Interface pinning and dynamics in random systems

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A detailed low-temperature treatment of the domain wall or interface pinning by imperfections in disordered systems with discrete symmetry of the order parameter is presented. Crossover behavior as well as analogies between pinning mechanisms in different systems is analyzed. Pinning may arise from random bonds, when the disordering agents do not break the local symmetry of the order parameter, or from random fields, when the disordering agents do break this symmetry. The interface roughness and response to an external driving force are discussed. The model is explained for dilute magnetic systems in a uniform field where the magnetic domain walls are pinned by random fields and/or random bonds. The results are, however, more general and apply also to interfaces in other systems, e.g., in fluid-fluid interfaces, (anti)ferroelectrics, solitons in incommensurate systems, etc. The interface roughness and pinning pressure (force per unit area) are estimated for weak and strong pinning and their scaling relations to length scale, temperature, frequency, and disorder strength (concentration) are given. The interface contribution to the static and dynamic susceptibility at low temperatures is evaluated. Because of pinning, the low-temperature dynamical susceptibility of disordered ferromagnets in or out of equilibrium carries a $[\ln(1/\omega)]^{2/\theta}$ frequency dependence in addition to the Debye relaxation behavior. In particular, $\theta=(d+1)/3$ for random-field systems, and $\theta(d=2)=1/3$ and $\theta(d=3)\approx 0.83$ for random-bond systems.

I. INTRODUCTION

Domain walls are, in general, the consequence of either competing interactions (sometimes also realized by the boundary conditions) or slowly decaying metastable configurations. In the first category are, e.g., magnetic systems, where the domain-wall energy is competing with the stray field energy.[1] The equilibrium domain configuration is such that the sum of the domain-wall energy and magnetic field energy is minimal. Stable interfaces can be observed also in other physically equivalent systems. For example, one can observe a wetting transition in the vicinity of the coexistence region of two phases, when a wall enclosing one phase becomes sufficiently attractive to the other phase. In this case, the interface is stabilized by the interactions with an external potential.[2] In the axial next-nearest-neighbor Ising (ANNNI) model or in incommensurate systems the domain-wall structure is determined by competing nearest-neighbor and next-nearest-neighbor interactions.[3,4] Different kinds of domain walls or interfaces then separate domains with different phase shifts. Examples are submonolayers of molecules or atoms on clean crystalline substrates.[5]

Slowly decaying metastable domain structures are observed, e.g., in disordered magnetic systems cooled from the paramagnetic into the ordered phase, when the domain walls appear as gradual freezing of dynamic fluctuations.[6,7] As the domain structure slowly decays toward a more stable configuration, the domain walls move in such a way that energetically unfavorable domains shrink and eventually disappear. In general, the domain walls or interfaces move or relax when an additional external pressure is exerted on the interfaces (ferromagnets in a variable magnetic field, fluid-fluid interfaces in porous systems[8]), or when the system is quickly cooled from the disordered into the ordered phase (magnetic systems). To simplify the notation, we will use in the following the term interface also as a synonym for domain walls.

Interface mobility in random systems is strongly hindered by pinning to impurities or other disordering agents. In the past, pinning has been studied for some specific systems.[9–27] A prototype example of interface

pinning is the dilute antiferromagnet (AFM) without or with applied external field. These systems were the subject of detailed experimental[6,27] and theoretical[7,13–18] investigations in the past. In dilute antiferromagnets, the domain-wall energy can be reduced if the wall runs through nonmagnetic impurities; this is random-bond pinning.[18] The wall can also be pinned by random fields when the domain magnetic moments are parallel to the local fluctuations of the random fields.[13–18] Attention has been paid to interface pinning in three-dimensional dilute antiferromagnets in a uniform field, also described by the random-field Ising model (RFIM).[6,7] Depending on the cooling conditions, these systems can be at low temperatures either in a stable long-range ordered state or in a metastable microdomain configuration. The microdomains are subject to shrinkage due to surface tension, but they can persist in these systems because of pinning of domain walls to random fields (RF's) and/or random bonds (RB's).[18] For example, broad domain walls in weakly anisotropic antiferromagnets are pinned only by fluctuations in random fields, whereas the narrow walls in strongly anisotropic antiferromagnets are pinned both by random fields and random bonds. Pinning is considered to be weak when the interface is pinned on the fluctuations in the density of the disordering agents and strong when the interface is pinned by many small clusters or isolated missing bonds.[11] Similarly, domain walls in ferroelectrics can be pinned by screw dislocations if the systems are also ferroelectric (e.g., Rochelle salt) and to edge dislocations if the systems are not ferroelastic, like triglycine sulfate.[28] Sufficiently strong electric fields depin the walls, which then move with a constant average velocity, depending on the field.

Another example of interface pinning appears in incommensurate phases, where a soliton, i.e., the interface between two almost commensurate domains, is driven by an external field, but can be pinned by distant random impurities.[20,29] As in the case of RFIM, the pinning is strong for narrow and weak for broad solitons. Furthermore, in real crystals with defects a dislocation line, which can be described as an elastic string, is locally pinned by strong defects.[30] Weak pinning and relaxation of defect lines has been considered by Ioffe and Vinokur.[31] In adsorbed monolayers the domain walls strongly interact with defects on the surface of the substrate crystal.[20] Similarly, the fluid-fluid interfaces in porous systems are pinned by interaction with a strongly irregular solid boundary.[8] Pinning in related systems like charge-density waves (CDW's) in the presence of impurities has been discussed in detail, e.g., by Fukuyama and Lee, by Klemm and Schrieffer,[11] and by others.[12]

The interface dynamics has also been studied in computer simulations and experimentally. Monte Carlo methods have been applied to verify the proposed mechanisms of interface growth, pinning, and relaxation in dilute magnetic systems.[23–26] Insight into the interface relaxation in dilute antiferromagnets has also been gained from measurements of the time-dependent remanent magnetization.[27]

The above examples indicate the importance of interface pinning in various systems. In general, an external field, or also the surface tension, produces a driving force on the interface. If the force is strong enough, the interface can be depinned after a time which depends on the pinning and driving forces, on the pinning energy, and on temperature. This leads to interface dynamics, which is the main subject of this paper.

The motivation of this paper is to provide a universal treatment of interface pinning and dynamics in various disordered systems with discrete symmetry of the order parameter at low temperatures and to stress the crossover between different pinning regimes. The frequency dependence of the interface relaxation and the effect of this relaxation on the dynamical susceptibility are estimated also. Two types of pinning are considered: (i) pinning in the presence of random bonds when the impurities do not break the local symmetry of the order parameter, and (ii) pinning in the presence of random fields when the impurities break the local symmetry. In Sec. II different mechanisms that contribute to the pinning of broad and narrow interfaces are discussed. Pinning forces, pinning energies, and interface relaxation are explained. In particular, we will use the magnetic example to describe the mechanisms of interface pinning and relaxation; the results are, however, applicable to all physically equivalent systems. Section II represents a generalization of the work of Ioffe and Vinokur, who considered weak pinning of dislocation lines in imperfect crystals.[31] The result of Sec. II will be used to evaluate the interface contribution to the order parameter and to the static susceptibility of weakly anisotropic ferromagnetic and antiferromagnetic systems. We will also discuss the energy dissipated in the course of the relaxation of the interfaces and the complex ac susceptibility.

II. INTERFACE PINNING

A. The model

The interface in d-dimensional systems is modeled as a $(d-1)$-dimensional elastic "drum head" of the intrinsic width ξ. In anisotropic Heisenberg systems, for example, $\xi \approx \pi(J/A)^{1/2}$, where J is the exchange energy and A is the uniaxial crystal-field anisotropy energy (see the Appendix). In strongly anisotropic Ising-type systems, $\xi \sim 1$, where the length scale is in units of lattice spacing. In incommensurate systems, on the other hand, ξ is the soliton width and is determined from the competition between the umklapp term, which favors commensurate structure, and the elastic energy, which favors incommensurate structure.[29,32] Of course, this description of the interface is only valid on length scales $L \gg \xi$.

The elastic energy of a slightly distorted interface is

$$\mathcal{H}_0 \approx \int d^{d-1}\mathbf{x}[\sigma + \tfrac{1}{2}\Gamma(\nabla z)^2] , \tag{2.1}$$

where σ is the interface tension, Γ is the interface stiffness, and z is the interface distortion from a flat reference plane. In general, the stiffness Γ is a complicated function of the temperature T, the concentration of disordering agents, and their strength.[33,34] Below the roughening transition temperature T_R, Γ is infinite and $\Gamma(\nabla z)^2$ in (2.1) has to be replaced by $\gamma|\nabla z|$, where $\gamma \approx J$

is the energy of a kink in the interface. However, it is believed that $T_R=0$ for *disordered* systems in $d\leq 3$.[19,34] For isotropic systems, $\Gamma=\sigma$, and for broad magnetic domain walls, $\Gamma\approx\sigma\approx J/\xi$.

The interaction energy of the interface with randomly distributed impurities can be written as

$$\mathcal{H}_1=-\int d^{d-1}\mathbf{x}\, V(\mathbf{x},z(\mathbf{x}))\ , \tag{2.2}$$

where the interaction potential V depends on the type of the impurities. We first consider impurities of the *random-bond* type, $V=V_{\rm RB}$, when the *impurities do not break the local (Ising) symmetry,* i.e., when there is no coupling of the impurities to the orientation of the order parameter. We write the interaction potential as a sum over all impurities:

$$V_{\rm RB}(\mathbf{x},z(\mathbf{x}))=\sum_j v_j a\delta_{a_\parallel}(\mathbf{x}-\mathbf{x}_j)\delta_a(z(\mathbf{x})-z_j)\ , \tag{2.3}$$

where $\vec{r}_j=(\mathbf{x}_j,z_j)$ is the position of the jth impurity, and v_j the potential energy which the interface can gain when it overlaps the impurity j. $\delta_a(x)$ reflects the shape of the interaction potential of range a between the interface and the impurity in the direction perpendicular to the interface and is assumed to be a "smeared-out" delta function of width a. Similarly, $\delta_{a_\parallel}^{(d-1)}(\mathbf{x})$ is a $(d-1)$-dimensional δ function of width $a_\parallel$ in the plane of the interface. In order to relate v_j and a to a microscopic model, we consider the impurities as spherical objects of diameter b, which can be either larger or smaller than the interface width ξ, and a short-range interaction potential. In that case $\delta_a(x)$ is of the order $1/a$ if the impurity and wall are in contact, and vanishes otherwise.

If $b<\xi$, the impurity diameter is small compared to the interface width and the interface can interact with the RB potential j only when the impurity j (in general, a locally disordered region) lies inside the interface. The range of the interaction is then determined by the interface width, $a\approx\xi$. For an estimate we imagine the interface as a magnetic domain wall and the impurity as a nonmagnetic region. In this case the energy of the domain wall, i.e., the interface tension, vanishes in the impurity region. As the energy density of the interface is $E_0\approx\sigma/\xi$, each impurity included in the interface makes an energy gain $v_j\approx E_0 b^d\approx c_j\sigma b^{d-1}(b/\xi)$, where c_j (and later c_j') is a constant of the order unity.

If $b>\xi$, the interface is narrow compared to the impurity size and the interface of the width ξ runs only through a narrow region of the impurity with the diameter b and, consequently, $a\approx b$. The gain in the interface energy is now $v_j\approx c_j' E_0 b^{d-1}\xi\approx c_j'\sigma b^{d-1}$.

In general, the potential energy of the interface interacting with the impurity j can be written as

$$v_j\approx c_j\sigma\frac{b}{a}b^{d-1}\ , \tag{2.4}$$

where the range of the interface-impurity interaction is $a\approx\max\{\xi,b\}$. We assume attractive interaction between the interface and impurity, $v_j>0$. In particular, v_j is of the order J in diluted ferromagnets if $b\sim\xi$.

The mean impurity strength v is defined via the configurational average

$$[V_{\rm RB}]_{\rm av}=\prod_j\int\frac{d\vec{r}_j}{\Omega}V_{\rm RB}=\frac{va}{D^d}\ ,$$

where Ω is the volume of the system and D is the mean impurity distance. The fluctuation of the random-bond impurity potential, which is important in the case of weak pinning, is

$$[V_{\rm RB}(\vec{r})V_{\rm RB}(\vec{r}')]_{\rm av}-[V_{\rm RB}(\vec{r})]_{\rm av}^2=\frac{v^2a^2}{D^d}\delta_{a_\parallel}(\mathbf{x}-\mathbf{x}')\delta_a(z-z')\ . \tag{2.5}$$

If the impurities break the symmetry of the local order parameter, the interaction of the interface with the impurities is of the *random-field* type and is nonlocal:[20]

$$V_{\rm RF}(\mathbf{x},z(\mathbf{x}))=\sum_j v_j\delta_{a_\parallel}(\mathbf{x}-\mathbf{x}_j)\int_0^{z(\mathbf{x})}dz'\,\delta_a(z'-z_j)+\text{const.} \tag{2.6}$$

In dilute antiferromagnets, $v_j=2M_0H_j$, where M_0 is the spontaneous (sublattice) magnetic moment and H_j is the random field at the position $\vec{r}_j$. The configurational average of the correlation function of the random-field potentials for large $|z'-z|$ is

$$[V_{\rm RF}(\vec{r})V_{\rm RF}(\vec{r}')]_{\rm av}-[V_{\rm RF}(\vec{r})]_{\rm av}^2-\frac{v^2}{D^d}\delta_{a_\parallel}(\mathbf{x}-\mathbf{x}')|z-z'|\ . \tag{2.7}$$

Here $v=2M_0H$ and H is the mean strength of the random field.

B. Weak and strong pinning

We now consider the effect of the *random potential* on the interface. In the case of an attractive random-bond potential, the interface will try to overlap with as many impurities as possible. As a result, the interface will be pinned either by local impurities or by their density fluctuations, and in both cases a finite force has to be applied in order to move a part of the interface.

For CDW's Fukuyama and Lee first pointed out that it is important to distinguish between *weak* and *strong pinning* according to the ratio ε of the impurity pinning energy and the elastic energy per impurity.[11] If $\varepsilon\gg 1$, the CDW is strongly pinned by isolated impurities, whereas for $\varepsilon\ll 1$ the single impurity potential is too weak to distort the CDW. In this case only fluctuations in the impurity density lead to pinning.

For interfaces the situation is slightly more complicated since besides the interimpurity spacing D, the effective width a of the interface also plays a role. In this paper we will mainly consider pinning of the interfaces on the *fluctuations* of the impurity density. If, in this case, the interface is displaced on the scale D by the interface width a, the average fluctuation of the energy of interaction with the impurities is $v(a/D)^{1/2}$. This has to be compared with the elastic energy $\Gamma a^2 D^{d-3}$. Therefore,

we introduce the ratio

$$\varepsilon=\frac{v(a/D)^{1/2}}{\Gamma a^2 D^{d-3}} \tag{2.8}$$

as a parameter that measures the pinning strength. A second parameter

$$\eta=\frac{a}{D}$$

describes the degree of disorder.

(i) For $\eta\gg 1$ the interfaces are *broad* compared with the impurity separation and therefore pinning on fluctuations in the impurity concentration always (for arbitrary ε) dominates over pinning by a single impurity. We call this the *weak pinning* regime. If, on the other hand, the impurities are strongly diluted, $\eta\ll 1$, the interface is again only weakly pinned when the number of impurities in the interface, $N(L)\approx(L^{d-1}a/D^d)$, is large, $N(L)\gg 1$, i.e., on large length scales $L\gg L_\eta$,

$$L_\eta\approx D\eta^{-1/(d-1)} . \tag{2.9}$$

L_η is the minimal length scale on which the interface can be weakly pinned on impurity density fluctuations. On scales $L\ll L_\eta$ the interface can only be pinned on single impurities. However, if $\varepsilon\eta^{-1/2}\ll 1$, a single impurity is not able to distort the wall by a; hence there is no pinning for $L\ll L_\eta$.

(ii) For $\eta\ll 1$ and $\varepsilon\gg\eta^{1/2}$, pinning due to isolated impurities becomes dominant on scales $D\lesssim L\ll L_\eta$. It seems to be natural to distinguish between an *intermediate* pinning regime, $\eta^{1/2}<\varepsilon<\eta^{-3/2}$, when the typical transverse distortion w of the interface is $a<w<D$, and a *strong pinning* regime, $\varepsilon>\eta^{-3/2}$, when the domain wall is spanned between neighboring randomly distributed impurities.

C. Interface roughness in the weak pinning case

In the weak pinning regime the broad interfaces take advantage of the impurity density fluctuations. This leads to a statistical width (roughness) of the interfaces, $w(L)$, which is defined as the configurational average of $\langle|z(x)-z(x+L)|^2\rangle^{1/2}$ and which scales with the length scale L as L^ζ.

Let us first consider the situation at $T=0$. A simple estimate for the roughness exponent ζ follows from the following Flory-type argument. The elastic part of $\mathcal{H}_0$ scales with the length as $\Gamma w^2 L^{d-3}$ and has to be compared with the fluctuation of $\mathcal{H}_1$, which is of the order

$$\delta E_{\rm imp}(L)=v\left[\frac{L^{d-1}a}{D^d}\right]^{1/2}\left|\frac{w}{a}\right|^{x_F/2} .$$

Here, $x_F^{\rm RF}=1$ and $x_F^{\rm RB}=-1$ for RF and RB systems, respectively. From the equality of terms, we obtain the Flory result for ζ:[21]

$$\zeta_F=\frac{5-d}{4-x_F},\quad x_F^{\rm RF}=-x_F^{\rm RB}=1 . \tag{2.10}$$

In a more refined theory which takes into account fluctuations on all length scales, the simple Flory argument is no longer valid and more elaborate renormalization-group techniques have to be applied.[22] Nevertheless, the results can be written in the same scaling form as before if x_F is replaced by a new exponent $x(d)$, which is for RF's again equal to $x^{\rm RF}=1$,[22] but approaches $x^{\rm RB}\approx -0.8$ for RB's as $d\to 5$.[19,22] $w(L)$ is then again obtained from the minimization of the total interface free energy $E(L,w)$, which is the sum of the elastic energy and energy gain $\delta E_{\rm imp}(L)$:

$$E(L,w)=\tfrac{1}{2}\Gamma w^2 L^{d-3}-v\left[\frac{L^{d-1}a}{D^d}\right]^{1/2}\left|\frac{w}{a}\right|^{x/2} . \tag{2.11}$$

Minimization of $E(L,w)$ yields, for the interface roughness $w(L)$ in $d<5$,

$$w\approx a(L/L_\varepsilon)^\zeta , \tag{2.12}$$

where

$$\zeta=\frac{5-d}{4-x(d)} ,$$

$$x^{\rm RF}(2<d\le 5)=1 , \tag{2.10'}$$

$$x^{\rm RB}(d\to 5)\approx -0.8 ;$$

$$L_\varepsilon=D\varepsilon^{-2/(5-d)} , \tag{2.13}$$

and ε is already defined in (2.8). L_ε is the minimal length scale on which the interface feels the potential barriers and is therefore rough. On scales $L<L_\varepsilon$, $w<a$ and the interface is flat and free, whereas the interface is weakly pinned when $L\gg L_\varepsilon$ and $L\gg L_\eta$ [see Eq. (2.9)]. Thus (2.12) is valid only on scales $L\gg\max(L_\eta,L_\varepsilon)$.

The considerations made so far apply to $T=0$. Thermal fluctuations additionally roughen the interface. In order to obtain results for *nonzero* temperatures, we consider first the random part $\mathcal{H}_1$ as a perturbation to $\mathcal{H}_0$. If disorder is neglected, the interface roughness, because of thermal fluctuations, is $w=w_T$ with

$$w_T\approx\left[\frac{T}{\Gamma}\right]^{1/2}L^{\zeta_T},\quad \zeta_T=(3-d)/2$$

(the temperature is in units of energy; $k_B=1$) for $d<3$, and $w_T\approx[(T/\Gamma)\ln L]^{1/2}$ in $d=3$ dimensions; $\zeta_T=0$ for $d>3$. With $z\propto w_T$, $\mathcal{H}_0/T$ is of order 1, whereas $\mathcal{H}_1/T$ scales as

$$\frac{\mathcal{H}_1}{T}\approx\frac{v}{T}\left[\frac{L^{d-1}a}{D^d}\left[\frac{w_T(L)}{a}\right]^{x_F}\right]^{1/2}$$
$$=\left[\frac{L}{L_T}\right]^{(4-x_F)\phi/4},\quad d\le 3 , \tag{2.14}$$

where the crossover exponent ϕ can be expressed in terms of the Flory exponent ζ_F:

$$\phi=2(\zeta_F-\zeta_T)=\frac{2-x_F}{4-x_F}(d-d_1) ,$$

$$d_1=\frac{2-3x_F}{2-x_F} . \tag{2.15}$$

On scales $L<L_T$ the influence of disorder is small so that

the interface fluctuations are described by the Gaussian Hamiltonian (2.1). Consequently, there is no renormalization of $\mathcal{H}_1$ on these scales and hence the exponents x_F and ζ_F appear in the definition of L_T. (See also the discussion at the end of this section, where these predictions are compared with the results of a renormalization-group (RG) calculation at finite T in $d=2$ dimensions.) On large scales $L>L_T$, however, weak RB disorder is relevant for $d>\frac{5}{3}$, whereas RF disorder is relevant for $d>-1$. The temperature-dependent length scale L_T is given by

$$L_T=L_\varepsilon\left[\frac{T}{T_\varepsilon}\right]^{1/\phi}, \quad d\le 3 , \tag{2.16}$$

where

$$T_\varepsilon=\Gamma a^2 L_\varepsilon^{d-3}=v\eta^{1/2}\varepsilon^{-(d-1)/(5-d)} \tag{2.17}$$

corresponds to the height of the barriers between metastable states on the scale L_ε, i.e., to the height of the smallest barriers. For $T>T_\varepsilon$, we expect that $w(L)$ can be written as

$$w(L)\approx w_T(L_T)\left[\frac{L}{L_T}\right]^{\zeta}, \quad L>L_T , \tag{2.12'}$$

whereas for $T\ll T_\varepsilon$, Eq. (2.12) applies. Equations (2.12) and (2.12') can be unified to

$$\begin{aligned} w(L)&\approx a_T\left[\frac{L}{L_{\varepsilon,T}}\right]^{\zeta} \\ &=a\left[1+\frac{T}{T_\varepsilon}\right]^{(\zeta_F-\zeta)/\phi}\left[\frac{L}{L_\varepsilon}\right]^{\zeta}, \end{aligned} \tag{2.12''}$$

where

$$L_{\varepsilon,T}=L_\varepsilon\left[1+\frac{T}{T_\varepsilon}\right]^{1/\phi}, \tag{2.18}$$

$$a_T=a\left[1+\frac{T}{T_\varepsilon}\right]^{\zeta_F/\phi},$$

and $\phi=(d+1)/3$ for RF's and $\phi=(3d-5)/5$ for RB's, $d\le 3$. For RF systems $\zeta=\zeta_F$ and hence the roughness amplitude is essentially temperature independent. Notice that Eq. (2.12'') is only valid on length scales $L>\max\{L_\eta,L_{\varepsilon,T}\}$.

The predictions for the amplitude of $w(L)$ in (2.12'') can be found also from more rigorous scaling considerations and from the integration of the RG flow equations.[35] In particular, for the RB case in $d=2$ dimensions with $\zeta_F=\frac{3}{5}$, $\zeta_T=\frac{1}{2}$, $\zeta=\frac{2}{3}$, and $(\zeta_F-\zeta)/\phi=-\frac{1}{3}$, we get $L_\varepsilon=D[\Gamma^{3/2}/(vD^{1/2})]^{2/3}$ and $T_\varepsilon=\Gamma^{1/3}a(v/D)^{2/3}$, in agreement with what was found in Ref. 35, where we denoted $\Delta=v^2a^2D^{-d}$. In the RB case in $d=3$ dimensions, we have $\zeta_F=\frac{2}{5}$, $\zeta_T=0$, $\phi=\frac{4}{5}$, $\zeta\approx 0.5\pm 0.08$,[36] $(\zeta_F-\zeta)/\phi\approx -0.125$, $L_\varepsilon=\Gamma(aD)^{3/2}/v$, and $T_\varepsilon\approx\Gamma a^2$, where we neglected logarithmic corrections.

D. Pinning forces, metastable domain radii, and creep velocity

In this section we mainly concentrate on weak pinning of the interface on impurity density fluctuations. We give the finite-temperature expressions for pinning pressure and creep velocity, when the interface is additionally roughened by thermal fluctuations.

In systems with weak pinning, the typical potential relief of the interface consists of many metastable minima at the distance $w(L)$, separated by energy barriers of height $E_B(L)$. The barriers $E_B(L)$ scale as the typical energy gain due to disorder, which is of the order of the elastic energy[37]

$$E_B(L)\simeq\Gamma L^{d-1}(w/L)^2, \quad [L\gg\max\{L_\eta,L_\varepsilon\}] . \tag{2.19}$$

For $T\ll T_\varepsilon$, $w(L)$ is given by (2.12) and hence $E_B(L)\approx T_\varepsilon(L/L_\varepsilon)^\theta$, where

$$\theta=2(\zeta-\zeta_T)=2\zeta+d-3 . \tag{2.20}$$

On *macroscopic* time scales $t>\tau(L)$, where

$$\tau(L)=t_0\exp[E_B(L)/T] , \tag{2.21}$$

the barriers on scale L are jumped over and are therefore ineffective for pinning. From (2.19) and (2.21) the minimal length scale L_t, on which the barriers are relevant, is

$$L_t\approx L_{\varepsilon,T}\left[\frac{T}{T+T_\varepsilon}\ln\left[\frac{t}{t_0}\right]\right]^{1/\theta} . \tag{2.22}$$

On *microscopic* time scales, on which $\ln(t/t_0)\approx 1$, if $T>T_\varepsilon$, we get $L_t\approx L_T$, as it should be since the same thermally activated processes are considered. To summarize, an interface is free on length scales $L\lesssim L_c(T,t)$:

$$L_c(T,t)=\max\{L_\eta,L_{\varepsilon,T},L_t\} , \tag{2.23}$$

where L_η is defined in (2.9) and $L_{\varepsilon,T}$ by (2.18). If $L_\eta<L_{\varepsilon,T}$, the minimal scale on which the interface is pinned, $L_c(T,t)$, can be written as

$$L_c(T,t)\approx L_{\varepsilon,T}\left[1+\frac{T}{T+T_\varepsilon}\ln\left[\frac{t}{t_0}\right]\right]^{1/\theta} . \tag{2.23'}$$

When the driven interface on the scale L is exposed to a driving force which is proportional to the area L^{d-1}, we introduce the pinning pressure, i.e., the pinning force per unit area, rather than the pinning force itself. The pinning pressure can be estimated as[7,18]

$$\begin{aligned} p(L)&\approx\frac{E_B(L)}{w(L)L^{d-1}} \\ &\approx\frac{T_\varepsilon}{aL_\varepsilon^{d-1}}\left[\frac{L}{L_\varepsilon}\right]^{\zeta-2} \quad (L\gtrsim L_\varepsilon) . \end{aligned} \tag{2.24}$$

As $\zeta-2<0$, the strongest pinning pressure is on the shortest length scale, $L=L_\varepsilon$, where $p(L_\varepsilon)=p_\varepsilon\approx T_\varepsilon/(aL_\varepsilon^{d-1})$. On length scales $L<L_\varepsilon$ the interface is free since there are no energy barriers.

At $T\gtrsim T_\varepsilon$ the interface roughness $w(L)$ is given by (2.12'); therefore, $E_B(L)\approx T(L/L_T)^\theta$, which has the

correct limit at the lower bound $L \approx L_T$. From this we get the pinning pressure

$$p(L) \approx p_\varepsilon \left[\frac{T}{T_\varepsilon} \right]^{(\zeta_F - 2)/\phi} \left[\frac{L}{L_T} \right]^{\zeta - 2} \quad (L \gtrsim L_T) . \tag{2.24'}$$

Both expressions, (2.24) and (2.24') can be unified to

$$p(L) \approx p_{\varepsilon,T} \left[\frac{L}{L_{\varepsilon,T}} \right]^{\zeta - 2} = p_\varepsilon \left[1 + \frac{T}{T_\varepsilon} \right]^{(\zeta_F - \zeta)/\phi} \left[\frac{L}{L_\varepsilon} \right]^{\zeta - 2} \quad (L > L_{\varepsilon,T}) , \tag{2.24''}$$

$$p_{\varepsilon,T} = \frac{T_\varepsilon}{a L_\varepsilon^{d-1}} \left[1 + \frac{T}{T_\varepsilon} \right]^{(\zeta_F - 2)/\phi} . \tag{2.25}$$

As expected, thermal fluctuations diminish the pinning pressure, since $\zeta_F < 2$. However, this reduction of $p(L)$ takes into account only fluctuations on *microscopic* time scales. From (2.24'') and (2.23') we get the maximal *time-dependent pinning pressure:*

$$p_c(t) \approx p_{\varepsilon,T} \left[1 + \frac{T}{T + T_\varepsilon} \ln \left[\frac{t}{t_0} \right] \right]^{(\zeta - 2)/\theta} . \tag{2.26}$$

If the system is initially in a metastable microdomain state, the domains can collapse on a time scale t only when the surface pressure $p \approx \sigma / R$ exceeds the pinning pressure $p_c(t)$. This is the case for small domain radii

$$R < R_c(t) = \frac{\sigma}{p_c(t)} . \tag{2.27}$$

Thus $R_c(t)$ determines the miminal size of metastable domains.

We will now discuss the creep velocity, i.e., the drift velocity of the interface exposed to a constant external pressure p_{ext}. At $T = 0$ the interface can move only when the pressure exceeds the threshold value p_ε. The situation for $T \neq 0$ has been discussed in some special cases by Feigel'man[38] (see also Ref. 39) and for $d = 1$ by Derrida and Pomeau.[40] In both cases the existence of a threshold has been found. Here we will show that the interface can move at low T also when $p_{\text{ext}} < p_\varepsilon$; in this case the interface motion is controlled by thermally activated depinning.

If an external pressure p_{ext} acts on the interface, it dominates the pinning pressure $p(L)$ [Eq. (2.24'')] on scales $L > L_{\text{ext}} = L_{\varepsilon,T}(p_{\varepsilon,T}/p_{\text{ext}})^{1/(2-\zeta)}$. On scales smaller than L_{ext} the motion of the interface is possible only via jumping over barriers. We obtain, as a generalization of an earlier result of Ioffe and Vinokur,[31] the *creep velocity* of the interface:

$$v(p_{\text{ext}}) \approx \frac{w(L_{\text{ext}})}{\tau(L_{\text{ext}})} = v_T \left[\frac{p_{\varepsilon,T}}{p_{\text{ext}}} \right]^{\zeta/(2-\zeta)} \times \exp \left[- \left[1 + \frac{T_\varepsilon}{T} \right] \left[\frac{p_{\varepsilon,T}}{p_{\text{ext}}} \right]^{\theta/(2-\zeta)} \right] , \tag{2.28}$$

where

$$v_T = \frac{a}{t_0} \left[1 + \frac{T}{T_\varepsilon} \right]^{\zeta_F/\phi} ,$$

and $p_{\varepsilon,T} \gg p_{\text{ext}}$. This result is valid in the weak pinning limit, when pinning originates from the fluctuations of the impurity density. It appears either on large length scales or when $\varepsilon \ll 1$. Equation (2.28) is a general expression for the creep velocity of a weakly pinned interface under pressure in random systems. For example, in ferromagnets the pressure on the interface, which can move with the creep velocity, is produced by a uniform external field H, so that $p_{\text{ext}} \approx m_0 H$.

In the strong pinning case, the pinning pressure is of the order $p_{\text{pin}} = v/(aD^{d-1})$. Thermal fluctuations depin the interface at the barrier v belonging to a single impurity after the time $t_v \approx t_0 \exp(v/T)$. Therefore, for long times the interface is again only weakly pinned by impurity density fluctuations.

III. SUSCEPTIBILITY

In the critical region, the susceptibility of diluted antiferromagnets has been studied theoretically by several authors[19,41,42] and experimentally by King *et al.*[43] and Kleemann *et al.*[44] Here we analyze the low-temperature magnetic susceptibility χ of dilute or weakly disordered ferromagnets or antiferromagnets. At low temperatures the bulk magnetization is saturated; therefore, the bulk susceptibility of the ordered phase is small, and the main contribution to χ comes from the interface motion, i.e., from the growth of the favorably oriented domains against the unfavorable domains under the influence of a magnetic field.

We evaluate the static susceptibility of diluted ferromagnets and antiferromagnets in a field. In the first case the external field acts as the driving force on the interface, whereas in diluted antiferromagnets the interfaces readjust on small length scales according to fluctuations in the random fields. The dynamic susceptibility is explained on the basis of the energy dissipation when the interfaces move across the potential barriers between different pinning sites.

A. Ferromagnetic susceptibility

Consider first the low-temperature interface susceptibility of a disordered ferromagnet when the main contribution to χ comes from the interface motion. As shown in Sec. II, on short length scales $L < L_c$ and at low fre-

quencies ω the interface is free; it reacts to the changes of the uniform external field in a time $t \ll 1/\omega$, where ω is the frequency of the applied magnetic field. This contributes a frequency-independent part to χ. On larger length scales the interface is pinned and relaxes slowly; this is the reason for a frequency-dependent interface contribution to $\chi(\omega)$. In the following we will determine the interface magnetization and therefrom the susceptibility. We consider the interface on a scale $L \lesssim L_c$, i.e., on a scale where it is free to respond to a small homogeneous field h, which produces a hump of height w. The energy of the hump,[37]

$$E(L,w)=\tfrac{1}{2}\Gamma w^2 L^{d-3}-m_0 h w L^{d-1} , \tag{3.1}$$

is minimal at $w=m_0 h L^2/\Gamma$. Here we assume that the driving pressure $m_0 h$ is much smaller than the pinning pressure p_c [Eq. (2.26)]. The field-induced interface magnetization is

$$M \approx m_0 L^{d-1} w \left[\frac{R}{L}\right]^{d-1} \frac{1}{R^d} \approx m_0^2 h L^2/R\Gamma , \tag{3.2}$$

where $1/R$ is the *interface density*. In a domain state, R is of the order of the domain radius whereas in incommensurate systems R denotes the spacing of the soliton lattice. The static (very-low-frequency) interface susceptiblity is thus

$$\chi=\frac{\partial M}{\partial h} \approx \frac{m_0^2 L^2}{R\Gamma} . \tag{3.3}$$

Although static, it depends on the frequency because the length scales, on which the interfaces can relax, grow logarithmically in time at low frequencies. The main contribution to the static susceptibility comes from the largest possible length scale, $L \approx L_c(t=1/\omega)$ [Eq. (2.23′)]:

$$\chi(\omega)=\chi_{\varepsilon,T}\left[1+\frac{T}{T+T_\varepsilon}\ln\left[\frac{1}{\omega t_0}\right]\right]^{2/\theta} , \tag{3.4}$$

$$\chi_{\varepsilon,T}=\frac{m_0^2 a L_\varepsilon^{d-1}}{T_\varepsilon}\frac{a}{R}\left[1+\frac{T}{T_\varepsilon}\right]^{2/\phi} . \tag{3.5}$$

$\chi_{\varepsilon,T}$ has a simple interpretation if we rewrite it in terms of the effective (T-dependent) transverse a_T and longitudinal $L_{\varepsilon,T}$ length scales [see Eq. (2.18)]:

$$\chi_{\varepsilon,T}=\frac{m_0^2 a_T L_{\varepsilon,T}^{d-1}}{T_{\varepsilon,T}}\frac{a_T}{R} . \tag{3.5′}$$

a_T/R is the volume fraction occupied by the domain wall and $m_0 a_T L_{\varepsilon,T}^{d-1}$ is the total magnetic moment of the minimal hump which can be formed by the wall. If we consider these minimal humps as elementary degrees of freedom, then $m_0^2 a_T L_{\varepsilon,T}^{d-1}/T_{\varepsilon,T}$ corresponds to the Curie susceptibility at the effective temperature, $T_{\varepsilon,T}=T_\varepsilon(1+T/T_\varepsilon)$. In a metastable microdomain state, R also depends on the disorder. If we use $R=R_c(t)$ [Eq. (2.27)] for the minimal size of the microdomains, we get from (3.4) and (3.5)

$$\chi(\omega)=\frac{m_0^2 a_T}{\sigma}\left[1+\frac{T}{T+T_\varepsilon}\ln\left[\frac{1}{\omega t_0}\right]\right]^{\zeta/\theta} , \tag{3.6}$$

which only weakly depends on the impurity disorder concentration. Equation (3.6) predicts a monotonous decrease of χ on a logarithmic frequency scale. This was recently confirmed by measurements of the ferroelectric domain walls in the disordred systems $SrTiO_3$:Ca and $KTaO_3$:Nb,[45] where a Cole-Cole-type response function was used to describe the entire accessible frequency range. However, its high-frequency tail corresponds to the small length scales, which are not accounted for in Eq. (3.6). Similar polydispersive behavior of domain-wall susceptibility of various ferroelectrics with impurities [Rochelle salt, triglycine sulfate, cesium dihydrogen phosphate, $KTaO_3$:Li, and $Pb\,Mg_{1/3}Nb_{2/3}O_3$] was recently reported by Shilnikov *et al.*[46]

B. Complex susceptibility

In this section we will calculate the dynamic susceptibility by a different approach; we will investigate the interface contribution to the energy dissipation and to the *complex* susceptibility of disordered ferromagnets at low temperature. The interfaces have a large number of different metastable configurations, and an external time-dependent (ac) magnetic field exerts a driving force on the interfaces which then fluctuate between their different energy minima, separated by energy barriers. The interface motion over the potential barriers is the main reason for the energy dissipation at low temperatures and low frequencies.

Consider the influence of an external time-dependent field $h(t)=h\exp(i\omega t)$ on an interface. Thermal motion over energy barriers E_B mediates transitions between wall configurations which have an energy difference $\Delta E \lesssim T$. Since the distribution of ΔE is smooth and has a width of the order E_B, there are only rare pairs of metastable interface configurations with $\Delta E \lesssim T$; we therefore treat the interface as an ensemble of noninteracting two-level systems (TLS's).[31]

The separation between the two minima of the TLS i is $w_i \sim w(L_i)$ and the energy difference is ΔE_i. Then the probability that in thermal equilibrium the system is in the higher-energy minimum is

$$n_0(\Delta E_i)=[\exp(\Delta E_i/T)+1]^{-1} . \tag{3.7}$$

The external field $h(t)$ disturbs the energy difference ΔE_i by $\delta E_i \approx m_0 h(t) L_i^{d-1} w_i$. The system therefore relaxes to the new time-dependent equilibrium configuration $n(\Delta E_i+\delta E_i)=n_0(\Delta E_i)+\delta n_i$. The time dependence of δn_i depends on the relaxation time $\tau_i \approx \tau_0 \exp(E_B(L_i)/T)$ of the TLS and on $\delta E_i(t)$. In a linear approximation,

$$\left[\frac{\partial}{\partial t}+\frac{1}{\tau_i}\right]\delta n_i+\frac{\partial n_0}{\partial \Delta E_i}\frac{\partial}{\partial t}\delta E_i=0 . \tag{3.8}$$

The power dissipated in this way by the TLS is[47]

$$P_i=-\mathrm{Re}\left\langle \delta m_i \frac{dH}{dt}\right\rangle , \tag{3.9}$$

where $\mathrm{Re}\langle\ \rangle$ denotes the real part of the time average and δm_i is the change of the magnetization induced by

the interface relaxation:

$$\delta m_i = m L_i^{d-1} w_i \, \delta n_i \ .$$

With this and the Fourier transform of Eq. (3.8), we get the power absorbed by the TLS i:

$$P_i \approx \frac{1}{4T}\left[\cosh\left(\frac{\Delta E_i}{2T}\right)\right]^{-2} (\delta E_i)^2 \frac{\omega^2\tau_i}{1+\omega^2\tau_i^2} \ . \tag{3.10}$$

The total power density dissipated by all TLS's is $P=(1/\Omega)\sum_i P_i$, where Ω denotes the volume. Since the distribution function for the ΔE_i is smooth and has a width of the order $E_B \gg T$, only the fraction T/E_B of the TLS contributes to the sum $\sum_i P_i$. Hence[31,37]

$$P \approx \frac{1}{RL^{d-1}} \frac{[\delta E(L)]^2}{E_B(L)} \frac{\omega^2\tau(L)}{1+\omega^2\tau^2(L)} \ , \tag{3.11}$$

where we have inserted $\Omega/(RL^{d-1})$ for the number of TLS's, and replaced L_i by a characteristic scale L, w_i by $w(L)$, τ_i by $\tau(L)$, etc. This can be understood as follows. The sum in (3.10) runs over all TLS's with an energy barrier E_B, which can be crossed by thermally activated hopping in a time $t \approx 1/\omega$. This limits the possible L_i to a narrow region around the typical value L. Below we will determine L by extremizing $\chi(L,\omega)$.

On the other hand, the energy dissipation is related to the imaginary part of the dynamic susceptibility[44]

$$P = \tfrac{1}{2}\omega\chi''(L,\omega)H^2 \ . \tag{3.12}$$

Comparing (3.11) and (3.12), we find the interface contribution to the complex dynamic susceptibility:

$$\chi(L,\omega) = \frac{m_0^2 L^2}{R\Gamma} \frac{1}{1+i\omega\tau(L)} \ , \tag{3.13}$$

with $\tau(L)$ given in (2.23). The main contribution to the real part $\chi'(\omega)$ comes from the length scale L, where $\partial\chi'(L,\omega)/\partial L = 0$, which yields $(\omega\tau)^{-2} \approx [\theta E_B(L)/T - 1]$. For low frequencies and temperatures this gives

$$L(\omega) \approx L_\varepsilon \left[\frac{T}{T_\varepsilon}\ln(1/\omega t_0)\right]^{1/\theta}$$

$$\approx L_c(t = 1/\omega) \ . \tag{3.14}$$

Thus, in the low-frequency limit, $\omega \to 0$ and for finite temperatures we get the same result for the real part of the complex susceptibility as we derived in Sec. III A. The dynamic interface susceptibility of disordered ferromagnets resembles the Debye relaxation, corrected for the logarithmic growth of the maximal length scale on which the TLS's relax.

C. Susceptibility of diluted antiferromagnet in a field

In the diluted antiferromagnet in a field, which is believed to be the experimental realization of the random-field Ising model, the external uniform field H in conjunction with the dilution generates the RF pinning centers. Again, we are interested only in the interface contribution to the magnetization and susceptibility, since the additional contributions which come from the bulk and volume magnetizations are almost constant at low temperature and their contribution to the susceptibility is small.

We start with the case of weak anisotropy or broad walls, where the pinning and hence the microdomain structure disappear for vanishing field. The interface configures in such a way that it gains excess mangetic energy. The interface roughness $w(L)$ on the length scale L is, according to (2.12''), for $x=1$ and $v \approx m_0 H$,

$$w(L) = \frac{(m_0 H)^{2/3}}{\Gamma^{2/3} D^{d/3}} L^{(5-d)/3} \ . \tag{3.15}$$

The local excess magnetization due to magnetic ion density fluctuation is $m = m_0[L^{d-1}w(L)]^{-1/2}$ and the interface contribution to the total magnetization density is

$$M \approx \frac{1}{R^d} m_0 [L^{d-1} w(L)]^{1/2} \left[\frac{R}{L}\right]^{d-1}$$

$$\approx \frac{1}{RD^{d/6}} \left[\frac{m_0^4 H}{\Gamma}\right]^{1/3} L^{2(2-d)/3} \ . \tag{3.16}$$

Here, in contrast to (3.2), the main contribution to M and hence to χ comes from the shortest scales (for $d>2$). The smallest scale, for which the calculation of $w(L)$ makes sense, is again given by L_ε [Eq. (2.13)] with $v = H m_0$. Thus, for $d=3$, M depends linearly on the external field H at low T when m_0 is saturated, and the interfaces give a linear contribution to the static susceptibility:

$$\chi = \frac{\partial M}{\partial H} = \frac{m_0^2}{(RD^{d/2} a \Gamma)} \ . \tag{3.17}$$

Here the microdomain size R is a history-dependent constant; in particular, it depends on the field at the time of cooling through the critical region. The susceptibility of dilute antiferromagnets shows no time dependence at low T, when the microdomain configurations are "frozen." As the temperature approaches T_c, however, the interface depinning leads again to a logarithmic time dependence of the remanent magnetization and of the susceptibility.[19,42]

IV. CONCLUSIONS

The model of interfaces interacting with randomly distributed pinning centers provides a basis to interpret the interface relaxation in various disordered systems. The model is used to evaluate the interface contribution to the dynamic susceptibility of random magnetic systems. The susceptibility is linear at low temperatures and fields, when the domain size R and magnetization m_0 are constant. At higher temperatures and fields, deviations from the linearity appear; however, at higher temperatures the bulk susceptibility, not considered in this model, starts to dominate. This observation is in agreement with Ref. 41, where the crossover from random-exchange to random-field critical behavior is analyzed, albeit in the critical region.

In this paper, χ has been calculated in two different approaches. First, the static (ferromagnetic) susceptibility is calculated in a model where the interface relaxes on

length scales that are shorter than the time-dependent length scale $L_c(t=1/\omega)$. In the second approach, the dynamic interface susceptibility is obtained from a model where the interface is modeled by an ensemble of two-level systems. Interface motion between the minima of the TLS's is the origin of the energy dissipation and of χ''. We have shown that the two different approaches are fully consistent, which can be considered as an argument for the validity of the proposed model.

We now give an order of magnitude estimate of the interface susceptibility χ of Heisenberg systems with weak uniaxial crystal-field anisotropy assuming $J\approx 10^{-2}$ eV$\approx 10^{-21}$ J, $A/J\approx 10^{-2}$, $m_0\approx 0.1$ V s/m^2, and 50% dilution, $D\approx 1$. In this case, $a_{T=0}\approx\xi\approx 10^{-8}$ m and $\Gamma\approx\sigma\approx J/\xi\approx 10^{-3}$ J/m^2. The low-temperature interface susceptibility of dilute ferromagnets [Eq. (3.4)] is then $\chi\approx 0.1\mu_0$, where μ_0 is the vacuum permeability. In particular, at low temperatures, when the bulk susceptibility is small, the interface contribution to the susceptibility is dominating. The above estimate is made for weakly pinned interfaces; in the case of strongly pinned interfaces, the susceptibility is substantially smaller.

The interface susceptibility of dilute antiferromagnets is the consequence of the rearrangements of the interfaces on short length scales. An estimate of Eq. (3.17), with the same parameters as used in the estimate of the ferromagnetic susceptibility, gives $\chi/\mu_0\approx 10^{-6}$. The interface susceptibility of broad-wall antiferromagnets is much smaller than the susceptibility of diluted ferromagnets and is probably also much smaller than the bulk antiferromagnetic susceptibility even at low temperatures. On the other hand, if the anisotropy A is large, the interfaces are fractal on short length scales and carry surface magnetization which gives a finite χ.[18]

Interface relaxation in magnetic systems can be experimentally studied by measuring either the time dependence of the excess magnetization or the susceptibility. Detailed measurements of the time dependence of the excess magnetization in dilute antiferromagnets have been performed by Leitão *et al.* and by Pollak *et al.*,[27] where a $[\ln(t)]^{-1}$ time dependence of the remanent magnetization in strongly anisotropic diluted antiferromagnets below T_c is reported.

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APPENDIX

In this appendix we estimate the influence of pinning in a diluted weakly anisotropic Heisenberg magnet. The Hamiltonian for the undiluted system is

$$\mathcal{H}=-J\sum_{\langle ij\rangle}S_iS_j-A\sum_i(S_i^z)^2\,, \tag{A1}$$

which can be rewritten in the continuum limit as

$$\tilde{\mathcal{H}}=\tfrac{1}{2}J\sum_{\langle ij\rangle}\int d^dr\left[\sum_{\alpha=1}^{n}(\vec{\nabla}S_\alpha)^2-AS_z^2\right]. \tag{A2}$$

Inserting in (A2) a domain-wall configuration $S_\alpha(\vec{r})=\hat{S}_\alpha(z-z(\mathbf{x}))$, $\tilde{\mathcal{H}}$ takes the form

$$\tilde{\mathcal{H}}\approx\sigma\int d^{d-1}x\sqrt{1+(\nabla z)^2}\,,$$

which for almost flat walls gives (2.1) with[37]

$$\sigma=\Gamma\approx J/\xi,\quad \xi\approx\sqrt{J/A}\,. \tag{A3}$$

Here we omit all numerical factors of the order 1. For weak anisotropy the width of the wall ξ is large compared with the lattice constant, which is equal to unity.

In diluted systems the extension b of the impurity is of the order 1 and hence much smaller than ξ. Then, according to (2.4),

$$v\approx\frac{J}{\xi^2}b^d\,. \tag{A4}$$

Insertion of (A4) in (2.8) yields

$$\varepsilon\approx\left[\frac{b}{D}\right]^d\left[\frac{D}{\xi}\right]^{5/2}. \tag{A5}$$

To consider an example in $d=3$ dimensions, we assume $\xi=10$ and 10% dilution, i.e., $D\approx 2$. Then $\varepsilon\approx 2\times 10^{-3}$, $L_\varepsilon\approx 10^3$, $\eta=5$, $L_\eta\approx 1$, $T_\varepsilon\approx 10J$, and the maximal pinning presure is $p_\varepsilon\approx 10^{-6}J$, where J is the exchange energy.

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