Generalized Franz-Keldysh theory of electromodulation

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We present a detailed generalization of the Franz-Keldysh theory of electromodulation which takes into account the presence of a dc surface electric field, $\mathcal{E}^s_{\mathrm{dc}}$. We demonstrate rigorously that when the ac modulating electric field is small compared to $\mathcal{E}^s_{\mathrm{dc}}$, the period of the Franz-Keldysh oscillations yields $\mathcal{E}^s_{\mathrm{dc}}$. This result is independent of the relation between the width of the space-charge region and the penetration depth of light. The effects of broadening also are considered.

I. INTRODUCTION

There has recently been considerable interest in the use of electromodulation[1-3] (EM) to study semiconductors[1-15] and semiconductor microstructures.[9-16] Electromodulation is the most useful of various optical modulation methods since it yields, in general, the sharpest structure[17] and is sensitive to surface (interface) electric fields.[6-9,13-15] While most of the work in EM has made use of the sharp, derivativelike features there is growing activity in the latter area, i.e., EM as a probe of surface or interface electric fields.[6-9,12,13] A very useful aspect of EM to study these effects is the observation of Franz-Keldysh oscillations (FKO).[1-3,6-9,14,15,18-20] Early work on FKO was reported using electroabsorption in the nonuniform field of the space-charge region of p-n junctions.[18,19] These authors pointed out that the period of the FKO was a direct measure of the maximum dc field in the p-n junction for the case of small ac modulation. However, no detailed theory in terms of Airy functions was presented. Bhattacharya *et al.*[6,7] observed that in the reflection mode (electroreflectance and photoreflectance) with small ac modulation the FKO were directly related to the maximum field in the surface space-charge region, i.e., the dc surface field $\mathcal{E}^s_{\mathrm{dc}}$. These authors presented a detailed theory which contained an error although the final result was correct. Bottka and co-workers[8-10] also have investigated the FKO related to the surface space-charge layer. However, they have erroneously concluded that the period of the FKO is related to the average dc field rather $\mathcal{E}^s_{\mathrm{dc}}$. The use of FKO is becoming significant in evaluating surface electric fields and related quantities such as Fermi-level pinning[6-10,13] and carrier concentrations.[6-10] Thus, it is important to establish the correct relationship between the period of the FKO and the field in the space-charge region.

In this paper we demonstrate rigorously that for low-field ac modulation ($\mathcal{E}_{\mathrm{ac}}$) in the presence of a large dc electric field the period of the FKO is a direct optical measure of the dc surface electric field, $\mathcal{E}^s_{\mathrm{dc}}$. The surface field can be due to Fermi-level pinning or can be externally applied. This result is independent of the relation between the width of the space-charge region (SCR) and the penetration depth of the light. The theory yields the relation between the magnitude of the electromodulation signal and the fields $\mathcal{E}_{\mathrm{ac}}$ and $\mathcal{E}^s_{\mathrm{dc}}$. The effects of finite modulation as well as the influence of broadening also are considered. Calculations for the case of GaAs with various doping levels will be presented.

II. THEORY

A. No broadening

1. Electroreflectance

In electroreflectance methods[1-4] such as Schottky barrier, metal-insulator-semiconductor, or semiconductor-electrolyte configurations, the electric field is modulated by $\pm\mathcal{E}_{\mathrm{ac}}$ about $\mathcal{E}_{\mathrm{dc}}$. In this case the variation in the dielectric function $\delta\epsilon^{\mathrm{ER}}(E,\mathcal{E}_{\mathrm{dc}},\mathcal{E}_{\mathrm{ac}})$ can be expressed as

$$\delta\epsilon^{\mathrm{ER}}(E,\mathcal{E}_{\mathrm{dc}},\mathcal{E}_{\mathrm{ac}})=\epsilon(E,\mathcal{E}_{\mathrm{dc}}+\mathcal{E}_{\mathrm{ac}})-\epsilon(E,\mathcal{E}_{\mathrm{dc}}-\mathcal{E}_{\mathrm{ac}}) \tag{1a}$$

$$=\Delta\epsilon(E,\mathcal{E}_{\mathrm{dc}}+\mathcal{E}_{\mathrm{ac}})-\Delta\epsilon(E,\mathcal{E}_{\mathrm{dc}}-\mathcal{E}_{\mathrm{ac}}) \ , \tag{1b}$$

where E is the photon energy and

$$\Delta\epsilon(E,\mathcal{E})=\epsilon(E,\mathcal{E})-\epsilon(E,0) \ . \tag{1c}$$

It has been shown that for an M_0 (three-dimensional) critical point the quantity $\Delta\epsilon(E,\mathcal{E})$ of Eq. (1c) can be written as[1-3]

$$\Delta\epsilon(E,\mathcal{E})=(B/E^2)(\hbar\theta)^{1/2}[G(\eta)+iF(\eta)] \ , \tag{2a}$$

where the quantity B (related to matrix element effects) is defined on p. 172 of Ref. 1. The parameters $\hbar\theta$ and η are

$$(\hbar\theta)^3=e^2\hbar^2\mathcal{E}^2/2\mu_{\parallel} \ , \tag{2b}$$

$$\eta=(E_0-E)/\hbar\theta \ , \tag{2c}$$

where $\mu_{\parallel}$ is the reduced effective mass in the direction of $\mathcal{E}$ and E_0 is the energy gap. The parameters $G(\eta)$ and $F(\eta)$ are electro-optic functions given by[1,3]

$$G(\eta)=\pi[\mathrm{Ai}'(\eta)\mathrm{Bi}'(\eta)-\eta\mathrm{Ai}(\eta)\mathrm{Bi}(\eta)]+\eta^{1/2}H(-\eta)\ , \tag{2d}$$

$$F(\eta)=\pi[\mathrm{Ai}'^2(\eta)-\eta\mathrm{Ai}^2(\eta)]-(-\eta)^{1/2}H(-\eta)\ . \tag{2e}$$

In Eqs. (2d) and (2e) $\mathrm{Ai}(\eta)$, $\mathrm{Bi}(\eta)$, $\mathrm{Ai}'(\eta)$, and $\mathrm{Bi}'(\eta)$ are Airy functions and their derivatives and $H(-\eta)$ is the unit step function.[1,3] By neglecting broadening effects we are in the regime where $\Gamma \ll \hbar\theta$, where Γ is a phenomenological broadening parameter.

We now consider the case in which $\mathcal{E}_{\mathrm{ac}} \ll \mathcal{E}_{\mathrm{dc}}$. We referred to this situation as the low-field Franz-Keldysh (LFFK) criterion. Several approximations will be made. We denote as z the distance into the SCR as measured from the surface and ρ $(=e|N_D-N_A|)$ as the net charge density, where N_D and N_A are the donor and acceptor concentrations, respectively. We assume that no significant free carriers are created in an electromodulation experiment, i.e., ρ remains a constant. For the SCR the abrupt junction approximation is made so that $\mathcal{E}(z)$ is a linear function of z given by[21]

$$\mathcal{E}(z)=\mathcal{E}^s_{\mathrm{dc}}[(W-z)/W] \tag{3a}$$

$$=(\rho/\epsilon_0)(W-z)\ , \tag{3b}$$

where $\mathcal{E}^s_{\mathrm{dc}}$ is the surface dc electric field, W is the width of the SCR and ϵ_0 is the static dielectric constant.

In the nonuniform field regime the quantity $\delta\epsilon(E,\mathcal{E}_{\mathrm{dc}},\mathcal{E}_{\mathrm{ac}})$ can be expressed as[3]

$$\delta\epsilon^{\mathrm{ER}}(E,\mathcal{E}_{\mathrm{dc}},\mathcal{E}_{\mathrm{ac}}) = -2i\kappa\left[\int_0^{W+\Delta W}\Delta\epsilon(E,\mathcal{E}(z-\Delta W))e^{i2\kappa z}dz - \int_0^{W-\Delta W}\Delta\epsilon(E,\mathcal{E}(z+\Delta W))e^{i2\kappa z}dz\right]\ , \tag{4}$$

where $\kappa(E)$ is the complex propagation vector of the light in the solid. The quantities $W+\Delta W$ and $W-\Delta W$ are the widths of the SCR for the case of $\mathcal{E}_{\mathrm{dc}}+\mathcal{E}_{\mathrm{ac}}$ and $\mathcal{E}_{\mathrm{dc}}-\mathcal{E}_{\mathrm{ac}}$, respectively. In Eq. (4) we have made use of the linear relation between $\mathcal{E}$ and z. In Ref. 8 the authors erroneously assigned the limits of this integral to be between 0 and W. Thus, they were in the regime of modulation from flat band which is not appropriate for LFFK.

By making an appropriate change of variables in Eq. (4) we can write

$$\delta\epsilon^{\mathrm{ER}}(E,\mathcal{E}_{\mathrm{dc}},\mathcal{E}_{\mathrm{ac}}) = -2i\kappa\left[e^{i2\kappa\Delta W}\int_{-\Delta W}^{W}\Delta\epsilon(E,\mathcal{E}(z))e^{i2\kappa z}dz - e^{-i2\kappa\Delta W}\int_{\Delta W}^{W}\Delta\epsilon(E,\mathcal{E}(z))e^{i2\kappa z}dz\right]\ . \tag{5}$$

We now make the low-modulation field approximation (LFFK), i.e., $\kappa\Delta W \ll 1$. Neglecting terms of order $(\Delta W)^2$ or higher, Eq. (5) can now be written as

$$\delta\epsilon^{\mathrm{ER}}(E,\mathcal{E}_{\mathrm{dc}},\mathcal{E}_{\mathrm{ac}}) = -2i\kappa\left[\int_{-\Delta W}^{\Delta W}\Delta\epsilon(E,\mathcal{E}(z))e^{i2\kappa z}dz + 4i\kappa\Delta W\int_0^W\Delta\epsilon(E,\mathcal{E}(z))e^{i2\kappa z}dz\right]\ . \tag{6}$$

In Refs. 6 and 7 Bhattacharya *et al.* assumed that $\kappa W \ll 1$.

From the linear relationship between $\mathcal{E}$ and z [Eq. (3)] we can write

$$z=W(1-\chi)\ , \tag{7a}$$

$$\chi=\mathcal{E}/\mathcal{E}^s_{\mathrm{dc}}\ , \tag{7b}$$

where W is the width of the SCR for the case of the dc surface electric field $\mathcal{E}^s_{\mathrm{dc}}$. Also we can write from Eq. (2c)

$$\eta=\eta^s_{\mathrm{dc}}/\chi^{2/3}\ , \tag{7c}$$

where

$$\eta^s_{\mathrm{dc}}=(E_0-E)/\hbar\theta^s_{\mathrm{dc}}\ , \tag{7d}$$

$$(\hbar\theta^s_{\mathrm{dc}})^3=e^2\hbar^2(\mathcal{E}^s_{\mathrm{dc}})^2/2\mu_\parallel\ . \tag{7e}$$

In addition we define

$$\xi=\mathcal{E}^s_{\mathrm{ac}}/\mathcal{E}^s_{\mathrm{dc}}=\Delta W/W\ . \tag{7f}$$

Using Eqs. (2a) and (7a)–(7f), the expression of Eq. (6b) can be rewritten as

$$\delta\epsilon^{\mathrm{ER}}(E,\mathcal{E}_{\mathrm{dc}},\mathcal{E}_{\mathrm{ac}})=[-4i\kappa(\epsilon_0/\rho)(\mathcal{E}^s_{\mathrm{dc}})^{1/3}(\mathcal{E}^s_{\mathrm{ac}})](J_s+J_{\mathrm{av}}) \tag{8a}$$

where

$$J_s(\eta^s_{\mathrm{dc}},\xi,0)=G_s(\eta^s_{\mathrm{dc}},\xi,0)+iF_s(\eta^s_{\mathrm{dc}},\xi,0)\ , \tag{8b}$$

$$J_{\mathrm{av}}(\eta^s_{\mathrm{dc}},\xi,0)=G_{\mathrm{av}}(\eta^s_{\mathrm{dc}},\xi,0)+iF_{\mathrm{av}}(\eta^s_{\mathrm{dc}},\xi,0)\ . \tag{8c}$$

The terms $J_s(\eta^s_{\mathrm{dc}},\xi,0)$ and $J_{\mathrm{av}}(\eta^s_{\mathrm{dc}},\xi,0)$ represent the contribution of the "surface" (J_s) and "average" (J_{av}) electric fields to the FKO for the unbroadened case ($\Gamma=0$). However, as we will show in the section below the period of the FKO from both these terms yields $\mathcal{E}^s_{\mathrm{dc}}$ as long as the modulation is in the LFFK regime. Complete expressions for G_s, F_s, G_{av}, and F_{av} are given in the Appendix.

2. *Photoreflectance*

In photoreflectance the electromodulation is from W to $W-\Delta W$ since the photoinjected electron-hole pairs lower the dc electric field.[22] Thus, Eq. (1b) becomes

$$\delta\epsilon^{PR}(E,\mathcal{E}_{\mathrm{dc}},\mathcal{E}_{\mathrm{ac}})=\Delta\epsilon(E,\mathcal{E}_{\mathrm{dc}})-\Delta\epsilon(E,\mathcal{E}_{\mathrm{dc}}-\mathcal{E}_{\mathrm{ac}})\ , \tag{9a}$$

$$\delta\epsilon^{PR}(E,\mathcal{E}_{\mathrm{dc}},\mathcal{E}_{\mathrm{ac}})=(\tfrac{1}{2})\delta\epsilon^{\mathrm{ER}}(E,\mathcal{E}_{\mathrm{dc}},\mathcal{E}_{\mathrm{ac}})\ . \tag{9b}$$

3. *Results for GaAs*

In order to evaluate G_s, G_{av}, F_s, and F_{av} for GaAs we use the following parameters: $V_{\mathrm{bi}}=0.7$ V,

TABLE I. Values of W, $\mathcal{E}^s_{dc}$, $\hbar\theta^s_{dc}$, aW, and $(\alpha_0/2)W$ for various net carrier concentrations $|N_D-N_A|$ for GaAs.

$\lvert N_D-N_A\rvert$ (cm^{-3})	W (Å)	$\mathcal{E}^s_{dc}$ (10^4 V/cm)	$\hbar\theta^s_{dc}$ (meV)	aW	$(\alpha_0/2)W$
1×10^{15}	10 200	1.3	5.1	27.5	0.15
1×10^{16}	3 220	4.3	11	8.7	0.071
1×10^{17}	1 020	13	23	2.8	0.033
1×10^{18}	322	43	51	0.87	0.015

$\epsilon_0=13.4\times8.85\times10^{-14}$ F/cm, $\mu_\parallel=0.055m_0$, where V_{bi} is the built-in surface potential due to Fermi-level pinning[23] and m_0 is the free-electron mass. The above value of $\mu_\parallel$ is related to transitions from the heavy-hole valence band.[24,25] The depletion width W and dc surface electric field ($\mathcal{E}^s_{dc}$) can now be expressed as[21,26,27]

$$W=[(2\epsilon_0/\rho)(V_{bi}-V_{ext}-kT/e)]^{1/2}, \tag{10a}$$

$$\mathcal{E}^s_{dc}=[(2\rho/\epsilon_0)(V_{bi}-V_{ext}-kT/e)]^{1/2}, \tag{10b}$$

$$\rho=e|N_D-N_A|, \tag{10c}$$

where V_{ext} is an externally applied voltage. We take $V_{ext}=0$.

In Table I we have listed W, $\mathcal{E}_s^{dc}$, $\hbar\theta_s^{dc}$, aW, and $(\alpha_0/2)W$ as a function of net-impurity concentration $|N_D-N_A|$ for GaAs. The parameters a and α_0 are defined in Eqs. (A3) and (A5) in the Appendix, respectively.

Plotted in Fig. 1 are G_s (solid line) and F_s (dotted line) as a function of $-\eta^s_{dc}$ for $\xi=0.01$ for the cases of $|N_D-N_A|=1\times10^{15}$ cm^{-3} [Fig. 1(a)] and 1×10^{18} cm^{-3} [Fig. 1(b)]. The two curves are almost identical in both period and amplitude.

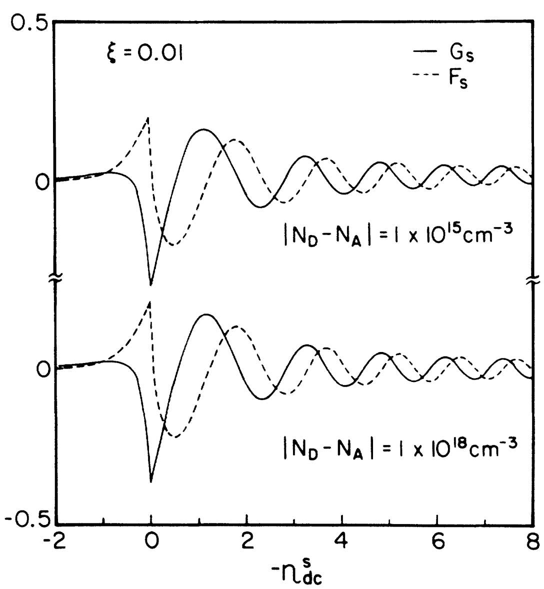

FIG. 1. Theoretical values of the unbroadened electro-optic functions G_s and F_s with $\xi=0.01$ for the case of $|N_D-N_A|=1\times10^{15}$ cm^{-3} and $|N_D-N_A|=1\times10^{18}$ cm^{-3}.

The extrema in the FKO are given by[28]

$$m\pi=\phi+(4/3)[(E_m-E_0)/\hbar\theta^s_{dc}]^{3/2}, \tag{11}$$

where m is the index of the mth extrema, ϕ is an arbitrary phase factor, and E_m is the photon energy of the mth oscillation.

Plotted in Fig. 2 is the quantity $(4/3\pi)[(E_m-E_0)/\hbar\theta^s_{dc}]^{3/2}$ as a function of m for G_s (open circles) and F_s (closed circles) as obtained from Figs. 1(a) and 1(b). The solid lines are least-squares fits to a linear function. All four curves yield a slope of 1.0. This demonstrates that over a wide range of depletion widths (see Table I) the period of the FKO of J_s for $\xi\ll1$ (LFFK regime) yields the dc surface electric field $\mathcal{E}^s_{dc}$.

Plotted in Figs. 3(a) and 3(b) are G_{av} (solid line) and F_{av} (dashed line) as a function of $-\eta^s_{dc}$ for $|N_D-N_A|=1\times10^{15}$ cm^{-3} and 1×10^{18} cm^{-3}, respectively. For the latter curve the FKO are rapidly damped out. In Figs. 4(a) and 4(b) we display $(4/3\pi)[(E_m-E_0)/\hbar\theta^s_{dc}]^{3/2}$ as a function of m as deduced from G_{av} (open circles) and F_{av} (closed circles) for the two different carrier concentrations. The solid lines are least-squares fits to a linear function. As in the case of Fig. 2 the slope is 1.0. Thus, in contrast to Refs. 8 and 10, we have demonstrated that the "average field" contribution to the FKO also yields the *dc surface electric field* and not the average

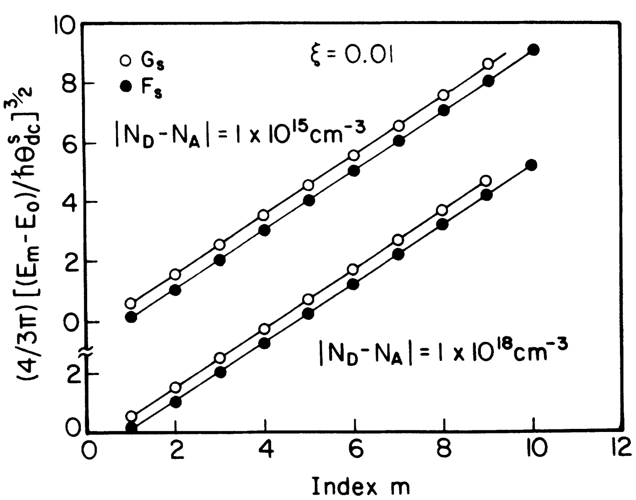

FIG. 2. The quantity $(4/3\pi)[(E_m-E_0)/\hbar\theta^s_{dc}]^{3/2}$ as a function of FKO index m for G_s and F_s with $\xi=0.01$ for $|N_D-N_A|=1\times10^{15}$ cm^{-3} and $|N_D-N_A|=1\times10^{18}$ cm^{-3}.

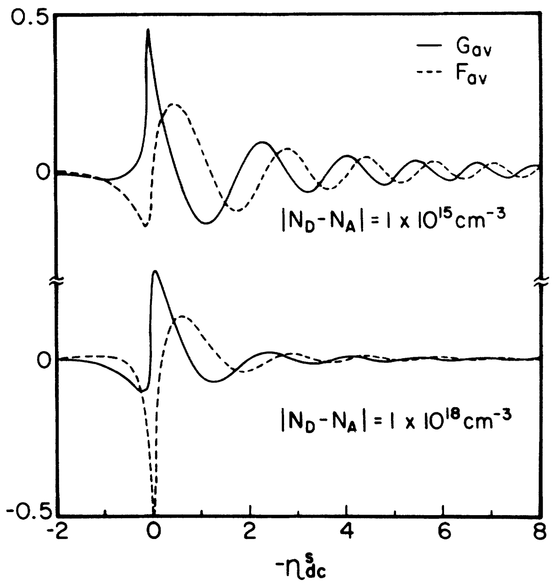

FIG. 3. Theoretical values of the unbroadened electro-optic functions G_{av} and F_{av} for the case of $|N_D-N_A|=1\times10^{15}$ cm^{-3} and $|N_D-N_A|=1\times10^{18}$ cm^{-3}.

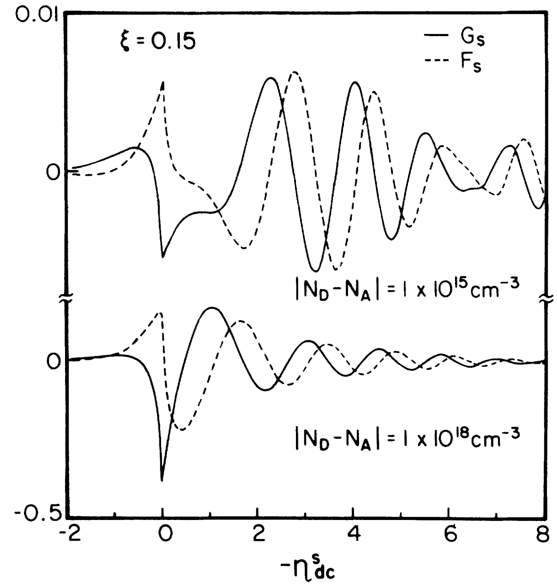

FIG. 5. Theoretical values of the unbroadened electro-optic functions G_s and F_s with $\xi=0.15$ for the case of $|N_D-N_A|=1\times10^{15}$ cm^{-3} and $|N_D-N_A|=1\times10^{18}$ cm^{-3}.

dc electric field.

We have also explored the situation where ξ is not small compared to unity. This affects only J_s and not J_{av}. Figures 5(a) and 5(b) display G_s (solid line) and F_s (dotted line) with $\xi=0.15$ for $|N_D-N_A|=1\times10^{15}$ cm^{-3} and 1×10^{18} cm^{-3}, respectively. The amplitude of F_s, G_s for the former case is considerably smaller than either F_{av}, G_{av} for $|N_D-N_A|=1\times10^{15}$ cm^{-3} [see Fig. 3(a)] or the other F_s, G_s. The situation of Fig. 5(a) ($\xi=0.15$) is starting to approach modulation from flatband and hence F_{av}, G_{av} will become more important in relation to F_s, G_s. The envelope function of Fig. 5(b) is considerably more damped in relation to Fig. 1(b). The reason for this dependence of the envelope function on ξ is the inhomogeneous nature of the field, as first pointed out in Ref. 7. Note also that for $-\eta^s_{dc} > 4$ the period of the FKO in Fig. 5(a) are somewhat different in relation to those of Fig. 5(b). This latter effect is due to the nature of the inhomogeneous field in relation to the penetration depth of the light. For $-\eta^s_{dc} < 4$ the period of the FKO are the same in Figs. 5(a), 5(b), 1(a), and 1(b). Thus, even for $\xi=0.15$ the first few FKO yield a correct value of the dc surface electric field.

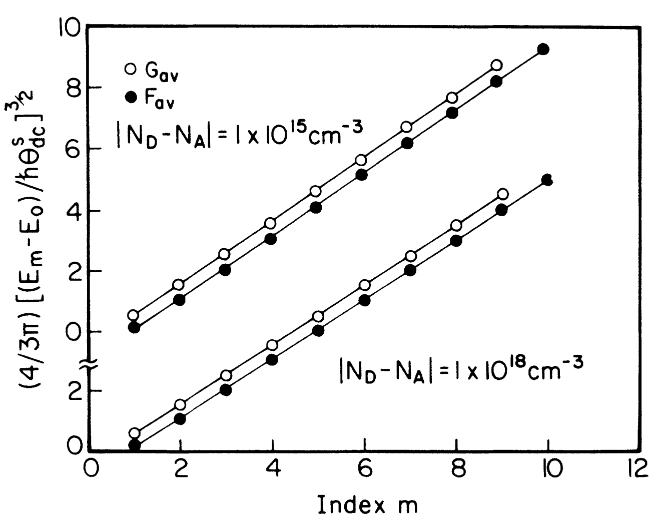

FIG. 4. The quantity $(4/3\pi)[(E_m-E_0)/\hbar\theta^s_{dc}]^{3/2}$ as a function of FKO index m for G_{av} and F_{av} for $|N_D-N_A|=1\times10^{15}$ cm^{-3} and $|N_D-N_A|=1\times10^{18}$ cm^{-3}.

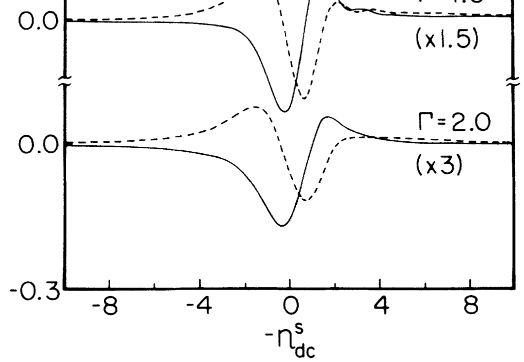

FIG. 6. Theoretical values of the broadened electro-optic functions G_s and F_s with $\xi=0.01$ for the case of $|N_D-N_A|=1\times10^{15}$ cm^{-3}.

B. Effects of Lorentzian broadening

The electric-field-induced change in the dielectric function ϵ in the presence of Lorentzian broadening may be obtained from the unbroadened change, $\delta\epsilon(E,\mathcal{E}_{dc},\mathcal{E}_{ac})$ $[=\delta\epsilon(E,\mathcal{E})]$, using[1]

$$\delta\epsilon(E,\mathcal{E},\Gamma)=(1/\pi)\int_{-\infty}^{\infty}\frac{\delta\epsilon(E',\mathcal{E})\Gamma}{(E-E')^2+\Gamma^2}dE' , \tag{12a}$$

where Γ represents the energy broadening or collision frequency. A contour integral of Eq. (12a) yields

$$\delta\epsilon(E,\mathcal{E},\Gamma)=\delta\epsilon(E+i\Gamma,\mathcal{E}) . \tag{12b}$$

The broadened quantities G_s, F_s, G_{av}, and F_{av} can thus be written as

$$G_i(\eta_{dc}^s,\xi,\Gamma)=(1/\pi)\int_{-\infty}^{\infty}\frac{G_i[(\eta_{dc}^s)',\xi,0]\Gamma}{(\eta_{dc}^s-\eta')^2+\Gamma^2}d\eta' , \tag{13a}$$

$$F_i(\eta_{dc}^s,\xi,\Gamma)=(1/\pi)\int_{-\infty}^{\infty}\frac{F_i[(\eta_{dc}^s)',\xi,0]\Gamma}{(\eta_{dc}^s-\eta')^2+\Gamma^2}d\eta' , \tag{13b}$$

where $i=s$ or av and Γ is in units of $\hbar\theta_{dc}^s$.

In order to illustrate the effects of broadening we have plotted in Fig. 6 the values of $G_s(\eta_{dc}^s,\xi,\Gamma)$ and $F_s(\eta_{dc}^s,\xi,\Gamma)$ for $|N_D-N_A|=1\times10^{15}$ cm^{-3} for $\xi=0.01$ and $\Gamma=1.0$, 1.5, and 2.0, in units of $\hbar\theta_{dc}^s=5.1$ meV (see Table I). The former quantity is represented by the solid line while the latter is the dashed line. For $\Gamma=1.0$ it is still possible to observe about eight or nine FKO. The effects of Γ are quite evident above $-\eta_{dc}^s=8$ as compared to Fig. 1. Only two or three FKO can still be seen for $\Gamma=1.5$ while for $\Gamma=2.0$ the FKO are completely damped out. In this case the electromodulation signal is in the third-derivative functional form regime.[3] Similar results have been obtained for $|N_D-N_A|=1\times10^{18}$ cm^{-3} for which $\hbar\theta_{dc}^s=51$ meV. The influence of the damping on the envelope function of the FKO could be used to evaluate the broadening parameter Γ.

In conclusion we have developed a detailed generalized theory of FKO in electromodulation which correctly takes into account of the presence of a dc surface electric field ($\mathcal{E}_{dc}^s$) when the ac electric-field modulation is small compared to $\mathcal{E}_{dc}^s$. The modulation of the dielectric function consists of two terms which represent the contributions of the "surface" (J_s) and "average" (J_{av}) electric fields to the FKO. Using a model numerical calculation in the vicinity of the direct band gap of GaAs we have demonstrated that both terms yield FKO related to the dc surface electric field independent of the penetration of the light in relation to the width of the SCR. This result follows from the fact that Airy functions can be expressed in terms of periodic sine and cosine functions. The period of such functions is not affected by the exponential envelope function related to the penetration depth of the light. Even for relatively large modulation ($\xi=0.15$) the first few FKO can be used to evaluate $\mathcal{E}_{dc}^s$. Our theory also relates the magnitude of the observed EM signal to $\mathcal{E}_{dc}^s$ and $\mathcal{E}_{ac}$. Broadening effects also have been considered.

APPENDIX

We take into account the relation between the penetration depth of the light and the width of the space charge region W. The complex propagation vector of the light κ can be written as

$$\kappa=(2\pi/\lambda)(n+ik) , \tag{A1}$$

$$\kappa=a+i(\alpha/2) , \tag{A2}$$

where

$$a=(2\pi n/\lambda) \tag{A3}$$

and λ is the wavelength of the light, n and k are the real and imaginary parts of the complex index of refraction, and α is the absorption coefficient.

We now consider the specific example of the fundamental (direct) band gap of GaAs(E_0). In the region of E_0 the quantity n does not vary very much with photon energy but there is a considerable change in α.[29] From Ref. 29 and taking a value of $E_0=1.42$ eV (8731 Å) we find that

$$a\approx3\times10^5\ \text{cm}^{-1} . \tag{A4}$$

For α as a function of photon energy E we use the expression

$$\alpha(E)=\alpha_0\left[\frac{E-E_0}{E_0}\right]^{1/2}H(E-E_0) , \tag{A5}$$

where $H(E-E_0)$ is the unit step function. The quantity α_0 can be evaluated from $\alpha(E)$ listed in Ref. 29. We find

$$\alpha_0\approx5\times10^4\ \text{cm}^{-1} . \tag{A6}$$

The quantity κW can be expressed as

$$\kappa W=aW+bW , \tag{A7}$$

where

$$b=(\alpha_0/2)(\hbar\theta_{dc}^s/E_0)^{1/2}(-\eta_{dc}^s)^{1/2}H(-\eta_{dc}^s) . \tag{A8}$$

The expressions for G_s, G_s, G_{av}, and F_{av} can be written as

$$G_s=(1/2\xi)\int_{1-\xi}^{1+\xi}\chi^{1/3}Me^{-2bW(1-\chi)}d\chi , \tag{A9}$$

$$F_s=(1/2\xi)\int_{1-\xi}^{1+\xi}\chi^{1/3}Ne^{-2bW(1-\chi)}d\chi , \tag{A10}$$

$$M=G\cos[2aW(1-\chi)]-F\sin[2aW(1-\chi)] , \tag{A11}$$

$$N=G\sin[2aW(1-\chi)]+F\cos[2aW(1-\chi)] , \tag{A12}$$

$$G_{av}=-2aWI_4-2bWI_3 , \tag{A13}$$

$$F_{av}=2aWI_3-2bWI_4 , \tag{A14}$$

$$I_3=\int_0^1\chi^{1/3}Me^{-2bW(1-\chi)}d\chi , \tag{A15}$$

$$I_4=\int_0^1\chi^{1/3}Ne^{-2bW(1-\chi)}d\chi , \tag{A16}$$

[1]See, for example, M. Cardona, *Modulation Spectroscopy* (Academic, New York, 1969), and references therein.
[2]See, for example, B. O. Seraphin, *Semiconductors and Semimetals,* edited by R. K. Willardson and A. C. Beer (Academic, New York, 1972), Vol. 9, p. 1, and references therein.
[3]See, for example, D. E. Aspnes, in *Handbook on Semiconductors,* edited by T. S. Moss (North-Holland, Amsterdam, 1980), Vol. 2, p. 109, and references therein.
[4]See, for example, F. H. Pollak, in *Proceedings of the Society of Photo-Optical Instrumentation Engineers, San Jose, 1981* (SPIE, Bellingham, 1981), Vol. 276, p. 142, and references therein.
[5]P. M. Raccah, J. W. Garland, Z. Zhang, V. Lee, D. Z. Xue, L. L. Ables, S. Ugur, and W. Wilensky, Phys. Rev. Lett. **53**, 1958 (1984).
[6]R. N. Bhattacharya, H. Shen, P. Parayanthal, F. H. Pollak, T. Coutts, and H. Aharoni, Solar Cells **21**, 371 (1987).
[7]R. N. Bhattacharya, H. Shen, P. Parayanthal, F. H. Pollak, T. Coutts, and H. Aharoni, Phys. Rev. B **37**, 4044 (1988); also, *Proceedings of the Society of Photo-Optical Instrumentation Engineers* (SPIE, Bellingham, 1987), Vol. 794, p. 81.
[8]R. Glosser and N. Bottka, in *Proceedings of the Society of Photo-Optical Instrumentation Engineers* (SPIE, Bellingham, 1987), Vol. 794, p. 88.
[9]N. Bottka, D. K. Gaskill, R. S. Sillmon, R. Henry, and R. Glosser, J. Electron. Mater. **17**, 161 (1988).
[10]D. K. Gaskill, N. Bottka, and R. S. Sillmon, J. Vac. Sci. Technol. B **6**, 1497 (1988).
[11]P. M. Raccah, J. W. Garland, S. E. Buttrill, Jr., L. Francke, and J. Jackson, Appl. Phys. Lett. **52**, 1584 (1988).
[12]H. Shen, S. H. Pan, Z. Hang, J. Leng, F. H. Pollak, J. M. Woodall, and R. N. Sacks, Appl. Phys. Lett. **53**, 1080 (1988).
[13]H. Shen, F. H. Pollak, J. M. Woodall, and R. N. Sacks, J. Vac. Sci. Technol. B **7**, 804 (1989).
[14]C. Van Hoof, K. Deneffe, J. DeBoeck, O. J. Arent, and G. Borghs, Appl. Phys. Lett. **54**, 608 (1989).
[15]H. Shen, F. H. Pollak, and J. M. Woodall, J. Vac. Sci. Technol. (to be published).
[16]F. H. Pollak and O. J. Glembocki, in *Proceedings of the Society of Photo-Optical Instrumentation Engineers* (SPIE, Bellingham, 1988), Vol. 946, p. 2.
[17]In bulk or thin-film materials low-field EM is related to the third derivative of the optical constants. However, as discussed in Ref. 16 for isolated quantum wells EM yields a first derivative.
[18]A. Frova and P. Handler, Phys. Rev. **137**, A1857 (1965).
[19]P. Handler, Phys. Rev. **137**, A1862 (1965).
[20]R. A. Batchelor, A. C. Brown, and A. Hammett, Phys. Rev. B **41**, 1401 (1990).
[21]See, for example, S. M. Sze, in *Physics of Semiconductor Devices,* 2nd ed. (Wiley, New York, 1981), p. 248.
[22]J. L. Shay, Phys. Rev. B **2**, 803 (1970).
[23]H. Hasegawa, H. Ishii, T. Sawada, T. Saitoh, S. Konishi, Y. Liu, and H. Ohno, J. Vac. Sci. Technol. B **6**, 1184 (1988).
[24]The dominance of the heavy-hole valence transitions in FKO was demonstrated by M. Chandreskhar and F. H. Pollak, Phys. Rev. B **15**, 2127 (1977).
[25]Values of the conduction and heavy-hole effective masses were taken from *Numerical Data and Functional Relationships in Science and Technology,* Vol. 17a of *Landolt-Bornstein,* edited by O. Madelung, M. Schulz, and H. Weiss (Springer, New York, 1982).
[26]In Refs. 8 and 10 Bottka and co-workers have erroneously given the net carrier concentration $\rho = e|N_D + N_A|$.
[27]If there is a photoinduced voltage the expression for $\mathcal{E}^s_{\mathrm{dc}}$ is given by

$$\mathcal{E}^s_{\mathrm{dc}} = [(2\rho/\epsilon_0)(V_{\mathrm{bi}} - V_{\mathrm{ext}} - V_p - kT/e)]^{1/2} ,$$

where V_p is the photoinduced voltage.
[28]Equation (9) of Ref. 7 incorrectly stated the expression for the extrema in the FKO as

$$m\pi = \phi + (16/3)[(E_m - E_0)/\hbar\theta^s_{\mathrm{dc}}]^{3/2} ,$$

although the correct equation was used to interpret the experimental results.
[29]B. O. Seraphin and H. E. Bennett, in *Semiconductors and Semimetals,* edited by R. K. Willardson and A. C. Beer (Academic, New York, 1967), Vol. 3, p. 499.