PHYSICAL REVIEW B VOLUME 42, NUMBER 1 1 JULY 1990

Solitonlike excitations in a spin chain with a biquadratic anisotropic exchange interaction

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(Received 16 January 1990)

Nonlinear solitonlike excitations in a spin chain with a biquadratic anisotropic exchange interaction are investigated using the coherent-state method combined with the Holstein-Primakoff bosonic representation of the spin operators. It is argued that the modified terms of the nonlinear Schrödinger equation are strongly restricted by the relation between the continuum approximation ($\eta = d/\lambda$, the degree of the long-wavelength approximation, where d is the lattice constant and λ is the characteristic wavelength of excitation) and the semiclassical approximation ($\epsilon = 1/\sqrt{S}$, the degree of the truncation of the operator expansion, where S is the spin length). When assuming that η and ϵ have the same order ($\eta \sim \epsilon$) after retaining terms of equivalent order $O(\epsilon^6)$, the motion of the Bose operator for the anisotropic case satisfies the nonlinear Schrödinger equation with cubic nonlinearity, and solitonlike excitations are obtained. The other two cases of the relations between η and ϵ ($\eta \sim \epsilon^{3/2}$ and $\eta \sim \epsilon^2$) are also discussed.

I. INTRODUCTION

Solitonlike excitations in quasi-one-dimensional magnets have generated a great deal of experimental[1–3] and theoretical[4–16] interest. The soliton solution for the spin chain has been studied by several different approaches. In the classical approach,[6–9] general single-soliton solutions are obtained for a continuum version of the classical linear Heisenberg chain. In a quantum spin system, a bosonic representation of the spin operators turns out to be a very suitable method for studying the solitary waves, because they allow one to include quantum corrections in a systematic way. In a spin-coherent representation,[17] one can work directly with the operators, make no approximation to the Hamiltonian, and can develop an exact nonlinear equation for the quantum system.[5] The other coherent-state treatments[10–16] use a severely truncated Holstein-Primakoff expansion[18] for $S_1^{\pm}$ and further approximate $\hat{H}$ by a Hamiltonian which is biquadratic in boson operators. Working in the coherent-state representation of Glauber,[19] and making the small-amplitude and long-wave approximations, one then finds solitary-wave profiles identical to classical solitons, which is the so-called semiclassical treatment.

In a quantum spin system, one can introduce the classical quantity $S_c = \hbar S$ and the condition $S_c = \lim_{\substack{\hbar \to 0 \\ S \to \infty}} (\hbar S)$ is the so-called semiclassical limit. We know that semiclassical treatments use a truncated Holstein-Primakoff expansion with a small parameter $\epsilon = 1/\sqrt{S}$ for obtaining a properly truncated Hamiltonian and that amplitudes of Glauber's coherent-state representation expand with the small parameter $\eta = d/\lambda$ (d is the lattice constant and λ is the characteristic wavelength) in the continuum limit. There is the argument that some papers,[10,11] on obtaining a nonlinear Schrödinger equation, suffered from a highly inconsistent conparison of terms (which in fact were of the same order of magnitude) within the framework of the Holstein-Primakoff representation. However, we note that those papers,[10–16] based on the Holstein-Primakoff representation, treated the truncated Hamiltonian as an "exact one" and then performed the continuum approximation independently and did not pay attention to the relation of two perturbations. In fact, for the coexistence of two perturbations, the relative ratio of two small parameters plays an important role in obtaining the proper nonlinear wave equation. Here we briefly mention the development of shallow-water wave theory to support this point of view.

In shallow-water wave theory, there are two small parameters: one is $\mu = kh$ used in the long-wavelength approximation (k is the wave number and h is the depth of the water), and the other one is $d = A/h$ used in the small-amplitude approximation (A is the amplitude of the water wave and h is the depth of the water). The relative ratio of the two small parameters is very important to obtain the reduced nonlinear wave equation. In history there are two quite different shallow-water wave theories. One is the Airy theory for $\mu \to 0$ and $\delta = O(1)$ and the other is the Boussinesq–Korteweg de Veries theory for $O(\delta) = O(\mu^2) \ll 1$. These two theories obtained quite different results for wave breaking in a suitable depth of the water. In 1953 this contradiction was solved by Ursell[20] who pointed out that the ratio $U_r = \delta/\mu^2$ (which is so-called Ursell number) determines the choice of which approximation is the main contribution. These different approximations correspond to different physical pictures.

The purpose of this paper is to focus on the relative ratio of two small parameters and investigate solitonlike excitations in a spin chain with a biquadratic exchange interaction. If we assume that η and ϵ have the same order ($\eta \sim \epsilon$), after retaining the terms of equivalent order $O(\epsilon^6)$, we find that the amplitude of the Glauber coherent-state representation satisfies the nonlinear Schrödinger equation with cubic nonlinearity and then obtains the solitonlike excitations. The other two cases of the relations between η and ϵ ($\eta \sim \epsilon^{3/2}$ and $\eta \sim \epsilon^2$) are also discussed. This paper is organized as follows. In Sec. II, we introduce the Hamiltonian of the system and express it in terms of the Holstein-Primakoff representation retaining the terms of order $O(\epsilon^8)$ and derive the equation of motion for the Bose operator. In Sec. II, using Glauber's coherent-state representation, we perform the continuum approximation and obtain the nonlinear Schrödinger equation for the amplitudes. In Sec. IV we discuss the role of the relative ratio of η and ϵ and the solitonlike excitations of the system. In Sec. V we make some comments on our treatment.

II. HAMILTONIAN OF THE SYSTEM AND THE EQUATION OF MOTION

The Hamiltonian describing the magnetic chain is

$$H=H_Z+H_E+H_A+H_B \ , \tag{1}$$

where

$$H_Z=-gu_B B\sum_i S_i^Z \tag{2}$$

is the Zeeman energy for external field B along the z axis. In addition,

$$H_E=-(J/2)\sum_i(\mathbf{S}_i\cdot\mathbf{S}_j)_\Delta \tag{3a}$$

is the exchange energy representing the anisotropic ferromagnetic interaction between nearest-neighbor spins $\mathbf{S}_i$ and $\mathbf{S}_j$, while exchange anisotropy is controlled by a parameter Δ and is defined as

$$(\mathbf{A}\cdot\mathbf{B})_\Delta=A^XB^X+A^YB^Y+(1-\Delta)A^ZB^Z \tag{3b}$$

and

$$H_A=-D\sum_i(S_i^Z)^2 \tag{4}$$

is the uniaxial crystal-field anisotropy, and

$$H_B=-(\nu J/2)\sum_i(\mathbf{S}_i\cdot\mathbf{S}_j)_\Delta^2 \tag{5}$$

is the biquadratic anisotropic exchange interaction between nearest-neighbor spins $\mathbf{S}_i$ and $\mathbf{S}_j$. For a high-spin system $(S>1)$, the biquadratic exchange interaction[21] should be considered. This interaction has been shown to give essential quantative modifications for the thermodynamics of the Heisenberg ferromagnet.[22,23] For $\Delta=0$, it is the case of isotropic exchange interaction which was studied by Ferrer.[13]

Introducing the dimensionless variables

$$\tilde{H}=H/J(S_c)^2 \ , \tag{6a}$$

$$\tilde{S}_i=S_i/S_c \ , \tag{6b}$$

$$\tilde{x}=x/\lambda \ , \tag{6c}$$

where λ is the characteristic wavelength of the excitation. Then the Hamiltonian of the system is written as

$$\tilde{H}=\tilde{H}_Z+\tilde{H}_E+\tilde{H}_A+\tilde{H}_B \ , \tag{7}$$

where

$$\tilde{H}_Z=-g_h\epsilon^2\sum_i S_i^Z \ , \tag{8a}$$

$$g_h=gu_BB/J \ , \tag{8b}$$

$$\tilde{H}_E=-\tfrac{1}{2}\sum_i(\mathbf{S}_i\cdot\mathbf{S}_{i+\rho})_\Delta \quad (\rho=\pm1) \ , \tag{9}$$

$$\tilde{H}_A=-g_a\sum_i(S_i^Z)^2 \ , \tag{10a}$$

$$g_a=D/J \ , \tag{10b}$$

$$\tilde{H}_B=-(g_\nu/2)\sum_i(S_i\cdot S_{i+\rho})_\Delta^2 \quad (\rho=\pm1) \ , \tag{11a}$$

and

$$g_\nu=\nu(S_c)^2 \ . \tag{11b}$$

Now we treat the Hamiltonian (7) in the Holstein-Primakoff representation for the spin operators

$$\begin{aligned}\tilde{S}_i^{\dagger}&=\sqrt{2}(1-\epsilon^2a_i^\dagger a_i)^{1/2}\epsilon a_i\\&=\sqrt{2}[1-\epsilon^2a_i^\dagger a_i/4-\epsilon^4a_i^\dagger a_ia_i^\dagger a_i/32\\&\quad-\epsilon^6a_i^\dagger a_ia_i^\dagger a_ia_i^\dagger a_i/128-O(\epsilon^8)]\epsilon a_i \ ,\end{aligned} \tag{12a}$$

$$\begin{aligned}\tilde{S}_i^{-}&=\sqrt{2}\epsilon a_i^\dagger(1-\epsilon^2a_i^\dagger a_i)^{1/2}\\&=\sqrt{2}\epsilon a_i^\dagger[1-\epsilon^2a_i^\dagger a_i/4-\epsilon^4a_i^\dagger a_ia_i^\dagger a_i/32\\&\quad-\epsilon^6a_i^\dagger a_ia_i^\dagger a_ia_i^\dagger a_i/128-O(\epsilon^8)] \ ,\end{aligned} \tag{12b}$$

and

$$\tilde{S}_i^Z=1-\epsilon^2a_i^\dagger a_i \ , \tag{12c}$$

where $\epsilon=1/\sqrt{S}$. By substituting Eq. (12) into Eqs. (7)–(11), we get, after retaining terms of order $O(\epsilon^8)$, the Hamiltonian

$$\tilde{H}=G(a_i^\dagger,a_i,a_{i+\rho}^\dagger,a_{i+\rho})+O(\epsilon^8) \ , \tag{13}$$

where G is the expression of $\tilde{H}$ in terms of $a_i^\dagger$, a_i, $a_{i+\rho}^\dagger$ and $a_{i+\rho}$ forms. The detailed expression is listed in Appendix A. We know that the Bose operator satisfies the following Heisenberg equation of motion:

$$i(\epsilon^2/JS_c)\partial a_j/\partial t=[a_j,\tilde{H}] \ . \tag{14}$$

Then we calculate the commutation $[a_j,\tilde{H}]$ as

$$[a_j,\tilde{H}_Z]=g_h\epsilon^4 a_j \ , \tag{15a}$$

$$[a_j,\tilde{H}_A]=2g_a\epsilon^2 a_j-g_a\epsilon^4 a_j-2g_a\epsilon^4 a_j^\dagger a_j a_j \ , \tag{15b}$$

$$\begin{aligned}[a_j,\tilde{H}_E]=&-(\tfrac{1}{2})\epsilon^2[(1+\Delta)\sum_\rho a_{j+\rho}+2(1-\Delta)a_j]\\ &+(\tfrac{1}{4})\epsilon^4\left[\sum_\rho(2a_j^\dagger a_j a_{j+\rho}+a_{j+\rho}^\dagger a_j a_j+a_{j+\rho}^\dagger a_{j+\rho}a_{j+\rho})-4(1-\Delta)\sum_\rho a_{j+\rho}^\dagger a_{j+\rho}a_j\right]\\ &+(\tfrac{1}{32})\epsilon^6\Big[\sum_\rho(3a_j^\dagger a_j^\dagger a_j a_j a_{j+\rho}+2a_j^\dagger a_j a_{j+\rho}+a_{j+\rho}^\dagger a_{j+\rho}^\dagger a_{j+\rho}a_{j+\rho}a_{j+\rho}+2a_{j+\rho}^\dagger a_j^\dagger a_j a_j a_j+a_{j+\rho}^\dagger a_{j+\rho}a_{j+\rho}+a_{j+\rho}^\dagger a_j a_j\\ &\quad-2a_{j+\rho}^\dagger a_{j+\rho}^\dagger a_{j+\rho}a_j a_j-4a_{j+\rho}^\dagger a_j^\dagger a_j a_{j+\rho}a_{j+\rho})\Big]+O(\epsilon^8)\ ,\end{aligned} \tag{15c}$$

$$\begin{aligned}[a_j,\tilde{H}_B]=&-\tfrac{1}{2}\epsilon^2 g_v[4(1-\Delta)\sum_\rho a_{j+\rho}-8(1-\Delta)^2 a_j]\\ &-\tfrac{1}{2}\epsilon^4 g_v\Big[[4+4(1-\Delta)^2]a_j-4(1-\Delta)\sum_\rho a_{j+\rho}+[8(1-\Delta)^2+4]\sum_\rho a_{j+\rho}^\dagger a_{j+\rho}a_j-10(1-\Delta)\sum_\rho a_j^\dagger a_j a_{j+\rho}\\ &\quad-5(1-\Delta)\sum_\rho a_{j+\rho}^\dagger a_j a_j+4\sum_\rho a_j^\dagger a_{j+\rho}a_{j+\rho}-5(1-\Delta)\sum_\rho a_{j+\rho}^\dagger a_{j+\rho}a_{j+\rho}+8(1-\Delta)^2 a_j^\dagger a_j a_j\Big]\\ &-\tfrac{1}{2}\epsilon^6 g_v[-(1-\Delta)B_1/32+B_2+B_3]+O(\epsilon^8)\ ,\end{aligned} \tag{15d}$$

where B_1, B_2, and B_3 are expressed in Appendix B. We now have the equation of motion expressed by bosonic operators a and $a^\dagger$,

$$i(\epsilon^2/JS_c)\partial a_j/\partial t=F(a_j^\dagger,a_j,a_{j+\rho}^\dagger,a_{j+\rho})\ , \tag{16}$$

where F is the expression of $[a_j,\tilde{H}]$ in terms of $a_j^\dagger$, a_j, $a_{j+\rho}^\dagger$ and $a_{j+\rho}$ forms.

III. NONLINEAR SCHRÖDINGER EQUATION

In order to solve Eq. (16) analytically, we introduce Glauber's coherent-state representation[19]

$$a_i^\dagger|\alpha\rangle=\alpha_i^*|\alpha\rangle\ , \tag{17a}$$

$$a_i|\alpha\rangle=\alpha_i|\alpha\rangle\ , \tag{17b}$$

$$|\alpha\rangle=\prod_i|\alpha(i)\rangle\ , \tag{17c}$$

where $|\alpha(i)\rangle$ is the coherent-state eigenvector for operator a_i and α_i is the coherent amplitude in this representation. The states $|\alpha(i)\rangle$ are nonorthogonal and overcomplete. The diagonal matrix element $\langle\alpha|\hat{A}|\alpha\rangle$ of an operator $\hat{A}$ is denoted by A. These elements are known to be good operator representatives.[5,24] We believe that no essential information is lost in this treatment in the condition of low-temperature excitations. Therefore, Eqs. (13) and (16) take the forms

$$\tilde{H}_s=\tilde{H}-\tilde{H}_0=\tilde{H}_s(\alpha_i,\alpha_{j+\rho},\alpha_i^*,\alpha_{j+\rho}^*)\ , \tag{18a}$$

$$\tilde{H}_0=-g_h\epsilon^2 N-g_a N-(1-\Delta)N-g_v(1-\Delta)^2 N\ , \tag{18b}$$

where N is the number of lattice sites, and

$$i\frac{\epsilon^2}{JS_c}\frac{\partial\alpha_j}{\partial t}=\tilde{F}(\alpha_j,\alpha_{j+\rho},\alpha_j^*,\alpha_{j+\rho}^*)\ . \tag{19}$$

We now perform the continuum limit for the system, where $\eta=d/\lambda$ is the small parameter representing the long-wave approximation:

$$\begin{aligned}&\alpha_i\to\alpha(x,t)\ ,\\ &\alpha_i^*\to\alpha^*(x,t)\ ,\\ &\sum_i\to\frac{1}{d}\int dx\to\frac{1}{\eta}\int d\tilde{x}\ ,\\ &\alpha_{j+\rho}\to\alpha+\eta\rho\alpha_{\tilde{x}}+\tfrac{1}{2}\eta^2\alpha_{\tilde{x}\tilde{x}}+\tfrac{1}{6}\eta^3\rho^3\alpha_{\tilde{x}\tilde{x}\tilde{x}}\\ &\qquad+\tfrac{1}{24}\eta^4\alpha_{\tilde{x}\tilde{x}\tilde{x}\tilde{x}}+O(\eta^5)\ ,\\ &\alpha_{j+\rho}^*\to\alpha^*+\eta\rho\alpha_{\tilde{x}}^*+\tfrac{1}{2}\eta^2\alpha_{\tilde{x}\tilde{x}}^*+\tfrac{1}{6}\eta^3\rho^3\alpha_{\tilde{x}\tilde{x}\tilde{x}}^*\\ &\qquad+\tfrac{1}{24}\eta^4\alpha_{\tilde{x}\tilde{x}\tilde{x}\tilde{x}}^*+O(\eta^5)\ .\end{aligned} \tag{20}$$

Having retained the terms $\eta^m\epsilon^n$ of order $O(m+n=8)$, Eq. (18) becomes

$$\tilde{H}_s=\langle\alpha|\tilde{H}_s|\alpha\rangle=\frac{1}{d}\int_{-\infty}^{+\infty}dx\,Y(x,t)=\frac{1}{\eta}\int_{-\infty}^{+\infty}d\tilde{x}\,\tilde{Y}(\tilde{x},t)\ , \tag{21}$$

where $\tilde{Y}(\tilde{x},t)$ is the reduced energy density and expressed as

$$\begin{aligned}\tilde{Y}(\tilde{x},t)=&C_i|\alpha|^2+C_2\alpha_{\tilde{x}}\alpha_{\tilde{x}}^*+C_3|\alpha|_{\tilde{x}\tilde{x}}^2\\ &+C_4(\alpha\alpha_{\tilde{x}\tilde{x}\tilde{x}\tilde{x}}^*+\alpha^*\alpha_{\tilde{x}\tilde{x}\tilde{x}\tilde{x}}-(1-\Delta)|\alpha|_{\tilde{x}\tilde{x}\tilde{x}\tilde{x}}^2)\\ &+C_5|\alpha|^4+C_6|\alpha|^2|\alpha|^2_{\tilde{x}\tilde{x}}\\ &+C_7|\alpha|^2\alpha_{\tilde{x}}\alpha_{\tilde{x}}^*+C_8|\alpha|_{\tilde{x}}^2|\alpha|_{\tilde{x}}^2+C_9|\alpha|^6\ ,\end{aligned} \tag{22a}$$

where

$$
\begin{aligned}
C_1 &= [g_h\epsilon^2 + 2g_a - 2 - 4\Delta(1-\Delta)g_v]\epsilon^2 \,, \\
C_2 &= [1 + 2g_v(1-\Delta)]\epsilon^2\eta^2 \,, \\
C_3 &= -\tfrac{1}{2}[1 + 2g_v(1-\Delta)]\epsilon^2\eta^2 \,, \\
C_4 &= \tfrac{1}{12}\eta^4\epsilon^2 \,, \\
C_5 &= \{-g_a + \Delta[1 + 2g_v(1-\Delta)] - 4\Delta^2 g_v\}\epsilon^4 \,, \\
C_6 &= \frac{\Delta}{2}\{[1 + 2g_v(1-\Delta)] - \Delta g_v\}\epsilon^4\eta^2 \,, \\
C_7 &= \{\Delta g_v - \tfrac{1}{2}[1 + 2g_v(1-\Delta)]\}\epsilon^4\eta^2 \,, \\
C_8 &= \{\tfrac{1}{4}[1 + 2g_v(1-\Delta)] - \Delta^2 g_v\}\epsilon^4\eta^2 \,, \\
C_9 &= 2\Delta^2 g_v\epsilon^6 \,.
\end{aligned} \tag{22b}
$$

The equation of motion (19) becomes

$$
\begin{aligned}
i\frac{1}{JS_c}\frac{\partial\alpha}{\partial t} = A_1\alpha - A_2\alpha_{\bar{x}\bar{x}} - A_3\alpha|\alpha|^2 \\
- A_4\alpha_{\bar{x}\bar{x}\bar{x}\bar{x}} + (A_5 + A_7)|\alpha|^2\alpha_{\bar{x}\bar{x}} \\
+ (A_6 + A_7)\alpha^2\alpha^*_{\bar{x}\bar{x}} + (2A_7 + A_8)\alpha\alpha_{\bar{x}}\alpha^*_{\bar{x}} \\
+ (A_8 + A_9)\alpha^*(\alpha_{\bar{x}})^2 + A_{10}\alpha|\alpha|^4 \,,
\end{aligned} \tag{23a}
$$

where

$$
\begin{aligned}
A_1 &= g_h\epsilon^2 + 2g_a - g_a\epsilon^2 - 2 - 4g_v\Delta(1-\Delta) \\
&\quad - 2g_v[1 + (1-\Delta)^2]\epsilon^2 + 4(1-\Delta)g_v\epsilon^2 \,, \\
A_2 &= [\tfrac{1}{2}(1+\Delta) + 2g_v(1-\Delta)(1-\epsilon^2)]\eta^2 \,, \\
A_3 &= 2g_a\epsilon^2 - 2\Delta\epsilon^2 - \frac{\epsilon^4}{8} - 20g_v(1-\Delta)\epsilon^2 + 4(1-\Delta)^2 g_v\epsilon^2 \\
&\quad + 4g_v\epsilon^2 - [\tfrac{1}{8}(1-\Delta) + 2\Delta^2]g_v\epsilon^4 \,, \\
A_4 &= \tfrac{1}{24}(1+\Delta)\eta^4 + \tfrac{1}{6}g_v(1-\Delta)\eta^4 \,, \\
A_5 &= [\tfrac{3}{4} + \tfrac{15}{2}g_v(1-\Delta) - 4g_v]\epsilon^2\eta^2 \,, \\
A_6 &= [\tfrac{1}{4} + \tfrac{5}{2}g_v(1-\Delta)]\epsilon^2\eta^2 \,, \\
A_7 &= [-(1-\Delta) + \tfrac{1}{4} + \tfrac{5}{2}(1-\Delta)g_v]\epsilon^2\eta^2 \,, \\
A_8 &= [\tfrac{3}{2} - 5(1-\Delta)g_v]\epsilon^2\eta^2 \,, \\
A_9 &= -4g_v\epsilon^2\eta^2 \,, \\
A_{10} &= 3\Delta^2\epsilon^4 g_v \,.
\end{aligned} \tag{23b}
$$

IV. THE ROLE OF RELATIVE RATIO OF η and ϵ AND THE SOLITON SOLUTION

We think that it is difficult to find the soliton solutions of Eq. (23). Here we discuss the degree of the long-wave approximation (listing three cases of the relative ratio of η and ϵ) and then continue to reduce Eq. (23).

(i) Firstly, we assume that η and ϵ have the same order ($\eta \sim \epsilon$), after retaining terms of equivalent order $O(\epsilon^6)$ of function $\tilde{F}$ in Eq. (19). The density $Y(x,t)$ [Eq. (22)] is reduced as

$$Y(x,t) = C_1|\alpha|^2 + C_2'\alpha_x\alpha_x^* + C_3'|\alpha|^2_{xx} + C_5|\alpha|^4 \,, \tag{24}$$

where C_1, C_2', C_3', and C_5 are expressed in Eq. (22b) with η replaced by the lattice constant d. The equation of motion (23) is reduced to

$$i\frac{1}{JS_c}\frac{\partial\alpha}{\partial t} = A_1\alpha - A_2'\alpha_{xx} - A_3'\alpha|\alpha|^2 \,, \tag{25a}$$

where A_1 and A_2' are expressed in Eq. (23b) with η replaced by the lattice constant d, and A_3' is

$$
\begin{aligned}
A_3' &= 2g_a\epsilon^2 - 2\Delta\epsilon^2 - 20g_v(1-\Delta)\epsilon^2 \\
&\quad + 4(1-\Delta)^2 g_v\epsilon^2 + 4g_v\epsilon^2 \,.
\end{aligned} \tag{25b}
$$

Through simple transformation, Eq. (25a) can reduce to the standard cubic nonlinear Schrödinger equation:

$$\alpha(x,t) = \left[\frac{2A_2'}{A_3'}\right]^{1/2} W(\xi,\tau)e^{-iA_1 t} \,, \tag{26a}$$

$$\xi = x \,, \tag{26b}$$

$$\tau = t/A_2' \,, \tag{26c}$$

$$iW_\tau + W_{\xi\xi} + 2|W|^2 W = 0 \,. \tag{26d}$$

Equation (26d) can be solved exactly by the inverse-scattering transform.[25] A single-soliton solution is

$$
\begin{aligned}
W(\xi,\tau) &= \beta\,\mathrm{sech}\beta(\xi - \xi_0 - 2k\tau) \\
&\quad \times \exp[ik\xi - i(k^2 - \beta^2)\tau - i\theta_0] \,.
\end{aligned} \tag{27a}
$$

That is,

$$
\begin{aligned}
\alpha(x,t) &= \left[\frac{2A_2'}{A_3'}\right]^{1/2} \beta\,\mathrm{sech}\beta(x - x_0 - 2kt/A_2') \\
&\quad \times \exp[i(kx - \omega t - \theta_0)] \,, \\
\omega &= A_1 + (k^2 - \beta^2) \,.
\end{aligned} \tag{27b}
$$

Equation (27b) is a wave packet traveling to the right with velocity $2k/A_2'$. If k is selected as zero, it localizes at position $x = x_0$ and oscillates with frequency ω. In the coherent-state representation, the energy of the system (1) is

$$
\begin{aligned}
E &= \langle\alpha|H\alpha\rangle/\langle\alpha|\alpha\rangle \\
&= JS_c^2\frac{1}{d}\int_{-\infty}^{+\infty} Y(x,t)dx \\
&= JS_c^2\frac{4A_2'\beta}{A_3'}\left[-\frac{C_1}{d} + \frac{C_2'}{d}\left[\frac{3}{5}\beta^2 + k^2\right]\right. \\
&\quad \left. + \frac{4}{3}\frac{C_5}{d}\frac{A_2'\beta^2}{A_3'}\right] \,.
\end{aligned} \tag{28}
$$

We can obtain the local magnetization distribution $\langle S^Z\rangle$ as

$$\langle S^Z \rangle = S_c(1-|\alpha|^2)$$

$$= S_c \left[1 - \frac{2A_2'}{A_3'} \beta^2 \mathrm{sech}^2 \beta(x - x_0 - 2kt/A_2') \right] . \quad (29)$$

It is interesting to note that if we normalize $\alpha(x,t)$, we obtain the excitation energy gap E_g. The normalization condition sets $\beta = A_3'/4A_2'$. With the elimination of β from the energy of the system (28) and setting $k=0$, we then obtain the excitation energy gap E_g:

$$E_g = \frac{JS_c}{d} \left[C_1 + \frac{1}{40} \frac{A_3'^2}{A_2'^2} C_2' + \frac{1}{12} \frac{A_3'}{A_2'} C_5 \right] , \quad (30a)$$

where

$$C_1 = \frac{1}{S}[gU_B B/JS + 2D/J - 2 - 4\Delta(1-\Delta)\nu S_c^2] ,$$

$$C_2' = [1 + 2\nu S_c^2(1-\Delta)] \frac{d^2}{S} ,$$

$$C_5 = \frac{1}{S^2}\{-D/J + \Delta[1+2\nu S_c^2(1-\Delta)] - 4\Delta^2 \nu S_c^2\} , \quad (30b)$$

$$A_2' = \left[\tfrac{1}{2}(1+\Delta) + 2\nu S_c^2(1-\Delta) \left[1 - \frac{1}{S} \right] \right] d^2 ,$$

$$A_3' = \frac{1}{S}[2D/J - 2\Delta - 20\nu S_c^2(1-\Delta) + 4(1-\Delta)^2 \nu S_c^2 + 4\nu S_c^2] .$$

(ii) Now we discuss more long-wave excitation and then assume that η and $\epsilon^{3/2}$ have the same order ($\eta \sim \epsilon^{3/2}$). After retaining terms of equivalent order $O(\epsilon^8)$ of function $\tilde{F}$ in Eq. (19), the equation of motion (23) is reduced as follows:

$$i\frac{1}{JS_c}\frac{\partial \alpha}{\partial t} = A_1\alpha - A_2\alpha_{\bar{x}\bar{x}} - A_3\alpha|\alpha|^2 + (A_5+A_7)|\alpha|^2\alpha_{\bar{x}\bar{x}} + (A_6+A_7)\alpha^2\alpha^*_{\bar{x}\bar{x}} + (2A_7+A_8)\alpha\alpha_{\bar{x}}\alpha^*_{\bar{x}} + (A_8+A_9)\alpha^*(\alpha_{\bar{x}})^2 + A_{10}\alpha|\alpha|^4 . \quad (31)$$

This nonlinear Schrödinger equation is similar to the result of Ferrer[13] (for the isptropic case, $A_{10}=0$). From this point of view, the study of Ferrer can be thought as the special case of $\eta \sim \epsilon^{3/2}$.

(iii) If we consider super-long-wave excitation, we can assume η and ϵ^2 have the same order ($\eta \sim \epsilon^2$). After retaining terms in equivalent order $O(\epsilon^8)$ of the function $\tilde{F}$ in Eq. (19), the equation of motion (23) is reduced to

$$i\frac{1}{JS_c}\frac{\partial \alpha}{\partial t} = A_1\alpha - A_2''\alpha_{\bar{x}\bar{x}} - A_3\alpha|\alpha|^2 + A_{10}\alpha|\alpha|^4 , \quad (32)$$

where A_1, A_3, and A_{10} are expressed by Eq. (23b) and A_2'' is

$$A_2'' = [\tfrac{1}{2}(1+\Delta) + 2g_\nu(1-\Delta)]\eta^2 . \quad (33)$$

The soliton solutions of nonlinear Schrödinger Eqs. (31) and (32) will be discussed in a future publication.

V. COMMENTS

We want to make some comments about the relevance and significance of our treatment of the present problem. We introduced the method of treating two small parameters in shallow-water wave theory into the magnetic system and settled the argument about the so-called highly inconsistent comparison of terms in obtaining the nonlinear Schrödinger equation.[10–14] We think that the result of Ferrer[13] can be thought of as a special case of $\eta \sim \epsilon^{3/2}$. From our detailed discussion of the special case $\eta \sim \epsilon$, we obtained single-soliton excitation and an energy gap in a spin chain with a biquadratic anisotropic exchange interaction. This demonstrates that the nonlinearities appear to be due to magnon-magnon interactions and give rise to magnon bound states. Theoretically, we could not determine which case is more important because different cases correspond to different physical pictures. Only from experimental and initial-excitation conditions of the magnetic system, we can estimate which case is more suitable. In our opinion, how external conditions influence the different intrinsic solitonlike excitations needs to be investigated further.

ACKNOWLEDGMENTS

This work was supported by the Foundation of Nature Science of China and by the Foundation of Doctoral Education of China.

APPENDIX A

The detailed expression of function G is as follows:

$$G(a_i^\dagger, a_i, a_{i+\rho}^\dagger, a_{i+\rho}) = \tilde{H}_Z + \tilde{H}_E + \tilde{H}_A + \tilde{H}_B , \quad (A1)$$

where

$$\tilde{H}_Z = -g_h\epsilon^2 N + g_h\epsilon^4 \sum_i a_i^\dagger a_i , \quad (A2)$$

$$\tilde{H}_A = -g_a N + 2g_a \epsilon^2 \sum_i a_i^\dagger a_i - g_a \epsilon^4 \sum_i a_i^\dagger a_i a_i^\dagger a_i \ , \tag{A3}$$

$$\begin{aligned}\tilde{H}_E = &-(1-\Delta)N - \frac{\epsilon^2}{2} \sum_{i,\rho} [(a_i a_{i+\rho}^\dagger + \text{H.c.}) - (1-\Delta)(a_i^\dagger a_i + a_{i+\rho}^\dagger a_{i+\rho})] \\ &- \frac{\epsilon^4}{2} \sum_{i,\rho} [(1-\Delta) a_i^\dagger a_i a_{i+\rho}^\dagger a_{i+\rho} - \tfrac{1}{4}(a_i a_{i+\rho}^\dagger a_{i+\rho}^\dagger a_{i+\rho} + a_i^\dagger a_i a_i a_{i+\rho}^\dagger + \text{H.c.})] \\ &+ \frac{\epsilon^6}{64} \sum_{i,\rho} [(a_i a_{i+\rho}^\dagger a_{i+\rho}^\dagger a_{i+\rho} a_{i+\rho}^\dagger a_{i+\rho} + a_i^\dagger a_i a_i^\dagger a_i a_i a_{i+\rho}^\dagger - 2a_i^\dagger a_i a_i a_{i+\rho}^\dagger a_{i+\rho}^\dagger a_{i+\rho} + \text{H.c.})] + O(\epsilon^8) \ ,\end{aligned} \tag{A4}$$

$$\begin{aligned}\tilde{H}_B = &-g_v(1-\Delta)^2 N - g_v \epsilon^2 \sum_{i,\rho} [(1-\Delta)(a_i a_{i+\rho}^\dagger + \text{H.c.}) - (1-\Delta)^2 (a_i^\dagger a_i + a_{i+\rho}^\dagger a_{i+\rho})] \\ &- \tfrac{1}{2} g_v \epsilon^4 \sum_{i,\rho} \{ 2(1-\Delta)^2 a_i^\dagger a_i a_{i+\rho}^\dagger a_{i+\rho} - \tfrac{1}{2}(1-\Delta)(a_i a_{i+\rho}^\dagger a_{i+\rho}^\dagger a_{i+\rho} + a_i^\dagger a_i a_i a_{i+\rho}^\dagger + \text{H.c.}) \\ &\qquad + [(a_i a_{i+\rho}^\dagger + \text{H.c.}) - (1-\Delta)(a_i^\dagger a_i + a_{i+\rho}^\dagger a_{i+\rho})]^2 \} \\ &- \tfrac{1}{2} g_v \epsilon^6 \sum_{i,\rho} \{ -\tfrac{1}{32}(1-\Delta)(a_i a_{i+\rho}^\dagger a_{i+\rho}^\dagger a_{i+\rho} a_{i+\rho}^\dagger a_{i+\rho} + a_i^\dagger a_i a_i^\dagger a_i a_i a_{i+\rho}^\dagger - 2a_i^\dagger a_i a_i a_{i+\rho}^\dagger a_{i+\rho}^\dagger a_{i+\rho} + \text{H.c.}) \\ &\qquad + [(a_i a_{i+\rho}^\dagger + \text{H.c.}) - (1-\Delta)(a_i^\dagger a_i + a_{i+\rho}^\dagger a_{i+\rho})] \\ &\qquad \times [(1-\Delta) a_i^\dagger a_i a_{i+\rho}^\dagger a_{i+\rho} - \tfrac{1}{4}(a_i a_{i+\rho}^\dagger a_{i+\rho}^\dagger a_{i+\rho} + a_i^\dagger a_i a_i a_{i+\rho}^\dagger + \text{H.c.})] \\ &\qquad + [(1-\Delta) a_i^\dagger a_i a_{i+\rho}^\dagger a_{i+\rho} - \tfrac{1}{4}(a_i a_{i+\rho}^\dagger a_{i+\rho}^\dagger a_{i+\rho} + a_i^\dagger a_i a_i a_{i+\rho}^\dagger + \text{H.c.})] \\ &\qquad \times [(a_i a_{i+\rho}^\dagger + \text{H.c.}) - (1-\Delta)(a_i^\dagger a_i + a_{i+\rho}^\dagger a_{i+\rho})] \} + O(\epsilon^8) \ ,\end{aligned} \tag{A5}$$

where N is the number of lattice sites.

APPENDIX B

Expressions B_1, B_2, and B_3 are as follows:

$$\begin{aligned}B_1 = 2 \sum_\rho (&3a_j^\dagger a_j^\dagger a_j a_j a_{j+\rho} + 2a_j^\dagger a_j a_{j+\rho} + a_{j+\rho}^\dagger a_{j+\rho}^\dagger a_{j+\rho} a_{j+\rho} a_{j+\rho} + 2a_{j+\rho}^\dagger a_j^\dagger a_j a_j a_j \\ &+ a_{j+\rho}^\dagger a_{j+\rho} a_{j+\rho} + a_{j+\rho}^\dagger a_j a_j - 2a_{j+\rho}^\dagger a_{j+\rho}^\dagger a_{j+\rho} a_j a_j - 4a_{j+\rho}^\dagger a_j^\dagger a_j a_{j+\rho} a_{j+\rho}) \ ,\end{aligned} \tag{B1}$$

$$\begin{aligned}B_2 = &\,2(1-\Delta) \sum_\rho (2a_{j+\rho}^\dagger a_j^\dagger a_{j+\rho} a_{j+\rho} a_j + a_{j+\rho}^\dagger a_{j+\rho}^\dagger a_{j+\rho} a_j a_j + 2a_j^\dagger a_j a_{j+\rho}) \\ &- 2(1-\Delta)^2 \sum_\rho (2a_{j+\rho}^\dagger a_j^\dagger a_{j+\rho} a_j a_j + a_{j+\rho}^\dagger a_{j+\rho}^\dagger a_{j+\rho} a_{j+\rho} a_j + 2a_{j+\rho}^\dagger a_{j+\rho} a_j) \\ &- \tfrac{1}{4} \sum_\rho (3a_j^\dagger a_j^\dagger a_j a_{j+\rho} a_{j+\rho} + 2a_{j+\rho}^\dagger a_j^\dagger a_{j+\rho} a_j a_j + a_{j+\rho}^\dagger a_{j+\rho}^\dagger a_{j+\rho} a_{j+\rho} a_j + 4a_{j+\rho}^\dagger a_{j+\rho} a_j) \\ &+ \frac{(1-\Delta)}{4} \sum_\rho (2a_{j+\rho}^\dagger a_j^\dagger a_{j+\rho} a_{j+\rho} a_j + a_{j+\rho}^\dagger a_{j+\rho}^\dagger a_{j+\rho} a_j a_j + 3a_j^\dagger a_j^\dagger a_{j+\rho} a_j a_j + 4a_j^\dagger a_{j+\rho} a_j) \\ &- \tfrac{1}{4} \sum_\rho (a_{j+\rho}^\dagger a_{j+\rho}^\dagger a_j a_j a_j + 2a_{j+\rho}^\dagger a_j^\dagger a_{j+\rho} a_{j+\rho} a_{j+\rho} + a_{j+\rho}^\dagger a_{j+\rho}^\dagger a_{j+\rho} a_{j+\rho} a_j \\ &\qquad + 2a_{j+\rho}^\dagger a_j^\dagger a_{j+\rho} a_j a_j + 2a_j^\dagger a_{j+\rho} a_{j+\rho} + 2a_j^\dagger a_j a_j) \\ &+ \frac{(1-\Delta)}{4} \sum_\rho (a_{j+\rho}^\dagger a_{j+\rho}^\dagger a_{j+\rho} a_{j+\rho} a_{j+\rho} + 2a_{j+\rho}^\dagger a_j^\dagger a_j a_j a_j + 2a_{j+\rho}^\dagger a_j^\dagger a_{j+\rho} a_{j+\rho} a_j \\ &\qquad + a_{j+\rho}^\dagger a_{j+\rho}^\dagger a_j a_j a_{j+\rho} + 2a_{j+\rho}^\dagger a_j a_j + 2a_{j+\rho}^\dagger a_{j+\rho} a_{j+\rho}) \\ &- \tfrac{1}{4} \sum_\rho (3a_j^\dagger a_j^\dagger a_{j+\rho} a_{j+\rho} a_j + 2a_{j+\rho}^\dagger a_j^\dagger a_{j+\rho} a_j a_j + a_{j+\rho}^\dagger a_{j+\rho}^\dagger a_{j+\rho} a_{j+\rho} a_j + 4a_{j+\rho}^\dagger a_{j+\rho} a_j) \\ &+ \frac{(1-\Delta)}{4} \sum_\rho (3a_j^\dagger a_j^\dagger a_{j+\rho} a_j a_j + 4a_j^\dagger a_{j+\rho} a_j + 2a_{j+\rho}^\dagger a_j^\dagger a_{j+\rho} a_{j+\rho} a_j + a_{j+\rho}^\dagger a_{j+\rho}^\dagger a_{j+\rho} a_j a_j) \\ &- \tfrac{1}{4} \sum_\rho (2a_{j+\rho}^\dagger a_j^\dagger a_j a_j a_{j+\rho} + a_{j+\rho}^\dagger a_{j+\rho}^\dagger a_{j+\rho} a_{j+\rho} a_j + 2a_j^\dagger a_{j+\rho} a_{j+\rho} + 2a_{j+\rho}^\dagger a_j^\dagger a_{j+\rho} a_{j+\rho} a_{j+\rho} + a_{j+\rho}^\dagger a_{j+\rho}^\dagger a_j a_j a_j) \\ &+ \frac{(1-\Delta)}{4} \sum_\rho (2a_{j+\rho}^\dagger a_j^\dagger a_{j+\rho} a_{j+\rho} a_j + a_{j+\rho}^\dagger a_{j+\rho}^\dagger a_{j+\rho} a_j a_j + 2a_j^\dagger a_{j+\rho}^\dagger a_j a_j a_j \\ &\qquad + a_{j+\rho}^\dagger a_{j+\rho}^\dagger a_{j+\rho} a_{j+\rho} a_{j+\rho} + 2a_{j+\rho}^\dagger a_{j+\rho} a_{j+\rho} + 2a_{j+\rho}^\dagger a_j a_j) \ ,\end{aligned} \tag{B2}$$

$$\begin{aligned}
B_3 = {} & 2(1-\Delta)\sum_{\rho}(2a^{\dagger}_{j+\rho}a^{\dagger}_{j}a_{j+\rho}a_{j+\rho}a_{j}+a^{\dagger}_{j+\rho}a^{\dagger}_{j+\rho}a_{j+\rho}a_{j}a_{j}+a^{\dagger}_{j+\rho}a_{j+\rho}a_{j+\rho}a_{j+\rho}+a^{\dagger}_{j+\rho}a_{j}a_{j}) \\
& -2(1-\Delta)^2\sum_{\rho}(2a^{\dagger}_{j+\rho}a^{\dagger}_{j}a_{j+\rho}a_{j}a_{j}+a^{\dagger}_{j+\rho}a^{\dagger}_{j+\rho}a_{j+\rho}a_{j+\rho}a_{j}+2a^{\dagger}_{j+\rho}a_{j+\rho}a_{j}) \\
& -\tfrac{1}{4}\sum_{\rho}(3a^{\dagger}_{j}a^{\dagger}_{j}a_{j}a_{j+\rho}a_{j+\rho}+2a^{\dagger}_{j}a_{j+\rho}a_{j+\rho}+2a^{\dagger}_{j+\rho}a^{\dagger}_{j}a_{j}a_{j}a_{j+\rho}+a^{\dagger}_{j+\rho}a^{\dagger}_{j+\rho}a_{j+\rho}a_{j+\rho}a_{j}+4a^{\dagger}_{j+\rho}a_{j+\rho}a_{j}) \\
& -\tfrac{1}{4}\sum_{\rho}(2a^{\dagger}_{j+\rho}a^{\dagger}_{j}a_{j+\rho}a_{j+\rho}a_{j+\rho}+2a^{\dagger}_{j+\rho}a^{\dagger}_{j}a_{j+\rho}a_{j}a_{j}+a^{\dagger}_{j+\rho}a^{\dagger}_{j+\rho}a_{j}a_{j}a_{j}+a^{\dagger}_{j+\rho}a^{\dagger}_{j+\rho}a_{j+\rho}a_{j+\rho}a_{j}+2a^{\dagger}_{j}a_{j}a_{j}) \\
& -\tfrac{1}{4}\sum_{\rho}(3a^{\dagger}_{j}a^{\dagger}_{j}a_{j+\rho}a_{j+\rho}a_{j}+2a^{\dagger}_{j+\rho}a^{\dagger}_{j}a_{j+\rho}a_{j}a_{j}+a^{\dagger}_{j+\rho}a^{\dagger}_{j+\rho}a_{j+\rho}a_{j+\rho}a_{j}+2a^{\dagger}_{j}a_{j+\rho}a_{j+\rho}+2a^{\dagger}_{j}a_{j}a_{j}) \\
& -\tfrac{1}{4}\sum_{\rho}(2a^{\dagger}_{j+\rho}a^{\dagger}_{j}a_{j+\rho}a_{j}a_{j}+2a^{\dagger}_{j+\rho}a^{\dagger}_{j}a_{j+\rho}a_{j+\rho}a_{j+\rho}+a^{\dagger}_{j+\rho}a^{\dagger}_{j+\rho}a_{j+\rho}a_{j+\rho}a_{j}+a^{\dagger}_{j+\rho}a^{\dagger}_{j+\rho}a_{j}a_{j}a_{j}+4a^{\dagger}_{j+\rho}a_{j+\rho}a_{j}) \\
& +\frac{(1-\Delta)}{4}\sum_{\rho}(3a^{\dagger}_{j}a^{\dagger}_{j}a_{j}a_{j}a_{j+\rho}+2a^{\dagger}_{j+\rho}a^{\dagger}_{j}a_{j}a_{j+\rho}a_{j+\rho}+a^{\dagger}_{j+\rho}a^{\dagger}_{j+\rho}a_{j+\rho}a_{j}a_{j}+4a^{\dagger}_{j}a_{j}a_{j+\rho}) \\
& +\frac{(1-\Delta)}{4}\sum_{\rho}(2a^{\dagger}_{j+\rho}a^{\dagger}_{j}a_{j}a_{j}a_{j}+2a^{\dagger}_{j+\rho}a^{\dagger}_{j}a_{j+\rho}a_{j+\rho}a_{j}+a^{\dagger}_{j+\rho}a^{\dagger}_{j+\rho}a_{j+\rho}a_{j+\rho}a_{j+\rho} \\
& \qquad +a^{\dagger}_{j+\rho}a^{\dagger}_{j+\rho}a_{j+\rho}a_{j}a_{j}+2a^{\dagger}_{j+\rho}a_{j+\rho}a_{j+\rho}+2a^{\dagger}_{j+\rho}a_{j}a_{j}) \\
& +\frac{(1-\Delta)}{4}\sum_{\rho}(3a^{\dagger}_{j}a_{j}a_{j}a_{j}a_{j+\rho}+2a^{\dagger}_{j+\rho}a^{\dagger}_{j}a_{j+\rho}a_{j+\rho}a_{j}+a^{\dagger}_{j+\rho}a^{\dagger}_{j+\rho}a_{j+\rho}a_{j}a_{j}+4a^{\dagger}_{j}a_{j}a_{j+\rho}) \\
& +\frac{(1-\Delta)}{4}\sum_{\rho}(2a^{\dagger}_{j+\rho}a^{\dagger}_{j}a_{j}a_{j+\rho}a_{j+\rho}+2a^{\dagger}_{j+\rho}a^{\dagger}_{j}a_{j}a_{j}a_{j}+a^{\dagger}_{j+\rho}a^{\dagger}_{j+\rho}a_{j+\rho}a_{j}a_{j} \\
& \qquad +a^{\dagger}_{j+\rho}a^{\dagger}_{j+\rho}a_{j+\rho}a_{j+\rho}a_{j+\rho}+2a^{\dagger}_{j+\rho}a_{j+\rho}a_{j+\rho}+2a^{\dagger}_{j+\rho}a_{j}a_{j}) \ . \qquad \text{(B3)}
\end{aligned}$$

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