PHYSICAL REVIEW B VOLUME 42, NUMBER 13 1 NOVEMBER 1990

Harris criterion for direct and orthogonal quenched randomness

A. Nihat Berker
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 23 March 1990)

Critical-behavior stability is studied, with respect to the introduction of quenched randomness that is direct (e.g., field randomness at field approach to criticality) or orthogonal (e.g., field randomness at temperature approach to criticality). Rather than resorting to renormalization-group theory, an argument is conducted by considering the effective uniform field per spin within a correlated region. By comparing the correlation lengths limited by this effective uniform field and by the actual uniform deviation from criticality, it is recovered that nonrandom critical behavior is not maintained under random fields when its susceptibility critical exponent γ is positive, and conversely.

Microscopic phenomenological arguments have yielded much insight, with little effort, to questions pertaining to the occurrence and nature of phase transitions under new conditions. Duly most celebrated examples are the Harris[1] criterion for random bonds and the Imry-Ma[2] argument for random fields. Harris asked whether the nature of a critical point is modified by quenched random bonds. ("Random bonds" most generally mean a distribution of local interactions that does not favor, at any locality, one of the coexisting ordered phases of the nonrandom system.) He compared, in the neighborhood of the critical point, the deviation of local critical temperatures between different correlated regions and the deviation of the actual temperature from the global critical temperature. He deduced that nonrandom critical behavior is maintained (not maintained) under random bonds when the nonrandom specific-heat critical exponents α is negative (positive). Imry and Ma considered the occurrence of phase transitions under quenched random fields. ("Random fields" most generally mean a distribution of local interactions that favors, differently at various localities, one of the coexisting ordered phases of the nonrandom system.) By comparing the energy costs of random fields under order and of domain boundaries under broken order, they deduced that the lower-critical spatial dimension for order under random fields is $d_l = 2$ for $n = 1$ component order parameters and $d_l = 4$ for $n \geq 2$ component order parameters. Today, neither of these two results are in doubt, and they provide valuable guidance.[3]

In the present paper, we ask Harris's question for random-field systems at dimensions $d > d_l$ at which the phase transition is allowed by the Imry-Ma argument: Is the nature of the nonrandom criticality modified by random fields? We give a microscopic phenomenological argument in the style of Harris.[1,4] We recover that the nonrandom critical behavior is not maintained when its susceptibility critical exponent γ is positive, and conversely. This result has been previously obtained by rather more technical studies employing renormalization-group theory[5,6] or correlation functions.[7] Our present argument, although delivered in random-field language, is general. It equally applies to (i) bond randomness at temperature deviation from criticality, or field randomness at uniform-field deviation from criticality and (ii) field randomness at temperature deviation from criticality, or bond randomness at uniform-field deviation from criticality. We shall call these different situations, (i) and (ii), direct and orthogonal randomness, respectively (Fig. 1). Orthogonal randomness has not been previously considered, to our knowledge.

Consider an originally nonrandom system that is at a small temperature or uniform-field deviation from its critical point, now perturbed by quenched random fields of root-mean-square strength $\bar{h}$. A correlated region includes ξ^d aligned spins, where ξ is the correlation length. The net random-field energy of the entire correlated region is $\pm\xi^{d/2}\bar{h}$, by the central-limit theorem. This is equivalent to having each spin, within the correlated region, subject to the same uniform field h, with

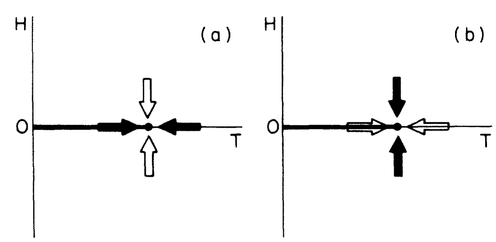

FIG. 1. Solid and open arrows respectively show criticality approaches for direct and orthogonal random perturbations: (a) random-bond perturbation, (b) random-field perturbation. The thick segment on the temperature axis is the coexistence line of the two phases with positive and negative magnetizations, respectively. This coexistence terminates at the critical point.

$$|h|=\xi^{-d/2}\bar{h} \ . \tag{1}$$

In general, a system that deviates, in temperature, by τ from criticality has correlation length

$$\xi_\tau = A_\tau |\tau|^{-\nu_\tau}, \quad \nu_\tau = 1/y_\tau > 0 \ , \tag{2}$$

where y_τ is the scaling exponent in the temperature direction.[8] On the other hand, a system that deviates, in uniform field, by h from critically has correlation length

$$\xi_h = A_h |h|^{-\nu_h}, \quad \nu_h = 1/y_y > 0 \ , \tag{3}$$

where y_h is the scaling exponent in the uniform-field direction. When both deviations are present, the smaller length of ξ_τ and ξ_h limits the correlations and therefore is the true correlation length ξ. In fact $\xi_\tau = \xi_h$ determines the "crossover" boundary $|h| \sim |\tau|^{y_h/y_\tau}$ between regions of temperature-approach and uniform-field-approach exponents.

Returning to our case of random-field perturbation, and substituting Eq. (1) into Eq. (3),

$$\xi_r = A_h \xi^{\nu_h d/2} \bar{h}^{-\nu_h} \ , \tag{4}$$

where ξ_r is the correlation length limited by the random fields and ξ is the true correlation length. For the random-field perturbation to be irrelevant, the correlation length limited by the uniform deviation, Eq. (2) or Eq. (3), must be the true correlation length, as it is in the unperturbed system. This requires ξ_r of Eq. (4), with the smaller of ξ_τ and ξ_h substituted for ξ, to be greater than this smaller of ξ_τ and ξ_h as given by Eq. (2) or Eq. (3). Thus, for temperature approach (i.e., when random fields constitute orthogonal randomness), this requirement is

$$A_h \xi_\tau^{\nu_h d/2} \bar{h}^{-\nu_h} \stackrel{?}{>} \xi_\tau \quad \text{as } \tau \to 0 \text{ and } \xi_\tau \to \infty \ , \tag{5}$$

or equivalently,

$$2 - d\nu_h \stackrel{?}{<} 0 \ . \tag{6}$$

The left side of this equation equals $\gamma\nu_h/\nu_\tau$, where γ is the critical exponent of the susceptibility, by standard scaling-exponent analysis at pure-system criticality.[8] For uniform-field approach (i.e., when random fields constitute direct randomness), the requirement is

$$A_h \xi_h^{\nu_h d/2} \bar{h}^{-\nu_h} \stackrel{?}{>} \xi_h \quad \text{as } h \to 0 \text{ and } \xi_h \to \infty \ , \tag{7}$$

which also reduces to Eq. (6). Thus, nonrandom criticality with $\gamma > 0$ is modified by quenched random-field perturbation, and conversely.

For most systems, the response function of the ordering density to its conjugate field is strongly singular at criticality, namely the susceptibility diverges ($\gamma > 0$).[9] Then, does the above analysis imply that some equivalent of uniform-field approach to criticality will be observed as $\tau \to 0$ for fixed $\bar{h} \neq 0$? The answer is no, as seen below. In this case, by our original analysis, ξ_r is the true correlation length and, by substitution into Eq. (4),

$$\xi_r \stackrel{?}{=} A_h \xi_r^{\nu_h d/2} \bar{h}^{-\nu_h} \quad \text{as } \xi_r \to \infty \ . \tag{8}$$

This is incorrect: Since in general $y_h = 1/\nu_h$ (a nonrandom system scaling exponent) does not equal $d/2$ (except at $d=6$), but is greater than $d/2$, there is an inconsistency in the derivation of Eq. (8). This can be narrowed to the application of Eq. (1). We conclude that for a criticality that occurs modified by randomness,[10] the cumulative effect of randomness on correlated regions is more complex than the simple Eq. (1).

Repetition of the analysis leading to Eq. (6) for quenched random-bond perturbations leads to the Harris criterion, whereby the nonrandom specific-heat exponent α determines whether nonrandom criticality is stable ($\alpha < 0$) or unstable ($\alpha > 0$) to random-bond perturbations, for both direct randomness (i.e., temperature approach) and orthogonal randomness (uniform-field approach).

This research was supported by the National Science Foundation under Grant No. DMR-87-19217 and by the Joint Services Electronics Program under Contract No. DAAL 03-89-C0001.

---

[1] A. B. Harris, J. Phys. C **7**, 1671 (1974).

[2] Y. Imry and S.-k. Ma, Phys. Rev. Lett. **35**, 1399 (1975).

[3] See, for example, K. Hui and A. N. Berker, Phys. Rev. Lett. **62**, 2507 (1989), where the Imry-Ma argument is extended to indicate that all Potts model transitions are second order in two dimensions. This is independently found in the rigorous results of M. Aizenman and J. Wehr, Phys. Rev. Lett. **62**, 2503 (1989).

[4] T. Nattermann and P. Rujan, Int. J. Mod. Phys. (to be published).

[5] A. Aharony, Phys. Rev. B **18**, 3318 (1978).

[6] D. Andelman and A. N. Berker, Phys. Rev. B **29**, 2630 (1984).

[7] Y. Shapir and A. Aharony, J. Phys. C **14**, L905 (1981).

[8] L. P. Kadanoff, Physics **2**, 263 (1966).

[9] On the other hand, this does indicate that spin-glass criticality will not be modified by random-field perturbations, since the uniform magnetization is not the ordering density and, indeed, experiments [J. A. Mydosh, in *Magnetism and Magnetic Materials—1974 (San Francisco)*, Proceedings of the 20th Annual Conference on Magnetism and Magnetic Materials, AIP Conf. Proc. No. 24, edited by C. D. Graham, G. H. Lander, and J. J. Rhyne (AIP, New York, 1975), p. 131] show a nondivergent (cusped) specific heat, namely $\gamma < 0$.

[10] For quenched random perturbations of a criticality that occurs already modified by quenched randomness, it was quite generally shown in Ref. 6 by renormalization-group theory that the "crossover exponent" ϕ does not equal the response function exponent, i.e., $\phi \neq \alpha$, for random-bond perturbation (Ref. 6) and $\phi \neq \gamma$ for random-field perturbation [A. Aharony, Europhys. Lett. **1**, 617 (1986)]. The latter has been experimentally confirmed in D. P. Belanger, A. R. King, and V. Jaccarino, Phys. Rev. B **34**, 452 (1986); C. Ramos, A. R.

King, and V. Jaccarino, Phys. Rev. B **37**, 5483 (1988). Since the sign of ϕ determines the relevance ($\phi > 0$) or irrelevance ($\phi < 0$) of the perturbation, one may think that the Harris criterion does not hold. However, for the calculated cases, it is seen that when α is negative ϕ is even more negative (Ref. 6), and when γ is positive, ϕ is even more positive [A. Aharony, Europhys. Lett. **1**, 617 (1986)].