PHYSICAL REVIEW B VOLUME 42, NUMBER 10 1 OCTOBER 1990

Magnetic quantization and the upper critical field of superconductors

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(Received 2 July 1990)

Recently, there has been renewed interest in the effect of orbital quantization on the upper critical field of superconductors, given the possibility of reentrant behavior at high fields. In this paper, the Gor'kov equations are solved numerically in the limit where only a few Landau levels are involved, including the effects of impurities and a nonzero g factor, for a set of parameters appropriate to a low-carrier-density superconductor. It is shown that these effects conspire to make the experimental observation of this effect possible, but unlikely.

I. HISTORY

In 1968, Gruenberg and Gunther published a paper analyzing the effects of orbital quantization on the upper critical field of type-II superconductors.[1] In it, they showed that Landau-level quantization had a dramatic effect on H_{c2}, causing T_c to oscillate as a function of field at very low temperatures. Moreover, they showed that at fields larger than H_{c2}, superconductivity was again possible, although at exponentially small temperatures. Although they indicated that T_c begins to increase again for fields much larger than H_{c2}, they did not carry the analysis any further because of their qualms about the validity of BCS theory at such high-field values. Finally, they showed that the effect of impurities and a g factor different from an even integer caused a rapid suppression of this reentrant behavior.

Recently, Tesanovic, Rasolt, and Xing[2] have reanalyzed this problem, giving particular attention to the limit where only one Landau level is involved. They showed that in this case, T_c increases with increasing field, exceeding its zero-field value, and that impurities would not affect this result. They also indicated that by allowing nonzero-Q pairing (Fulde-Ferrell state[3]), g factors different from zero would also not affect this result (except at high enough fields where Pauli limiting would occur). They also demonstrated that the standard flux-lattice solution is preserved, with the lattice constant being set by the orbit radius of the Landau level.

A more pessimistic view has been offered by Rieck, Scharnberg, and Klemm.[4] They showed that some of the results of Ref. 2 were influenced by an assumption that the density of states (DOS) was constant within a pairing width about the chemical potential, μ. Since the DOS in a magnetic field is strongly energy dependent, correcting for this changes the results somewhat, as they demonstrate in several nice figures. In particular, they find that a g factor differing from an even integer strongly quenches the reentrant behavior, leading to the conclusion that the chances of seeing this effect are very slim. They also indicated that the strong effect of impurities found in Ref. 1 was due to an assumption that the impurities were pair breakers, and thus agree in that aspect with the authors of Ref. 2.

In this paper, the Gor'kov equations are solved numerically in the limit where only a few Landau levels are involved for a parameter set appropriate to a low-carrier-density superconductor. Solutions are generated including the effect of impurity broadening of the DOS as well as allowing a nonzero g factor with nonzero-Q pairing. From this, some conclusions are rendered concerning the chances of observing reentrant behavior at high fields.

II. THEORY

The Gor'kov equation for H_{c2} including magnetic quantization is [Eq. (2.6), Ref. 1]

$$1=VeH/8\pi\hbar c\sum_{r,l}(\tfrac{1}{2})^{r+l}(r+l)!/r!l!\int dk/2\pi[\tanh(\beta\epsilon_{r+}/2)+\tanh(\beta\epsilon_{l-}/2)]/(\epsilon_{r+}+\epsilon_{l-})\,, \tag{1}$$

where V is the pair potential, r and l are Landau-level indices running from 0 to ∞, k is the z component of the momentum, and

$$\epsilon_{r\sigma}=(r+\tfrac{1}{2})\hbar\omega_c+k^2/2m+\sigma g/4\hbar\omega_c-\mu\,,$$

with ω_c being the cyclotron frequency and g the g factor. Note that there is a factor of 2 difference from Ref. 1 [an error was made when going from Eq. (2.4) to (2.6)]. The k integral is restricted to be such that $k^2/2m$ is within a pairing width of μ. For fields high enough where only one Landau level is involved (with $g=0$), this equation is identical to the zero-field BCS gap equation, except that the constant DOS is replaced by $N(E,H)/2$, where $N(E,H)$ is the DOS of one Landau parabola. One immediately notices that for high temperatures, T_c is proportional to H (if one ignores the field and temperature dependence of μ). This is due to the fact that the degeneracy of the Landau level, and thus the number of states available for pairing, linearly increases with H. For a realistic calculation, though, Eq. (1) has to be numerically integrated within a pairing width, thus taking into account the strongly energy-dependent behavior of $N(E,H)$. Moreover, μ, which depends strongly on H and T, has to be determined numerically at nonzero T [for $T=0$ in the quantum limit, μ is equal to $\frac{4}{9}E_F^3/(\hbar\omega_c)^2$ where E_F is the zero-field Fermi energy]. When more than one Landau level is involved, several levels are within a pairing range, leading to a sum of numerical integrals to be done.

At a first approximation level, there is no effect due to nonmagnetic impurities because of Anderson's theorem.[2,4,5] This conclusion ignores the broadening which is important due to the singularity in the DOS at the bottom of a Landau parabola. One can model this broadening by assuming the following replacement for $E^{-1/2}$ in $N(E,H)$:

$$\theta(E)\{[E'+(E'^2+\Gamma^2)^{1/2}]/2(E'^2+\Gamma^2)\}^{1/2}, \qquad (2)$$

where Γ is the linewidth and $E'=E-\Gamma$.[6]

When the g factor deviates from an even integer, one finds a rapid suppression of T_c since the degeneracy of the up- and down-spin Landau parabolas is lifted. This can be compensated by allowing for pairing at nonzero-Q values.[2,4] This involves replacing k by $k \pm Q/2$ with $+$ for spin up and $-$ for spin down. In general, one now has two integrations to perform per Landau-level pair, with Q being varied to maximize the right-hand side of Eq. (1).

Finally, a parameter set appropriate to a low-carrier-density superconductor needs to be chosen. For our purpose here, we assume a zero field E_F of 100 K, a Debye width of ± 20 K about μ, and a BCS coupling constant ($NV/2$ where N is the zero field DOS) of 0.3. This leads to a zero field T_c of 810 mK. μ in a field is calculated assuming that the density of electrons, n, remains the same (for the case here, $n = 3.63\times 10^{18}\ e^-/\text{cm}^3$). This parameter set is somewhat optimistic when compared to low-carrier-density superconductors such as $SrTiO_3$, which has a T_c of only 100 mK for n as low as $1\times 10^{19}\ e^-/\text{cm}^3$.[7]

III. RESULTS

In Fig. 1, results are shown in the quantum limit ($\hbar\omega_c > 60T$) for $g=0$ and linewidth broadenings of 0, 1, and 5 K. Note the divergence of T_c with H. By the time H reaches 400 T, μ is below the bottom of the Landau parabola, and therefore the validity of the BCS approximation is highly questionable (at higher fields around 900 T, T_c begins to decrease again since the integration range goes to zero as μ approaches -20 K). Impurity broadening suppresses T_c somewhat. If impurities are treated as pair breakers, as in Ref. 1, the suppression effect is much more dramatic. Still, T_c substantially exceeds the zero field T_c even with broadening.

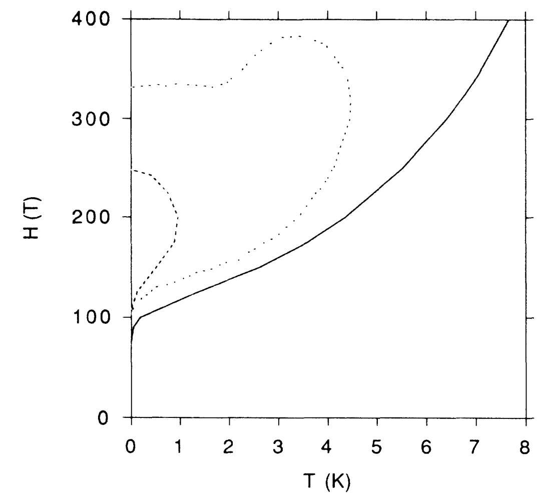

FIG. 2. H_{c2} (T) vs T (K) in the quantum limit for $g=0$ (solid curve), $g=0.05$ (dotted curve), and $g=0.1$ (dashed curve).

In Fig. 2, the effect of including a nonzero g factor is demonstrated. As one can see, this has a very large impact on the results. At lower fields, the transition is somewhat suppressed, whereas at higher fields the curve becomes Pauli limited. For $g=0.05$, we find the unusual result that $Q=0$ for the higher-temperature part of the curve (for $g=0.1$, the entire curve has nonzero Q).

Next, we investigate the oscillatory region where more than one Landau level is involved. In Fig. 3, results for $g=0$ with $\Gamma=0$ and 1 K are shown for fields between 20 and 100 T. Note that the largest peak (involving pairing in the $l=1$ Landau level) has a T_c somewhat lower than the zero field T_c. A broadening of 1 K leads to a 50% de-

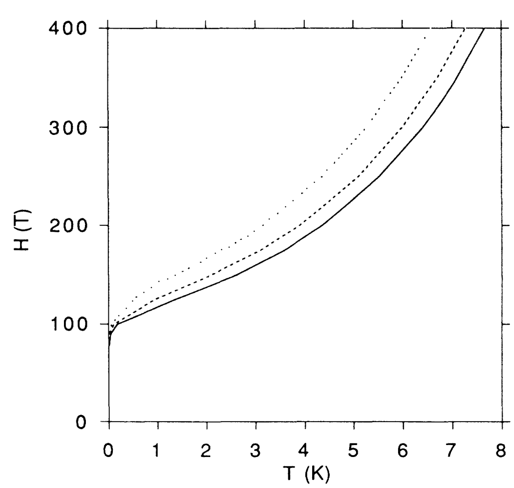

FIG. 1. H_{c2} (T) vs T (K) in the quantum limit for $g=0$ with broadenings of 0 K (solid curve), 1 K (dashed curve), and 5 K (dotted curve).

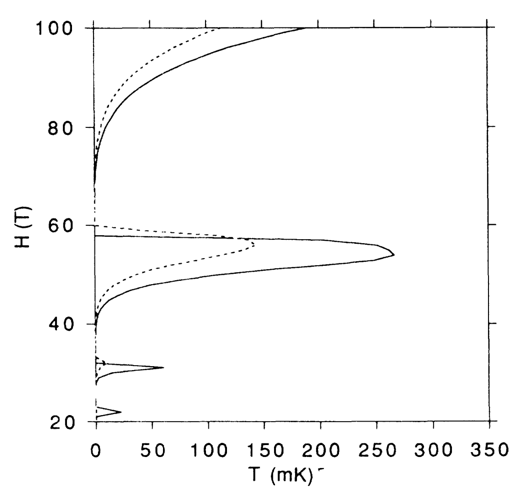

FIG. 3. H_{c2} (T) vs T (mK) for $g=0$ and fields between 20 and 100 T with broadenings of 0 K (solid curve) and 1 K (dashed curve).

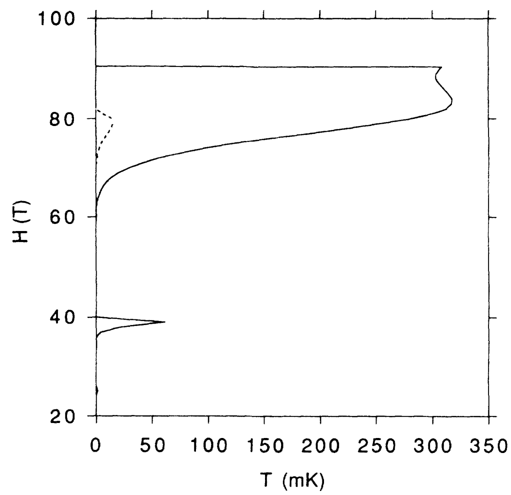

FIG. 4. H_{c2} (T) vs T (mK) for $g=2$ (solid curve) and $g=2.05$ (dashed curve).

crease in T_c. In Fig. 4, we show results for $g=2$ and $g=2.05$. The large peak corresponds to pairing between the $l=1$ up and $l=0$ down Landau levels. The unusual shape of the large peak does not appear to be an artifact (for $g=2.01$, the peak breaks into two peaks). One can also see from the figure that a g factor deviating from two has a large effect on the transition, with the transition temperature being suppressed by a factor of 100 when going from $g=2$ to 2.05.

IV. DISCUSSION

Given the above results, what are the chances of seeing this unusual reentrant behavior? A good example of a very-low-carrier-density superconductor is $SrTiO_3$,[7] where superconductivity occurs down to $10^{19}e^-/cm^3$ and where de Haas–van Alphen oscillations have been seen[8] with a g factor reported to be two.[9] Even for the lowest carrier concentration, the Fermi energy is in excess of 100 K,[10] so the largest peak would be in a field range of order 100 T (the other peaks would most likely be washed out). Moreover, since T_c for this concentration is only 100 mK, significant broadening and deviation of the g factor from exactly two would tend to drive T_c down to less than a millikelvin (even assuming that this field range could be reached by current technology, such as by explosively driven fields). At higher concentrations, T_c increases and a second band crosses the Fermi energy.[7,10] Since this band would have a lower effective Fermi energy, one could possibly be in an accessible field range for reentrant behavior. The problem is that the relative coupling constant of the second band is small since the ratio of its DOS to the total DOS is small. This implies a small T_c since the reentrant condition would not be satisfied for the first band.

In summary, reentrant superconductivity at high fields due to magnetic quantization has been discussed in relation to numerical parameters appropriate for an optimistic low-carrier-density superconductor. From this, it is concluded that there is a possibility of seeing such an effect, although the possibility is somewhat unlikely due to constraints imposed by being in an accessible field range (low enough Fermi energy), having a g factor close to an even integer, and not having too dirty a sample. This assumes that the BCS treatment is applicable for such high fields, which remains to be determined.

ACKNOWLEDGMENTS

The author would like to especially thank Allan MacDonald and Kurt Scharnberg for numerous discussions on this matter. This work was supported by the U.S. Department of Energy, Office of Basic Energy Sciences, under Contract No. W-31-109-ENG-38.

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