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Low-temperature energy relaxation in $Al_xGa_{1-x}As/GaAs$ heterojunctions

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The total average energy-loss rate of a two-dimensional electron gas in an $Al_xGa_{1-x}As/GaAs$ heterojunction at low temperature is calculated for various densities and lattice temperatures. We take into account the inelastic-scattering mechanisms due to acoustic phonons via deformation-potential and piezoelectric coupling, and the low-energy phonon modes associated with the coupling of quasiparticle excitations to the LO phonons. At high carrier densities we show that the inclusion of these phonon modes can explain the experimental data of Manion *et al.* [Phys. Rev. B **35**, 9203 (1987)] for the energy-loss rates at low temperatures without using an enhanced value of the deformation-potential constant.

A great deal of experimental and theoretical effort has been focused on transport and power-loss measurements in $Al_xGa_{1-x}As/GaAs$ heterojunctions at low temperatures.[1-6] Undoubtedly, these studies have been partly motivated by the current controversy surrounding the magnitude of the fundamental coupling constants of the electron-phonon interaction[7] in a two-dimensional electron gas (2D EG). Since the mobile carriers are to a great extent confined to the GaAs side of the heterojunction, we expect the relevant electron-phonon coupling constants to be those of bulk GaAs. However, a number of studies[1,2,4,5] suggest that the deformation-potential constant D is 11–16 eV, which is significantly larger than the accepted bulk value[8] of 7 eV. It has also been recently suggested that the piezoelectric constant h_{14} also needs to be enhanced in an $Al_xGa_{1-x}As/GaAs$ heterojunction.[5]

Most determinations of the deformation-potential constant D have been derived from the temperature dependence of the mobility at low temperatures. Depending on the adopted theoretical model,[5,6] values in the range 7–13 eV for D have been quoted in the literature. Enhanced values of D have also been independently inferred from energy-relaxation experiments in heterojunctions at low temperatures.[1,2] Energy-relaxation measurements should provide a more reliable estimate of D, since the elastic scattering arising from remote ionized impurities have no effect on the energy-loss rate.

At low temperatures below 40 K it has conventionally been accepted that the only relevant inelastic-scattering mechanisms are the acoustic phonons via deformation-potential and piezoelectric coupling. However, it has been shown recently that the coupling of quasiparticle excitations to the LO phonons provides an additional mechanism for energy relaxation,[9] which may be particularly important at low electron temperatures and densities. This naturally leads us to consider a theoretical model where the total power loss in a heterojunction at low temperature is due to both the acoustic phonons and renormalized LO phonons, thereby possibly eliminating the need to artificially enhance the value of D from its bulk value. In this paper, we quantitatively investigate the consequences of such a model and compare our results with the existing experimental data.

We assume that the electrons only occupy the lowest electric subband, and that the normal z component of the electron confinement is accurately described by the Fang-Howard-Stern variational wave function.[10] We work within the "electron temperature" model in which the effect of the applied field is to separately keep the 2D EG in equilibrium at an elevated temperature T_e from the lattice system, which is at temperature T_L. Following Ref. 1, the average energy loss per electron via acoustic-phonon scattering can be evaluated by calculating the energy gained by the lattice system from the electrons and dividing by the total number of electrons. The average energy-loss rate via acoustic-phonon scattering is then,

$$\left\langle \frac{dE}{dt} \right\rangle = -\frac{1}{N_e} \sum_{\mathbf{q}} \hbar\omega_{\mathbf{q}} \frac{dN_{\mathbf{q}}}{dt} , \tag{1}$$

where N_e is the total number of electrons, $\hbar\omega_{\mathbf{q}}$ is the energy of a phonon with wave vector $\mathbf{q}$, and dN_q/dt is the rate of change of the phonon distribution function. From Fermi's "golden rule," $dN_{\mathbf{q}}/dt$ may be written as

$$\frac{dN_{\mathbf{q}}}{dt} = 2\frac{2\pi}{\hbar} \sum_{k} |I(q_z)|^2 \frac{1}{\epsilon^2(q_{\parallel}, T_e)} |M(q)|^2 \delta(E_{\mathbf{k}} + \hbar\omega_{\mathbf{q}} - E_{\mathbf{k}+\mathbf{q}}) \times \{(N_{\mathbf{q}}+1) f(E_{\mathbf{k}+\mathbf{q}_{\parallel}})[1 - f(E_{\mathbf{k}})] - N_{\mathbf{q}} f(E_{\mathbf{k}})[1 - f(E_{\mathbf{k}+\mathbf{q}_{\parallel}})]\} , \tag{2}$$

where the summation is over the two-dimensional electron wave vector $\mathbf{k}$. The phonon wave vectors which are parallel and perpendicular to the heterolayer interface are denoted by $q_\parallel$ and q_z, respectively. $I(q_z)$ is the overlap integral due to the lowest electric subband.[11]

The three-dimensional matrix element for acoustic-phonon scattering is denoted by $M(q)$.[1,11] $N_\mathbf{q}$ and $f(E)$ are the Bose and Fermi distribution functions, respectively. Finally, $E_\mathbf{k}=\hbar^2k^2/2m^*$ and the phonon frequency $\omega_\mathbf{q}$ is given by the dispersion relation $\omega_\mathbf{q}=uq=u(q_\parallel^2+q_z^2)^{1/2}$, where u is either the longitudinal or transverse velocity of sound in GaAs. The above equations can be reduced to the following expression for the average energy-loss rate:[12]

$$\left\langle \frac{dE}{dt} \right\rangle = \frac{2u(2m^*)^{1/2}}{\hbar E_F(2\pi)^2}\int_0^\infty dq\, q^2\left\{\exp\left[\left(\frac{1}{k_BT_L}-\frac{1}{k_BT_e}\right)\hbar\omega_\mathbf{q}\right]-1\right\} \times\int_0^{\pi/2}d\theta|I(q_z)|^2\frac{|M(q)|^2}{\epsilon^2(q_\parallel,T_e)}\int_{E_{\min}}^\infty dE_\mathbf{k}E_\mathbf{k}^{-1/2}f(E_\mathbf{k}) \times[1-f(E_\mathbf{k}+\hbar\omega_\mathbf{q})]\left[1-\frac{(\hbar\omega_\mathbf{q}-E_{q_\parallel})^2}{4E_\mathbf{k}E_{q_\parallel}}\right]^{-1/2}, \quad (3)$$

where E_F is the Fermi energy and $m^*=0.067m_0$ is the effective mass in GaAs. The minimum energy in the integration, $E_{\min}$, is determined by energy and momentum conservation in the plane and is given by $E_{\min}=(\hbar\omega_\mathbf{q}-E_{q_\parallel})^2/4E_{q_\parallel}$. The screening factor $\epsilon(q_\parallel,T_e)$ at finite (nonzero) temperature is calculated with use of the Maldague formula[13] within the random-phase approximation (RPA).[14] Equation (3) can be solved numerically to find the dependence of $\langle dE/dt\rangle$ on T_L, T_e, and N_s due to the inelastic scattering via acoustic phonons.

The bare LO phonons play no role at low temperatures due to their relatively high energy. However, due to the many-body effects arising from the coupling of the LO phonons to the quasiparticle excitations of the 2D EG, the spectral function of the many-body renormalized phonon propagator gives rise to low-energy "quasiparticle-like" LO-phonon modes, which provide an additional channel for energy relaxation at low temperatures, competing with the usual acoustic-phonon energy-loss mechanism. The renormalized LO-phonon propagator is given by[15] ($\hbar=1$)

$$D(q,\omega)=\frac{2\omega_{\rm LO}}{\omega^2-\omega_{\rm LO}^2-2\omega_{\rm LO}M_\mathbf{q}^2\chi(q,\omega)}, \quad (4)$$

where $\omega_{\rm LO}=36.8$ meV is the LO-phonon frequency, and $M_\mathbf{q}^2$ is the bare two-dimensional Fröhlich coupling constant in the bulk phonon approximation.[9] The effects due to screening from the electron gas are contained in the reducible polarizability function $\chi(q,\omega)=\chi^0(q,\omega)/\epsilon(q,\omega)$. Here

$$\chi^0(q,\omega)=-2\sum_\mathbf{k}\frac{f(E_{\mathbf{k}+\mathbf{q}})-f(E_\mathbf{k})}{\hbar\omega+i\gamma-E_{\mathbf{k}+\mathbf{q}}+E_\mathbf{k}} \quad (5)$$

and

$$\epsilon(q,\omega)=1-V_\mathbf{q}\chi^0(q,\omega) \quad (6)$$

where $V_\mathbf{q}=2\pi e^2/\kappa_\infty q$, and $\kappa_\infty=10.9$ is the optical dielectric constant. A finite value of γ phenomenologically takes into account the electron scattering by impurities.

The finite thickness of the heterolayer can be simply taken into account by the replacement $M_\mathbf{q}^2\rightarrow M_\mathbf{q}^2f_\mathbf{q}$ and $V_\mathbf{q}\rightarrow f_\mathbf{q}V_\mathbf{q}$ in Eqs. (4) and (6), where $f_\mathbf{q}$ is the form factor associated with the variational subband wave function.[9] From Fermi's "golden rule," and the fluctuation-dissipation theorem, we arrive at the following expression for the energy-loss per carrier via emission of the dressed LO phonons:[16]

$$P=\frac{1}{N_e}\sum_\mathbf{q}M_\mathbf{q}^2\int\frac{d\omega}{\pi}\hbar\omega[n_{T_L}(\omega)-n_{T_e}(\omega)] \times{\rm Im}\chi(q,\omega)\,{\rm Im}D(q,\omega), \quad (7)$$

where $n_T(\omega)$ is the Bose occupation factor at temperature T. Hot-phonon effects can be neglected in the experimental range of interest in this paper.

The energy-loss rate in $Al_xGa_{1-x}As/GaAs$ heterojunctions at low temperatures has experimentally been investigated by Manion *et al.* (hereafter, MAECH).[1] They reported on the energy-loss rate as a function of T_e for a single fixed carrier density of $N_s=6\times10^{11}$ cm^{-2} for different values of T_L. Assuming only acoustic-phonon scattering they derived an enhanced value of 16 eV for D.

Using Eqs. (3) and (7), we calculate the energy loss due to acoustic phonons and the renormalized LO phonons for carrier densities $N_s=(0.6\text{–}6)\times10^{11}$ cm^{-2}, at different lattice temperatures, as a function of the electron temperature T_e. For a given N_s we numerically evaluate the energy loss solely from the acoustic-phonon theory using an enhanced value of D. In addition, we also calculate the energy-loss rate from the combined theory which takes into account the contribution from the renormalized LO phonons, using the bulk GaAs value for D. Our results for $N_s=6\times10^{11}$ cm^{-2} and $T_L=4.2$, 3.4, and 5.5 K are presented in Figs. 1(a)–1(c). We see that the curve which corresponds to the total contribution from the acoustic and renormalized LO phonons using $D=7$ eV fits the data of MAECH remarkably well in the electron temperature range, precisely where the experimental data is taken. Therefore, at high carrier densities we see that it is possible to fit the experimental data of MAECH without an enhanced value of D, as long as the low-energy phonon modes associated with the renormalized LO phonons are taken into account. From Fig. 1 we clearly see an enhanced energy-loss rate and stronger dependence on T_e for the combined theory than for the purely acoustic-phonon theory. In order to check definitively which theoretical model is a valid explanation for the data of MAECH, one needs to extend the electron temperature

range over which the data is taken. In Fig. 2 we show the dependence of the energy-loss rate on carrier density for a fixed electron temperature, $T_e = 6$ K. Clearly, the curves corresponding to the purely acoustic-phonon theory, irrespective of the value for D used, is rather featureless and the energy-loss rate is essentially independent of the carrier density. Although the combined theory agrees with the acoustic-phonon theory at high densities, we see a marked discrepancy which increases as the carrier density is decreased. As we can see in Fig. 3 the agreement between the combined and acoustic-phonon theory becomes much poorer as the carrier density is decreased. The inclusion of the many-body effects greatly enhances the energy-loss rate as the density is decreased. This effect cannot be determined from the data set of MAECH since their study did not incorporate a range of densities. The density dependence of the energy-loss rate has been studied by Hirakawa and Sakaki (HS) (Ref. 2) for a single fixed lattice temperature $T_L = 4.2$ K. It is not clear why the energy-loss rates found by MAECH are approximately twice that found by HS. Assuming only acoustic-phonon scattering, HS derived an enhanced value of 11 eV for D. Their data is consistent with the acoustic-phonon theory and does not show a strong dependence of the energy-loss rate on carrier density. The data of HS have also been explained without an enhanced value of D

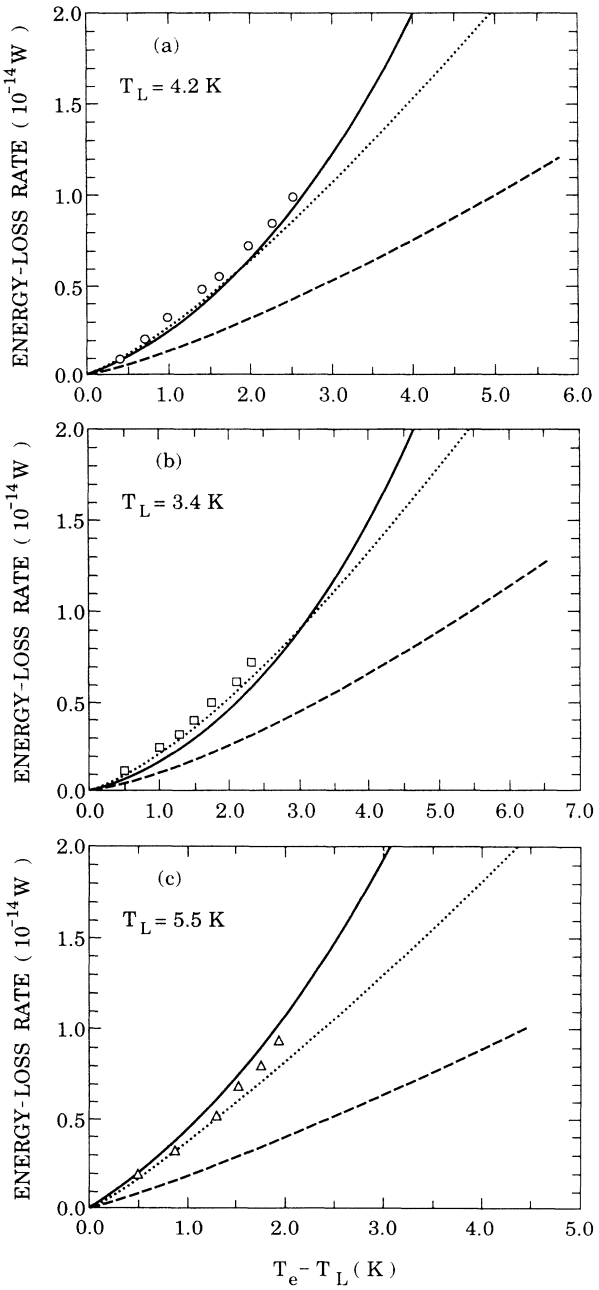

FIG. 1. Energy-loss rate vs electron temperature for $N_s = 6 \times 10^{11}$ cm^{-2} for (a) $T_L = 4.2$ K, (b) $T_L = 3.4$ K, and (c) $T_L = 5.5$ K. The dotted ($D = 16$ eV) and dashed ($D = 11$ eV) lines are computed from the acoustic-phonon theory only. The solid line is computed from the combined theory (i.e., acoustic and renormalized LO phonons as explained in the text) with $D = 7$ eV. Data points are from Manion *et al.* (Ref. 1).

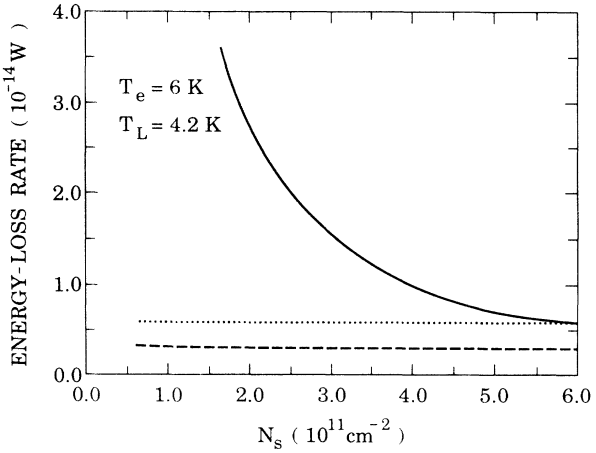

FIG. 2. Energy-loss rate vs carrier density for $T_e = 6$ K. The dotted ($D = 16$ eV) and dashed ($D = 11$ eV) lines are computed from the acoustic-phonon theory only. The solid line is computed from the combined theory with $D = 7$ eV.

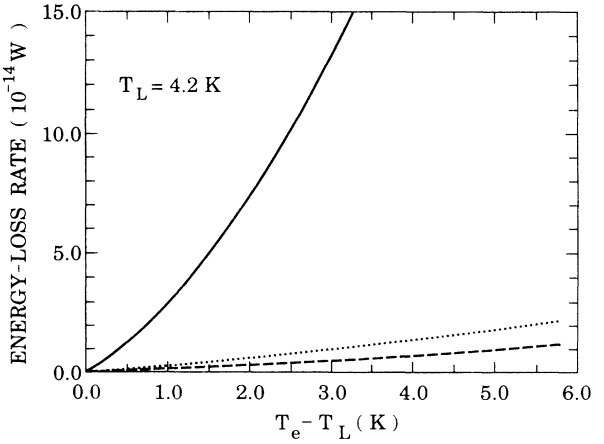

FIG. 3. Energy-loss rate vs electron temperature for $N_s = 1 \times 10^{11}$ cm^{-2} and $T_L = 4.2$ K. The dotted ($D = 16$ eV) and dashed ($D = 11$ eV) lines are computed from the acoustic-phonon theory only. The solid line is computed from the combined theory with $D = 7$ eV.

by Okuyama and Tokuda[6] within the acoustic-phonon theory, with screening only for the piezoelectric coupled scattering.

In conclusion, we have shown that in the high-carrier-density range the inclusion of the renormalized LO-phonon contribution to the standard acoustic-phonon theory with $D=7$ eV is in close agreement with the data of MAECH. The theory which takes into account the low-energy phonon modes exhibits a much stronger dependence on N_s and T_e than the standard acoustic-phonon theory with an enhanced value of D. In order to check which model is indeed valid at low temperatures, it is necessary to extend the electron-temperature range over which MAECH obtained their data. In view of the current experimental ambiguities in the energy-loss results, we propose detailed N_s- and T_e-dependent electron-energy-loss measurements to resolve this important issue.

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