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Distribution of pinning energies and the resistive transition in superconducting films

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The effect of the distribution of pinning energies on the current-voltage characteristics, magnetic flux creep, and critical currents of superconducting films in a transversal magnetic field H has been considered. In the case of macroscopic inhomogeneities of the pinning forces, the flux creep has been formulated as a percolation transition in the vortex system. The model proposed gives the strong dependence of the critical current on H at low H in the absence of weak links in a superconductor, the increase of the flux-creep activation energy with temperature T at low T and H, and the nonlogarithmic time decay of the magnetization in the flux-creep regime.

The recent studies of the resistive transition,[1] magnetic flux creep,[2] and, especially, electric-noise measurements[3] indicate a wide distribution of pinning energies E_p in high-T_c superconductors with $E_p \sim 0.03$–1 eV and the dispersion $\Delta E \sim E_p$. This fact proves to be essential for the dependence of the magnetic-flux-creep rate on temperature T and magnetic field H, which is important for the current-carrying capacity of high-T_c materials.

In this paper the resistive states of a superconducting film containing randomly distributed pinning centers with different pinning energies E_p have been considered. The film is assumed to be thin enough, when the distortion of the vortex line is negligible and the value E_p is equal to the difference of the energies of the pinned and unpinned vortices. Two characteristic cases are examined. In the first case the sizes of the pinning centers, a, are assumed to be small as compared to a spacing l_p between them and the density of the pinning centers to be uniform on the scales considerably exceeding l_p. The second case corresponds to macroscopic space modulations of the pinning forces. For these models the current-voltage characteristic and the flux-creep activation energy U have been found within the framework of a self-consistent mean-field approach.

To describe the resistive state in the case of uniform density of the pinning centers, let us divide the vortex density B/ϕ_0 into the mean density of the pinned vortices n_b and the mean density of the free vortices n_f depinned due to thermal fluctuations. Assume the differential resistivity of the superconductor $\rho(j)$ is determined by the equilibrium density of the free vortices by analogy with the flux-flow resistivity ρ_f as $\rho(j)=\rho_f n_f \phi_0/B$. Here j is the average current density, $\rho_f \sim \rho_n B/H_{c2}$, B is the magnetic induction, ϕ_0 is the magnetic flux quantum, H_{c2} is the upper critical field, and ρ_n is the resistivity in the normal state. Then the average electric field $\mathcal{E}$ associated with the viscous motion of the free vortices is given by

$$\mathcal{E}=\rho_f \phi_0 B^{-1}\int_0^j n_f(j')dj' . \tag{1}$$

This model differs somewhat from the approach based on the thermally assisted hopping of vortices between pinning centers.[4] However, such a model seems to be more adequate just in the case of "point" pinning centers ($a \ll l_p$) and low B (see below) for which the motion of the free vortex between pinning centers at $j=0$ is not directly due to the interaction of the vortex with them, but rather has a diffusion character. At $j>0$, this motion is determined by the balance of the Lorentz and viscous forces; the vortex-vortex interaction, correlated vortex motion, and collisions of vortices with pinning centers are assumed to be taken into account in the value ρ_f. This model also describes the crossover between the flux-creep and the flux-flow regimes, for instance, the critical-state model corresponds to $n_f=B/\phi_0$ at $j>j_c$ and $n_f=0$ at $j<j_c$, which yields $j=j_c+\rho_f^{-1}\mathcal{E}$ at $j>j_c$ where j_c is the critical-current density. The fluctuations result in the smearing of the jump in $n_f(j)$ at $j=j_c$ and the arising of thermally activated vortices at $j<j_c$ where the value n_f considerably depends on T, B, and j.

To calculate $n_f(j)$ let us define the distribution function of the pinning energies $f(E)$ so that the product $f(E)dE$ would be equal to the density of the vortex positions having the depinning energies E_p with $E<E_p<E+dE$ and

$$\int_0^\infty f(E)dE=n_p . \tag{2}$$

Here n_p is the total density of the pinning positions corresponding to all local minima of vortex energies per unit area. Examine the low-field region $B<H_p \sim \phi_0 n_p$ for which the number of the pinning positions exceeds the number of vortices. Then the interaction between vortices, modifying the bare pinning energies, may be taken into account in the mean-field approximation, which results in the dependence of $f(E)$ and n_p on B and j at $B>H_{c1}$ where H_{c1} is the lower critical field. At $B>H_p$ all pinning positions are occupied and the rest of the vortices are pinned due to the collective interaction.[5] Here, we do not discuss the specific mechanisms of pinning so the values E_p, n_p, and $f(E)$ are assumed to be phenomenological functions which depend on B, T, and j (for qualitative estimates one may assume $n_p \sim l_p^{-2}$, so $H_p \sim 0.1$–10 T if $l_p \sim 10^2$–10^3 Å).

In the thermodynamic equilibrium these positions are occupied by the vortices, beginning with the positions having maximum values of E_p. To describe the relaxation of

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a metastable vortex configuration, let us separate this process into two stages. At the first stage the local redistribution of vortices on the scales of order $l_p \ll l$ occurs and the thermodynamic equilibrium in the vortex system determined by local values of j and B is settled [$l \sim (\phi_0/B)^{1/2}$ is the vortex spacing]. After that, the magnetic flux creep, being accompanied by slow decay of macroscopic currents, begins. Such separation is possible only in the case $l_p \ll l$ for which the local redistribution of vortices does not change the macroscopic magnetization. We assume the first stage is fast enough as compared to the second one and so do not consider here metastable filling of pinning positions.

Notice that at $B < H_p$ each pinning position can be occupied by only one vortex, which is due to the interaction energy of two vortices, being at a distance less than the magnetic penetration depth, is of the order of the energy of a lone vortex W_0. The value W_0 exceeds E_p even in the case of the strongest pinning of the vortex core by normal precipitates or local variations of the film thickness as the vortex core itself gives the small contribution to W_0.[6] Considering the filled pinning positions as a localized vortex states, one can conclude that the pinned vortices obey some "exclusion" principle due to which of these states cannot be filled simultaneously by more than one vortex.

The equilibrium filling of the pinning positions can be found from the standard expression for the thermodynamic potential Ω (Ref. 7)

$$\Omega = -kT \ln \sum_N \{\exp[(\mu + E_p)/kT]\}^N . \qquad (3)$$

Here N is the number of vortices which can occupy the state with the energy $-E_p$ counted off from the energy of unpinned vortex, the vortex interaction in the mean-field approximation is taken into account in the dependence of E_p on B and j, μ is the chemical potential in the vortex system, and k is the Boltzmann constant. If the function $f(E)$ is finite at any E, than the probability that two different pinning centers have the same value E_p is negligible and so the number N in Eq. (3) can be equal either to 0 or 1. As a result, the minimization of Eq. (3) with respect to μ yields the "fermion"-energy distribution $p(E)$ (Ref. 7) for pinned vortices

$$p(E) = 1/(e^{(U-E)/kT}+1), \quad n_b = \int_0^\infty p(E) f(E)\, dE , \qquad (4)$$

where $f(E)$ plays the role of the density of localized vortex states and $\mu = -U$.

The "exclusion" principle is not valid for the free vortices which can occupy, without restrictions, the degenerate state $E = 0$ for which the maximum density of all possible space positions of a vortex H_{c2}/ϕ_0 considerably exceeds both the total vortex density B/ϕ_0 at $B < H_p$ and the density of pinning positions n_p. Hence, it follows that the number N can run all integer values from zero to infinity and so the summation in Eq. (3) and the differentiation of the result with respect to μ yields the Bose distribution[7] for the free vortices, i.e.,

$$n_f = (H_{c2}/\phi_0)/[\exp(U/kT) - 1] . \qquad (5)$$

The equation for U follows from the condition of the vortex number conservation $n_b + n_f = B/\phi_0$:

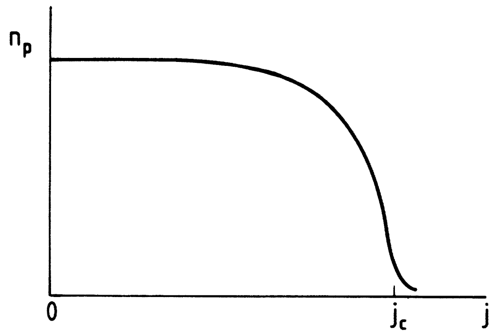

FIG. 1. The density of pinning positions vs j.

$$B = \frac{H_{c2}}{e^{U/kT}-1} + \phi_0 \int_0^\infty \frac{dE\, f(E,B,T,j)}{e^{(U-E)/kT}+1} . \qquad (6)$$

At $U \gg kT$, the value U is the activation energy which depends on T, B, and j. The density of the pinning positions n_p decrease with the increase of j due to the Lorentz forces as it is sketched in Fig. 1. At $j > j_c$ the largest part of the pinning positions disappears, which corresponds to the resistive transition to the flux-flow regime where $n_p \rightarrow 0$ and $U = kT \ln(H_{c2}/B + 1)$.

In the flux-creep regime $kT \ll U$ the stepwise Fermi function $p(E)$, in Eq. (6), varies at $E \simeq U$ much more sharply than the smooth dependence of $f(E)$. Then the integral in Eq. (6) can be evaluated by the standard procedure used in the low-T thermodynamics of the electron gas (see, e.g., Ref. 7), which yields

$$U(T,B,j) = U_0 - \pi^2 k^2 T^2 f'(U_0)/6 f(U_0) , \qquad (7)$$

$$B = \phi_0 \int_{U_0}^\infty f(E,T,B,j)\, dE . \qquad (8)$$

Here $U_0(T,B,j)$ is the minimum depinning energy for the filled positions (Fig. 2), the prime denotes the differentiation with respect to E, and the last term in Eq. (7) is assumed to be small.

At $U \gg kT$ the current-voltage characteristic can be calculated explicitly, assuming the energy $U(j)$ decreases with j. Then at $j \gg j_c kT/U$ the main contribution to $\mathcal{E}$ gives the narrow vicinity of the point $j' = j$ in Eq. (1),

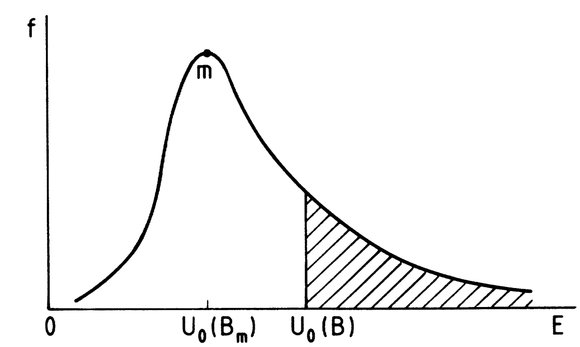

FIG. 2. The qualitative dependence of $f(E)$. The area of the hatched domain is proportional to the total density of vortices.

where one can put $U(j')=U(j)+(j'-j)dU/dj$ and tend the lower limit to $-\infty$. The result is

$$\mathcal{E}=\beta\rho_n kT\,|dU/dj|^{-1}\exp(-U/kT)\,, \tag{9}$$

with $\beta=\rho_f H_{c2}/\rho_n B$. At $j \ll j_c kT/U(j)$ the current-voltage characteristic is linear with the resistivity given by

$$\rho=\beta\rho_n \exp(-U(0)/kT)\,, \tag{10}$$

where the pre-exponent depends weakly on B in accordance with the results of Ref. 1.

Generally, at $j \gg j_c kT/U$ the dependence $U(j)$ determined by Eq. (6) is nonlinear, which correlates with the nonlinearity of $U(j)$ observed in high-T_c films[8] (see also Ref. 9). Consider, for example, the simple case of depinning energies decrease similarly with the increase of j as $E_p(j)=y(j)E_p(0)$, where $y(j)$ is a function vanishing at $j=j_c$. Then all parameters with the dimension of energy in $f(E)$ should be multiplied by $y(j)$, which is equivalent to the scaling $f(E,j)=y(j)^{-1}f[E/y(j)]$ with $n_p(j)$ $=$const. As a result, Eq. (7) becomes

$$U=U_0 y(j)-\pi^2k^2T^2f'(U_0)/6y(j)f(U_0)\,,$$

with U_0 corresponding to $j=0$. Thus, the activation energy $U(j)$ proves to be nonlinear even if the local values $E_p(j)$ depend linearly on j, i.e., $y=1-j/j_c$. In this case the nonlinearity of $U(j)$ increases with T and j.

This nonlinearity manifests itself in the nonlogarithmic time decay of the induced current $J(t)$ in the flux-creep regime. The process is described by the electrodynamic equation $LdJ/dt+V(J)=0$, where L is the inductivity of the superconducting circuit and $V(J)$ is the resistive part of the voltage determined by Eq. (9). Acting in the same manner as when obtaining Eq. (9), one finds

$$U[j(t)]=kT\ln(t/t_0),\quad t \gg t_0\,, \tag{11}$$

where $t_0=L/\beta R$, and R is the resistance of the circuit in the normal state.

The resistive transition in a sample with macroscopic inhomogeneities in $j_c(\mathbf{r})$ and $U(\mathbf{r})$ has the specific features as compared to the case discussed above. In the flux-creep regime $(U \gg kT)$ the relatively weak inhomogeneities in $U(\mathbf{r})$ can result in strong inhomogeneities in $\rho(\mathbf{r})$, which is accompanied by the arising of some domains with high-local-creep rate surrounded by domains with low-creep rate. The existence of such high-creep drops inside a superconductor essentially influences the macroscopic flux creep only if their concentration exceeds the percolation threshold, when the vortices or the vortex bundles[4] can move along the corresponding network of the percolation channels through the whole cross section of the specimen (see Fig. 3). Notice that only a small part of all high-creep drops belongs to the so-called infinite percolation cluster[10] contributing to the macroscopic flux creep, which, in particular, should be taken into account when reconstructing the distribution function of the activation energies $F(W)$ (Ref. 2) by means of the measured dependence $U(B)$.

The above arguments can be illustrated in the framework of the effective-medium theory which gives the following equation for average resistivity $\langle\rho\rangle$ in the two-

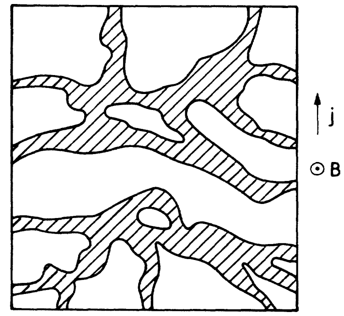

FIG. 3. The percolation network of the high-creep channels (hatched) in a superconducting film.

dimensional (2D) case[10]

$$1=2\int_0^\infty dW\,F(W)\rho(W)/[\rho(W)+\langle\rho\rangle]\,. \tag{12}$$

Here $\rho(W)$ is the local resistivity and the integral of $F(W)$ over W from 0 to ∞ is equal to unity. For $\rho(W)=\rho_0\exp(-W/kT)$ with $\rho_0=$const, Eq. (12) becomes

$$1=2\int_0^\infty dW\,F(W)/(e^{(W-U)/kT}+1)\,, \tag{13}$$

$$\langle\rho\rangle=\rho_0\exp(-U/kT)\,. \tag{14}$$

At $U \gg kT$ the integral (13) containing the Fermi function $p(W)$ can be transformed in the same manner as Eq. (6), which yields

$$U=U_c-\pi^2k^2T^2F'(U_c)/6F(U_c)\,, \tag{15}$$

$$1=2\int_0^{U_c}F(W)dW\,. \tag{16}$$

Here U_c is the percolation threshold for the potential $U(\mathbf{r})$. At $j>j_c kT/U$ the problem becomes more complicated as it reduces to the analysis of current distribution in randomly inhomogeneous nonlinear media.[11]

Thus, the resistive flow of the free vortices can both be uniform $(l_p \ll l)$ and considerably nonuniform (see Fig. 3). In both cases, however, the value U is determined by the form of $f(E)$ and $F(W)$ so the existence of low-energy tails or peaks in $f(E)$ or $F(W)$ due to the effect of granularity, chemical inhomogeneities, etc., results in the essential decrease of U.

The dependences of U on T, B, and j are determined by Eqs. (6) and (13). For instance, at low T the superconducting parameters are independent of T so the function $f(E)$ may also be assumed to be independent of T. Let us introduce the field B_m at which the energy $U_0(B_m)$ corresponds to the maximum in $f(E)$ as shown in Fig. 2. Then the activation energy $U(T)$ increases with T for $B<B_m$ and decreases with T for $B>B_m$ due to the last term in Eq. (7). For macroscopic inhomogeneities, the sign of the last term in Eq. (15) is not so clear in physical meaning and depends on the shape of $F(W)$. In particular, for 2D cases there is the exact result[12] $U_c=\langle U(\mathbf{r})\rangle$, $F'(U_c)=0$ if the function $F(W)$ is symmetrical with respect to the point $W=\langle U\rangle$. The case $F'(U_c)<0$ takes place for the nonsymmetrical function $F(W)$ with a pronounced high-energy tail (see Refs. 2 and 3).

The relation between U and the activation energy U_m measured in flux-creep experiments $U_m = -kTj_c d\ln t/dj$ (Refs. 13–15) is given by $U_m = j_c|dU/dj|$ [see Eq. (11)]. For nonlinear $U(j)$ the energy U_m depends on j, however, in the case of slow decay of $j(t)$ in the flux-creep regime one can put $U_m = U_m(\bar{j})$, where $\bar{j}$ is an average value of $j(t)$ within the measured interval $t_1 < t < t_2$, that is, $j(t_2) < \bar{j} < j(t_1)$ and $j(t_1) - j(t_2) \ll j$. Notice the energy U_m was observed to increase with T at low T (Refs. 13–15), for instance, the data of Ref. 15 for oriented grained $YBa_2Cu_3O_x$ fit the formula $U_m = U_0 + AT^2$ with $U_0 = 30$ meV and $A = 0.244$ meV/K^2 (5 K $< T <$ 30 K), in accordance with the predicted low-T dependence of $U(T)$ given by Eqs. (7) and (15).

As it follows from Fig. 2, the value $U(B)$ decreases at $B < H_p$ on the scale of $B \sim H_p \ll H_{c2}$. The opposite case $B > H_p$ corresponds to the essentially collective pinning for which $U(B)$ varies on the scale of $B \sim H_{c2} \gg H_p$.[9,16] Thus, the distribution of pinning energies leads to the appearance of low-B cusp in $U(B)$ at $B < H_p$. This results in the corresponding low-B cusp in the measured critical current density $\tilde{j}_c$ due to flux creep, the cusp increasing with T

$$\tilde{j}_c = \{1 - [kT/U_m(T,B)]\ln(\mathcal{E}_0/\mathcal{E}_m)\} j_c , \tag{17}$$

where $\tilde{j}_c = j_c(T=0)$, $\mathcal{E}_0$ is a constant of the material, $\mathcal{E}_m$ is the threshold voltage criterium, and $U(j)$ is assumed, for simplicity, to be linear. The correlation between strong dependences of $\tilde{j}_c(B)$ and $U_m(B)$ at $B < 0.5$ T and the increase of low-B cusp in $\tilde{j}_c(B)$ as T increases were observed in Ref. 14 for oriented grained $YBa_2Cu_3O_x$.

Finally, the phenomenological model, taking into account the effect of distribution of pinning energies on the resistive state of superconductors, has been proposed. The results obtained are insensitive to specific pinning mechanisms, which allows us to conclude about some general features of the resistive transition and of the dependences $j_c(T,B)$ and $U(T,B)$.

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