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Apparent critical currents and rf steps in a second-order proximity-induced Josephson effect

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Numerical $I(V)$ results for superconductor–insulator–normal-metal (S-I-N) tunnel junctions suggest that a unified treatment of the weak excess currents (pair-field susceptibility) and proximity-induced Josephson-effect (PJE) phenomena [apparent critical currents, rf steps, and Fraunhofer-like $I_c(B)$] is provided by the Geshkenbein and Sokol time-dependent Ginzburg-Landau theory of second-order Josephson effects in S-I-N tunnel junctions. The numerical results and other considerations speak against a recent suggestion that a phase-slip center in the superconducting tip may play a role in PJE point-contact experiments. These considerations confirm the utility of proximity Josephson-effect experiments to probe unconventional superconductivity in materials such as UBe_{13}, as first pointed out by Han *et al.*

I. INTRODUCTION

Experiments on superconductor–insulator–normal-metal (S-I-N) tunneling point contacts (S=Nb, Ta; N=Ta, In, Mo, UBe_{13}; I a transmissive tunnel barrier)[1–3] reveal several features resembling a (proximity induced) Josephson effect for $T_{cn} < T < T^* < T_{cs}$, where $T_{cn}(T_{cs})$ is the critical temperature of the normal (superconducting) member of the point-contact tunnel junction, and T^* is an apparent junction critical temperature. These features, some of which had earlier been reported in references cited in Ref. 1, include: (1) A low resistance $J(V)$ branch centered at $V=0$, terminating with increasing V in a sharp reduction of slope, dI/dV, to create an apparent critical current density $J_c'(T)$. In point-contact tunneling experiments on normal metals the small $V=0$ resistance was originally treated[1–3] as a series spreading resistance $R_s=\rho/d$, with ρ the resistivity in N and d the contact diameter. (In the reinterpretation, it is no longer clear to what extent an inherent junction resistance may contribute to the small series resistance; see below). Although the slope transition broadens with temperature, a T dependence of $J_c'(T)$ can be defined which resembles that typical of a conventional Josephson junction with $T_c=T^*<T_{cs}$; (2) apparent Shapiro steps of voltage spacing $h\nu/2e$; (3) evidence for $J=J_c'\sin(\phi)$, where ϕ is the difference in phases of superconducting pairs in N and in S, via a Fraunhofer-like $J_c'(B)$, with B the impressed magnetic field; and (4) an $I_c'(R)$ product (with R the normal-state junction resistance) substantially reduced below ideal values for a first-order Josephson effect.

These proximity-induced Josephson-effect (PJE) experiments on tunneling point contacts, and the earlier thin-films pair-field susceptibility (PFS) experiments,[4–7] which yielded a small excess current near $V=0$, evidently reflect a tendency for superconducting pairing to occur in the surface region of N by superconducting proximity with S. The phases of the superconducting pair wave functions in N,S are θ_N,θ_S, with $(\theta_N-\theta_S)=\phi$.

Initial analysis[1] of the superconducting proximity effect in the S-I-N tunnel junction, using time-independent Ginzburg-Landau theory, gave a phase-dependent lowering of the free energy

$$F=\begin{cases}-F_0\cos^2(\phi), & -\frac{\pi}{2}<|\phi|<\frac{\pi}{2}\\ 0, & |\phi|>\frac{\pi}{2}\end{cases} \quad (1)$$

by formation of a proximity-induced superconducting layer when $|\phi|<\pi/2$, whose depth was of the order of the coherence length ξ in N. A peculiar property was the stability of the induced superconductivity only when the phase difference was less than $\pi/2$. On the basis of this free energy, a true Josephson supercurrent $J(\phi)=(2e/h)dF/d\phi$ was initially proposed (1). $J(\phi)$ is nonsinusoidal ($\sin2\phi$ for $|\phi|<\pi/2$, zero for $\pi/2<|\phi|<3\pi/2$) but extended with period 2π. This periodicity was taken consistent with conventional rf steps of width $h\nu/2e$, as observed.

The experimental features (1)–(4) enumerated above are accounted for by the PJE model,[1–3] which, however, ignores any series resistance or other complications from pair to quasiparticle current conversion behind the induced superconducting region in the N layer.

Subsequently, Thuneberg and Ambegaokar[8] (TA) reported confirmation of the proximity-induced Josephson effect, in a microscopic analysis (see also Ref. 9) that explicitly included the $V=0$ supercurrent. TA's initial analysis gave half-spaced rf steps, on the assumption (with short relaxation times) that the induced superconducting state forms and disappears twice during the period of the microwave field. This feature arises because the induced order exists only for $|\phi|<\pi/2$.

A different treatment, including numerical modeling of the PJE experimental results, was provided independently by Geshkenbein and Sokol[10] (GS), in a thorough application of time-dependent Ginsburg-Landau theory (TDGL) to the one-dimensional S-I-N sandwich. The barrier of width a is characterized by a conductivity σ smaller than the bulk normal-state conductivity, σ_0, of the S and N regions, themselves identical except for the occurrence of bulk superconductivity in S alone. In reduced units[11] the TDGL equation for the complex order parameter Ψ is

$$u(\dot{\psi}+i\mu\psi)=\psi''+[\theta(X)-|\psi|^2]\psi\,, \tag{2}$$

where the dot and primes represent, respectively, the time and x derivatives, μ is the chemical potential, and

$$\theta(x)=\begin{cases}1, & x<0 \text{ (in } S)\\ -\tau^2, & x>0 \text{ (in } N)\,,\end{cases} \tag{3}$$

with

$$\tau^2=T^2/(T_c^2-T^2)\,. \tag{4}$$

Here u is a pair-breaking parameter conventionally taken as 12.0 corresponding to gapless superconductivity.

The current density j (normalized as in Ref. 11) consists of superconducting and normal components (j_s+j_n)

$$j=\mathrm{Im}\psi^*\psi'-\mu'\,. \tag{5}$$

A useful boundary condition is deduced by Geshkenbein and Sokol from Eqs. (2) and (5)

$$\psi_S'=\psi_N'=D(\psi_N-\psi_S)\,. \tag{6}$$

The Josephson relation

$$j_s'=j_0'\sin(\theta_N-\theta_S) \tag{7}$$

is obtained by GS [following Eq. (13) therein]. Adopt the usual notation $\Psi_S=\sqrt{n_s}\exp[i\theta_S], \Psi_N=\sqrt{n_n}\exp[i\theta_N]$, with n_s (n_n) the pair density in $S(N)$, for current carrying pair wave functions (complex order parameters). Evaluate Eq. (5), making use of the boundary condition (6) to substitute for Ψ', to find $j_0'=\sqrt{n_S n_N}D$. Here dimensionless $D=\xi/(\sigma_0 RA)$ (where ξ is the superconducting coherence length, σ_0 the conductivity of the bulk, R the resistance of the junction of area A) sets the scale of barrier transmissivity, described by T^2 in the tunneling Hamiltonian theory. Note that $D<1$ can be expressed also as $D=(\xi/a)(\sigma/\sigma_0)$, where the barrier width a is much smaller than the coherence length ξ. The prime notation for j_s' warns that this is not a true Josephson supercurrent. The GS supercurrent j_s' occurs only for finite V and is time independent. In fact, the phase difference ϕ in Eq. (7) develops only for finite voltage. In the GS theory, as in the PFS model discussed in Ref. 6, neither the conventional $V=0$ supercurrent nor an ac supercurrent at finite voltage is found.

The GS supercurrent density j_s, in case $D\ll 2\tau^2\sqrt{\mu}$, is

$$j_s'(V)=\frac{D^2}{\tau\sqrt{2}}\frac{\{[1+(uV/\tau^2)^2]^{1/2}-1\}^{1/2}}{[1+(uV/\tau^2)^2]^{1/2}}\,. \tag{8}$$

This time-independent current is of order D^2, analogous to a second-order Josephson effect.[12] For strong barrier transmissivity $D>0.1$ this contribution is sharply nonlinear as in the PJE experiments (below) and qualitatively resembles the $I(V)$ of a resistively shunted Josephson junction with a small series resistance. Thus the slope dI/dV at $V=0$ remains finite and tends to decrease with increasing temperature. While behavior reproducing the PJE experiments[1-3] is predicted by the GS theory for large $D>0.1$, PFS-like results[4-7] are found for appropriate weak coupling $D<0.01$. Here the excess current is small and smoothly varying, much as observed in the PFS experiments. The PFS excess currents[4,5] had earlier been interpreted using the TDGL theory, see especially Kadin and Goldman,[6] as a second-order Josephson effect.

A summary of the PFS work, comments on the relation of the PFS and PJE experimental regimes, and on the inadequacy of the PJE model, have recently been provided.[7] It is argued in Ref. 7 that the PJE model[3] is incorrect because (i) it neglects the resistance and voltage drop associated with conversion of pairs to quasiparticles in N, behind the induced superconducting region and (ii) because it, and the initial analysis of TA, incorrectly assumes, for finite V, that the phase difference between the induced order parameter and that in S increases as $\phi=(2eV/\hbar)t$, leading to a genuine ac supercurrent. In the PFS work, in GS, and in recent work of TA,[9] the phase of the induced order parameter basically follows that of the order in S, with a time independent phase-lag developing as the voltage is increased. It is further argued[7] (iii) that second-order Josephson-effect models[6,7,10] arising from TDGL theory are incapable of giving the apparent critical current and distinct rf steps as seen in the PJE data. In view of this, it is argued that the PJE and PFS effects must arise from different physics. Namely, (iv) it is suggested[7] that the PJE results are peculiar to S point contacts to N, and arise from an assumed phase-slip center (PSC) in the tip. As proposed, (see Fig. 5 of Ref. 7) an inherent PSC occurs in the superconducting tip at a location, near the N-S interface, where the current density exceeds the *bulk* J_c.[13,14] The proposed PSC must be close enough to the N interface, on the order of the coherence length ξ, to provide the observed depression of the junction T_c^* below that of the S metal, and must be regular enough in geometry that a Fraunhofer-like $I_c(B)$ can be observed (Fig. 2 of Ref. 1).

The main purpose of this Rapid Communication is to demonstrate that the second-order Josephson-effect analysis of GS based on TDGL theory is capable of providing a unified understanding of the relatively rich PJE phenomena first reported by Han *et al.*,[1,2] as enumerated in points (1)-(4) above; as well as the PFS excess current.[4-7] Thus of the points above raised in Ref. 7, we suggest that (iii) is incorrect, and (iv), although it cannot be excluded in every case, is very unlikely. The PJE phenomena, as we show below, are in good agreement with the GS theory, leaving no need to postulate a more complex and unlikely situation.

II. RESULTS AND DISCUSSION

Figure 1 shows a family of $I(V)$ curves calculated from Eq. (16) of GS, Ref. 10, using parameter values which are reasonable for the PJE experiments. Namely, the coupling $D=\xi/(\sigma_0 RA)=0.2$, $u=12$, and $T_{cs}=4.48$ K. Temperatures are $T=1.20$, 1.61, 2.02, 2.43, 2.84, and 3.25 K (in order of decreasing apparent critical current). In this and the following calculations the pair-breaking parameter u is set at 12, a conventional value in the TDGL theory corresponding to gapless superconductivity, and current and voltage scales are normalized as in Ref.

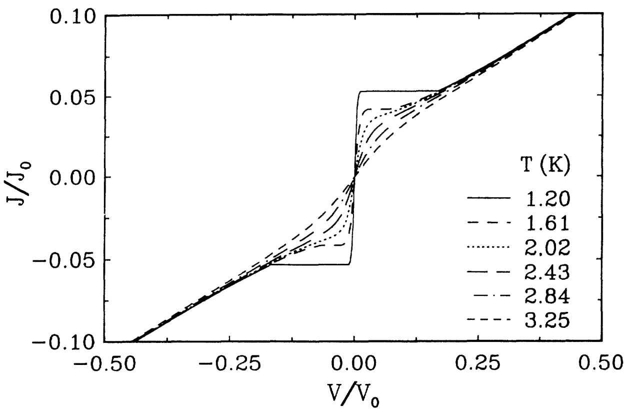

FIG. 1. Calculated $I(V)$ curves obtained from the theory of Geshkenbein and Sokol [Eq. (16) of Ref. 10] using parameter values $D=0.2$, $T_{cs}=4.48$ K, and $u=12$ (see text), appropriate to Ta point-contact tunnel junctions on Mo or UBe_{13}. Current and voltage scales normalized as in Ref. 11.

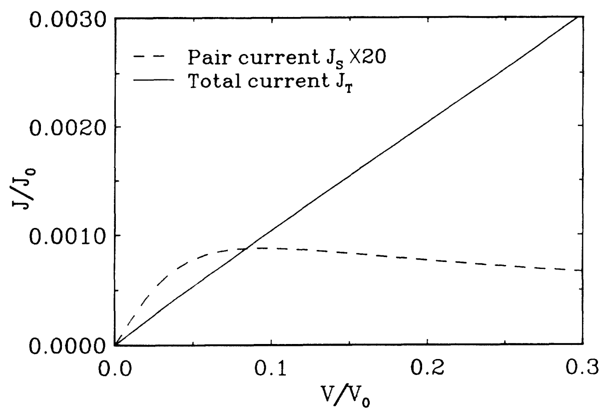

FIG. 3. Calculation of $I(V)$ using theory of Geshkenbein and Sokol [Eq. (16) of Ref. 10] and parameter values $D=0.01$, $u=12$, and $T=4.48$ K appropriate to the pair-field susceptibility regime, in which small excess currents are observed. Similar curves are reported in Refs. 4 and 5.

11. These parameters are chosen to approximate the S-I-N point-contact junction experiments with a Ta tip contacting Mo or UBe_{13}. In the one-dimensional GS theory, in agreement with the PFS model,[6,7] a true Josephson supercurrent is not found. Thus a small apparent "series resistance" arises inherently in the GS theory, and would add to the small contribution of the spreading resistance known to occur in the point-contact geometry.

Figure 2 demonstrates, contrary to statements in Ref. 7, that sharp rf steps can arise in the GS second-order Josephson-effect theory.[10] The rf steps have been calculated using Eqs. (15) and (20) of GS, with parameter values $u=12$, $D=0.16$, $T_{cs}=4.48$ K, $A=0.25$, and $\omega=0.08$. The results generally agree with rf steps observed using a Ta tip on UBe_{13}, published in Fig. 3 of Ref. 2 and Fig. 3 of Ref. 3.

The rf steps predicted by Eq. (20) of Ref. 10 have the conventional Shapiro step spacing $h\nu/2e$. In the GS

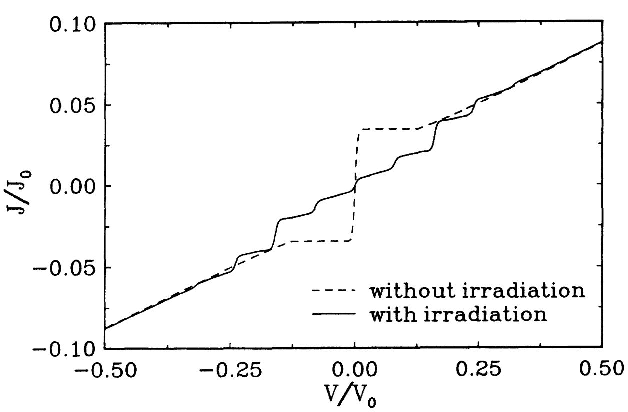

FIG. 2. Calculated effects of microwave irradiation of $\omega=0.08$ in units of $2eV_0/\hbar$ and amplitude of $V_{ac}=0.25V_0$ on $I(V)$ for parameter values $D=0.16$, $T_{cs}=4.48$ K, and $u=12$ (see text) simulating a Ta point-contact tunnel junction on Mo or UBe_{13}, at $T=1.20$ K. The numerical results are obtained from Eqs. (16) and (20) of Ref. 10. The rf steps are of conventional spacing $\Delta V=\hbar\omega/2e$, and the pattern is similar to that reported in Fig. 3 of Ref. 2.

theory, however, the steps are not a consequence of an oscillating supercurrent of frequency $2eV/h$, but rather arise as an image of the sharp nonlinearity in the $I(V)$ near $V=0$, shifted by multiples of the photon energy $\hbar\omega$, as explained by Tien and Gordon[15] and Tucker and Millea.[16] These steps are weighted by the square of the Bessel function n, rather than the first power as in conventional Shapiro steps.

Figure 3 shows a weak, broadly peaked excess current, characteristic of the PFS regime, as predicted by the GS theory with smaller values of the coupling parameter D. Here parameter values in GS Eq. (15) are $u=12$, and $D=0.01$. The smaller values of D used to model the PFS results are in qualitative accord with the smaller barrier transmissivity in these thin film junctions.

The most reliable regime for comparing the barrier transmissivity in the PJE and PFS experiments is the low-temperature limit in which both members of the junctions are superconducting, and true Josephson j_c values proportional to $1/RA$ are obtained. The range of such true critical current densities in the PFS experiments[4,5] is 1-10 A/cm^2 while in the PJE experiments [(1)-(3)] the estimated typical value is 10^3 A/cm^2.[13] Thus $1/RA$ values, which are roughly 100-1000 larger in PJE than in PFS, are implied. The corresponding variation in $D=\xi/(\sigma_0 RA)$ is less certain by the appearance of the further parameters ξ,σ_0. However, it seems that typical D values at least a factor of 16-20 smaller than used in modeling the PJE are reasonable for PFS.

We suggest that differing barrier transmissivity is the most likely reason for the characteristic differences in the PJE and PFS results, and, specifically, that the physical origin of both effects is the same.

A Fraunhofer-like $j_c'(B)$ appears to be implied by the GS theory, as observed,[1] as a consequence of the Josephson expression for the supercurrent density across the contact, $j=j_0'\sin(\theta_N-\theta_S)$ given by GS [following Eq. (13)]. Because the depth of the induced superconducting layer in N is limited to the order of a coherence length in N, it will not completely screen a small magnetic field. Hence one expects the magnetic field period ΔB of the Fraunhofer pattern $j_c'(B)$ for a small junction to be given by a relation similar to that for a small rectangular Josephson junction with one electrode having a film thickness less than the corresponding penetration depth:[17,18]

$$\Delta B = \Phi_0/L[\lambda_S + \lambda_N \tanh(\xi_N/2\lambda_N) + a] . \quad (9)$$

Here $\lambda_{S,N}$ represents the superconducting penetration depth in $S(N)$, L is the length of the junction, and Φ_0 is the flux quantum.

III. SUMMARY

We suggest that the time-dependent Ginzburg-Landau treatment of the *N-I-S* junction with transmissive tunnel barrier by Geshkenbein and Sokol[10] is capable of reproducing all of the features of the proximity-induced Josephson-effect point-contact tunneling experiments, as well as the weak excess currents seen in the pair-field susceptibility experiments reviewed in Ref. 7.

We suggest that the phase-slip center postulated in the recent review,[7] although it cannot be ruled out in every case, is an unnecessary complication.[13] In fact, the low current densities, relative to bulk J_c values, and the smooth temperature dependences of J_c' (Refs. 1 and 2) are inconsistent with this suggestion.

The rf steps observed in the PJE experiments at spacing $V = h\nu/2e$ as modeled by GS,[10] arise in the GS theory when parameter values are appropriate for a sharp nonlinearity (apparent critical current) in $I(V)$ centered at $V=0$. The origin of the steps is not an oscillating supercurrent at finite V. Rather, the steps arise in a fashion similar to those near the sum gap in an irradiated *S-I-S* quasiparticle tunnel junction, treated in detail by Tien and Gordon[15] and by Tucker and Millea.[16] The voltage difference between two adjacent current steps is $h\nu/2e$. This reflects the fact that the nonlinearity of the *I-V* characteristics originates from the pair current, in which the charge carrier is $2e$.

These effects in their entirety arise from proximity induced superconductivity at the surface of the N region. If, in a PJE experiment, superconducting order of a competing nature appears in the bulk of N at T_{cn},[2] a first-order Josephson current between the two bulk order parameters may be forbidden by symmetry and/or by suppression of the competing order parameter at the surface of N by the induced superconductivity from S.[2] In this case PJE current with all of the characteristics noted above will still exist, with some suppression below T_{cn} from competition of the induced and bulk order parameters in N. This suppression, as noted,[2] becomes a tool to study the interaction of the differing order parameters. For this reason, we believe that studies such as reported in Refs. 2 and 3 remain a useful probe of unconventional superconductivity.

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[1]S. Han *et al.*, Phys. Rev. B **32**, 7567 (1985).

[2]S. Han *et al.*, Phys. Rev. Lett. **57**, 238 (1986); see also A. M. Kadin and A. M. Goldman, *ibid.* **58**, 2275 (1987); E. L. Wolf, A. J. Millis, and S. Han, *ibid.* **58**, 2276 (1987).

[3]E. L. Wolf *et al.*, J. Appl. Phys. **61**, 3899 (1987).

[4]J. T. Anderson and A. M. Goldman, Phys. Rev. Lett. **25**, 743 (1970).

[5]R. V. Carlson and A. M. Goldman, J. Low Temp. Phys. **25**, 67 (1976).

[6]A. M. Kadin and A. M. Goldman, Phys. Rev. B **25**, 6701 (1982).

[7]A. M. Kadin, Phys. Rev. B **41**, 4072 (1990).

[8]E. V. Thuneberg and V. Ambegaokar, Phys Rev. Lett. **60**, 365 (1988).

[9]E. V. Thuneberg and V. Ambegaokar, Phys. Rev. Lett. **62**, 2335 (1989); V. Ambegaokar (private communication); E. V. Thuneberg and V. Ambegaokar (unpublished).

[10]V. B. Geshkenbein and A. V. Sokol, Zh. Eksp. Teor. Fiz. **94**, 259 (1988) [Sov. Phys. JETP **67**, 362 (1988)].

[11]L. Kramer and A. Baratoff, Phys. Rev. Lett. **38**, 518 (1977).

[12]In a first-order Josephson effect J_c is of order T^2, with T the tunneling matrix element; or of order D in the GS theory. Thus Eq. (8) describes a supercurrent analogous to a *second-order* Josephson effect, equivalent to J of order T^4 in transfer Hamiltonian theory.

[13]The experimental PJE papers give reasonable evidence that the current density J near the contact is reduced by the tunnel barrier to values substantially below the bulk critical current density J_c which must be reached for the postulated inherent phase slip center (Ref. 7). The values of I_c/A, with A the junction area (at low temperatures $T \ll T_{cs}, T_{cn}$ with both electrodes superconducting), in PJE experiments are typically in the range 10^2–10^3 A/cm^2. While I_c is directly measured, the areas [inferred from the Fraunhofer-like $I_c(B)$, from microscopy, and from measured spreading resistances, as explained in Refs. 1 and 2] are not known precisely. Nevertheless, it is clear that the I_c/A values, although imperfectly known, are small compared to the critical current density of the bulk tip metals used. For example, $J_c(T=2\text{ K})$ for Ta is in excess of 10^6 A/cm^2.

[14]It is possible that the postulated PSC might occur at a grain boundary in the tip. This might well allow phase slip to occur at current densities lower than the bulk J_c, as required by the experiments. We cannot rule this out in rare cases, but at present do not understand how a grain boundary could occur with high probability in the constrained position and orientation of the proposed PSC model. We observe the tips to be characteristically convex, so that a constriction is definitely rare. It has also been suggested that experiments be carried out in the reverse geometry, N point upon S, which should certainly be done.

[15]P. K. Tien and J. P. Gordon, Phys. Rev. **129**, 647 (1963).

[16]J. R. Tucker and M. F. Millea, Appl. Phys. Lett. **33**, 611 (1978).

[17]A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect* (Wiley, New York, 1982), p. 76.

[18]M. Weinacht, Phys. Status Solidi **32**, K169-172 (1969).