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Local chiral symmetry and charge-density waves in one-dimensional conductors

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Symmetry-related features of charge-density-wave transport phenomena are studied using a non-mean-field effective Lagrangian approach. It is pointed out that a local chiral symmetry (based on the Kač-Moody algebra) emerges in the low-energy structure of one-dimensional electron-phonon systems. From this symmetry follow directly power-law correlations of both electrons and phonons. The Peierls instability is suppressed owing to one-dimensional fluctuations. Still the charge-density wave arises and the chiral anomaly can account for acceleration of a sliding charge-density wave along with a phonon-drag effect. The problem of pinning of charge-density waves is discussed in relation to explicit breakings of the chiral symmetry.

I. INTRODUCTION

Symmetries are an important concept in condensed-matter physics, as well as particle physics, especially in describing collective phenomena such as superconductivity and superfluidity. Charge-density waves[1] (CDW) in some low-dimensional organic alloys, being electron-phonon collective modes, are also deeply tied to the symmetries of microscopic electron-phonon systems. It is only recently, however, that such a connection has been clearly recognized.[2-4] Notable among various theoretical attempts[1] to explain the CDW is an effective-Lagrangian approach[3-5] which interprets the formation and acceleration of the incommensurate charge-density wave (ICDW) in terms of a chiral symmetry and the chiral anomaly of the one-dimensional electron-phonon system.

Early investigations of CDW systems rely mostly on mean-field treatments, which presuppose long-range order via a vacuum expectation value of the phonon field $\langle \phi(x) \rangle \neq 0$. Mean-field treatments, however, are known to be inappropriate for one-dimensional systems for which fluctuations are generally important.[6] Further, the nonzero expectation value $\langle \phi(x) \rangle \neq 0$, which drives spontaneous breakdown of chiral symmetry, appears to be in conflict with Coleman's theorem[7] known for (relativistic) quantum field theory in $(1+1)$ dimensions.

The purpose of this paper is to present a new non-mean-field effective Lagrangian approach to one-dimensional ICDW systems. Special emphasis is placed on clarifying the symmetry-related features of the CDW transport phenomenon. The present paper is a (self-contained) continuation of our previous paper.[8] It is shown by combined use of bosonization and an operator-product expansion that the low-energy structure of the one-dimensional electron-phonon system is effectively described by a modified version of the Thirring model, which is exactly solvable. Characteristic to this low-energy theory are a local chiral symmetry (Kač-Moody symmetry) and conformal symmetry, which imply the existence of a massless mode, the phason. Correspondingly, in sharp contrast to the mean-field result, both electrons and phonons show long-range power-law correlations and the electron spectrum has no gap at the Fermi energy; the chiral symmetry turns out to be unbroken for zero and finite temperatures. Still the CDW arises as an electron-phonon collective mode, and the chiral anomaly can account for the sliding of the CDW along with a phonon drag effect. We discuss the problem of pinning of the CDW in connection with explicit breakings of chiral symmetry.

In Sec. II we relate the ICDW system to a solvable model and point out the emergence of a local chiral symmetry in the long-wavelength structure of the electron-phonon system. In Sec. III we derive power-law correlations of electrons and phonons, and study the symmetry features underlying them. In Sec. IV we show that both sliding of the ICDW and a plasma mode in the ICDW system are naturally understood in terms of the chiral anomaly. Temperature and pinning effects are discussed in Secs. V and VI, respectively. Section VII is devoted to concluding remarks.

II. ONE-DIMENSIONAL ELECTRON-PHONON SYSTEM AND A LOW-ENERGY EFFECTIVE LAGRANGIAN

A one-dimensional electron-phonon system bears a formal resemblance to a $(1+1)$-dimensional relativistic quantum field theory.[2-4] The left- and right-moving modes ψ_L and ψ_R of the nonrelativistic electron field near the Fermi surface may be regarded as the chiral components of the Dirac fermion $\psi=(\psi_L,\psi_R)^t$, with the Lagrangian

$$\psi_L^\dagger i(\partial_t - v_F\partial_x)\psi_L + \psi_R^\dagger i(\partial_t + v_F\partial_x)\psi_R \ , \qquad (2.1)$$

where $v_F = p_F/m$ is the Fermi velocity; we set $c=\hbar=1$. The Fermi momentum p_F is assumed to be incommensurate with the lattice. The ψ_L and ψ_R, whose energies and momenta are measured relative to $p_F^2/(2m)$ and $\pm p_F$, respectively, are slowly varying. They are scattered by the lattice phonon modes ϕ and $\phi^\dagger$ of momenta $\pm 2p_F$, via the interaction

$$G_0(\psi_L^\dagger \psi_R \phi^\dagger + \psi_R^\dagger \psi_L \phi) , \tag{2.2}$$

where G_0 is the electron-phonon coupling constant. The phonon Lagrangian is

$$L_{\rm ph} = \phi^\dagger(-\partial_t^2 + v_Q^2\partial_x^2 - \omega_0^2)\phi , \tag{2.3}$$

where ω_0 denotes the optical-phonon frequency $\omega(Q)$ at $Q=2p_F$ and $v_Q^2 = \frac{1}{2}[\omega^2(Q=2p_F)]''$.

The sum of Eqs. (2.1)–(2.3) can be cast into the "relativistic" form

$$L_{\rm eff}^{(0)} = \bar\psi i\gamma^\mu(\partial_\mu + ieA_\mu)\psi + (G/\sqrt{2})(\phi_1\bar\psi\psi + i\phi_2\bar\psi\gamma_5\psi) + L_{\rm ph} , \tag{2.4}$$

where $\psi=(\psi_L,\psi_R)^t$ and $\bar\psi \equiv \psi^\dagger\gamma^0 = (\psi_R^\dagger, \psi_L^\dagger)$; and $\phi=(1/\sqrt{2})(\phi_1+i\phi_2)$ in terms of real fields ϕ_1 and ϕ_2. Our "relativistic" (two-vector) notation is $x^\mu = (x^0,x^1) \equiv (t, x/v_F)$, $\partial_\mu = (\partial_0,\partial_1) \equiv \partial/\partial x^\mu$, $\gamma^\mu = (\gamma^0,\gamma^1) \equiv (\tau_1, i\tau_2)$, and $\gamma_5 \equiv -\gamma^0\gamma^1 = \tau_3$ in terms of the Pauli matrices τ_i. The Dirac matrices obey $\{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu}$ and $\gamma^\mu\gamma^5 = \epsilon^{\mu\nu}\gamma_\nu$, with the "metric" $g^{\mu\nu} = \text{diag}(1,-1)$ and the antisymmetric tensor $\epsilon^{\mu\nu}$, $\epsilon^{01} = \epsilon_{10} = 1$. To take full advantage of relativistic notation, we write the action as $\int d^2x\, L_{\rm eff}^{(0)}$ with $d^2x \equiv dx^0dx^1 = (1/v_F)dt\,dx$; accordingly, the rescaling $\sqrt{v_F}\psi \to \psi$, $\sqrt{v_F}\phi \to \phi$, $G_0/\sqrt{v_F} = G$ is understood in passing from Eqs. (2.1)–(2.3) to Eq. (2.4).

The charge ρ and current J of the electron form the "relativistic" (rescaled) current $j^\mu = \bar\psi\gamma^\mu\psi = (j^0,j^1) = (v_F\rho, J)$. Its coupling to an external electric potential (Φ, A) has been introduced in Eq. (2.4), with $A^\mu = (A^0, A^1) \equiv (\Phi, v_FA)$. As noted earlier,[4] $L_{\rm eff}^{(0)}$ is invariant under the chiral transformations $\psi(x) \to e^{i(\alpha+\beta\gamma_5)}\psi(x)$ and $\phi(x) \to e^{-2i\beta}\phi(x)$. Our main task is to study how this chiral symmetry is realized in this model, especially for low-lying excitations.

Using the well-known bosonization rules[9–12] for a massless free fermion,

$$\bar\psi\gamma^\mu\psi \to -\frac{1}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_\nu\eta,\quad \bar\psi\gamma^\mu\gamma_5\psi \to -\frac{1}{\sqrt{\pi}}\partial^\mu\eta ,$$
$$\bar\psi\psi \to \frac{\mu}{\pi}:\!\cos(\sqrt{4\pi}\eta)\!:,\quad \bar\psi\gamma_5\psi \to i\frac{\mu}{\pi}:\!\sin(\sqrt{4\pi}\eta)\!: , \tag{2.5}$$

we write Eq. (2.4) in the equivalent bosonic form

$$L^{\rm boson} = \tfrac{1}{2}(\partial_\nu\eta)^2 + \frac{G\mu}{2\pi}(\phi :\!e^{i\sqrt{4\pi}\eta}\!: + \phi^\dagger :\!e^{-i\sqrt{4\pi}\eta}\!:) + (e/\sqrt{\pi})A^\mu\epsilon_{\mu\nu}\partial^\nu\eta + L_{\rm ph} , \tag{2.6}$$

where $\eta(x)$ is a massless (pseudoscalar) boson field with the chiral transformation law $\eta(x) \to \eta(x) + \beta/\sqrt{\pi}$. Here, μ is an infrared cutoff defining the massless propagator $\langle \mathcal{T}[\eta(x)\eta(0)]\rangle = -(4\pi)^{-1}\ln[\mu^2(-x^2+i0)]$ with $x^2 = x^\nu x_\nu = x_0^2 - x_1^2$. The normal ordering $::$ is defined with respect to the positive- and negative-frequency parts $\eta^{(\pm)}$. Since we use the free-field bosonization rules, we are working in the interaction picture and treat the phonon-η coupling as a perturbation. Appendix A summarizes some basic commutation relations of free fields in two dimensions.

Let us now eliminate the phonon field and write the phonon-exchange interaction in the form

$$L^{\phi\eta} = \left[\frac{G\mu}{2\pi}\right]^2 \times \int d^2\epsilon\, D(\epsilon) :\!e^{i\sqrt{4\pi}\eta(x+\epsilon/2)}\!::\!e^{-i\sqrt{4\pi}\eta(x-\epsilon/2)}\!: , \tag{2.7}$$

where the phonon propagator

$$D(\epsilon) \equiv \langle \epsilon|(\omega_0^2 - \partial_0^2 + a^2\partial_1^2 - i0)^{-1}|0\rangle$$

is given by

$$D(\epsilon) = i(2\pi a)^{-1}K_0(\omega_0(-\bar\epsilon^2 + i0)^{1/2}) , \tag{2.8}$$

where $a \equiv v_Q/v_F$ and $\bar\epsilon^\mu \equiv (\epsilon^0, \epsilon^1/a)$. The modified Bessel function $K_0(z) = -\gamma - \ln(z/2) + O(z^2)$, where γ is Euler's constant. The operator product in Eq. (2.7) is rewritten as

$$L^{\phi\eta} = \left[\frac{G}{2\pi}\right]^2 \int d^2\epsilon[D(\epsilon)/\epsilon^2] \times :\!\exp\{i\sqrt{4\pi}[\eta(x+\tfrac{1}{2}\epsilon) - \eta(x-\tfrac{1}{2}\epsilon)]\}\!:$$
$$= \text{const} + \frac{G^2}{2\pi}\int d^2\epsilon\, D(\epsilon)\frac{\epsilon^\mu\epsilon^\nu}{\epsilon^2}(\partial_\mu\eta)(\partial_\nu\eta) + \cdots . \tag{2.9}$$

The constant term is logarithmically (ultraviolet) divergent. The omitted terms contain four derivatives or more, and are not important for long wavelengths. In the "Lorentz invariant" case,[13] $a=1$, $\int d^2\epsilon(\epsilon^\mu\epsilon^\nu/\epsilon^2)D(\epsilon) = -i\frac{1}{2}g^{\mu\nu}/\omega_0^2$. For $a \neq 1$ this integral measures the difference in the temporal and spatial variations of $D(\epsilon)$. Carrying out the integral gives[14]

$$L^{\phi\eta} = \frac{1}{2\pi}[g_1(\partial_0\eta)^2 - g_0(\partial_1\eta)^2] + \cdots , \tag{2.10}$$

where

$$g_0 \equiv 2a(1+a)^{-1}g = 2[v_Q/(v_Q+v_F)]g ,$$
$$g_1 \equiv 2(1+a)^{-1}g = 2[v_F/(v_Q+v_F)]g , \tag{2.11}$$
$$g \equiv G^2/(2\omega_0^2) = G_0^2/(2\omega_0^2 v_F) > 0 .$$

The phonon exchange at short distances has yielded the long-wavelength effective interaction (2.10), which, in terms of ψ, is rewritten as a four-fermion interaction. Thus the long-wavelength feature of the original Lagrangian $L_{\rm eff}^{(0)}$ is effectively described by a modified Thirring model, with the Lagrangian

$$L_{\rm eff} = \bar\psi i\gamma^\mu(\partial_\mu + ieA_\mu)\psi - \tfrac{1}{2}(g_0 j_0^2 - g_1 j_1^2) . \tag{2.12}$$

(The Thirring model corresponds to the $a=1$ case, where $g_0 = g_1$.) The bosonic counterpart of this is given by

$$L_{\rm eff}^{\rm boson}=\tfrac{1}{2}Z(\partial_0\eta)^2-\tfrac{1}{2}X(\partial_1\eta)^2+(e/\sqrt{\pi})A^\mu\epsilon_{\mu\nu}\partial^\nu\eta\ , \tag{2.13}$$

with $Z=1+g_1/\pi$ and $X=1+g_0/\pi$.

Both Lagrangians are chiral invariant. Remarkably, larger symmetries arise for $e=0$. To see this, let us introduce some notation. Along with the rescaled coordinates $\hat{x}^\mu\equiv(x^0,x^1\sqrt{Z/X})$ and $\hat{\partial}_\mu=\partial/\partial\hat{x}^\mu=(\partial_0,\sqrt{X/Z}\,\partial_1)$, the rescaled field $\xi(x)=(ZX)^{1/4}\eta(x)$ recovers the standard normalization, with the action

$$\int d^2x\, L_{\rm eff}^{\rm boson}(e=0)=\int d^2\hat{x}\,\tfrac{1}{2}(\hat{\partial}_\mu\xi)(\hat{\partial}^\mu\xi)\ , \tag{2.14}$$

where $d^2\hat{x}=\sqrt{Z/X}\,d^2x$. Now the conformal symmetry of the action is manifest. Still a larger symmetry exists: The $\xi(x)$, obeying the two-dimensional massless field equation $\hat{\partial}_\mu\hat{\partial}^\mu\xi(x)=0$, can be written as a superposition of left- and right-moving modes. As usual, one can introduce the dual field $\tilde{\xi}(x)$ by $\hat{\partial}_\mu\tilde{\xi}=\epsilon_{\mu\nu}\hat{\partial}^\nu\xi$ or $\tilde{\xi}(x)=\sqrt{Z/X}\int_{-\infty}^{x}dz^1\,\xi(x^0,z^1)$. Then $\xi(x)\pm\tilde{\xi}(x)$ represent the left- and right-moving modes of $\xi(x)$ (depending only on $\hat{x}^\pm=x^0\pm\sqrt{Z/X}\,x^1$, respectively). This clear separation of left- and right-moving modes is understood as a local chiral symmetry peculiar to two dimensions, known as the Kač-Moody (KM) symmetry.[15] The action (2.14) is invariant under the transformations

$$\xi(x)\to\xi(x)+f_+(\hat{x}^+)-f_-(\hat{x}^-)\ , \tag{2.15}$$

where $f_\pm$ are arbitrary functions of $\hat{x}^\pm$. Note that the (global) chiral and conformal transformations are only part of these (infinite-dimensional) Kač-Moody transformations.

The fermion Lagrangian (2.12) (with $e=0$) of course shares the same KM symmetry, though not very apparent. One can verify that $L_{\rm eff}$ is invariant under the following transformations of the fields and current:

$$\psi_L\to e^{if_L}\psi_L,\ \psi_R\to e^{if_R}\psi_R\ ,\qquad j_\mu\to j_\mu-(2\pi)^{-1}\epsilon_{\mu\nu}\partial^\nu(f_L-f_R)\ , \tag{2.16}$$

with the local phases $f_{L,R}$ of the form

$$f_L=\sqrt{\pi}[(Y+Y^{-1})f_+(\hat{x}^+)+(Y-Y^{-1})f_-(\hat{x}^-)]\ ,\qquad f_R=\sqrt{\pi}[(Y-Y^{-1})f_+(\hat{x}^+)+(Y+Y^{-1})f_-(\hat{x}^-)]\ , \tag{2.17}$$

where

$$Y\equiv(ZX)^{1/4}=[(1+g_1/\pi)(1+g_0/\pi)]^{1/4}\ . \tag{2.18}$$

This KM symmetry is a new symmetry of the low-energy effective theory; it is not present in the original Lagrangian (2.4).

III. LONG-WAVELENGTH FEATURES OF THE ELECTRON-PHONON SYSTEM

In this section we set $A^\mu=0$ in Eq. (2.12), and study the long-wavelength features of the (pure) electron-phonon system by solving this modified Thirring model. The $a\neq1$ case turns out solvable in much the same (though more complicated) way as the $a=1$ case.

The current algebra plays a central role in solving the Thirring model.[9,16–18] Let us therefore begin with the definition of the current $j_\mu=\bar{\psi}\gamma_\mu\psi$. Charge conservation implies $\partial_\mu j^\mu=0$; hence one can write j^μ as a curl. Note that the KM transformation laws (2.15) and (2.16) are compatible if the vector current $j_\mu=\bar{\psi}\gamma_\mu\psi$ of the full theory takes the same form as the free-field current; we thus take[19]

$$j_\mu(x)=-(1/\sqrt{\pi})\epsilon_{\mu\nu}\partial^\nu\eta(x)=-(1/Y)(1/\sqrt{\pi})\epsilon_{\mu\nu}\partial^\nu\xi(x)\ , \tag{3.1}$$

with $\xi(x)=Y\eta(x)$. We remark that, although the current j^μ remains unmodified, the current algebra gets rescaled by factor $1/Z$ in the full theory, i.e., $[j_0(x),j_1(0)]=i(1/Z)(1/\pi)\delta'(x^1)$ for $x^0=0$. The chiral symmetry of the modified Thirring model implies another conserved current, the axial current $j_5^\mu=\bar{\psi}\gamma^\mu\gamma_5\psi$, which we shall study later.

One can reconstruct the fermion field ψ out of the free-boson field ξ. We quote the result first:

$$\psi(x)=\ :\exp\{i\sqrt{\pi}[Y\tilde{\xi}(x)+(1/Y)\xi(x)\gamma_5]\}:u\ , \tag{3.2}$$

where $u=(u_L,u_R)^t$ is a two-component constant. The normal ordering :: is defined with respect to the positive- and negative-frequency parts $\xi^{(\pm)}$ and $\tilde{\xi}^{(\pm)}$. It is immediately verifiable that, with the current identification in Eq. (3.1), this $\psi(x)$ obeys the (properly normal-ordered) equation of motion following from the four-fermion Lagrangian (2.12).

The operator solution $\psi(x)$ in Eq. (3.2) satisfies the following operator product expansion:

$$\psi_L(x)\psi_L^\dagger(y)=\rho_L(x-y)\,:\exp(\,i\sqrt{\pi}\{Y[\tilde{\xi}(x)-\tilde{\xi}(y)]+(1/Y)[\xi(x)-\xi(y)]\}):\ , \tag{3.3}$$

$$\rho_L(z)=(|u_L|^2/i\mu)(\hat{z}^+-i0)^{-1}h(z)\ , \tag{3.4}$$

$$h(z)=1/[-\mu^2(\hat{z}^+-i0)(\hat{z}^--i0)]^{\lambda^2}\ , \tag{3.5}$$

with $\lambda\equiv\frac{1}{2}(Y-Y^{-1})$; $\hat{z}^\pm\equiv\hat{x}^\pm-\hat{y}^\pm=x^0-y^0\pm\sqrt{Z/X}\,(x^1-y^1)$, etc. Note that $\lambda>0$ and $Y>1$ for $g>0$. For $\psi_R(x)\psi_R^\dagger(y)$, replace $1/Y\to-1/Y$ in Eq. (3.3) and $\rho_L(z)\to\rho_R(z)=(|u_R|^2/i\mu)(\hat{z}^--i0)^{-1}h(z)$.

The overall normalization of ψ is fixed so as to reproduce Eq. (3.1). We employ the split-point regularization of the vector current, $j_\mu(x;\epsilon)\equiv\bar{\psi}(x+\frac{1}{2}\epsilon)\gamma_\mu\psi(x-\frac{1}{2}\epsilon)$. Some care is needed in taking the limit $\epsilon^\mu\to0$: To obtain the "Lorentz" covariant bosonic expression (3.1), we have to regularize $j_\mu(x;\epsilon)$ in a noncovariant way. We first average over the directions of ϵ^0 and ϵ^1 with their dispersions kept so that

$$(\epsilon_0)^2=-(b^2/a)(\epsilon_1)^2\ , \tag{3.6}$$

where $b^2\equiv\sqrt{Z/X}$, and then let $(\epsilon_0)^2=(1+a/b^2)^{-1}\epsilon^\mu\epsilon_\mu\to-0$. This reproduces Eq. (3.1) and fixes $u_{L,R}$,

$$|u_L|^2=|u_R|^2=[\mu/2\pi h(\epsilon)](1+ab^2)/(a+X)\ . \tag{3.7}$$

With this choice, $\rho_{L,R}(z)$ are independent of the infrared cutoff μ.

The operator ψ is transformed correctly under chiral transformations. The j^0 generates the vector-U(1) transformations

$$[j_0(x), \psi(y)] = -\psi(y)\delta(x^1-y^1), \text{ for } x^0=y^0 , \quad (3.8)$$

while j_1 generates the axial transformations

$$[j_1(x), \psi(y)] = -(1/Z)\gamma^5\psi(y)\delta(x^1-y^1) , \quad (3.9)$$

$$[j_1(x), \xi(y)] = -(i/\sqrt{\pi})(Y/Z)\delta(x^1-y^1) , \quad \text{for } x^0=y^0 . \quad (3.10)$$

This suggests the identification $j_5^0 = Zj_1$ of the axial current j_5^μ. The conservation law $\partial_\mu j_5^\mu = 0$ is consistent with Eq. (3.1) and the field equation $\partial_\mu \partial^\mu \xi = 0$ only when $j_5^\mu = (j_5^0, j_5^1) \propto (Zj_1, -Xj_0)$. Therefore the duality relation $j_5^\mu = \epsilon^{\mu\nu} j_\nu$ now gets modified for $a \neq 1$; and the axial current has to be regularized in a way different from the vector current. We define the axial current as $j_5^\mu(x;\epsilon) = \bar\psi(x + \frac{1}{2}\rho\epsilon)\gamma^\mu\gamma_5\psi(x - \frac{1}{2}\rho\epsilon)$, and take this time the dispersions of ϵ^μ so that

$$(\epsilon_0)^2 = -ab^6(\epsilon_1)^2 . \quad (3.11)$$

The regularized current then takes the form

$$(j_5^0(x), j_5^1(x)) = (\rho^{-(\lambda^2+1)}/Y^2)(Zj_1(x), -Xj_0(x)) = (Zj_1(x), -Xj_0(x)) , \quad (3.12)$$

upon adjusting the scale factor $\rho^{-(\lambda^2+1)} = Y^2$.

Equation (3.3) reveals the power-law behavior of the (long-wavelength) electron correlation functions

$$\langle \psi_L(x)\psi_L^\dagger(0) \rangle \propto (\hat{x}^+ - i0)^{-1}(\hat{x}^2)^{-\lambda^2} ; \quad (3.13)$$

for $\langle \psi_R(x)\psi_R^\dagger(0)\rangle$ set $x^+ \to x^-$ in the above. Thus the scaling dimension of the electron field ψ is given by

$$d[\psi] = \tfrac{1}{2} + \tfrac{1}{4}(Y - Y^{-1})^2 . \quad (3.14)$$

The $\psi(x)$ anticommutes, $\{\psi(x), \psi(0)\} = 0$ for $x^0 = 0$, while the anomalous dimension of ψ makes $\{\psi, \psi^\dagger\}$ noncanonical, e.g., $\{\psi_L(x), \psi_L^\dagger(0)\} \neq \delta(x^1)$. This noncanonical structure poses no problem because our operator solution ψ represents only the long-wavelength features of the electron exactly. Since the original electron-phonon interaction is super-renormalizable, the electron and the phonon would be free particles at short distances, recovering their canonical nature.

It is illuminating to check the exact result (3.13) in low-order perturbation theory. A direct calculation shows that the $O(g)$ correction to the electron propagator depicted in Fig. 1 has the structure $\propto (G^2/\omega_0^2)(1+a)^{-1}(\gamma^0 p_0 + a\gamma^1 p_1)$ for $p^\mu \to 0$. This leads to the inverse propagator of the form $\sqrt{Z}\gamma^0 p_0 + \sqrt{X}\gamma^1 p_1$ to $O(g)$, which is consistent with Eq. (3.13). This simple check would be a justification of the somewhat formal procedure reducing Eq. (2.6) to Eq. (2.12).

The power-law correlation of electrons shows that the electron spectrum has no gap at the Fermi energy. Consequently, the phonon field must have a vanishing vacuum expectation value $\langle \phi(x) \rangle = 0$. It is possible to see this more directly. The equation of motion of ϕ,

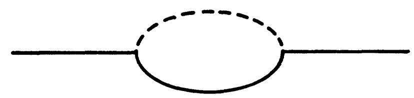

FIG. 1. $O(g)$ self-energy correction to the electron propagator. Solid lines denote electrons while the dashed line denotes a phonon.

$$(-\partial_0^2 + a^2\partial_1^2 - \omega_0^2)\phi(x) + G\psi_L^\dagger(x)\psi_R(x) = 0 , \quad (3.15)$$

relates $\langle \phi(x) \rangle$ to the vacuum condensate $\langle \psi_L^\dagger \psi_R \rangle$. Consider the corresponding regularized quantity

$$I(x) \equiv \psi_L^\dagger(x + \tfrac{1}{2}\epsilon)\psi_R(x - \tfrac{1}{2}\epsilon) \propto \mu^\kappa(-1/\hat\epsilon^2)^\sigma :e^{-i\sqrt{4\pi}\xi(x)/Y}:, \text{ as } \epsilon^\mu \to 0 , \quad (3.16)$$

where $\kappa = 1/Y^2$ and $\sigma = \frac{1}{2}(1-\kappa)$. Therefore, $\langle \phi(x) \rangle \propto \langle I(x) \rangle$ vanishes if we let the infrared cutoff $\mu \to 0$ keeping the ultraviolet cutoff $\epsilon^\mu \neq 0$. Although $\langle I \rangle = 0$, $I(x)$ has a nonvanishing correlation (for $\mu \to 0$):

$$\langle I(x)I^\dagger(0) \rangle \propto (-1/\hat\epsilon^2)^{2\sigma}(-1/\hat{x}^2)^\kappa , \quad (3.17)$$

where $\hat\epsilon^\mu = (\epsilon^0, \sqrt{Z/X}\,\epsilon^1)$. This entails a drastic change in the phonon correlation: The free-phonon correlation falls off exponentially for large separations. The exponential falloff, however, is overtaken by the power-law behavior of the quantum correction in Eq. (3.17). Thus the phonons have a power-law long-range correlation with the scaling dimension $d[\phi] = \kappa = 1/Y^2$.

Let us here recall the mean-field result:[3,4] The phonon-field expectation value $\langle \phi(x) \rangle \neq 0$, which drives spontaneous breakdown of chiral symmetry, opens up a gap in the electron energy spectrum, leading to a short-range correlation of electrons. The ICDW is attributed to a massless mode, the Nambu-Goldstone mode, carried by the phase of the phonon field; this apparently associates the ICDW to the phonon sector.

Our picture of the ICDW, which emerges from the exact solution to the effective Lagrangian (2.12), is fundamentally different from the mean-field result. The electrons have a long-range power-law correlation and no Peierls's energy gap arises. One-dimensional fluctuations make $\langle \phi(x) \rangle$ vanish and suppress Peierls's instability. The ICDW is associated with the current j^μ, or with the boson field $\xi(x)$, which naturally is identified with the phason.[20,21] The phason ξ is a massless mode propagating with velocity $v_{\text{phason}} = v_F\sqrt{X/Z}$. Although the phason appears to originate primarily from the electron sector, the phonon sector also contributes via the effective low-energy interaction in Eq. (2.12). The collective nature of the phason or the ICDW is clearly seen from the fact that both the electron and the phonon correlation functions in Eqs. (3.13) and (3.17) depend on the common arguments $\hat{x}^\pm = x^0 \pm \sqrt{Z/X}\,x^1$.

In the present model, the chiral symmetry is manifest, as characterized by $\langle\phi\rangle=0$ and $\langle\phi\phi^{\dagger}\rangle\neq 0$. One can show by counting of scaling dimensions that any local operators O of nonzero chiral charge, composed of ψ and $\psi^{\dagger}$, have a vanishing vacuum expectation value for $\mu\to 0$. Hence $\langle[Q_5, O]\rangle=0$, where Q_5 is the axial charge, and the chiral symmetry is unbroken.

The pattern of symmetry realization, however, looks different in the equivalent bosonic (i.e., the phason-phonon) representation of the model. There the phason ξ fails to commute with the axial charge, $[Q_5,\xi]\neq 0$; see Eq. (3.10). Thus the axial symmetry is spontaneously broken and the phason is an associated Nambu-Goldstone (NG) boson. This situation, though puzzling, is not a contradiction for the following reason: In the original electron-phonon representation of the model, the phason ξ is a massless collective mode carried by the current, and it can never be isolated as an elementary massless boson. [Note that $\xi\sim\cos^{-1}(\pi\bar{\psi}\psi/\mu)$ is not a well-defined operator for $\mu\to 0$.] There is no contradiction with Coleman's theorem[7] here. The unbroken chiral symmetry of the fermion theory appears spontaneously broken in its bosonized version where ξ arises as an *elementary* NG boson.

That the ξ is a NG boson in the bosonized version provides a reason why a massless mode arises in the electron-phonon system with unbroken chiral symmetry. The more direct and fundamental reason, we emphasize, is the Kač-Moody symmetry, which implies the presence of massless modes in the currents.

Recent developments[15] in conformal field theory have revealed that the conformal (and KM) algebra in two dimensions is so restrictive that a wide class of conformal models turns out solvable. As a matter of fact, the power-law behavior of the correlation functions in Eqs. (3.13) and (3.17) directly follows from the KM symmetry. Instead of using the representation theory of the KM and conformal algebras, however, we have chosen to use bosonization here, which can handle the complications due to "noncovariance" (for $a\neq 1$) and which gives rise to a clear physical picture of the phason.

IV. SLIDING OF CHARGE-DENSITY WAVES IN AN ELECTRIC FIELD

In this section we study how the ICDW responds to an applied electric field. The equation of motion of the phason following from Eq. (2.13),

$$(\partial_t^2 - v_{\text{phason}}^2\partial_x^2)\xi = (Y/Z)(e/\sqrt{\pi})v_F E \tag{4.1}$$

with $v_{\text{phason}}=v_F\sqrt{X/Z}$, shows that an applied electric field $E=(1/v_F)\epsilon^{\mu\nu}\partial_\mu A_\nu$ accelerates the phason, giving rise to a sliding CDW, as noted earlier.[4]

As is well known,[22] in the presence of the electric potential A^μ the axial-current conservation $\partial_\mu j_5^\mu$ becomes anomalous while the vector current j^μ continues to be conserved. The anomalous axial-current conservation law is equivalent to the phason equation (4.1). Indeed, Eq. (4.1) is combined with Eq. (3.12) to yield

$$\partial_\mu j_5^\mu = Z\partial_0 j_1 - X\partial_1 j_0 = -(e/\pi)\epsilon^{\mu\nu}\partial_\mu A_\nu \ . \tag{4.2}$$

An alternative derivation of the axial anomaly is given in Appendix B.

For a spatially homogeneous field $E(\omega)e^{-i\omega t}$, Eq. (4.2) reproduces the low-frequency form of the conductivity derived by Lee, Rice, and Anderson,[20] $\sigma(\omega)=eJ(\omega)/E(\omega)=ie^2n/(m^*\omega)$, where $n=p_F/\pi$ is the number density of electrons (per spin). In our case the effective electron mass is given by

$$m^* = Zm = [1+2(g/\pi)v_F/(v_F+v_Q)]m > m \ . \tag{4.3}$$

Our solution therefore contains the phonon-drag effect, which depends simply on the electron-phonon coupling $g=\frac{1}{2}G^2/\omega_0^2$ and v_Q/v_F.

Not only the sliding of the ICDW but also a plasma mode in the ICDW medium can be understood in terms of the axial anomaly. Let us add the Maxwell term $\frac{1}{2}v_F E^2$ to $L_{\text{eff}}^{\text{boson}}$ in Eq. (2.13) and study the electric field induced by a displacement of the phason mode. The Maxwell equation now reads $E=-(ev_F/\sqrt{\pi})\eta$, yielding the equation

$$(\partial_t^2 - v_{\text{phason}}^2\partial_x^2)E = -(e^2n/mZ)E \ . \tag{4.4}$$

Thus the plasma frequency is $\omega_p=(e^2n/m^*)^{1/2}$ while the charge screening length is characterized by $\lambda_{\text{sc}}=v_{\text{phason}}/\omega_p$.

V. TEMPERATURE EFFECTS

We have so far studied the electron-phonon system at zero temperature. It is not difficult to extend our analysis to finite temperature. To a free massless propagator in Minkowski space

$$\langle \mathcal{T}[\eta(x)\eta(0)]\rangle = -(1/4\pi)\ln[\mu^2(-x^2+i0)] \tag{5.1}$$

corresponds the temperature Green's function

$$\langle\eta(x)\eta(0)\rangle_\beta = -(1/4\pi)\ln\left[(i\mu)^2(\beta/\pi)^2\sinh\left[\frac{\pi}{\beta}x^+\right]\times\sinh\left[\frac{\pi}{\beta}x^-\right]\right], \tag{5.2}$$

where $\beta=1/(k_BT)$ denotes the inverse temperature; μ is an infrared cutoff. (They coincide at short distances $x^+x^-=x^2\to -0$.) Accordingly, the $D(\epsilon)/\epsilon^2$ in Eq. (2.9) is replaced by

$$D(\epsilon)_\beta/\{(\beta/\pi)^2\sinh[(\pi/\beta)\epsilon^+]\sinh[(\pi/\beta)\epsilon^-]\} \ ,$$

where $D(\epsilon)_\beta$ stands for the free-phonon temperature Green's function. The g_0 and g_1 in Eq. (2.11) will deviate from their zero-temperature values. Fortunately the explicit evaluation of $g_0(\beta)$ and $g_1(\beta)$ is not needed for the derivation of the correlation functions. The operator solution (3.2) is also valid at finite temperature, and the electron correlation function takes the form

$$\langle\psi_L(x)\psi_L^\dagger(0)\rangle_\beta \propto \left[1\Big/\sinh\left[\frac{\pi}{\beta}\hat{x}^+\right]\right]^{1+\lambda^2}\left[1\Big/\sinh\left[\frac{\pi}{\beta}\hat{x}^-\right]\right]^{\lambda^2}; \tag{5.3}$$

analogously for $\langle \psi_R \psi_R^\dagger \rangle_\beta$. The exponent $\lambda^2 = \frac{1}{4}(Y - 1/Y)^2$ will depend on T through $g_0(\beta)$ and $g_1(\beta)$. Thus the electron correlation function falls off exponentially at finite temperature, with the correlation length $\sim v_{\text{phason}} \beta/\pi$ growing like $1/T$ for $T \to 0$. Its exponential falloff turns into a power-law falloff at $T=0$. This is in accord with a general theorem[23] that one-dimensional systems undergo no phase transitions for $T > 0$.

As in the zero-temperature case, one can argue that the order parameter $\langle \phi(x) \rangle_\beta$ vanishes as $\mu \to 0$; hence the chiral symmetry continues to be unbroken at finite temperature.

The anomalous axial-current conservation law (4.2) is unmodified at finite temperature, except that Z and X become T dependent. Again it implies the occurrence of a sliding ICDW; the conductivity $\sigma(\omega) = ie^2 n/(m^* \omega)$ depends on T through $m^* = Zm$.

VI. PINNING OF CHARGE-DENSITY WAVES

In the foregoing sections we have seen that the chiral symmetry hardly gets broken spontaneously in one dimension and that purely one-dimensional systems with chiral symmetry generally support a superconducting ICDW for $T \geq 0$. The metal-to-insulator transitions of real quasi-one-dimensional conductors should thus be attributed to the formation of an explicit chiral breaking below the critical temperature. Interactions among CDW chains are considered to form three-dimensional order that triggers the phase transition;[20] such order can be regarded as a chiral breaking. Formation of an effective chiral breaking through interchain interactions has been discussed using mean-field methods.[24] For the description of the CDW dynamics as well as phase transitions in real materials it is necessary to take into account pinning of the CDW via impurities, commensurability, or interchain interactions. In this section we present a very simplified treatment of impurity pinning of the ICDW to emphasize the importance of chiral symmetry in understanding pinning phenomena.

The interaction of the charge density with the impurity potential $V_{\text{imp}}(x)$ is written as

$$V_{\text{imp}}(x)[j_0 + \cos(2p_F x)\bar{\psi}\psi - i\sin(2p_F x)\bar{\psi}\gamma_5\psi] \; . \qquad (6.1)$$

The first forward-scattering term maintains chiral symmetry; it is clear that $\partial_x V_{\text{imp}}$ acts like an electric field (discussed in Sec. IV) that accelerates or decelerates the ICDW locally. The second and third terms, which represent backward scattering, break the axial symmetry. For simplicity, let us suppose that impurities are both randomly and densely distributed; i.e., we consider the case of weak pinning.[21] Then replace $V_{\text{imp}}(x)\cos(2p_F x)$ by the coarse-grained average $M = \langle V_{\text{imp}}(x)\cos(2p_F x) \rangle_{\text{av}}$, where the averaging is made over some characteristic distance over which (slowly varying) $\bar{\psi}\psi$ changes its sign. For the ICDW of long wavelengths the impurity effect is now represented as an effective "relativistic" mass term $M\bar{\psi}\psi$. The $\bar{\psi}\psi$ is the simplest of chiral breaking terms of positive parity, composed of the electron fields alone. One practical way to simulate the pinning effects (due to impurities, commensurability, or interchain interactions) is to introduce such low-energy chiral-breaking effective interactions. The $\bar{\psi}\psi$, being the lowest-dimensional operator of such, will describe the leading pinning effect in the long-wavelength behavior of the ICDW.

Let us draw some consequences of pinning by including an effective "mass" term to the original Lagrangian (2.4), $L_{\text{eff}}^{(0)} \to L_{\text{eff}}^{(0)} - M\bar{\psi}\psi$. The mass term drastically changes the long-wavelength features of the model: The electron spectrum has a gap $\propto 2M$ about the Fermi level. A simple perturbation calculation shows that the phonon field $\phi(x) \equiv |\phi(x)| e^{i\chi(x)}$ develops a vacuum expectation value $\langle |\phi(x)| \rangle$ of $O((G/\omega_0^2)M \ln M)$ and that its phase $\chi(x)$ acquires a mass of $O(M)$. Thus the pinning effect overtakes one-dimensional fluctuations, leading to the gap $\propto 2M$ and the order $\langle \phi \rangle \neq 0$.

As before, one can construct, via bosonization, the low-energy effective theory, which is the (modified) massive Thirring model,

$$L_{\text{eff}} = \bar{\psi}(i\gamma^\mu \partial_\mu - M)\psi - \tfrac{1}{2}(g_0 j_0^2 - g_1 j_1^2) \; , \qquad (6.2)$$

or its bosonic equivalent, the sine-Gordon model,

$$L_{\text{eff}}^{\text{boson}} = \tfrac{1}{2}Z(\partial_0 \eta)^2 - \tfrac{1}{2}X(\partial_1 \eta)^2 - (M\mu/\pi) :\cos(\sqrt{4\pi}\eta): . \qquad (6.3)$$

The (tree-level) mass m_ξ of ξ is given by $m_\xi^2 = -4\mu M/Z$. From the short-distance behavior of the η propagator [note Eq. (2.8)], one finds that $\mu = -\frac{1}{2}e^\gamma m_\xi$. Thus the μ, which previously was an infrared cutoff, is now nonzero,[25]

$$\mu = e^{2\gamma} M/Z \; . \qquad (6.4)$$

The coupling constants g_0 and g_1 are functions of M/ω_0 now. For $M/\omega_0 \ll 1$, one may use the values in Eq. (2.11); the next corrections[26] to g_0 and g_1 are of the form $g(M/\omega_0)^2[\ln(M/\omega_0) + \text{const}]$.

The equivalence of the massive Thirring model and the sine-Gordon model has been studied extensively.[10,11] For $a=1$, our fermion operator (3.2) and Eq. (6.4) correctly reproduce Mandelstam's result.[11]

It is clear from Eq. (6.3) that the mass term breaks the Kač-Moody symmetry (hence both axial and conformal symmetries as well) explicitly and that there remains no massless (long-range) mode in the theory. Both electrons and phonons have only short-range correlations. The correlation lengths, however, grow up indefinitely as $M \to 0$, and the $M=0$ limit realizes the previous chiral- and conformal-symmetric case of Sec. III; the spontaneously broken chiral symmetry of the mean-field treatment is thereby never realized.

In the presence of an applied electric field, the phason equation reads

$$(\partial_t^2 - v_{\text{phason}}^2 \partial_x^2)\xi + m_\xi^2 (Y/\sqrt{4\pi})\sin(\sqrt{4\pi}\xi/Y) = (Y/Z)(e/\sqrt{\pi})v_F E \; . \qquad (6.5)$$

This is easily seen to be equivalent to the anomalous conservation law of the axial current

$$\partial_\mu j_5^\mu = 2iM\bar{\psi}\gamma_5\psi - (e/\pi)\epsilon^{\mu\nu}\partial_\mu A_\nu \ . \tag{6.6}$$

The conductivity now reads $\sigma(\omega)=i(e^2n/m^*)\omega/(\omega^2-m_\xi^2)$ for weak E. The sliding of the ICDW is suppressed owing to pinning, resulting in a vanishing static conductivity. It is also clear from Eq. (6.5) that pinning makes the plasma frequency of the ICDW medium higher, $\omega_p=[(e^2n/m^*)+m_\xi^2]^{1/2}$.

VII. CONCLUDING REMARKS

In this paper we have studied the symmetry properties of one-dimensional ICDW systems. We have seen that the formation and acceleration of the ICDW is deeply tied to the symmetries (and their quantum modification, anomalies) of the electron-phonon system. In particular, we have noted that a local chiral symmetry (the Kač-Moody symmetry) emerges in the low-energy structure of the one-dimensional electron-phonon system. Both chiral and conformal symmetries are part of this local chiral symmetry. The Kač-Moody symmetry not only signals the existence of massless modes but also determines the essential features of the theory. In consequence, both electrons and phonons show long-range power-law correlations. The chiral symmetry remains unbroken, with $\langle\phi\rangle=0$ and $\langle\phi\phi^\dagger\rangle\neq 0$. Fluctuations suppress long-range order, and Peierls's instability disappears. Still the ICDW arises as a collective electron-phonon mode (the phason), which is the massless mode implied by the Kač-Moody symmetry, and slides bodily through the lattice under the influence of an applied electric field. The Peierls-Fröhlich mechanism[27] is here realized in a manner peculiar to one dimension.

It will be useful here to comment on some earlier non-mean-field studies of one-dimensional conductors (based on the Luttinger model, "g-ology" models,[12,28] etc.). There, one starts with phenomenological effective Lagrangians with no explicit account of phonons; bosonization is extensively used and the emergence of a massless mode has been known. Our approach differs from earlier work in that the low-energy effective Lagrangian (a modified Thirring model) is derived here from the microscopic electron-phonon system by taking proper account of phonon exchanges. In addition, our analysis is focused on a detailed study of the symmetry-related aspects of the CDW transport phenomenon, which has not been fully discussed in earlier work. In particular, our analysis has shown that the phason is a massless mode associated with the Kač-Moody symmetry, and that general features of pinning phenomena may be understood in terms of the manner in which this symmetry gets spoiled via pinning mechanisms.

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APPENDIX A

In this Appendix we summarize some basic commutation relations of a massless field in two dimensions.[17] A massless scalar field $\eta(x)$, obeying the equation $\partial_\mu\partial^\mu\eta=0$, is decomposed into the left- and right-moving modes,

$$X(x^+)=\sqrt{\pi}[\eta(x)+\tilde{\eta}(x)],\quad \bar{X}(x^-)=\sqrt{\pi}[\eta(x)-\tilde{\eta}(x)] \ , \tag{A1}$$

where $x^\pm=x^0\pm x^1$; $\tilde{\eta}$ denotes the dual field defined by $\partial_\mu\tilde{\eta}=\epsilon_{\mu\nu}\partial^\nu\eta$.

The $\eta(x)$ (normalized according to the action $\int d^2x\,\frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta$) obeys the commutation relation

$$[\eta(x),\,\eta(0)]=iD(x)=-i\tfrac{1}{2}\epsilon(x^0)\theta(x^2) \ . \tag{A2}$$

The η and $\tilde{\eta}$ are decomposed into the positive- and negative-frequency parts ($\eta=\eta^{(+)}+\eta^{(-)}$, etc.) in the usual way. One has to introduce an infrared cutoff μ (or the size of the system $\sim 1/\mu$) to define the commutation relations of $\eta^{(\pm)}$ and $\tilde{\eta}^{(\pm)}$. They are neatly written in terms of $X^{(\pm)}=\sqrt{\pi}(\eta^{(\pm)}+\tilde{\eta}^{(\pm)})$ and $\bar{X}^{(\pm)}=\sqrt{\pi}(\eta^{(\pm)}-\eta^{(\pm)})$ as

$$[X(x^+)^{(\pm)},X(0)^{(\mp)}]=\mp\ln[(\pm i\mu)(x^+\mp i0)] \ , \tag{A3}$$

$$[X^{(\pm)},X^{(\pm)}]=[\bar{X}^{(\pm)},\bar{X}^{(\pm)}]=0 \ , \tag{A4}$$

$$[X^{(\pm)},\bar{X}^{(\pm)}]=[X^{(\pm)},\bar{X}^{(\mp)}]=-i\pi/4 \ . \tag{A5}$$

For $[\bar{X}^{(\pm)}(x^-),\bar{X}^{(\mp)}(0)]$, replace $x^+\to x^-$ in Eq. (A3).

APPENDIX B

In this Appendix we derive the axial anomaly in Eq. (4.2) by an operator method of Jackiw and Johnson.[29] The external potential A^μ is treated as a perturbation to the modified Thirring Lagrangian (2.12). We regularize both the vector and axial currents in a gauge-invariant way:

$$\begin{aligned} j^\mu(x;\epsilon)&=\bar{\psi}(x+\tfrac{1}{2}\epsilon)\gamma^\mu\Gamma[A;\epsilon]\psi(x-\tfrac{1}{2}\epsilon) \ ,\\ j_5^\mu(x;\epsilon)&=\bar{\psi}(x+\tfrac{1}{2}\rho\epsilon)\gamma^\mu\gamma_5\Gamma[A;\rho\epsilon]\psi(x-\tfrac{1}{2}\rho\epsilon) \ , \end{aligned} \tag{B1}$$

where $\Gamma[A,\epsilon]=\exp(-ie\int dz^\mu A_\mu)$ with the line integral connecting $\bar{\psi}$ and ψ. Formally the duality relation $j_5^\mu(x;\epsilon)=\epsilon^{\mu\nu}j_\nu(x;\rho\epsilon)$ holds. Note, however, that the $j_\nu(x;\rho\epsilon)$ here is regularized according to the prescription (3.11); it differs from the vector current defined by the prescription (3.6).

Using the equation of motion of ψ following from Eq. (2.12), one can cast the axial-current divergence in the form

$$\partial_\lambda j_5^\lambda(x;\epsilon)=i\rho\epsilon_\nu[eF^{\nu\mu}+g_\mu(\partial^\nu j^\mu)]j_\mu^5(x;\epsilon)\quad\text{as }\epsilon^\mu\to 0 \ , \tag{B2}$$

where $g_\mu=(g_0,g_1)$; the sum over μ, though somewhat unusual in notation, is implied in the usual sense. We substitute for $j_5^\mu(x;\epsilon)$ on the right-hand side its vacuum expectation value

$$\langle j_5^\mu(x;\epsilon)\rangle = (\rho^{-(\lambda^2+1)}/i\pi) \times (1+ab^2)/(a+X)\epsilon^{\mu\nu}\hat{\epsilon}_\nu/\hat{\epsilon}^2 \;, \qquad \text{(B3)}$$

and let $\epsilon^\mu \rightarrow 0$, noting Eqs. (3.11) and (3.12). Some algebra then leads to the axial-current conservation law (4.2). In the same way one can verify the vector-current conservation $\partial_\mu j^\mu(x;\epsilon)=0$.

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$$I=-(i/\omega_0^2)\int d\theta\cos^2\theta/[\cos^2\theta+(1/a^2)\sin^2\theta]$$
$$=-i(2\pi/\omega_0^2)a/(1+a)\;.$$

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