Metallic non-Fermi-liquid fixed point in two and higher dimensions

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(Received 14 May 1990)

Quasi-one-dimensional electron systems in two and higher dimensions are studied. It is shown that the interchain electron hopping is renormalized to zero in the infrared limit if the intrachain repulsive interaction is strong enough. In this case the higher-dimensional electron system flows to metallic non-Fermi-liquid fixed points—a Luttinger liquid. Some experimental consequences, in particular the transport properties, of this infrared fixed point are studied. It would be interesting to observe this state in quasi-one-dimensional electron systems.

Electron systems are known to have four infrared fixed points which respect the spin rotation and the translation symmetries. Those fixed points are insulator, superconductor, Fermi liquid (metal) and Luttinger Liquid (LL) (also a metal). Up to now, the LL fixed point is only known (theoretically) to exist in one dimension.[1] Recently Anderson[2] suggested that the high-T_c superconductors are described by a two-dimensional LL based on some experimental and theoretical considerations. In this paper we will show the existence of a new kind of metallic fixed points in two and higher dimensions. These fixed points resemble the one-dimensional LL and will be called the LL in higher dimensions. We will give a sufficient condition for an electron system to flow to a LL fixed point. Our strategy is to consider quasi-one-dimensional systems and to treat interchain hopping and interchain interactions as perturbations. We ask whether those interchain perturbations as relevant or not in the infrared limit. If all the interchain interactions and the interchain hopping are irrelevant operators, the LL fixed point on the chain will survive in the higher dimensions. We will also study some experimental consequences of the high-dimensional LL.

Let us first briefly summarize some results of one-dimensional LL. In one dimension, LL are generic fixed points for interacting electrons. Thus we may use a *weakly* interacting one-dimensional electron system

$$H=t\sum_{nm}c_m^\dagger c_n + \sum_{k_1+k_2=k_3+k_4} V(k_1,k_2,k_3,k_4)(c_{k_1}^\dagger c_{k_3})(c_{k_2}^\dagger c_{k_4}) \tag{1}$$

to understand LL fixed point. The Fermi momentum k_F is assumed not to commensurate with the lattice. For simplicity we will start with spinless electrons. The low energies excitations are near the two Fermi points. Following Ref. 3 we may write the electron operator as a sum of the right moving and the left moving electron operators

$$a^{-1/2}c_n \approx e^{ik_F an}\psi_L(an)+e^{-ik_F an}\psi_R(an) . \tag{2}$$

$\psi_{L,R}(x)$ are smooth functions of x and describe the electrons near the two Fermi points. In the continuum limit the free-electron Hamiltonian ($V=0$) has a form

$$H_{\text{free}}=\int dx\, v_F\left[\psi_L^\dagger i\frac{d}{dx}\psi_L-\psi_R^\dagger i\frac{d}{dx}\psi_R\right] . \tag{3}$$

The best way to study the low-energy properties of interacting one-dimensional electron systems is to use current algebra. It is known that one-dimensional spinless electrons are described by the U(1) Kac-Moody (KM) algebra:[1]

$$\begin{aligned}&[J_{L,R;k},J_{L,R;k'}]=\mp\frac{1}{2\pi}k\delta_{k+k'} ,\\ &[J_{L,k},J_{R,k'}]=0 ,\\ &[H_{\text{free}},J_{L,R;k}]=\mp v_F k J_{L,R;k} .\end{aligned} \tag{4}$$

$J_{L,R;k}$ are the electron densities for the right moving and the left moving excitations:

$$J_{L,R;k}=\int dx\frac{1}{\sqrt{L}}e^{-ikx}L\psi_{L,R}^\dagger(x)\psi_{L,R}(x) . \tag{5}$$

They are also proportional to the electrical currents of the left movers and the right movers. It can be shown that the Hamiltonian satisfying (4) has a form quadratic in currents:

$$H_{\text{free}}=\pi v_F\int dx(J_R J_R+J_L J_L) . \tag{6}$$

Equation 4 completely determines the low-energy dynamics of the system.

The electron operators satisfy the algebra

$$\begin{aligned}&[J_{L,R}(x),\psi_{L,R}(x')]=\delta(x-x')\psi_{L,R}(x) ,\\ &[J_{L,R}(x),\psi_{R,L}(x')]=0 ,\\ &[H_{\text{free}},\psi_{L,R}(k)]=\mp v_F k\psi_{L,R}(k) .\end{aligned} \tag{7}$$

The electron operators can be written in terms of the current operators

$$\psi_{L,R}=\eta:e^{\pm i\Phi_{L,R}}: , \tag{8}$$

where η is a cutoff-dependent constant and $\Phi_{L,R}$ is given

by $J_{L,R}=(1/2\pi)\partial_x\Phi_{L,R}$. One can check explicitly that $\psi_{L,R}$ in (8) satisfy the algebra (7). Using the correlation function of $\Phi_{L,R}$ [which is determined by (4)],

$$\langle \Phi_{L,R}(t,x)\Phi_{L,R}(0)\rangle = \ln(x\pm v_F t) , \tag{9}$$

we find the electron propagator to be

$$\langle \psi^\dagger_{L,R}(t,x)\psi_{L,R}(0)\rangle \propto (x\pm v_F t)^{-1} ,$$

or in the momentum space,

$$\langle \psi^\dagger_{L,R}(\omega,k)\psi_{L,R}(\omega,k)\rangle = \frac{1}{\omega\pm v_F k} . \tag{10}$$

Thus the electron operators are indeed given by (8).

The interaction term in (1) can also be written in terms of the current operator (in the continuum limit):

$$\delta H = \int dx[\pi\delta v(J_L J_L + J_R J_R)+\lambda_1 J_L J_R] , \tag{11}$$

where

$$\lambda_1 = 2V(0)-2\mathrm{Re}V(2k_F)$$

and $V(q)$ is the two-body potential. The first term in (11) just renormalize the velocity $v_F\to\tilde{v}=v_F+\delta v$. The second term is responsible for the LL behavior of the interacting electrons. The total Hamiltonian is

$$H = H_{\text{free}}+\delta H = \int dx[\pi\tilde{v}(J_L J_L + J_R J_R)+\lambda_1 J_L J_R] . \tag{12}$$

The Hamiltonian (12) can be diagonalized by introducing

$$j_{L,R} = \mathrm{ch}(\theta)J_{L,R}+\mathrm{sh}(\theta)J_{R,L} . \tag{13}$$

We find that

$$H = \int dx\ \pi v(j_L j_L + j_R j_R) \tag{14}$$

if we choose $\mathrm{th}(2\theta)=\lambda_1/2\pi\tilde{v}$. Here v is given by $v=\tilde{v}/\mathrm{ch}(2\theta)$. One can easily check that $j_{L,R}$ also form a U(1) KM algebra:

$$\begin{aligned}
&[j_{L,R;k}, j_{L,R;k'}] = \mp\frac{1}{2\pi}k\delta_{k+k'} ,\\
&[j_{L,k}, j_{R,k'}] = 0 ,\\
&[H, j_{L,R;k}] = \mp vk j_{L,R;k} .
\end{aligned} \tag{15}$$

Therefore the interacting electron system is exact soluble in the infrared limit. The low-lying excitations have only two velocities $\pm v$. In terms of new fields $j_{L,R}$ the electron operators have the form

$$\psi_{L,R} = \eta{:}\exp\pm i[\mathrm{ch}(\theta)\phi_{L,R}-\mathrm{sh}(\theta)\phi_{R,L}]{:} , \tag{16}$$

where $\phi_{L,R}$ are given by $j_{L,R}=(1/2\pi)\partial_x\phi_{L,R}$. Note due to the mixing (13), the electron operators contain both the right moving and the left moving excitations. We find that

$$\langle \psi^\dagger_{L,R}(t,x)\psi_{L,R}(0)\rangle \propto \frac{1}{(x\pm vt)(x^2-v^2t^2)^g} , \tag{17}$$

where $g=\mathrm{sh}^2(\theta)$. In the momentum space we have

$$\langle \psi^\dagger_{L,R}(\omega,k)\psi_{L,R}(\omega,k)\rangle = \frac{\omega_0^{-2g}(v^2k^2-\omega^2)^g}{\omega\pm vk} . \tag{18}$$

From (18) it is clear that the interacting electrons are not described by FL theories, since the propagators contain no single-particle poles.

Now let us consider a two-dimensional electron system

$$\begin{aligned}
H = &\sum_n (t_x c^\dagger_{n+\hat{x}}c_n - t_y c^\dagger_{n+\hat{y}}c_n)\\
&+\sum_{m,n} V(m-n)(c^\dagger_m c_m)(c^\dagger_n c_n) .
\end{aligned} \tag{19}$$

First let us assume the potential $V(m-n)$ to have zero range in the y direction:

$$V(m-n)=\delta(m_y-n_y)V_x(m_x-n_x) , \tag{20}$$

and concentrate on the effects of the y hopping term. When $t_y=0$ the Hamiltonian (19) becomes decoupled one-dimensional chains and is exact soluble in the infrared limit. The ground state is described by LL. In following we will consider the situation when $t_y \ll t_x$, V. In this case we may treat

$$\begin{aligned}
H_y &= \sum_n t_y c^\dagger_{n+\hat{y}}c_n\\
&= t_y\int dx\sum_{n_y}[\psi^\dagger_L(n_y+1)\psi_L(n_y)+\psi^\dagger_R(n_y+1)\psi_R(n_y)]
\end{aligned}$$

as a perturbation. We ask whether the LL at $t_y=0$ is stable or not against such a perturbation. This question can be answered in a renormalization-group (RG) approach. Because the unperturbed theory has no dispersion in the y direction, we will consider the following RG scaling:

$$(x,y,t)\to(\eta x,t,\eta t) . \tag{21}$$

From (17) we see that the operator $\psi_{L,R}$ scales like $\psi_{L,R}\to\eta^{-1/2-g}\psi_{L,R}$. This implies that $\psi_{L,R}$ has a dimension $\frac{1}{2}+g$ under the RG scaline (21). (x and t have a RG dimension -1 while y has a RG dimension 0.) An operator is relevant, marginal or irrelevant if its RG dimension is less, equal or greater than 2, respectively. Because H_y has a dimension $1+2g$, it is an irrelevant perturbation if (and only if) $g>\frac{1}{2}$. In this case t_y flows to zero in the infrared limit. The system behaves like decoupled chains in the low-energy limit and ground state remains to be a LL. When $g<\frac{1}{2}$, H_y is relevant and t_y getting larger and larger in the infrared limit. Eventually the renormalized t_y becomes comparable with the renormalized t_x and V [because the velocity is unchanged under the RG scaling (21), the renormalized t_x and V is of the order v/a^*, where a^* is the cutoff scale]. In this case the renormalized electron system has strong hopping in both the x and y directions. The perturbative analysis break down and we do not know what is the true ground state.

Now let us assume that $t_y=0$ and study the effects of weak interchain two-body interactions. In the continuum limit (in x direction) such an interaction has a form

$$\delta H=\int dx \sum_{n_y,m_y} \{V_1(n_y-m_y)[J_L(n_y)J_L(m_y)+J_R(n_y)J_R(m_y)]+V_2(n_y-m_y)[J_L(n_y)J_R(m_y)+J_R(n_y)J_L(m_y)]$$
$$+V_3(n_y-m_y)[\psi_L^\dagger(n_y)\psi_R(n_y)\psi_R^\dagger(m_y)\psi_L(m_y)+\text{H.c.}]\} . \tag{22}$$

Let us first concentrate on the operator $\psi_L^\dagger(n_y)\psi_R(n_y)\psi_R^\dagger(m_y)\psi_L(m_y)$. In terms of the bosonic field $\phi_{L,R}$, the operator $\psi_R^\dagger(m_y)\psi_L(m_y)$ has a form

$$\psi_R^\dagger\psi_L \propto \exp i\{[\text{ch}(\theta)-\text{sh}(\theta)]\phi_L+[\text{ch}(\theta)-\text{sh}(\theta)]\phi_R\} . \tag{23}$$

Therefore $\psi_R^\dagger(m_y)\psi_L(m_y)$ has a dimension

$$[\text{ch}(\theta)-\text{sh}(\theta)]^2=1-2\text{sh}(\theta)[\text{ch}(\theta)-\text{sh}(\theta)] .$$

This implies that the V_3 term in (22) is irrelevant if and only if the intrachain interaction is attractive [to be exact $\lambda_1=2V(0)-2\text{Re}V(2k_F)<0$]. In this case $\theta<0$ and the dimension of the V_3 term is greater than 2. We find that the LL ground state is stable against a small interchain interaction if the intrachain interaction is attractive. The V_1 and V_2 terms in (22) are marginal operators. It may modify θ a little bit. But when θ is finite and V_1, V_2 are small, the first two terms in (23) cannot change a irrelevant operator to an relevant one (or *vice versa).* The V_3 term remain to be irrelevant even in presence of the V_1 and V_2 terms.

After dropping the irrelevant V_3 term the total Hamiltonian becomes

$$H=\int dx\sum_{n_y}\{\pi\bar{v}[J_L(n_y)J_L(n_y)+J_R(n_y)J_R(n_y)]+\lambda_1 J_L(n_y)J_R(n_y)\}$$
$$+\int dx \sum_{n_y,m_y} \{V_1(n_y-m_y)[J_L(n_y)J_L(m_y)+J_R(n_y)J_R(m_y)]+V_2(n_y-m_y)[J_L(n_y)J_R(m_y)+J_R(n_y)J_L(m_y)]\} \tag{24}$$

The currents satisfy the following algebra:

$$[J_{L,R;k}(n_y),J_{L,R;k'}(m_y)]=\mp\frac{1}{2\pi}k\delta_{k+k'}\delta_{n_y,m_y} ,$$
$$[J_{L,k}(n_y),J_{R,k'}(m_y)]=0 . \tag{25}$$

The above Hamiltonian can be diagonalized by a generalized transformation of (13):

$$j_R(k_y)=\sum_{n_y}[U^{RR}_{k_y,n_y}J_R(n_y)+U^{RL}_{k_y,n_y}J_L(n_y)] ,$$
$$j_L(k_y)=\sum_{n_y}[U^{LR}_{k_y,n_y}J_R(n_y)+U^{LL}_{k_y,n_y}J_L(n_y)] . \tag{26}$$

The matrix U can be chosen to preserve the algebra (25) and at the same time diagonalize the Hamiltonian. Therefore (24) is exactly soluble. We find that ground state of (24) is a LL (i.e., a non-Fermi-liquid state with gapless charge fluctuations). We would like to remark that although the electron cannot hop from one chain to another (since $t_y=0$), the charge density disturbance in one chain can cause a charge disturbance in other chains due to the interchain interaction. Therefore the charge fluctuations can propagate in both x and y directions. But j_y always remain to be zero.

Summarizing the above results we conclude that the ground state of a two-dimensional spinless electron system is a LL if (a) the interchain hoping and the interchain interactions are weak, (b) the interchain interaction is strong enough such that $g>\frac{1}{2}$ and (c) the intrachain interaction is attractive ($\lambda_1<0$). In other situations (i.e., when a–c are not all satisfied), the ground-state properties are unclear because of the strong interactions. The above results obviously also apply the higher-dimensional systems.

The condition (a) is required because we are using perturbative RG theory. The condition (b) implies that all the interchain hopping are irrelevant. If the condition (c) is satisfied, there will be no relevant interchain interactions. Therefore all the perturbations will remain to be small as we go to the infrared limit, if a–c are satisfied. Most interchain interactions are irrelevant except the V_1 and the V_2 terms in (22) which are marginal operators. To describe the infrared fixed point we need to include those marginal interactions in the total Hamiltonian. It turns out that the total Hamiltonian is exactly soluble (after dropping the irrelevant interaction and hopping operators). We find the system flow to a new infrared fixed point—LL—which contains gapless charge fluctuations but is distinct from the Fermi liquid and the superfluid. The LL fixed points in high-dimensional systems studied here span a infinite dimensional manifold. The different LL fixed points are labeled by two functions $V_1(n)$ and $V_2(n)$ together with two real parameters v and g. We would like to emphasize that the LL liquid fixed points studied above are generic because all the perturbations about those fixed points are either irrelevant or marginal which just connect to another LL fixed point. There are no unstable directions around the LL fixed point.

In order to get a clearer picture about the two-dimensional LL, we will try to calculate the electron propagator. For simplicity we will ignore the interchain two-body interaction and only consider the effects of the y hopping term. Because $T(\psi^\dagger\psi)-:\psi^\dagger\psi:$ is not a c number, the Wick theorem does not apply. The electron propagator is not given by the Feynman diagram in Fig. 1. The vertex corrections like the one in Fig. 2 are not in-

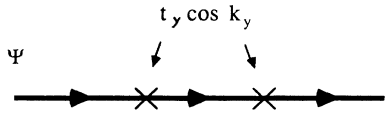

FIG. 1. The approximate electron propagator with the interchain hopping. The vertex corrections are not included.

cluded in Fig. 1. This makes it very difficult to calculate the exact electron propagator. The best we can do at the moment is to ignore the vertex corrections and to study the approximate electron propagator given by Fig. 1:

$$G^{(2D)}_{L,R}(\omega,k)\approx\langle\psi^{\dagger}_{L,R}(\omega,k)\psi_{L,R}(\omega,k)\rangle = \frac{\omega_0^{-2g}(v^2k_x^2-\omega^2)^g}{(\omega\pm vk_x)+t_y\cos(ak_y)\omega_0^{-2g}(v^2k_x^2-\omega^2)^g} . \tag{27}$$

The propagator (27) is at least correct to the first and second orders in t_y. Note that when $V=g=0$ (27) becomes the exact free-electron propagator (to all the orders in t_y), and when $t_y=0$ (27) becomes the propagator for decoupled chains. This gives us some confidence that (27) is probably a reasonable starting point.

Let us analyze the properties of a two-dimensional electron system described by the propagator (27). The Fermi surface is determined by the poles of $G^{(2D)}(\omega=0,k)$. The Fermi momenta are given by the equation

$$\pm vk_x\omega_0^{2g}(v^2k_x^2)^{-g}+t_y\cos(ak_y)=0 . \tag{28}$$

When $g>\frac{1}{2}$ we find that (28) has no solutions for small k_x and small t_y. In this case $G^{(2D)}$ has no poles for small ω and there is no Fermi surface. The ground state of (19) is not a FL. Comparing to the first term in the denominator of the propagator (27), the second term can be ignored in the low energy and the long-wavelength limit. This implies that H_y is irrelevant and agrees with the RG results obtained before. Equation (19) becomes decoupled one-dimensional chains at low energies if $g>\frac{1}{2}$. The above result is not surprising. When g is large the electron spectral weight becomes so low near the Fermi energy that the electrons can hardly have any coherent interchain hopping. Because the similarity between (18) and (27), we will call the ground state of (19) with $g>\frac{1}{2}$ Luttinger liquid. Some properties of (19) with $g>\frac{1}{2}$ are similar to the two-dimensional Luttinger-liquid behaviors proposed by Anderson.

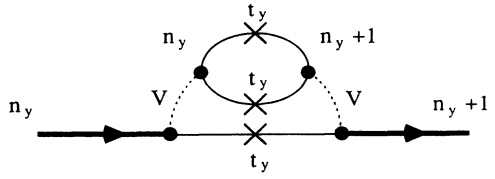

FIG. 2. The lowest-order vertex corrections appear at the order $(t_y)^3$.

When $g<\frac{1}{2}$, H_y is relevant. In this case (28) has solutions and $G^{(2d)}$ has quasiparticle poles at small ω. The residuals at the Fermi surface are given by $Z=\omega_0^{-2g}(v^2k_x^2)^g$. Z is nonzero except at $k_y=\pm\pi/2a$. The single-particle poles have nontrivial dispersion in both the x direction and the y direction. The electrons can hop coherently in two dimensions. The appearance of the quasiparticle poles at low energies and the existence of the Fermi surface imply that the ground state is a FL. However, this result is less reliable. When the vertex corrections at higher orders in t_y are included, the FL state might be unstable and becomes, e.g., a CDW state or a new LL state.

From (15) we see that (19) with zero t_y contains only two kinds of excitations at low energies, i.e., the right movers with velocity v and the left movers with velocity $-v$. The electron operators are composite operators of the right and the left movers. This is reflected in that the electron propagators have no poles. As we turn on the t_y hopping, a pole is developed in the electron propagators if $g<\frac{1}{2}$. The pole in some sense can be regarded as a bound state of the right movers and the left movers. The right movers and the left movers by themselves are confined in each individual chain in the x direction. Only some proper bound states of the right movers and the left movers correspond to the physical electrons and can coherently hop in the y directions. When $g<\frac{1}{2}$ an infintesimal y-hopping term is sufficient to make the electron-like bound states and induce a coherent hopping in the y direction. When $g>\frac{1}{2}$ a small y-hopping term cannot induce the electron-like bound states. In this case the LL is stable against small hopping terms in the y direction.

The above results about the spinless electrons can be easily generalized to spin-$\frac{1}{2}$ electrons. In one dimension the interacting spin-$\frac{1}{2}$ electrons are described by the U(1)⊗SU(2) KM algebra at low energies. The U(1) KM algebra describes the charge fluctuations and the SU(2) KM algebra describes the spin fluctuations. At low energies, the charge fluctuations and the spin fluctuations decouple.[1,3]

Here we will describe the spin-$\frac{1}{2}$ electron system by the currents of the spin-up electrons and the spin-down electrons,

$$J_{\sigma;R,L}=:\psi^{\dagger}_{\sigma;L,R}(x)\psi_{\sigma;L,R}(x): \quad \sigma=+,- . \tag{29}$$

The currents $J_{\sigma;R,L}$ satisfy the U(1)×U(1) KM algebra:

$$[J_{\sigma;L,R;k},J_{\sigma';L,R;k'}]=\mp\frac{1}{2\pi}k\delta_{k+k'}\delta_{\sigma,\sigma'},$$
$$[J_{\sigma;L,k},J_{\sigma';R,k'}]=0 . \tag{30}$$

In this formalism the spin-rotation symmetry is not manifest.

For free electron system with velocities of the spin and the charge fluctuations are the same. We have

$$[H_{\text{free}},J_{\sigma;L,R;k}]=\mp v_F k J_{\sigma;L,R;k} \ . \tag{31}$$

(30) and (31) give a complete description of the electron system in the low-energy limit. In terms of the total charge current $J_c=\sum_\sigma J_\sigma$ and the total $2S_z$ spin current $J_s=\sum_\sigma \sigma J_\sigma$, the algebra (30) can be written as

$$[J_{c;L,R;k},J_{c;L,R;k'}]=\mp\frac{1}{2\pi}k\delta_{k+k'} \ ,$$
$$[J_{s;L,R;k},J_{s;L,R;k'}]=\mp\frac{2}{2\pi}k\delta_{k+k'} \ , \tag{32}$$
$$\text{others}=0 \ .$$

The Hamiltonian has a form

$$H_{\text{free}}=\tfrac{1}{2}\pi v_F\int dx(J_{cR}J_{cR}+J_{cL}J_{cL}+J_{sR}J_{sR}+J_{sL}J_{sL}). \tag{33}$$

The electron operators can be written in terms of the current operators

$$\psi_{\sigma;L,R}=:e^{\pm i(\Phi_{c;L,R}+\sigma\Phi_{s,L,R})/2}: \ ,$$

where

$$J_{c;L,R}=(1/2\pi)\partial_x\Phi_{c;L,R}$$

and

$$J_{s;L,R}=\frac{1}{2\pi}\partial_x\Phi_{s,L,R} \ .$$

The operator $:e^{i\Phi c;L,R/2}:$ (the holon) carries an unit electric charge and $:e^{i\sigma\Phi_{s;L,R}/2}:$ (the spinon) carries spin $S_z=\sigma\frac{1}{2}$. The correlation function between two spinon operators $:e^{i\sigma\Phi_{s;L,R}/2}:$ has a form $[1/(x\pm v_F t)^{1/2}\delta_{\sigma,\sigma'}$. The correlation function between two holon operators $:e^{i\Phi_{c;L,R}/2}:$ also has the form $[1/(x\pm v_F)t]^{1/2}$. Thus we find the free-electron propagator to be

$$G_{\text{free};L,R(t,x)}=\left[\frac{1}{x\pm v_F t}\right]^{1/2}\left[\frac{1}{x\pm v_F t}\right]^{1/2} \tag{34}$$

as expected.

The interaction in the spin-$\frac{1}{2}$ electron system is much more complicated than the interactions in the spinless electron system. Here we will only consider a simple situation where the electron interactions only involves small momentum transfer. In this case the interaction Hamiltonian δH only contains the charge current

$$\delta H=\int dx[\pi\delta v(J_{cL}J_{cL}+J_{cR}J_{cR})+\lambda_1 J_{cL}J_{cR}] \ . \tag{35}$$

Here

$$\lambda_1=2V(0)-\text{Re}V(2k_F)\approx 2V(0) \ .$$

As we turn on the electron interactions the velocity of the charge fluctuations v_h and the velocity of the spin fluctuations v_s are no longer the same. The charged operator $:e^{i\Phi_{c,L,R}/2}:$ is replaced by

$$:\exp i[\text{ch}(\theta)\phi_{c;L,R}-\text{sh}(\theta)\phi_{c;R,L}]/2: \ ,$$

due to the mixing between the right moving current and the left moving current. The electron propagators now become

$$G_{L,R}(t,x)=G_{1;L,R}(t,x)G_{2;L,R}(t,x) \ ,$$
$$G_{1;L,R}(t,x)\propto\left[\left[\frac{1}{x\pm v_s t}\right]\left[\frac{1}{x\pm v_h t}\right]\right]^{1/2} \ , \tag{36}$$
$$G_{2;L,R}(t,x)\propto\left[\frac{1}{x^2\pm v_h^2 t^2)}\right]^g \ ,$$

where $g=sh^2(\theta)/2$. In the momentum space we have

$$G_{1;L,R}(\omega,k)\propto\frac{1}{[(v_s k\pm\omega)(v_h k\pm\omega)]^{1/2}} \ ,$$
$$G_{2;L,R}(\omega,k)\propto(v_h^2k^2-\omega^2)^{g-1} \ . \tag{37}$$

The total propagator $G_{L,R}(\omega,k)$ is quite complicated in the momentum space. It has a simple form in two cases: $g=0$ and $v_s=v_h$. When $g=0$, we have $G_{L,R}=G_{1;L,R}$. When $v_s=v_h=v$, $G_{L,R}$ is given by (18). For generic situations we would like to approximate $G_{L,R}$ by

$$G_{L,R}(\omega,k)\approx\frac{\omega_0^{-2g}(v_h^2k^2-\omega^2)^g}{[(v_s k\pm\omega)(v_h k\pm\omega)]^{1/2}} \ . \tag{38}$$

(38) has several correct features: (a) (38) is exact at $g=0$ and/or at $v_s=v_h$. (b) the spectral weight of (38) is nonzero only when (ω,k) can be regarded as the total energy and the momentum of several right (or left) moving spin excitations and the charge excitations. In this case (ω,k) has a form

$$(\omega,k)=(\pm v_s k_1+v_h k_2-v_h k_3,\sum_i k_i) \ ,$$

where $\pm k_1,k_2$ and $-k_3$ are greater than zero. The $\pm$ signs are for G_L and G_R, respectively. (c) (38) satisfies the scaling condition

$$G_{L,R}(\eta\omega,\eta k)=\eta^{2g-1}G_{L,R}(\omega,k) \ .$$

This scaling condition can be directly obtained from (36). Because of those properties, we expect (38) correctly represents the qualitative features of the exact electron Green function. From (c) we find that the exact electron Green function takes a simple form at $\omega=0$:

$$G_{L,R}(\omega=0,k)\propto k^{-1}|k|^{2g} \ . \tag{39}$$

Therefore (38) can be made exact at $\omega=0$ if we choose ω_0 properly.

To calculate $G_{L,R}$ exactly, let us consider the retarded Green functions. The spectral weight $A_i^{(R)}$ of the retarded propagators $G_i^{(R)}$, $i=1,2$ are given by

$$G_{i;L,R}^{(R)}(\omega,k)=\int d\varepsilon\frac{A_{i;L,R}^{(R)}(\varepsilon,k)}{\omega-\varepsilon+i\delta} \ .$$

The retarded spectral weight satisfies $A^{(R)}(\varepsilon)=0$ for $\varepsilon<0$. From (37) we find that

$$A^{(R)}_{1;L,R}(\omega,k)\propto \frac{1}{[(-v_s k\mp\omega)(v_h k\pm\omega)]^{1/2}} \times\Theta((-v_s k\mp\omega)(v_h k\pm\omega))\Theta(\omega)\ , \tag{40}$$

$$A^{(R)}_{2;L,R}(\omega,k)\propto(\omega^2-v_h^2k^2)^{g-1}|\sin(g\pi)|\Theta(\omega^2-v_h^2k^2)\Theta(\omega)\ .$$

Note (40) is valid only for small ω and k. From $A_l^{(R)}$ we are able to calculate the spectral weight $A^{(R)}$ of the retarded Green function $G^{(R)}_{L,R}$:

$$A^{(R)}_{L,R}=\int\frac{d\nu\,dq}{2\pi}A^{(R)}_{1;L,R}\left(\frac{\varepsilon}{2}+\nu,k-q\right)A^{(R)}_{2;L,R}\left(\frac{\varepsilon}{2}-\nu,q\right). \tag{41}$$

Unfortunately (41) is complicated and I cannot obtain an analytic result. However, (41) is probably useful for numerical calculations.

It would be interesting to calculate the spin-spin correlation function $\langle S_z(x,t)S_z(0)\rangle$ in our model and to compare with the results obtained in Refs. 4 and 2. The spin operator in the continuum limit takes the following form:

$$a^{-1}S_z=a^{-1}c^\dagger\sigma_z c \approx J_{sL}+J_{sR}+e^{2ik_Fx}\psi_R^\dagger\sigma_z\psi_L+e^{-2ik_Fx}\psi_L^\dagger\sigma_z\psi_R\ . \tag{42}$$

The first two terms contribute to the spin-spin correlation function a term proportional to $(x-v_st)^{-2}$. The contribution from the last two terms can be evaluated by using the KM algebra. In terms of the current operators we may rewrite $\psi_R^\dagger\sigma_z\psi_K$ as

$$\begin{aligned}\psi_R^\dagger\sigma_z\psi_L&=\sum_\sigma{:}e^{i\Phi_{cR}/2}e^{\sigma i\Phi_{sR}/2}{::}e^{i\Phi_{cL}/2}e^{\sigma i\Phi_{sL}/2}{:}\\&=\sum_\sigma{:}e^{\sigma i\Phi_{sR}/2}e^{\sigma i\Phi_{sL}/2}{::}e^{i[\mathrm{ch}(\theta)\phi_{cR}-\mathrm{sh}(\theta)\phi_{cL}]/2}{::}e^{i[\mathrm{ch}(\theta)\phi_{cL}-\mathrm{sh}(\theta)\phi_{cR}]/2}{:}\\&=\eta\sum_\sigma{:}e^{-\sigma i\Phi_{sR}/2}e^{\sigma i\Phi_{sL}/2}{::}e^{(i/2)[\mathrm{ch}(\theta)-\mathrm{sh}(\theta)]\phi_{cL}}{::}e^{(i/2)[\mathrm{ch}(\theta)-\mathrm{sh}(\theta)]\phi_{cR}}{:}\ ,\end{aligned} \tag{43}$$

where η is a cutoff-dependent constant. From (43) we find that the contribution to the spin-spin correlation function from the last two terms in (42) takes the following form:

$$\frac{1}{(x^2-v_s^2t^2)^{1/2}}\frac{1}{(x^2-v_h^2t^2)^\alpha}e^{i4k_Fx}+\text{H.c.}\ , \tag{44}$$

where

$$\begin{aligned}\alpha&=\tfrac12+\mathrm{sh}^2(\theta)-\mathrm{ch}(\theta)\mathrm{sh}(\theta)\\&=\tfrac12+2g-sgn(\theta)\sqrt{2g+4g^2}\ .\end{aligned} \tag{45}$$

If $\theta<0$ we see that the contributions from the first two terms in (42) dominate at long distances and $\langle S_z(r)S_z(0)\rangle\sim r^{-2}$. If $\theta>0$ the contributions from the last two terms in (42) dominate at long distances and $\langle S_z(r)S_z(0)\rangle\sim\cos(4k_Fr)r^{-1-2\alpha}$. We would like to remark that the exponent 2 in J_s correlation function and the exponent $\frac12$ in the first term in (44) are determined by the spin fluctuations and are exact even after we include the interaction between spin fluctuations. Those exponents are completely fixed by the SU(2) KM algebra, which is expected to be exact at low energies as long as the spin rotation symmetry remains unbroken.

It is shown that $2g=\frac18$ for the $U=\infty$ Hubbard model.[2,4] In this case $\alpha=1$ if $\theta<0$ and $\alpha=\frac14$ if $\theta>0$. For on-site repulsion interaction, λ_1, and hence θ, are greater than zero. We find

$$\langle S_z(r)S_z(0)\rangle\sim\cos(4k_Fr)r^{-3/2}\ .$$

Thus the result in Refs. 4 and 2 can be recovered from our calculation.

Now let us study the two-dimensional spin-$\frac12$ electron system described by (19). First we will use the RG approach to study effects of the y-hopping term. From (27) we see that ψ has a RG dimension $\frac12+g$ under the scaling (21). Therefore the y hopping term is irrelevant if and only if $g>\frac12$. Therefore when $g>\frac12$ the two-dimensional electron system remain to be a LL after turning on a small y-hopping term. The approximate electron propagator given in Fig. 1 is found to be

$$G^{(2D)}_{L,R}(\omega,k)\approx\frac{\omega_0^{-2g}(v_h^2k_x^2-\omega^2)^g}{\sqrt{(\omega\pm v_sk_x)(\omega\pm v_hk_x)}+t_y\cos(ak_y)\omega_0^{-2g}(v_h^2k_x^2-\omega^2)^g}\ . \tag{46}$$

Again when $g>\frac12$, $G^{(2D)}$ has no poles at small ω and the two-dimensional electron system behave as a LL. This agrees with the RG result.

Now let us consider the effects of the interchain interactions assuming $t_y=0$. Again we will only consider the long-range interactions. The interaction Hamiltonian in this case contain charge current only:

$$\delta H=\int dx \sum_{n_y,m_y} \{V_1(n_y-m_y)[J_{cL}(n_y)J_{cL}(m_y)+J_{cR}(n_y)J_{cR}(m_y)]+V_2(n_y-m_y)[J_{cL}(n_y)J_{cR}(m_y)+J_{cR}(n_y)J_{cL}(m_y)]\}. \tag{47}$$

(47) can be diagonalized as before and the interacting electron system is exactly soluble. We find that the ground state is a LL. Therefore the LL state is stable against long-range interchain interactions.

The short-range interchain interactions are very complicated. Their effects are unclear at the moment. We do not know for the spin-$\frac{1}{2}$ electron system whether the LL state of the decoupled chains is stable or not against short-range interchain interactions.

The LL studied in this paper appear in quasi-one-dimensional systems $t_y \ll t_x$. When $V=0$ the ground state is a FL and (19) behave like a metal in both the x and y directions. As we turn on an intrachain interaction V, the renormalized y-hopping amplitude t_y^* becomes smaller and smaller. When V exceeds a critical value, $t_y^*=0$ and the hopping in the y direction is completely turned off at low energies. In this case (19) behaves like an "insulator" in the y direction [i.e., the conductance in y direction $\sigma_{yy}(T)\rightarrow 0$ as $T\rightarrow 0$] and the ground state becomes a LL. We see that a LL may behave like a metal in some directions and an "insulator" in others. This resembles the opposite T dependence of the conductances σ_{ab} and σ_c in the high-T_c superconductors.[5] Such an opposite T dependence suggests that the electrons in the CuO plane probably do not form a Fermi liquid.[2,6]

Let us discuss the T dependence of σ_{yy} in more detail. In real samples the intrachain electron relaxation time τ is finite due to impurities and phonons. From (46) we see that the interchain electron hopping rate is of order

$$\tau_{\rm hop}^{-1}(\varepsilon)=t_y(\varepsilon/\omega_0)^{2g} ,$$

where ε is the energy of the electron. The renormalized bandwidth in the y direct is of order

$$\varepsilon_y=t_y(t_y/\omega_0)^{2g/(1-2g)}\Theta(1-2g) ,$$

which satisfies $\tau_{\rm hop}^{-1}(\varepsilon_y)=\varepsilon_y$. When

$$\tau \ll \tau_{\rm hop}(\varepsilon_0) , \tag{48}$$

where ε_0 is the largest one of τ^{-1}, T and ε_y, the electron transport in the y direction can be described by interchain tunneling. The interchain tunneling current is given by[7]

$$I=2e\sum_k t_y^2 \int \frac{d\omega}{2\pi} A(k,\omega)A(k,\omega+eV)[n_F(\omega)-n_F(\omega+eV)] , \tag{49}$$

where $A(k,\omega)$ is the electron spectral function in a chain, n_F is the Fermi distribution function and V is the voltage difference between neighboring chains. Because the spectral function satisfies a scaling condition

$$A(\eta k,\eta\omega)=\eta^{2g-1}A(k,\omega) ,$$

we have

$$\sum_k A^2(k,\omega)\propto\omega^{4g-1} .$$

From (49) we see that

$$\sigma_{yy}(T)=\left.\frac{dI}{dV}\right|_{V=0}\propto T^{4g} . \tag{50}$$

Therefore the exponent in the electron propagator (36) can be directly determined by measuring the temperature dependence of $\sigma_{yy}(T)$ provided that the condition (48) is satisfied.

We would like to stress that the above result is obtained by using the interchain tunneling picture which is valid only when the condition (48) is satisfied. If instead

$$\tau > \tau_{\rm hop}(\varepsilon_0) , \tag{51}$$

we can still estimate σ_{yy} by using the Drude model. In this case $\tau_{\rm hop}^{-1}(\varepsilon_0)$ can be regarded as the renormalized y hopping amplitude t_y^* which determines the effective mass in the y direction. The ratio between σ_{xx} and σ_{yy} is given by

$$\frac{\sigma_{yy}}{\sigma_{xx}}=\frac{m_{xx}^*}{m_{yy}^*}=\frac{a^2/t_x}{\tau_{\rm hop}(\varepsilon_0)d^2}\propto\varepsilon_0^{2g}$$

or

$$\sigma_{yy}\propto\frac{\tau}{m_{yy}^*}\propto\tau\varepsilon_0^{2g} , \tag{52}$$

where m^*'s are the renormalized electron effective masses, d is the interchain distance and a is the lattice constant of the chain. If $T>(\tau^{-1}(T),\varepsilon_y)$ we have $\sigma_{yy}/\sigma_{xx}\propto T^{2g}$. If $\tau^{-2}(T)>(T,\varepsilon_y)$ we have $\sigma_{yy}\propto\tau^{1-2g}$ and $\sigma_{xx}\propto\tau$. In this case σ_{xx} and σ_{yy} have opposite T dependence only when $g>\frac{1}{2}$. If $\varepsilon_y>(\tau^{-1}(T),T),\sigma_{yy}/\sigma_{xx}$ should be a constant. We see σ_{yy} may different T dependences under different conditions. The above analysis also apply to high-T_c superconductors. It is interesting to see which conditions [(48) or (51)] are satisfied by the high-T_c samples. We may also control the impurity concentration to make the samples satisfy (48) or (51).

In this paper we studied quasi-one-dimensional electron systems. The low-energy effects of various interchain perturbations were discussed. The interchain hopping terms were shown to be irrelevant if the interchain interaction is strong enough such that $g>\frac{1}{2}$. In this case the system flow to a metallic non-Fermi-liquid fixed points—the LL fixed points even in two and higher dimensions. The LL states studied in this paper behave like a metal in one direction and an insulator in other directions. This indicates that the motion of the electrons in LL is confined in one direction. In our model, the very existence of the LL is closely related to this dynamical confinement. In a more general LL state, the

motion of electrons may be confined in several discrete directions. If this happens, the LL may behave like metal in all the directions.

We studied the transport properties of the higher-dimensional LL states. We found that the conductance perpendicular to the chains, $\sigma_{yy}(T)$, has a power-law dependence on temperature. Measuring $\sigma_{yy}(T)$ can determine the exponent g in the electron propagator. Even when $g<\frac{1}{2}$, σ_{xx} and σ_{yy} still show some kind scaling properties if T or τ^{-1} is larger than the renormalized bandwidth in the y direction,

$$\varepsilon_y = t_y (t_y/\omega_0)^{2g/(1-2g)}\Theta(1-2g) .$$

We also studied low-energy effects of the interchain interactions in a spinless electron system. We found that all interchain interactions are inrelevant if the intrachain attractive interaction is attractive. In this case the LL state is robust against arbitrary interchain perturbations provided that $g>\frac{1}{2}$. However for a electron system with repulsive intrachain interaction, the interchain interactions in general may induce CDW or SDW states at zero temperature. But above the CDW or SDW transition temperature the system might resemble the LL state at finite temperatures.

The LL, as a generic infrared fixed point, should appear in nature. Based on the discussions in this paper, the LL are likely to appear in quasi-one-dimensional electron systems. Recently the edge state of the two-dimensional fractional quantum Hall (FQH) states are shown to be (chiral) LL with the anomalous dimension $g=(l-1)/2$. Here $1/l$ is the filling fraction. Thus two-dimensional LL may also appear on the surface of the recently discovered three-dimensional FQH states.[8]

ACKNOWLEDGMENTS

I would like to thank P. W. Anderson, Y. Ren, Z. Z. Wang, and R. C. Yu for many helpful discussions. This work is supported in part by Grant No. DE-FG02-90-ER40542.

[1] J. M. Luttinger, J. Math. Phys. **15**, 609 (1963); D. C. Mattis and E. H. Lieb, J. Math. Phys. **6**, 304 (1965); E. H. Lied and D. C. Mattis, *Mathematical Physics in One Dimension* (Academic, New York, 1966), Chap. 4; A. Luther and I. Peschel, Phys. Rev. B **9**, 2911 (1974); Phys. Rev. Lett. **32**, 992 (1974); A. Luther and V. J. Emery, *ibid.* **33**, 589 (1974); A. Luther, Phys. Rev. B **19**, 320 (1979); F. D. M. Haldane, J. Phys. C **14**, 2585 (1981); G. D. Mahan, *Many-Particle Physics* (Plenum, New York, 1981), p. 311.

[2] P. W. Anderson, Phys. Rev. Lett. **64**, 1839 (1990); (unpublished); P. W. Anderson and Y. Ren (unpublished).

[3] I. Affleck, Nucl. Phys. **B265**, 409 (1986); I. Affleck and F. D. M. Haldane, Phys. Rev. **36**, 5291 (1987).

[4] A. Parola and S. Sorella, Phys. Rev. Lett. **64**, 1831 (1990).

[5] S. W. Tozer *et al.*, Phys. Rev. Lett. **59**, 1768 (1987); S. J. Hagen *et al.*, Phys. Rev. B **37**, 7928 (1988); S. Martin *et al.*, Phys. Rev. Lett. **60**, 2194 (1988); D. A. Brawner *et al.*, Phys. Rev. B **40**, 9329 (1989).

[6] P. W. Anderson and Z. Zhou, Phys. Rev. Lett. **60**, 132 (1988).

[7] J. R. Schreiffer, D. J. Scalapino, and J. W. Wilkins, Phys. Rev. Lett. **10**, 336 (1963).

[8] R. V. Chamberlin *et al.*, Phys. Rev. Lett. **60**, 1189 (1988); J. R. Cooper *et al.*, *ibid.* **63**, 1984 (1989); S. T. Hannahs *et al.*, *ibid.* **63**, 1988 (1989).