Enhancement and suppression of the transition temperature of a three-dimensional XY ferromagnet by control of vortex-loop fugacity

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A previous vortex-loop scaling analysis for the three-dimensional XY model is extended to include an external loop-segment chemical potential $\mu=\lambda k_B T$. The loop fugacity y_0 is suppressed, $y_0 \to y_0 e^{-2\pi\lambda}$, for $\lambda>0$, enhancing the transition temperature $T_c(\lambda)$, in agreement with the Monte Carlo work of Kohring, Shrock, and Wills. One also gets the suppression of transition temperatures $T_c(e^2)$ of lattice superconductors by electromagnetic charge e^2, by mapping onto this loop-fugacity model. A possible approach to superconductor high-T_c enhancement, by tailored suppression of topological excitations, is briefly conjectured.

I. INTRODUCTION

The raising of transition temperatures is of much current interest in the context of high-T_c superconductors.[1] The three-dimensional (3D) planar ferromagnet or XY model is isomorphic to 3D Josephson-junction arrays and is closely related to 3D lattice superconductor models[2,3] and may be relevant for high-T_c phenomena. Thermally generated topological excitations (vortex loops) have been conjectured to play an essential role in the 3D XY transition.[4]

The 2D XY and superconductor-film transition are controlled[5] by 2D topological excitations (vortex points). A 3D XY vortex-loop scaling approach[6,7] closely following the 2D XY vortex-point scaling of Kosterlitz[8] has been developed. The transition involves an added expansion of loop size by nested-loop screening as temperature T increases, with a size blowout at $T=T_c$ of the dominant loop diameter $\xi_- \sim (T_c-T)^{-\nu}$. The transition temperature depends on the bare-loop fugacity $y_0=y_0(K_0)$, which is controlled solely by the bare coupling $K_0 \equiv J/k_B T>0$.

Kohring, Shrock, and Wills,[9] in Monte Carlo (MC) work, have introduced an externally controlled chemical potential $\mu=\lambda k_B T$ for vortices in the angular $-\pi>\theta_i \leq \pi$ variables on a cubic lattice (i). They find that, for $\lambda>0$ (suppression of vorticity), T_c is enhanced, with no transition out of the ordered state for $\lambda>\lambda_c=0.55$.

A similar enhancement of T_c for the suppression of the appropriate topological excitations is found for the Heisenberg ferromagnet,[10] and for lattice gauge models[11] that are similar to those emerging[12] from high-T_c Hubbard Hamiltonians.

In this work, we show that the external (scaled) chemical potential $\lambda>0$ simply suppresses the loop fugacity $y_0 \to y_0 e^{-2\pi\lambda}$ of the previous[7] scaling analysis. The previous results then yield a backward-bending transition line λ-K_0, i.e., an enhancement of the transition temperature $T_c(\lambda)>T_c(0)$. A complementary lattice-superconductor problem with a fluctuating electromagnetic field (and coupling e^2) can be mapped onto this loop-fugacity model through $K_0 \to e^2/4\pi^2$, $\lambda \to (4K_0)^{-1}$. A backward-bending e^2-K_0^{-1} transition line then results, i.e., $T_c(e^2)<T_c(0)$, as in the MC results of Dasgupta and Halperin.[2] Inverted XY behavior,[2] found on the e^2-K_0^{-1} line, should occur at the $K_0=0, \lambda=\lambda_c$ point in the K_0-λ diagram.

In Sec. II we relate the 3D XY model with a vorticity weight λ to a vortex-loop model with a modified fugacity $y_0(K_0,\lambda)$. The λ-K_0 line follows. In Sec. III the lattice superconductor problem also maps onto a loop model with $y_0(e^2,K_0^{-1})$ loop fugacity. The e^2-K_0^{-1} line follows. Section IV has a summary and comments on possible further work.

II. VORTEX-LOOP MODEL WITH CHEMICAL POTENTIAL

The 3D XY model, with a chemical potential for vorticity, is[9] ($K_0 \equiv J/k_B T$, $\lambda \equiv \mu/k_B T$)

$$\beta H = -K_0 \sum_{\langle ij \rangle} \cos\theta_{ij} + \lambda \sum_p \left| \sum\!\!\!\!\!\!\bigcirc\, ^{(p)}\theta_{ij}/2\pi \right| , \tag{2.1}$$

where $\theta_{ij} \equiv \theta_i - \theta_j$ is the phase difference and $-\pi<\theta_i \leq \pi$ on cubic lattice sites $\{i\}$. The λ term involves a sum of θ_{ij} around a plaquette p, and a sum over all such plaquettes. The modulus ensures the same contribution $\sim\lambda$, regardless of the sign of the vorticity $S^{(p)}$:

$$\sum\!\!\!\!\!\!\bigcirc\, ^{(p)}\theta_{ij}/2\pi = S^{(p)}=0,\pm 1 . \tag{2.2}$$

Defining a bond variable

$$S_{ij}=S^{(p)} \tag{2.3}$$

for bonds ij around a plaquette p, the last term can be written as

$$\lambda \sum_p \left| \sum\!\!\!\!\!\!\bigcirc^{(p)} \theta_{ij}/2\pi \right| = \lambda \sum_p S^{(p)} \sum\!\!\!\!\!\!\bigcirc^{(p)} \theta_{ij}/2\pi$$

$$= 2\lambda \sum_{\langle ij \rangle} S_{ij} \frac{\theta_{ij}}{2\pi} . \tag{2.4}$$

A factor of 2 appears going from plaquette sums to bond sums, since counting every plaquette means each shared bond between neighbors appears twice. Thus, the partition function can be written

$$Z_\theta = \prod_i \int_{-\pi}^{\pi} \frac{d\theta_i}{2\pi} \sum_{\{S_{ij}\}} \exp\left[K_0 \sum_{\langle ij \rangle} \cos\theta_{ij} - \frac{2\lambda}{2\pi} \sum_{\langle ij \rangle} S_{ij}\theta_{ij} \right]$$

$$\times \prod_p \delta_{2\pi S^{(p)}, \sum\!\!\!\!\!\!\bigcirc^{(p)} \theta_{ij}} . \tag{2.5}$$

This can be mapped via a dual transform[13] onto vortex loops with an external fugacity, as now shown. The reader interested in the application of this map can go on from (2.18) below.

Expanding both terms, the Kronecker δ is $(-\infty < Q^{(p)} < \infty)$

$$\delta_{2\pi S^{(p)}, \sum\!\!\!\!\!\!\bigcirc^{(p)} \theta_{ij}} \propto \sum_{\{Q^{(p)}\}} \exp\left[2\pi i \sum_p \left[S^{(p)} - \sum\!\!\!\!\!\!\bigcirc^{(p)} \frac{\theta_{ij}}{2\pi} \right] Q^{(p)} \right]$$

$$= \sum_{\{Q^{(p)}\}} \exp\left[2\pi i \sum_p S^{(p)} Q^{(p)} \right] \exp\left[-2i \sum_{\langle ij \rangle} Q_{ij} S_{ij} \right] , \tag{2.6}$$

where $Q_{ij} = Q^{(p)}$ round vorticity bearing plaquettes and zero otherwise, analogous to (2.3). The Boltzmann-factor Fourier expansion is

$$\left\{ \exp\left[K_0 \cos\theta_{ij} - \frac{\lambda}{\pi} S_{ij}\theta_{ij} \right] \right\} \exp(-2iQ_{ij}\theta_{ij}) = \sum_{n_{ij}=-\infty}^{\infty} \exp[V(n_{ij};\lambda)] \exp[i(n_{ij} - 2Q_{ij})\theta_{ij}]$$

$$= \sum_{n_{ij}} e^{V(n_{ij}+2Q_{ij};\lambda)} e^{in_{ij}\theta_{ij}} , \tag{2.7}$$

where $e^{V(x)}$ is the Fourier coefficient of the weight factor in curly brackets and $n_{ij} = 0, \pm 1, \pm 2, \ldots$ is the Fourier label. For low temperatures, $K_0^{-1} \ll 1$, using the inverse Fourier transformation[13]

$$e^{V(n_{ij};\lambda)} \approx I(0,\lambda) \exp[I(n_{ij},\lambda)/I(0,\lambda) - 1] ,$$

where

$$\frac{I(n_{ij},\lambda)}{I(0,\lambda)} - 1 \equiv \int_{-\pi}^{\pi} \frac{d\theta_i}{2\pi} \int_{-\pi}^{\pi} \frac{d\theta_j}{2\pi} e^{K_0 \cos\theta_{ij}} e^{-\lambda S_{ij}\theta_{ij}/\pi} [e^{-in_{ij}\theta_{ij}} - 1]/I(0,\lambda)$$

$$\simeq -\frac{n_{ij}^2}{2K_0} + \frac{i\lambda}{\pi K_0} S_{ij} n_{ij} , \tag{2.8}$$

where $\lambda/K_0 = \mu/J \ll 1$ has been taken.

Using (2.6)–(2.8) and doing the θ_i integration, one gets

$$Z_\theta = Z_\theta(K_0,\lambda) = \sum_{\{n_{ij}\}} \sum_{\{Q^{(p)}\}} \sum_{\{S_{ij}\}} \exp\left[2\pi i \sum_p S^{(p)} Q^{(p)} \right] \exp\left[-\sum_{\langle ij \rangle} n_{ij}^2/2K_0 \right] \exp\left[-2 \sum_{\langle ij \rangle} Q_{ij}^2/K_0 \right] \exp\left[-\sum_{\langle ij \rangle} n_{ij} Q_{ij}/K_0 \right]$$

$$\times \exp\left[i \frac{\lambda}{\pi K_0} \sum_{\langle ij \rangle} S_{ij}(2Q_{ij} + n_{ij}) \right] \prod_i \delta_{\sum_{\hat{r}=1}^{6} n_{i+\hat{r}}, 0} . \tag{2.9}$$

As usual,[7,13] the θ_i integration gives a zero-divergence constraint on the Fourier label n_{ij}. In terms of a vector notation $n_{ij} \to n_{\mu,i}$ for ij in the $\mu = 1,2,3$ direction, the Kronecker δ constraint enforces

$$\sum_{\mu=1}^{3} \Delta_\mu n_{\mu,i} = 0 \quad \forall i , \tag{2.10}$$

where Δ_μ is a discrete divergence in the μ direction. This means the $n_{\mu,i}$ field forms closed loops at finite temperatures (ignoring improbable system-spanning lines). For future reference we note, as done by Dasgupta and Halperin,[2] that the $K_0 \neq 0$, $S^{(p)} = 0$ case corresponds to loops interacting only by a contact interaction

$$Z_\theta(K_0, \lambda = 0) = \sum_{(n_{\mu,i})}{}' \exp\left[-\sum_{\mu,i} n_{\mu,i}^2/2K_0 \right] , \tag{2.11}$$

where irrelevant factors have been dropped and the prime refers to the closure condition (2.10).

Going over to a dual lattice to satisfy (2.10) as an identity, the closed-loop original-lattice fields are expressed as curls of dual-lattice fields [on a cubic lattice shifted by $(\frac{1}{2},\frac{1}{2},\frac{1}{2})$]:

$$\begin{aligned} n_{\mu,i} &\to \sum_{\nu,\lambda} \epsilon_{\mu\nu\lambda}\Delta_\nu N_\lambda(i) , \\ Q_{\mu,i} &\to \sum_{\nu,\lambda} \epsilon_{\mu\nu\lambda}\Delta_\nu P_\lambda(i) , \\ S_{\mu,i} &\to \sum_{\nu,\lambda} \epsilon_{\mu\nu\lambda}\Delta_\nu S_\lambda(i) . \end{aligned} \tag{2.12}$$

Here variables with arguments like $N_\lambda(i)$ refer to the dual lattice, while variables with subscripts like $n_{\mu,i}$ refer to the original lattice. Since dual variables pierce through the original lattice plaquettes, and $Q^{(p)},S^{(p)}$ are plaquette variables $P_\lambda(i)=Q^{(p)}$, $S_\lambda(i)=S^{(p)}$ for plaquettes P, perpendicular to the dual-lattice direction λ. (The λ direction label is not to be confused with the chemical-potential variable, or $\sqrt{-1}$ with the site label i.)

Using the Poisson summation formula as usual,[7,13]

$$Z_\theta = \sum_{\{S_\mu(i)\}} \sum_{\{P_\lambda(i)\}} \left\{ \exp\left[4\pi i \sum_{\mu,i} S_\mu(i)P_\mu(i)\right] \exp\left[-2\sum_{\mu,i}(\epsilon\Delta P)^2_\mu(i)/K_0\right] \exp\left[\frac{2i\lambda}{\pi K_0}\sum P_\mu(i)(\epsilon\Delta)^2 S_\mu(i)\right]\right\}$$
$$\times \sum_{\{j_\mu(i)\}}{}' \int_{-\infty}^{\infty} d\phi_\mu \exp\left[-\sum(\epsilon\Delta\phi)^2_\mu(i)/2K_0\right] \exp\left[2\pi i \sum_{\mu,i} \tilde{J}_\mu(i)\phi_\mu(i)\right] , \tag{2.13}$$

where $(\epsilon\Delta)^2 \equiv -\Delta^2$, and we define

$$\tilde{J}_\mu(r) \equiv J_\mu(r) + \frac{\lambda}{2\pi^2 K_0}(-\Delta^2)S_\mu(r) - \frac{i}{\pi K_0}(-\Delta^2)P_\mu(r) . \tag{2.14}$$

A gauge transform $\phi_\mu(i)\to\phi_\mu(i)+\Delta_\mu\chi(i)$ leads to a conservation constraint on $\tilde{J}_\mu$. Since **S** is related to **J** as shown later, the conservation constraint is a loop condition $\mathbf{\Delta}\cdot\mathbf{J}=0$, denoted by the prime on the J sum.

Doing the Gaussian $\{\phi_\mu(i)\}$ integration yields

$$Z_\theta \simeq Z_J = \sum_{\{S_\mu(i)\}} \sum_{\{P_\lambda(i)\}} \sum_{\{J_\mu(i)\}}{}' \exp\left[-\frac{\pi K_0}{2}\sum_{\substack{\mu \\ r,r'}} \tilde{J}_\mu(r)U(r-r')\tilde{J}_\mu(r')\right]\{\ \} , \tag{2.15}$$

where the curly bracket is as in (2.13). Here,

$$\sum_{\mu=1}^{2} \Delta_\mu^2 U(r) = -4\pi\delta_{r,0} \tag{2.16}$$

and the spin-wave exchange interaction is $U(r)\simeq a_0/r$, where a_0 is the lattice constant [more precisely, $U(r)$ is the 3D lattice Greens function].

Neglecting the terms $\lambda/K_0 \ll 1$ with $\lambda<1$ for consistency with the previous $K_0^{-1}\ll 1$ expansion, one gets, from (2.14) and (2.16),

$$Z_J \simeq \sum_{\{J_\mu(i)\}}{}' \sum_{\{S_\mu(i)\}} \prod_{\mu,i} \delta_{J_\mu(i),S_\mu(i)} \exp\left[-\frac{\pi K_0}{2}\sum_{\mu,r,r'} J_\mu(r)U(r-r')J_\mu(r')\right] \exp\left[-2\lambda\sum_{\mu,i} J_\mu(i)S_\mu(i)\right] , \tag{2.17}$$

i.e., the θ vorticity, related to **S**, is the same as the **J** variable consistent with our previous backward-dual result.[14] For $J_\mu(r)=0,\pm1$ dominant values, one gets the final result

$$Z \simeq \sum_{\{J_\mu(i)\}}{}' \exp\left[-\frac{\pi K_0}{2}\sum_{\mu,r,r'} J_\mu(r)U(r-r')J_\mu(r')\right] \times \exp\left[-2\lambda\sum_{\mu,r} J^2_\mu(r)\right] . \tag{2.18}$$

This is just the vortex-loop model with an externally controlled fugacity factor, i.e., the loop-fugacity model.

The bare- (circular) loop fugacity at the smallest scale is modified by the second term as

$$y_0 \to y_0(K_0,\lambda) = y_0(K_0,0)e^{-2\pi\lambda} , \tag{2.19}$$

where[7]

$$y_0(K_0,0) = y_0 = e^{-5.631K_0} .$$

Comparing (2.18) and (2.11), one has the "self-duality" result

$$Z_\theta(K_0,\ \lambda=0) = Z_J(K_0=0,\ \lambda=(4K_0)^{-1}) \tag{2.20}$$

and hence a phase transition occurs moving along the λ axis, at $\lambda=\lambda_c$, where, for[7,9] $K_{0c}=0.453$,

$$\lambda_c = (4K_{0c})^{-1} = 0.552 . \tag{2.21}$$

This is close to the series value and MC value[9] of

$\lambda_c = 0.55 \pm 0.005$. (There is an element of arbitrariness in connecting large-scale continuum results to the appropriate square-lattice original scale, as mentioned in Ref. 7, where a plaquette relation between the bare core cutoff a_c and lattice size a_0 is chosen. The numerical agreements are thus, in some sense, fortuitous.)

Since $K_0^{-1} \sim T$ while $\lambda \sim T^{-1}$, it is clear that the transition along the ($K_0 = 0$, $\lambda \neq 0$) axis will have an inverted XY nature[2] with the nonsingular specific-heat asymmetries in T switched about T_c, as compared with the ($K_0 \neq 0$, $\lambda = 0$) transition. Since K_0 controls the dominant long-range behavior in (2.18), the rest of the K_0-λ line for $K_0 \neq 0$ must be[2] of the XY type.

The scaling analysis proceeds as[7] in the $\lambda = 0$ case, but with the bare fugacity reduced as in (2.19). The physical picture is as before.[7] As the temperature is raised from zero, larger, but still simple, loops are thermally generated; these can accommodate more nested screening loops, allowing the loops to grow still larger. A size blowout of the dominant tumbling loop excitation of diameter $\xi_- \sim (T_c - T)^{-\nu}$ occurs at T_c when screening sets in by vector cancellations between random segments $\{\mathbf{J}(r)\}$. Above T_c one has long, random, screened interaction loops. A loop-crinkling ansatz relates the irregular scale or core size a_c below T_c to the self-avoiding random-walk exponent $x \approx 0.6$ in 3D. T_c is determined as before,[7] with only the bare fugacity changed by the short-ranged (and critically irrelevant) chemical-potential term; the exponents are unchanged.

The transition temperature $T_c = J/K_0$, in terms of small deviations $\tilde{K} \equiv (K_0 - K^*)/K^*$ and $\tilde{y}_0 = (y_0 - y^*)/y^*$ from fixed point $(K^*, y^*) = (0.3875, 0.0624)$, is given by[7]

$$\frac{K_0}{K^*} - 1 = \frac{|\alpha_-|}{6(1 - x/L^*)} \left[\frac{y_0(K_0,\lambda)}{y^*} - 1 \right] . \tag{2.22}$$

Here $|\alpha_-| = 2.4888$ is the negative eigenvalue of the loop stability matrix $L^* \equiv 1 - x \ln K^*$, and $x \approx 0.6$ is the 3D self-avoiding walk exponent. With $\lambda = 0$ and $K_0 \approx K_{0c}$, the deviations are small, $\tilde{K} \approx 0.17$ and $\tilde{y} \approx 0.25$. Equation (2.22) can be written, for $K_{0c} = 0.453$, as

$$\lambda = 0.896(K_{0c} - K_0) - 0.159 \ln[1 - 0.292(K_{0c} - K_0)]$$
$$\approx 0.942(K_{0c} - K_0) . \tag{2.23}$$

The transition temperature is thus linearly enhanced by the loop chemical potential $\mu > 0$,

$$T_c \simeq (J + 1.07\mu)/k_B K_{0c} . \tag{2.24}$$

Figure 1 shows the MC data and the K_0-λ transition line of (2.23). Since, for $\lambda \sim \lambda_c$, one has $\tilde{K} \sim 1$ and the

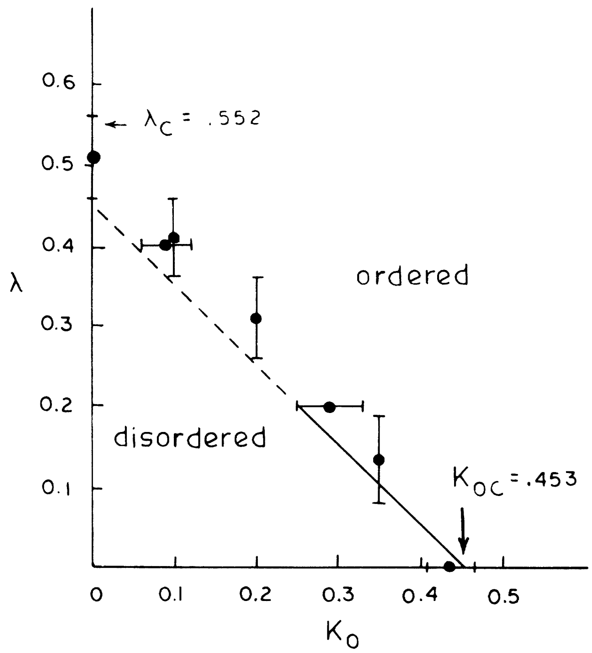

FIG. 1. K_0-λ phase boundary for vortex-loop-fugacity model with Monte Carlo data points of Ref. 9. The solid line is the boundary within the (linear) regime of validity, the dashed line is the extrapolation; the arrows denote critical points $(K_{0c}, 0)$, related to $(0, \lambda_c)$ by self-duality.

linearized form (2.22) breaks down, the upper part of the line is shown dashed. There is reasonable agreement in the regime of validity of the loop-scaling calculation of the critical temperature. The intersection is $\lambda_c \simeq 0.428$ for the dashed linear extrapolation while the self-duality arguments give the better value of (2.21). Thus, the $\{\mathbf{J}(r)\}$ loop description,[6,7] complementary to the $\{\theta_i\}$ angular description, is lent further support.

Kohring *et al.*[9] find a residual degeneracy and a reduced magnetization at $\lambda > \lambda_c$, $K_0 = 0$. In the loop picture, this corresponds to a subcritical fugacity

$$y_0(K_0 = 0, \lambda) < y_0(0, \lambda_c) = 0.078 .$$

Their other plots, e.g., total loop density versus T, would involve an integration over all scales of y_l, while the results are valid for scales $\sim \xi_-$. Therefore, we do not pursue that here.

III. LATTICE-SUPERCONDUCTOR MODEL

The lattice-superconductor model involves both the $\{\theta_i\}$ phase and gauge-field $\{A_{\mu,i}\}$ fluctuations and is

$$Z_s = Z_s(K_0, e^2) = \prod_{\mu,i} \int_{-\pi}^{\pi} \frac{d\theta_i}{2\pi} \int_{-\infty}^{\infty} dA_{\mu,i} \exp \left[K_0 \sum_{\mu,i} \cos(\Delta_\mu \theta_i - A_{\mu,i}) \right] \exp \left[-\sum_i (\mathbf{\Delta} \times \mathbf{A})^2 / 2e^2 \right] , \tag{3.1}$$

where e is the (scaled) electromagnetic coupling that, in these units, depends on temperature $e^2 \sim T$.

In an early work, Peskin[3] had shown that there is a transition along the e^2 line for $K_0 = 0$. Dasgupta and Halperin[2] had, in MC work on the equivalent Villain model, found a second-order backward-bending $e^2 - K_0^{-1}$ line with inverted

XY behavior except at the XY point $e^2=0$, $K_0=K_{0c}$. Kleinert[3] had mapped the model onto a Ginzburg-Landau form and obtained a mean-field $e^2-K_0^{-1}$ transition line. It is also possible to do a topological mean-field theory[15] on the vortex-loop model itself, with spin ordering corresponding to the loops orienting and canceling from the bulk as $T\to 0$. In the following, one finds that the lattice superconductor $(e^2-K_0^{-1})$ can be mapped onto the loop-fugacity model (K_0,λ) and the critical line $T_c(e^2)$ obtained here from the scaling approach.[7]

Integrating over $\{\theta_i\}$ variables after the usual Fourier expansion

$$Z_s(K_0,e^2)=\sum_{\{n_{\mu,i}\}}{}' \exp\left[-\sum_{\mu,i} n_{\mu,i}^2/2K_0\right]\prod_{\mu,i}\int_{-\infty}^{\infty} dA_{\mu,i}\exp\left[-i\sum_{\mu,i} n_{\mu,i}A_{\mu,i}\right]\exp\left[-\frac{1}{2e^2}\sum_i(\mathbf{\Delta}\times\mathbf{A}_i)^2\right], \tag{3.2}$$

where the prime denotes the zero-divergence restriction of Eq. (2.10), i.e., original-lattice loops.[2] Doing the Gaussian integration over the vector potential $A_{\mu,i}$, a (original-lattice) loop-fugacity model results,

$$Z_s(K_0,e^2)=\sum_{\{n_{\mu,i}\}}{}' \exp\left[-\frac{\pi}{2}\frac{e^2}{4\pi^2}\sum_{\substack{\mu,\\ r,r'}} n_{\mu,r}n_{\mu,r'}U(r-r')\right]\exp\left[-\frac{1}{2K_0}\sum n_{\mu,r}^2\right]. \tag{3.3}$$

Here $U(r-r')$ is the 3D (original-lattice) Greens function, as in (2.16), and comes from the exchange of a "photon" rather than a "spin wave."

Comparing (3.3) and (2.18), one has the correspondence

$$K_0\to e^2/4\pi^2,\quad \lambda\to(4K_0)^{-1}, \tag{3.4}$$

or

$$Z_s(K_0,e^2)=Z_J(e^2/4\pi^2,1/4K_0), \tag{3.5}$$

the duality[2,3] first found by Peskin.

From (2.19) the bare-loop fugacity is

$$y_0(e^2/4\pi^2,1/4K_0)=e^{-5.631(e^2/4\pi^2)}e^{-\pi/2K_0} \tag{3.6}$$

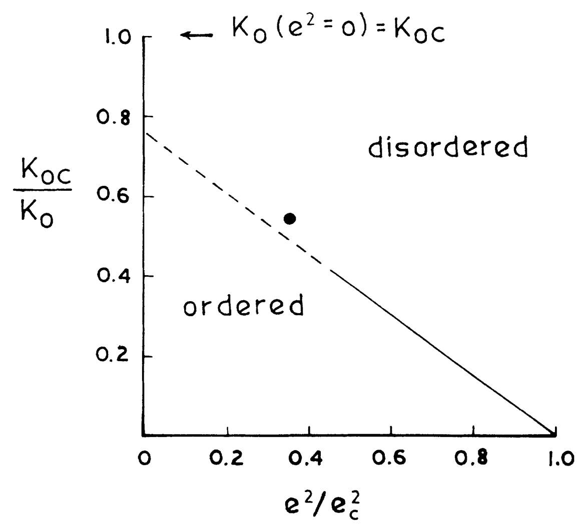

FIG. 2. e^2-K_0^{-1} phase boundary for the lattice superconductor with the Monte Carlo data point $e^2=5$ of Ref. 2. The solid line is the boundary within the (linear) regime of validity, the dashed line is the extrapolation, the axis critical points, related by self-duality, are scaled to unity.

with the critical value

$$e_c^2\equiv 4\pi^2K_{0c}\simeq 17.88 \tag{3.7}$$

for $K_{0c}=0.453$. The transition line of (2.22) is, in this case,

$$\frac{K_{0c}}{K_0}=0.736\left[1-\frac{e^2}{e_c^2}\right]-0.2888\ln\left[1-0.132\left[1-\frac{e^2}{e_c^2}\right]\right]$$
$$\simeq 0.774\left[1-\frac{e^2}{e_c^2}\right]. \tag{3.8}$$

Once again the analysis is valid for small deviations

$$\tilde{e}^2\equiv(e^2-4\pi^2K^*)/4\pi^2K^*\ll 1,$$
$$\tilde{y}_0\equiv(y_0-y^*)/y^*\ll 1,$$

as indicated by the solid line of Fig. 2. For $K_0^{-1}=0$, one has the point $e^2/e_c^2=1$.

MC work by Dasgupta and Halperin[2] for the Villain model, where $K_{0c}=0.33$, finds that, for $e^2\equiv 5$, $K_0^{-1}=1.62$. Our analysis uses the cosine interaction, that asymptotically has the same topological excitation interaction as the Villain model, but may differ at the original scale. Our choice of core size $a_c=a_c(a_0)$ that determines K_{0c} was with the cosine interaction vortices in mind.[7] Thus, for comparison with the Villain results, scaled variables e^2/e_c^2, K_{0c}/K_0 are used with appropriate K_{0c}. The MC Villain-model data point is then $e^2/e_c^2=0.383$, $K_{0c}/K_0=0.535$, indicated by a dot in Fig. 2. One finds from (3.8), for $e^2=5, K_{0c}=0.453$, that the theoretical curve falls nearby at $e^2/e_c^2=0.28$, $K_{0c}/K_0=0.558$. Since the lattice-superconductor model is mapped by (3.4) onto the loop-fugacity model, further MC work on the former for other e^2/e_c^2 points might be compared with existing $K_0-\lambda$ MC data of the latter.

IV. SUMMARY AND FURTHER PROBLEMS

An externally controlled chemical potential $\lambda k_BT\equiv\mu>0$ for 3D XY vorticity is shown to decrease the vortex-loop fugacity and enhance the transition temperature T_c. Self-duality arguments relate the transition at ($K_0=K_{0c}$, $\lambda=0$) to the transition at ($K_0=0$, $\lambda=\lambda_c$), with the theoretical result $\lambda_c=(4K_{0c})^{-1}=0.552$ agreeing closely with the MC value 0.55 ± 0.005. The K_0-λ transition line is close to the MC data in the region of validity of the linearized fixed-point analysis. Inverted XY behavior is predicted at ($K_0=0$, $\lambda=\lambda_c$).

The lattice-superconductor model with an electromagnetic coupling parameter e can be mapped onto the loop-fugacity model. The backward-bending e^2-K_0^{-1} (T_c suppression) line maps onto the backward-bending (T_c enhancement) line. Further MC e^2-K_0^{-1} data on the lattice superconductor (for both $K_0>0$ and $K_0<0$) and comparison with the K_0-λ data would be useful.

Capacitive 2D Josephson-junction arrays[16] are also possible (2+1)D applications of the 3D loop ideas. The phase boundaries of the (4+1)D lattice gauge and Abelian Higgs models, and the effect of topological excitation suppression, have been explored[11] in MC work. A topological and scaling analysis of these gauge models would clearly be of interest.

It is amusing to note that metal-oxide-metal structure of Josephson-junction arrays[16] has some conceptual resonance with the copper-oxygen-copper structure of the high-T_c materials, where the coherence length is of atomic-spacing size. Apart from the fact that arrays consider $2e$ pairs rather than electrons, the coupling K_0 is like a tunneling parameter t, the grain-charging energy E_{CG} is like an on-site Hubbard repulsion u, the junction charging energy E_{CJ} is like an extended Hubbard nearest-neighbor coupling V. Resonating-valence-bond ideas map the Hubbard model onto local pair amplitudes with bond variables (gauge fields) rather than the site variables θ_i of the XY model. The phase boundaries of the 3D Josephson array (insulator-superconductor) in terms of charging energies-dissipation-anisotropy may provide insight into such other models.[2] A similar MC and topological scaling investigation of Hubbard-related (2+1)D lattice gauge models[12] would be of interest.

As Anderson[17] has emphasized, the generalized helicity modulus is an ordering probe for several phase transitions. The superfluid fraction ρ_s/ρ_0, for example, is unity at $T=0$ and reduced to zero as $T\rightarrow T_c$ by thermal excitations. In the topological viewpoint, these are vortex points (2D) or loops (3D), and a suppression of topological fugacities means having to go to a higher temperature to reach the critical fugacity value at which a size blowout occurs. The viewpoint is complementary to the usual focus on factors that enhance the order parameter. One focuses instead, on parameters $\{\lambda_i\}$ that suppress disorder parameters. For high-T_c superconductors, this suggests a focusing on how possible $\{\lambda_i\}$ parameters like electronegativity[1] (atomic charging), dissipative local mode coupling, composition, and structure, might suppress or raise the relevant topological fugacities, and raise or suppress T_c in the families of candidate materials.

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