# Hydrodynamics of an antiferromagnet with fermions

A. V. Balatsky*
*Department of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801*
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The long-wavelength description of a doped two-dimensional antiferromagnet (AFM) is considered. By assuming the existence of local AFM correlations, the hydrodynamics of the AFM and fermionic degrees of freedom is given. The broken time-reversal symmetry in the AFM leads to the spinor nature of doped fermions, and we find that the total "pseudospin" of fermions is conserved. This leads to the existence of additional hydrodynamic diffusive modes. This pseudospin index leads to possible "singlet" and "triplet" superconducting condensates in magnetically correlated materials. At low temperatures we find nonhydrodynamic damping of spin waves due to scattering by fermions.

## INTRODUCTION

After the discovery of high-$T_c$ superconductivity, experiments were made that showed the importance of localized spins to the whole picture of superconductivity.[1,2] It is commonly believed that the magnetic properties of copper oxides are important for possible mechanism of superconductivity.[3-5] Apart from the microscopic analysis of the Hubbard model and its generalizations (see, for example, Refs. 3 and 4), the phenomenological approach, based on the symmetry consideration of the ground state can be used. An important step in this direction was made by Chakravarty, Nelson, and Halperin,[6] when they showed, using experimental results, that an $s=\frac{1}{2}$ two-dimensional (2D) quantum antiferromagnet (QAFM) is well described by a nonlinear $\sigma$ model (NL$\sigma$). This mapping leads to a description of the long-wavelength dynamics of the QAFM in terms of only a few collective variables, i.e., staggered and net magnetization. Shraiman and Siggia[7] constructed the semiclassical Hamiltonian for holes in a 2D QAFM, suggesting that the doped state of a QAFM can be described in terms of these collective variables and a fermionic wave function. These results can be obtained from the strongly correlated Hubbard model, using the slave-boson technique. Numerical calculations showed that the minimum of the effective band for holes lies at the $X$ point of the Brillouin zone, i.e, at $k=(\pi/2,\pi/2)$. At that time the electron doped superconductors with quite different band structures were discovered (see, for a review, Ref. 8). Recently it was pointed out,[9] that a Fermi-liquid description of the doped antiferromagnet requires an explicit statement about the number of carriers involved in the Fermi sea. It seems that the picture with the minimum of the band at the $X$ point corresponds to $x$ number of carriers in the Luttinger theorem, while $1+x$ corresponds to the Fermi sea, centered at the $\Gamma$ point (here $x$ is the doping concentration). These two situations can correspond to the same AFM ground state. So, from a general point of view, it is important to distinguish between the symmetry statements and the consequences of the position of the minimum of the band of doped particles, concentration, etc. In fact, the position of the band minimum is not a universal feature and varies from model to model. For the $t$-$t'$-$J$ model at large $t'$, the band minimum is at the $\Gamma$ point.[4,10] So we will consider these two possibilities ($\Gamma$ versus $X$ point), and I will argue that the properties of the doped antiferromagnet are sufficiently the same. In this paper we shall examine a doped antiferromagnet using primarily the symmetry of the ground state (which is supposed to have strong AFM correlations). I will show that local AFM correlations lead to a double-valued (spinor) representation of the fermion wave function for the doped particle in an antiferromagnet. This fact, in combination with the symmetry properties of the collective variables of an antiferromagnet, leads unambiguously to a hydrodynamical Hamiltonian for fermions, which is identical to the Hamiltonian derived from the Hubbard model.[7] An initial low-energy description of an undoped antiferromagnet is given in terms of the AFM order parameter $\mathbf{n}$, $\mathbf{n}^2=1$, and the magnetic moment $\mathbf{m}$, which is the conjugate momentum. The vector $\mathbf{n}$ lies on the unit sphere $S^2=\mathrm{SU}(2)/\mathrm{U}_n(1)$, where SU(2) is the group of rotations in the spin space and $\mathrm{U}_n(1)$ is the unbroken group of rotations around $\mathbf{n}$. It follows that the rotations around $\mathbf{n}$ in the pure antiferromagnet do not generate a new state, whereas the fermion wave function does change under these rotations. Indeed we find that a spinor representation leads self-consistently to opposite charges for the two components of the spinor with respect to rotations around $\mathbf{n}$. This fact is a general consequence of the uniaxial anisotropy in the spin space ($\mathbf{n}$ vector) and broken time-reversal invariance (double valuedness). I argue that this will take place even in the weak coupling limit for the spin-density wave (SDW).[11] It is shown that in the AFM correlated metals the total group consists of $\mathrm{U}_n(1)+\mathrm{U}_{\mathrm{EM}}(1)$, where the $\mathrm{U}_{\mathrm{EM}}(1)$ is the electromagnetic group. So we observe that there can be two types of superconducting condensates: One is the analog of the spin-singlet condensate and the other is the

 

analog of the spin triplet, depending on their transformations under $U_n(1)$.

We suggest the existence of a characteristic relaxation time $\tau_R$, and build the hydrodynamic description of coupled magnetic and fermionic degrees of freedom. Symmetry analysis enables us to find a new hydrodynamic diffusive mode in the doped antiferromagnet, which corresponds to the conserved fermion "pseudospin." We also consider the coupling of spin waves to the fermionic degrees of freedom. This coupling produces extra damping of the spin waves, which can be measured experimentally. At low temperatures, when $\tau_R$ diverges, we find nonhydrodynamic damping of spin waves due to the imaginary part of the fermion polarization operator.

The plan of this article is as follows: In the next section we consider the symmetry properties of pure and doped antiferromagnet, and in the section following that we construct the hydrodynamics of an AFM with fermions. Some of the results of this article were published previously in Refs. 11 and 12.

## SYMMETRY PROPERTIES

Here we shall consider the symmetry properties of fermionic excitations in an AFM using the established mapping of a 2D QAFM on a NL$\sigma$ model.[6] For this purpose we need to use the symmetry of the ground state of the 2D QAFM (henceforth we set $T=0$). Fermions, doped into an antiferromagnet, destroy long-range AFM order. It is clear, however, that locally they feel the AFM correlations, and these correlations put strong restrictions on the possible local symmetry properties of the fermions. Another important issue is that in the symmetry considerations, of course, the only important symmetry is the symmetry of the ground state. So this approach is valid not only for the Hubbard model, but also for any microscopic model with long-range AFM order in the ground state. Since we know that the ordered state of the antiferromagnet in the long-wavelength limit is well described by NL$\sigma$, we will keep this correspondence in the doped case as well. Then, from the symmetry properties of two sublattice Heisenberg antiferromagnets, we get the group of invariances of the ground state:

$$G_{\mathrm{AFM}}=\{P,RT,T^2,\mathrm{U}(1)_{\mathbf{n}}\} , \tag{1}$$

where $P$ is the point group of the crystal, $RT$ is the combined element of translation on one site ($T$) and the time-reversal operator ($R$), conserved in the AFM, as well as $T^2$.[12,13] The order parameter $\mathbf{n}$ is the point on the sphere $S^2=\mathrm{SU}(2)/\mathrm{U}(1)_{\mathbf{n}}$, where SU(2) is the initial group of rotations in the spin space and $\mathrm{U}(1)_{\mathbf{n}}$ is the unbroken group of rotations around $\mathbf{n}$. So, formally, any rotations around $\mathbf{n}$ do not produce changes in the state of the AFM. For any nonfermionic quantity $\mathrm{U}(1)_{\mathbf{n}}$ is unimportant. Any excitations of the ground state are described by some irreducible representation (IR) of $G_{\mathrm{AFM}}$. From what follows, if we dope fermions with antiferromagnets and do not destroy the ground state of the AFM, then fermions are described by the IR of $G_{\mathrm{AFM}}$. As can be easily seen from a microscopic approach to the Heisenberg AFM,[4,5,7,13] the fermions in an AFM organize a two-component object $\boldsymbol{\psi}=(\psi_1,\psi_2)$ where the subscripts 1 and 2 correspond in microscopic calculations, using the Schwinger boson decomposition, to $A$ and $B$ sublattices. What should be noted is that the two-component nature of fermion excitations is already seen in the weak-coupling Hubbard model with a SDW.[11] So, as we shall see, the two-component structure of $\boldsymbol{\psi}$ is a general result of the $RT$ symmetry of an AFM. Indeed, one can find the double valued or spinor IR of the $RT$ group

$$RT_a\psi_1=e^{i\mathbf{k}\cdot\mathbf{a}}\psi_2^* , \tag{2a}$$

$$RT_a\psi_2=-e^{i\mathbf{k}\cdot\mathbf{a}}\psi_1^* . \tag{2b}$$

Representation (2) is analogous to the double-valued representation of usual time reversal for spin-$\frac{1}{2}$ particles.[13] It is $(RT)^2=-1$, and components $\psi_1$ and $\psi_2$ are analogous to the Kramers doublet.[14] Note also that any field in AFM, braked $RT$ (e.g., $\mathbf{m}$ denoting magnetization vector) lifts the double degeneracy (Kramers theorem) between $\psi_1$ and $\psi_2$.

For an AFM without spin-orbital coupling, the IR of another subgroup of $G_{\mathrm{AFM}}$ is

$$\begin{aligned} &T_a^2\boldsymbol{\psi}=e^{2i\mathbf{k}\cdot\mathbf{a}}\boldsymbol{\psi}, \quad T_a^2\mathbf{n}=P\mathbf{n}=\mathbf{n} , \\ &RT\mathbf{m}=-\mathbf{m}, \quad RT\mathbf{n}=\mathbf{n} . \end{aligned} \tag{3}$$

Of course $\psi$ transforms under some IR of $P$ also, but we are interested in the consequences of magnetic ordering, so we fix some IR of $P$. Note, however, that the IR of $P$ at different points of Brillouin zone (BZ) are different, which can bring extra effects.[14]

The only representation of $\mathrm{U}(1)_{\mathbf{n}}$ for two particles $\psi_1$ and $\psi_2$ can be written as

$$\mathrm{U}(1)_{\mathbf{n}}\boldsymbol{\psi}\rightarrow\exp(i\boldsymbol{\tau}\cdot\mathbf{n}\varphi)\boldsymbol{\psi} , \tag{4}$$

where $\varphi$ is the angle of rotation in the spin space around $\mathbf{n}$, and $\boldsymbol{\tau}$ are the Pauli matrices.[12,13] In (4) the anisotropy in spin space, i.e., the existence of $\mathbf{n}$, is taken into account as well as the two-component nature of $\boldsymbol{\psi}$. So we find that fermions in the AFM have charge with respect to $\mathrm{U}(1)_{\mathbf{n}}$. This fact was first noted by Wiegmann,[5] but he considered only the case of local $\mathrm{U}(1)_{\mathbf{n}}$, which naturally leads to the appearance of a gauge field in the AFM. In contrast, we shall suggest that this group is global, i.e., $\varphi$ is constant, and thus we can put $\varphi=0$ in further consideration.

This symmetry consideration enables us to make some prediction about the nature of superconducting transition in AFM based materials. Indeed, fermionic excitations are then described by two U(1) groups:

$$U(1)_{\mathbf{n}}\times U(1)_{\mathrm{EM}} , \tag{5}$$

where $\mathrm{U}(1)_{\mathrm{EM}}$ is the electromagnetic group. Then, quite generally, one can imagine two types of superconducting states: (a) $\Delta_a=\langle\psi_1\psi_1\rangle$, which breaks both $\mathrm{U}(1)_{\mathbf{n}}$ and $\mathrm{U}(1)_{\mathrm{EM}}$ and (b) $\Delta_b=\langle\psi_1\psi_2\rangle$, which breaks only $\mathrm{U}(1)_{\mathrm{EM}}$. The state of type $\Delta_a$ is analogous to triplet pairing, and the state of type $\Delta_b$ is analogous to singlet pairing. So, generally, the properties of these superconductors are

quite different, but we shall not consider this question here.

## HYDRODYNAMICS

To build up the hydrodynamics we must make some assumptions. First of all we shall consider only the metallic regime of electrons in an AFM, and thus the concentration of particles must be larger than the critical concentrations are for the metal-insulator transition.[15] On the other hand, we assume that even in the disordered phase (without long-range order) the AFM coherence length is large. This enables us to use the local AFM order, which feels the particle in the vicinity of its position.. So, the second condition that should be satisfied, is $k_F\xi \gg 1$, where $k_F$ is the Fermi wave vector for doped particles and $\xi$ is the AFM coherence length. We also assume that the adequate long-wavelength description of the AFM in these conditions can still be given in terms of the NL$\sigma$ model, as in the undoped case.[6] It has been shown that in the 2D AFM with the attraction between doped particles and spin waves a polaron state can be formed.[12] So, in principal, we should avoid the problems with localization of particles, as well as phase separation. Here we assume that this is not the case in our model. The main assumption is that the long-wavelength dynamics of a doped AFM is described by only a few degrees of freedom: $\mathbf{m}$ the local magnetization, $\mathbf{n}$ the local value of the order parameter, $\rho=\langle\psi_\alpha^\dagger\psi_\alpha\rangle$ the density of particles (angular brackets $\langle\ \rangle$ indicate thermodynamic average), since the total number of doped particles is conserved, $J_\mu=-(1/2m)\langle\psi_\alpha^\dagger i\partial_\mu\psi_\alpha+\text{c.c.}\rangle$ mass current, $\boldsymbol{\rho}=\langle\psi_\alpha^\dagger\tau_{\alpha\beta}\psi_\beta\rangle$ "pseudospin" (henceforth we shall omit the "pseudo"), and the corresponding spin current $\mathbf{J}_\mu=-(1/2m)\langle(\psi^\dagger\tau i\partial_\mu\psi+\text{c.c.})\rangle$; the reason for considering $\boldsymbol{\rho}$ as the hydrodynamic variable is that in linearized approximation $\boldsymbol{\rho}$ is locally conserved, i.e., in the lowest order in gradients and neglecting nonlinearities there is no coupling between the fluctuational part of spin density and NL$\sigma$ degrees of freedom so that the total excess of spin $\mathbf{Q}=\int\boldsymbol{\rho}\,d^2x$ is conserved and is the generator of spin rotations for fermions (see below). Of course in the complete theory only total spin should be conserved.

So, under some perturbation of the system, and after some characteristic relaxation time $\tau_R$, one can describe the state of the system by local values of these hydrodynamic variables. This suggestion provides enough information to construct the closed hydrodynamics of AFM with fermions. In zero doping the AFM is described by the NL$\sigma$ model with the Hamiltonian,

$$H_{\mathrm{NL}\sigma}=\tfrac{1}{2}\int\chi_{ij}^{-1}m_im_j+\rho_s(\partial_\mu\mathbf{n})^2d^2x\ , \tag{6a}$$

where $m_i=(m_x,m_y,m_z)$ are components of the generator of rotations in the spin space, $\chi_{ij}^{-1}=\chi_\parallel^{-1}n_in_j+\chi_\perp^{-1}\delta_{ij}^\perp$ is the susceptibility tensor, $\delta_\perp^{ij}=(\delta^{ij}-n_in_j)$, $\mu=x,y$, and $\mathbf{m}$ and $\mathbf{n}$ form the algebra of Poisson brackets (PB's):

$$\begin{aligned}&\{m_i(x)m_j(y)\}=\epsilon_{ijk}m_k\delta(x-y)\ ,\\&\{m_i(x)n_j(u)\}=\epsilon_{ijk}n_k\delta(x-y)\ ,\\&\{n_in_j\}=0\ .\end{aligned} \tag{6b}$$

The Hamiltonian (6a) describes the long-wavelength dynamics of the AFM and reproduces the spectrum of spin waves (SW's): $\omega=(\rho_s\chi_\perp^{-1})^{1/2}k$.[16]

After adding fermions in the small doping limit, we restrict the consideration to the first order in concentrations and to the second order in gradients. The possible nonsuperconducting fermionic configurations in the real-space representation are $\rho,\psi_\alpha^\dagger\partial_\mu^2\psi_\alpha,\boldsymbol{\rho},\mathbf{J}_\mu,J_\mu$. It is not too hard to see that in accordance with the IR [(2) and (3)] the first two terms are pure scalars, the third transforms as $\mathbf{m}$ (i.e., $RT\mathbf{m}=-\mathbf{m}$), and the last two transform as scalar gradients under $G_{\mathrm{AFM}}$, except that the spin current is also a vector in spin space.[11] Therefore, one can write the hydrodynamical Hamiltonian for fermions (see, for example, Ref. 7):

$$H=H_{\mathrm{NL}\sigma}+H_0^F-g_1\psi^\dagger\tau\psi\mathbf{m}-g_2\mathbf{J}_\mu\mathbf{V}_\mu+g_3\psi^\dagger\psi\mathbf{V}_\mu^2\ , \tag{7}$$

where $H_0^F$ is the Hamiltonian for the pure fermionic subsystem, containing only even powers of $\partial_\mu$, which we choose to be $H_0^F=-\psi^\dagger(\partial_\mu^2/2m)\psi$ and $\mathbf{V}_\mu=\mathbf{n}\wedge\partial_\mu\mathbf{n}$ is the standard magnetization current in the NL$\sigma$ model.[17]

To find the equations of motion for the Hamiltonian (7) we use the PB method[18] for $\mathbf{m}$ and $\mathbf{n}$, and use the Heisenberg equation $i\dot{\psi}=[H,\psi]$ for fermionic operators. We suggest that the PB's between classical variables remain as in (6b) and $\psi$ operators obey the standard anticommutation rules:

$$\{\psi_\alpha^\dagger(r)\psi_\beta(r')\}=\delta_{\alpha\beta}\delta(r-r'),\ \{\psi_\alpha(r)\psi_\beta(r')\}=0\ .$$

We also assume that $\psi$ commutes with $\mathbf{m},\mathbf{n}$: $[\psi,\mathbf{m}]=[\psi,\mathbf{n}]=0$. We can distinguish between fermions and NL$\sigma$ degrees of freedom only in the linearized limit, which allows us to use these commutators. The equations of motion for NL$\sigma$ are given by PB's: $\{H,\mathbf{m}\}$, etc. Using the above, we come to the system of equations of motion:

$$\frac{\partial\mathbf{m}}{\partial t}=\rho_s\partial_\mu\mathbf{V}_\mu+g_1(\boldsymbol{\rho}\wedge\mathbf{m})-g_2\partial_\mu\mathbf{J}_\mu+\chi^{-1}K_m\partial_\mu^2\mathbf{m}\ , \tag{8a}$$

$$\frac{\partial\mathbf{V}_\mu}{\partial t}=\chi_\perp^{-1}\partial_\mu\mathbf{m}-g_1\partial_\mu\boldsymbol{\rho}+\rho_sK_v\partial_\mu\partial_\nu\mathbf{V}_\nu\ , \tag{8b}$$

$$\frac{\partial\boldsymbol{\rho}}{\partial t}=-2g_1(\boldsymbol{\rho}\wedge\mathbf{m})-2g_2(\mathbf{J}_\mu\wedge\mathbf{V}_\mu)-\partial_\mu\mathbf{J}_\mu+\partial_\mu\left[\frac{g_2}{m}\mathbf{V}_\mu\rho\right]\ , \tag{8c}$$

$$\frac{\partial\rho}{\partial t}=-\partial_\mu J_\mu+\partial_\mu(g_2/m\,\mathbf{V}_\mu\boldsymbol{\rho})\ , \tag{8d}$$

where we omit higher-order gradient terms. Coefficients $K_m$ and $K_v$ described the diffusion of magnetization and the damping of the spin waves in the pure AFM. In the limit $g_1=g_2=0$ we recover the hydrodynamic equations for the pure antiferromagnet (see the classic articles[19]). We omit from our consideration the energy diffusion equation, since it is decoupled from the other degrees of freedom.

The system (8) is nonlinear, and it follows that evolu-

tion of $\mathbf{m}$ and $\mathbf{V}_\mu$ depend on the fermionic degrees of freedom. The fermion density and spin density can be changed not only by a divergence of the correspondent current, but also by coupling to a magnetic subsystem. For example, the spin density $\boldsymbol{\rho}$ rotates around $\mathbf{m}$, but these rotations are described by nonlinear terms in (8). The next step is to limit ourselves to the linearized hydrodynamic equations. This system is still incomplete in the sense that we need the equation of motion for the currents. In order to get this we will use the linear response theory. Decomposing the spin current and spin density into two pieces,

$$\mathbf{J}_\mu=\mathbf{J}_{\mu c}+\mathbf{J}_{\mu f} ,$$

$$\boldsymbol{\rho}=\boldsymbol{\rho}_c+\boldsymbol{\rho}_f ,$$

we can get $\mathbf{J}_{\mu c},\boldsymbol{\rho}_c$ from the linear response theory (see the Appendix) and their dynamics are governed by the NL$\sigma$ variables:

$$\boldsymbol{\rho}_c=\beta\mathbf{m} , \tag{9a}$$

$$\mathbf{J}_{\mu c}=-\alpha\mathbf{V}_\mu-K_\rho\partial_\mu\boldsymbol{\rho} , \tag{9b}$$

and $\alpha=-g_2 n/m$, $\beta=g_1 m/\pi$. We also add the diffusion term $K_\rho$ in $\mathbf{J}_\mu$. The reason for doing this is that in the first term of the Hamiltonian (7) the nonequilibrium spin density can be transported away by the emission of the magnons. Since pure magnetization cannot propagate in AFM, we suggest that $\boldsymbol{\rho}$ propagates diffusively.[20]

Then, as we mentioned above, the spin density $\boldsymbol{\rho}_f$ obeys the conservation equation $\partial_t\boldsymbol{\rho}_f=\partial_\mu K_\rho(\partial_\mu\boldsymbol{\rho})$. The quantities $\mathbf{J}_{\mu f},\boldsymbol{\rho}_f$ described the fluctuations around the value, given by (9) and caused, for example, by quantum fluctuations. In order to get the equations of motion for the fluctuating part of current $\mathbf{J}_{\mu f}$, we will use the Heisenberg equations for operator $\mathbf{J}_{\mu f}$,

$$\partial_t\mathbf{J}_{\mu f}=\partial_\mu\left[\frac{g_1}{m}\mathbf{m}\rho\right]+2g_1(\mathbf{m}\wedge\mathbf{J}_{\mu f}) .$$

The linearized system of hydrodynamic Eqs. (8) and (9) can be written then as

$$\partial_t\mathbf{m}=\tilde{\rho}_s\partial_\mu\mathbf{V}_\mu+g_2K_\rho\partial_\mu^2\boldsymbol{\rho}+\beta g_2K_\rho\partial_\mu^2\mathbf{m}-g_2\partial_\mu\mathbf{J}_\mu , \tag{10a}$$

$$\partial_t\mathbf{V}_\mu=\tilde{\chi}_\perp^{-1}\partial_\mu\mathbf{m}-g_1\partial_\mu\boldsymbol{\rho} , \tag{10b}$$

$$\partial_t\mathbf{J}_\mu=\partial_\mu\left[\frac{g_1}{m}\mathbf{m}\rho\right] , \tag{10c}$$

$$\partial_t\boldsymbol{\rho}=K_\rho\partial_\mu^2\boldsymbol{\rho}+\beta K_\rho\partial_\mu^2\mathbf{m} , \tag{10d}$$

where $\tilde{\rho}_s=(\rho_s+\alpha g_2)$, $\tilde{\chi}_\perp^{-1}=\chi_\perp^{-1}-\beta g_1$, and we omit the subscript $f$ for the fluctuating part of spin density and spin current. We shall omit coefficients $K_m=K_v=0$, since they lead to higher-order corrections. For example, in the case of the SW mode, the damping of spin waves is given by $D=\chi^{-1}K_m+\rho_sK_v$.[19]

System (10) is the central result of this section. Note that in the linearized limit the fermion density decouples, and we do not consider it further. In the matrix form the equations of motion after Fourier transformation are

$$i\omega U=\begin{bmatrix}-\beta g_2K_\rho k^2 & \tilde{\rho}_s ik & -ig_2k & -g_2K_\rho k^2\\ ik\tilde{\chi}_\perp^{-1} & 0 & 0 & -ig_1k\\ ik\dfrac{g_1}{m}\rho & 0 & 0 & 0\\ -\beta g_2K_\rho k^2 & 0 & 0 & -K_\rho k^2\end{bmatrix}U , \tag{11a}$$

$$U=(m^z,V^z,J^z,\rho^z) , \tag{11b}$$

and we choose fluctuations in the spin space along the $z$ axis. In the leading order the eigenvalue equations are

$$c^2=\tilde{c}^2+\alpha g_1 , \tag{12}$$

$$(\omega+iK_\rho k^2)(-\omega^2+c^2-i\omega\beta g_2K_\rho k^2)+\tilde{\rho}_s\beta g_1K_\rho k^4=0 . \tag{13}$$

As we can see at $\alpha=\beta=0$ the spectrum consists of two modes:

(a) SW with dispersion,

$$\omega^2=c_0^2k^2,\quad c_0^2=\rho_s\chi_\perp^{-1} , \tag{14a}$$

(b) fermion spin-diffusion mode (FM),

$$\omega=iK_\rho k^2 . \tag{14b}$$

This result can be anticipated from Eqs. (10), since at $\alpha=\beta=0$ limit the spin diffusion mode completely decouples from SW sector. The solution of the reduced eigenvalue problem for nonzero $\alpha,\beta$ is

(a) SW

$$\omega^2=c^2k^2 , \tag{15}$$

(b) FM

$$\omega=-iK_\rho\left[1-\frac{\beta g_1\tilde{\rho}_s}{c^2}\right]k^2 . \tag{16}$$

The damping of SW is always proportional to $k^2$ and is modified by the presence of the $K_\rho$ term:

$$\tilde{D}=-\tfrac{1}{2}\beta K_\rho(g_1\tilde{\rho}_s/c^2+g_2) . \tag{17}$$

Since we do not know exact expressions for the coefficients $g_1,g_2$, the quantitative consideration can be given in the asymptotic region of strong softening of the SW $c\to 0$, providing that the second term in (17) is smaller than the first one. Then noting that $\beta g_1=\chi_\perp^{-1}-\tilde{\chi}_\perp^{-1}$ we get

$$\tilde{D}=\tfrac{1}{2}K_\rho(c_0^2-c^2)/c^2 , \tag{18}$$

and the effective spin-diffusion constant is $K_\rho c_0^2/c^2$. This formula allows us to estimate the effect of fermions on the damping rate of SW in terms of experimentally measured quantities.

So we find that for an antiferromagnet in the presence of fermions an additional hydrodynamic mode, FM, appears. This mode corresponds to spin diffusion. As fol-

lows from (15) and (17), the spectrum of SW's is modified due to interaction with fermions. This fact is in quantitative agreement with the results of Ref. 12 where it was shown that the SW spectrum softens due to particle-hole polarization, and the effect of fermions is presented in both stiffness and in the susceptibility. Note that we get a new type of hydrodynamic damping of the SW's due to interaction with fermions. When the spectrum of the SW's becomes very soft, e.g., $C^2 \to 0$, the window for SW's shrinks, since $\tilde{D} \to \infty$. This means that for strong interaction between fermions and SW's, the propagation of SW's is strongly damped due to particle-hole polarization. This damping of SW's is proportional to the concentration of particles $n$, since $\alpha \sim n$, and $\beta$ is nonzero due to deviation of the Fermi surface from the $X$ point in the BZ and thus is also proportional to density.

The results for the damping coefficient of SW's and for the spin diffusion coefficients do not depend on the location of the minimum of the band, since they are formulated in terms of hydrodynamic coefficients $C^2$, $\tilde{C}^2$, $K\rho$, etc. The values of these coefficients depend, of course, on the microscopic model and on the location of the minimum of the band. In Ref. 7 it has been argued that the single-particle excitation spectrum of the $t$-$J$ model is unstable towards the spiral instability even on the level of one doped hole. This is the artifact of the mean-field approximation, and the normal Fermi-liquid state should be stable in some interval of doping, as is clear from the experiment. From this it follows that, for the band minimum at the $X$ point at small doping, our results apply as well as to the $\Gamma$-point case, since we consider only the fluctuations around the ground state irrespective of the location of the minimum of the band.

Until now we have considered only the hydrodynamic regime of our model. In the case $T \to 0$, when the relaxation time $\tau_R$ diverges and we can neglect the collisions, or in high frequency, we come to the nonhydrodynamic domain of SW and FM, analogous to the zero sound. In this region the damping of SW becomes linear in momentum. The dispersion of SW is given by Eq. (15), where $\alpha,\beta$ are imaginary (see the Appendix):

$$\omega = ck(\cos\phi/2 + i\sin\phi/2) ,$$
$$\phi = \phi_1 + \phi_2, \quad \tan\phi_1 = \mathrm{Im}(\alpha g_2)/\mathrm{Re}(\tilde{\rho}_s + \alpha g_2) , \tag{19}$$
$$\tan\phi_2 = \mathrm{Im}(-\beta g_1)/\mathrm{Re}(\chi_\perp^{-1} - \beta g_1) .$$

If we set $\beta = 0$ and neglect the effect of the SW softening, we come to the estimation of the nonhydrodynamic damping, mentioned by Shraiman and Siggia,[7] $\mathrm{Im}c = (g_2^2 n/m)(c_0/v_F\rho_s)$. When $C_0/V_F \to 1$, the system is close to resonance, which leads to the divergence of the damping of SW. It follows that spin waves become overdamped and cannot propagate, and, as follows from the formula (A6), the damping of SW's is linear for any $\alpha$ at small frequencies.

## COMPARISON WITH EXPERIMENT

As follows from our results, the interaction of SW's with fermions leads to the SW softening $C^2 = C_0^2 + \alpha g_2 - g_1\beta\bar{\rho}_s$ and to the new mechanism of the damping of SW's. Since there is clear experimentation using inelastic neutron scattering in pure $La_2CuO_4$ and in 5% doped $La_{2-0.05}Ba_{0.05}CuO_4$,[21] we can compare the experimental measurements of the SW velocity and the decay rate with our results.

Experimentally it has been shown that in pure $La_2CuO_4$ the SW spectrum with negligible damping provides a good fit to the measured cross section for neutrons with energy 30 meV with the SW velocity $C_0 = 0.85$ eV Å, in the whole range of temperatures from 5 to 200 K. In the doped case the broadening of the scattering peak was observed. Analysis of the effect of doping by two fitting procedures was proposed in Ref. 21: In one the fitting parameter was the SW velocity assuming negligible damping, and this approach gives $C = 0.63$ eV Å, or they fit the decay rate of SW's at initial SW velocity $C_0$. In the latter case the decay rate $\mathrm{Im}\omega = k^2 D = 22$ meV at the characteristic wave vector $k = 0.1$ Å$^{-1}$ at 5 K. Of course these results do not clarify the mutual interplay between the softening and damping of the SW's, but we will assume that actual values of renormalized velocity and damping rate are given by the numbers close to those obtained in the experiment. Then by using Eq. (18) we get

$$k^2 D = k^2 K_\rho (C_0^2 - C^2)/2C^2, \quad K_\rho = 5.365 \text{ Å}^2\text{eV} . \tag{20}$$

Since $K\rho$ described the spin diffusion, we can estimate

$$K_\rho = \tfrac{1}{3} v_F l_{\mathrm{sp}} , \tag{21}$$

where $v_F$ is the Fermi velocity and $l_{\mathrm{sp}}$ is the mean free path for the spin scattering. Since by assumption the main contribution to the damping comes from the scattering of fermions by magnons, it is natural to assume that the interaction of carriers with magnons leads to the mass mean free path of the same order, i.e., $l = l_{\mathrm{sp}}$. Then we find the relaxation time

$$\tau = l/v_F = l^2/(3K_\rho) = 0.41 \times 10^{-14} \text{ sec} , \tag{22}$$

where we assume that $l$ is of order of two lattice sites ($a = 5.37$ Å) as is known from the electron transport measurements, and our estimate for the relaxation time is in the good agreement with the scattering time $\tau = 10^{-14}$ sec,[15] especially if we take into account uncertainties in the $l_{\mathrm{sp}}$, in the value we used of $D$ and relation between $D$ and $K_\rho$. Note also that both relaxation times are close to the characteristic time of fluctuations of the Cu spins with the exchange constant $J = 0.16$ meV,[21] $\tau = h/J = 2.58 \times 10^{-14}$ sec. Thus we showed that relaxation processes in the spin scattering and the electron transport channels turn out to be close for the samples with small doping. Assuming that all SW damping comes from the interaction with fermions via Eq. (18), and using a characteristic mean free path, the interaction with fermions can explain strong SW damping, observed experimentally.

## CONCLUSION

Using the symmetry properties of the short-range Néel ground state of an AFM we obtain the properties of

doped fermions in the AFM. We find the long-wavelength Hamiltonian for fermions, which coincides with the one derived from the one-band Hubbard model. This Hamiltonian provides a useful framework for studying different kinds of instabilities, such as spiral instabilities. It follows from our results that the normal Fermi-liquid state in the AFM correlated metals contains nontrivial effects. For example, we find a new hydrodynamic mode that corresponds to the conserved pseudospin of the doped particles. We also find that mutual interaction of fermions and magnetic degrees of freedom lead to the additional damping of SW's. This damping can be large enough to make SW propagation impossible. Indeed, our analysis applies at wave vectors $k_\xi < k < k_D$, where $k_\xi$ is defined as the wave vector of the AFM coherence length, and $k_D = c_{SW}/D$ is defined as the wave vector of strong damping of SW. Since, as we have seen, the corrections to $D$ can be large, it means that in principal the window for SW propagation shrinks to zero as $D$ increases. Of course, any real estimate of $k_D$ depends on the values of the unknown hydrodynamic coefficients $g_1, g_2, K_\rho$. Experimentally, the effect of doping on the SW can be studied by measuring inelastic neutron scattering. The doping-dependent contribution to $D$ can be observed.

In the low-temperature limit, when collisions are negligible, the excitations become nonhydrodynamic. The damping of SW becomes linear in momentum, and spin waves become overdamped at $V_F/C_0 \approx 1$ due to Landau damping. Experimentally the most interesting changes in the properties of SW and FM can be observed at the resonance condition $V_F = C_0$. We also estimate the lifetime for the fermions from the decay rate for SW due to interaction with fermions. We find reasonably good agreement with the lifetime found from the conductivity measurements, which implies that the damping of the SW in doped case comes from the interaction with fermions.

We also find that there are two different possibilities for the superconducting order parameter in these metals. One is the analog of the spin singlet and the other is the analog of the spin-triplet superconductors. The singlet superconductor corresponds to the pairing of $\Psi$ and $RT\Psi$ in analogy with the pairing of $\Psi$ and time-reversal $R\Psi$ in the usual superconductors.[22] Using the analog of the Anderson theorem for the $RT$ type of pairing one can find that nonmagnetic impurities are as strong as magnetic for the usual $R$ type of superconductors.[4] This is probably the reason why the doping interval for superconductors is so narrow, especially in the electron-doped superconductors $Nd_{2-x}Ce_xCuO_4$.[9,10] Among the unanswered questions should be noted the question about the description of the doped paramagnetic state, when $k_\xi < 1$, i.e., when particles really are in the paramagnetic state. Probably, the analogous hydrodynamic description of the spiral states can be done, and it should show new effects.

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## APPENDIX

In this appendix we shall show that from linear response theory follows unambiguously the relation between $\alpha$ and the spin current-current correlation. The linear coupling between $\mathbf{J}_\mu$ and $\mathbf{V}_\mu$ is given by term with $g_2$ in the Hamiltonian

$$H = H_0 - g_2 \mathbf{J}_\mu \cdot \mathbf{V}_\mu , \tag{A1}$$

where $H_0$ does not contain linear coupling between $\mathbf{J}_\mu$ and $\mathbf{V}_\mu$. Then we find (see Ref. 23)

$$\delta\langle J^i_\mu(r,t)\rangle = -\int_{-\infty}^{t} \alpha^{ij}_{\mu\nu}(r,t|r',t') \times \delta V^j_\nu(r',t')d(r')dt' , \tag{A2}$$

where

$$\alpha^{ij}_{\mu\nu}(rt|r't') = g_2\langle \tfrac{1}{2}[J^i_\mu(r,t), J^j_\nu(v',t')]\rangle = -\frac{g_2}{2}\delta^{ij}\delta_{\mu\nu}\frac{K_F^2}{m^2}\int \Pi_0(q_1\omega)e^{iq(r-r')-i\omega(t-t')}dq\,d\omega , \tag{A3}$$

and where $\Pi_0(q,\omega)$ is the usual 2D fermion polarization bubble. At $\omega \ll v_F q$, $q < 2k_F$ we get $\Pi_0 \simeq m/2$. Then, since for hydrodynamical description we can take the limiting value of $\Pi_0$ ($\omega \to 0$, $q \to 0$) in (A3), we find

$$\alpha^{ij}_{\mu\nu} \simeq -g_2(n/m)\delta_{\mu\nu}\delta_{ij} = \alpha\delta_{\mu\nu}\delta_{ij} . \tag{A4}$$

This expression for $\alpha$ leads exactly to the expression for the SW velocity renormalization due to interaction with fermions [see Eqs. (15), and see also Ref. 12]:

$$\tilde{c}^2 = \chi_\perp^{-1}(\rho_s - g_2^2 n/m) . \tag{A5}$$

In the case of low temperatures we should take into account the imaginary part

$$\mathrm{Im}\Pi_0(q,\omega) = \frac{m}{2\pi}\frac{a}{(1-a^2)^{1/2}} ,$$

where $a = \omega/(qV_F)$. This part is nonzero only for $a < 1$. For $C_0/V_F \to 1$ the imaginary part becomes dominant. In this case we can consider the imaginary $\alpha$:

$$\alpha = ig_2\frac{n}{m}\frac{a}{(1-a^2)^{1/2}}, \quad a \approx C_0/V_F . \tag{A6}$$

In the same manner we can find for $\beta$:

$$\beta = g_1\Pi_0(q,\omega) = g_1 m/\pi + ig_1\mathrm{Im}\Pi_0(q,\omega) . \tag{A7}$$

*Permanent address: Landau Institute for Theoretical Physics, Moscow, U.S.S.R.

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