 0.115, 0.333, 1.092, 4.083, and 7.307. Also shown are recent data obtained on $YBa_2Cu_3O_7$ by Mannhart *et al.*[14]

ponent gap over the temperature.

It is convenient in presenting further results to introduce deviations from BCS according to the definition[10]

$$B(T) \equiv B^{\rm BCS}(T)\eta_B(T)\,, \qquad (7)$$

where B is any quantity of interest and $B^{\rm BCS}$ is its BCS value. We will be interested here in $\eta_{\lambda_l}(0)$ and $\eta_{\lambda_l}(T_c)$ presented in Fig. 2, solid and dashed curves, respectively, as a function of T_c/ω_{ln}. Since the limit $T_c/\omega_{ln} \rightarrow 0$ corresponds to the BCS limit, η is one in this case. As the coupling increases, $\eta_{\lambda_l}(0)$ grows steadily to achieve a value of approximately 1.45 at $T_c/\omega_{ln}=4.0$. On the other hand, $\eta_{\lambda_l}(T_c)$ exhibits a more complex behavior. It first decreases below one towards a minimum of 0.90 before increasing to be over 1.10 at $T_c/\omega_{ln}=4.0$. So far we have presented results only for a δ function spectral density. In conventional superconductors, the electron-phonon spectral density $\alpha^2F(\omega)$ is often known from inversion of tunneling data.[12] In that case, $\eta_{\lambda_l}(T)$ can be calculated directly from the Eliashberg equations. We have done this for V, In, Nb(*A*), V_3Si, La, Pb, Hg, $Nb_3Ge(1)$, and $Pb_{0.8}Bi_{0.2}$ (Ref. 13) as representative systems and our numerical results are presented in Fig. 3 (solid circles) in the order listed above which is also the order of increasing T_c/ω_{ln}. A phenomenological visual fit to the curves in Fig. 3 (solid curves) are

$$\eta_{\lambda_l}(0) = \left[1 + 5\left(\frac{T_c}{\omega_{ln}}\right)^2 \ln\left(\frac{\omega_{ln}}{3.8T_c}\right) + 0.4\left(\frac{T}{\omega_{ln}}\right)\right], \qquad (8)$$

$$\eta_{\lambda_l}(T_c) = \left[1 - 2.5\left(\frac{T_c}{\omega_{ln}}\right)^2 \ln\left(\frac{\omega_{ln}}{1.9T_c}\right)\right]. \qquad (9)$$

For details of the spectra used, the reader is referred to Ref. 13 where forms similar to those used here [Eqs. (8) and (9)] are derived for thermodynamic properties.

If one is interested in the dc Josephson current instead of the penetration depth, the quantities of interest are

$$\eta_{J_c}(0) \equiv [\eta_{\lambda_l}(0)]^{-2}\,, \qquad (10)$$

$$\eta_{J_c}(T_c) \equiv [\eta_{\lambda_l}(T_c)]^{-2}\,, \qquad (11)$$

which are presented in Fig. 4, solid and dashed curves, respectively. It is of interest to note that $\eta_{J_c}(0)$ initially

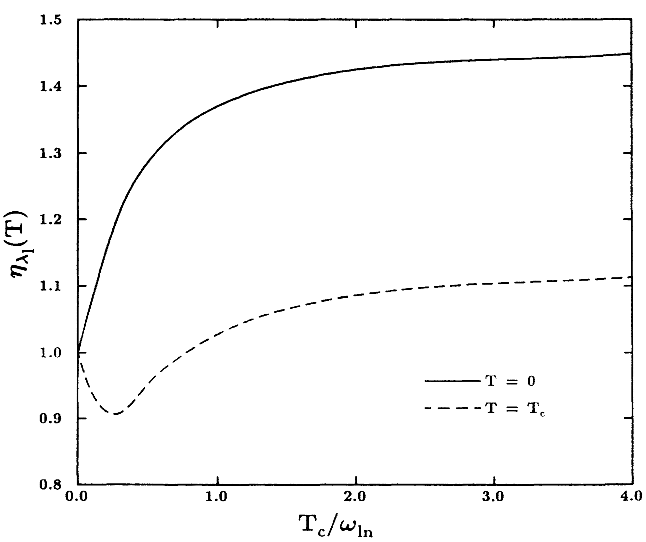

FIG. 2. Strong-coupling correction $\eta_{\lambda_l}(T)$ for the local limit penetration depth $\lambda_l(T)$ at $T=0$ (solid curve) and $T=T_c$ (dashed curve), as a function of coupling strength T_c/ω_{ln}.

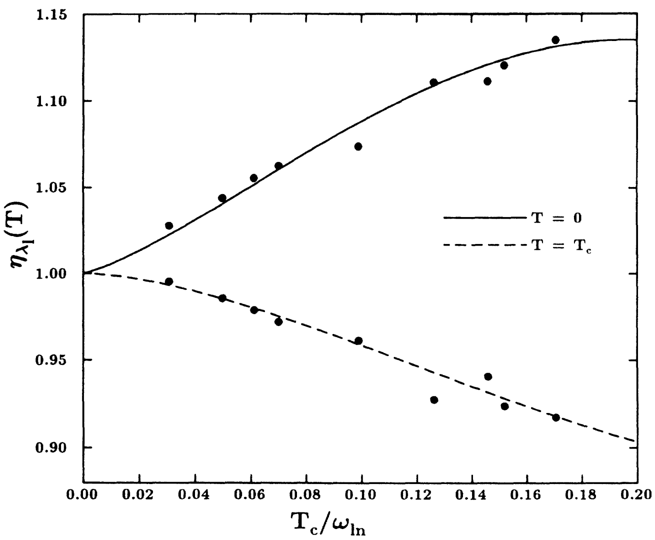

FIG. 3. The strong-coupling correction $\eta_{\lambda_l}(T)$ for $T=0$ (solid curve) and $T=T_c$ (dashed curve) for real materials (solid circle). The curves are the best visual fit to the data that we could get and are given by Eqs. (8) and (9), respectively.

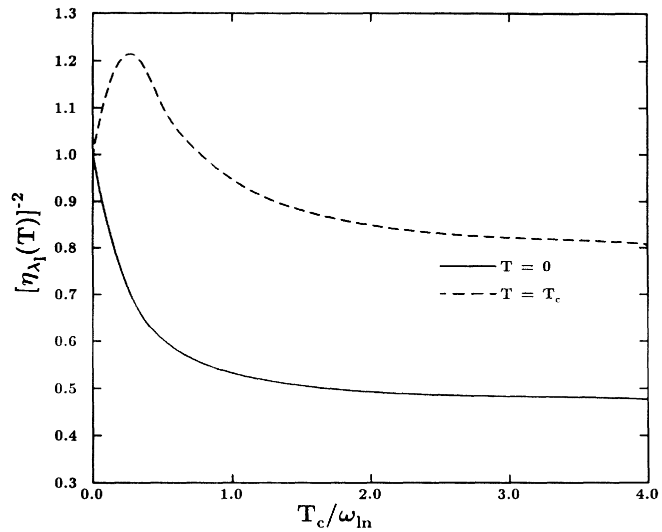

FIG. 4. Strong-coupling corrections $\eta_{J_c}(T)$ for the dc Josephson critical current $J_c(T)$ at $T=0$ (solid curve) and $T=T_c$ (dashed curve), as a function of coupling strength T_c/ω_{ln}.

drops very rapidly as T_c/ω_{ln} increases from zero, but then it levels off for higher values of T_c/ω_{ln}, attaining a magnitude somewhat below 0.5 at $T_c/\omega_{ln}=4.0$. Even for very strong coupling, the reduction from BCS is significant but not very large.

Finally, a fit to the real materials data presented in Fig. 3 gives the following form in the region $0.0\leq T_c/\omega_{ln} \lesssim 0.2$:

$$\eta_{J_c}(0)=\left[1-14.0\left(\frac{T_c}{\omega_{ln}}\right)^2\ln\left(\frac{\omega_{ln}}{4.3T_c}\right)-0.5\left(\frac{T_c}{\omega_{ln}}\right)\right] \tag{12}$$

and

$$\eta_{J_c}(T_c)=\left[1-5.0\left(\frac{T_c}{\omega_{ln}}\right)^2\ln\left(\frac{\omega_{ln}}{1.6T_c}\right)\right]. \tag{13}$$

Returning to Fig. 1, we have placed recent data on it $J_c(T)$ for $YBa_2Cu_3O_7$ obtained by Mannhart *et al.*[14] It is clear that strong coupling cannot explain the observed temperature variation. It also cannot explain the observed decrease of $J_c(0)$ over its BCS value of an order of magnitude. Figure 4 shows that we can get, at most, a factor of 2.

IV. CONCLUSION

The temperature variation of the dc Josephson critical current was found to be similar to that predicted by Ambegaokar and Baratoff[4] based on BCS theory even when the coupling strength is very large. For T_c/ω_{ln} of order 4 to 7, there are, however, some important differences especially at low reduced temperature $T/T_c=t$ where it is found that the curve remains flat only for a very short range of t. This is true even though our calculations are for an s-wave superconductor. This feature is characteristic of very strong coupling and is not to be taken as indicative that the gap has zeros on the Fermi surface as it would have for p or d symmetry. While it is found that the strong-coupling corrections to $J_c(0)$ reduce its value from BCS, the reduction is only slightly more than a factor of 2 in the range investigated. In a recent Letter, Mannhart *et al.*[14] speculate that the order of magnitude reduction in $J_c(0)$ observed in $YBa_2Cu_3O_7$ films might be due to strong coupling although other explanations are also offered. We conclude from our calculations that strong coupling cannot be the answer. Also, the observed temperature variation is not reproduced.

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