

Finite-size effects and anisotropic melting of the vortex solid in high-temperature superconductors

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A mean-field model of anisotropic melting of the vortex solid in high-temperature superconductors is proposed. For a slab sample with dimensions $l_{ab} \gg l_c$, where $2l_{ab}$ and $2l_c$ are the average diameter of the ab plane and the c axis thickness, respectively, large thermal fluctuations and finite-size effects may result in anisotropic two-dimensional melting at crossover temperatures $T_X(H)$ below the three-dimensional-melting transition $T_M(H)$. Thus a quasi-two-dimensionally ordered vortex-liquid phase may exist in $T_X(H) < T < T_M(H)$. Generally, $T_X(H)$ decreases with the decreasing sample thickness, increasing magnetic field, and larger Ginzburg-Landau parameter $\kappa (\equiv \lambda/\xi)$. In the limit of $\frac{1}{2} H_{c2} \ll H < H_{c2}$, the geometric anisotropy plays a more important role in determining $T_X(H)$ than the electronic-mass anisotropy.

Despite intense efforts in trying to understand the mixed-state properties of high-temperature superconducting oxides,¹⁻⁶ it is an unsettled issue whether the low-temperature phase is a true zero-resistance, "vortex-solid" state, or may be described by the conventional flux-creep model, which asserts nonzero resistance at any finite temperature. Even if such a "vortex-solid" state does exist, it is also unresolved whether the vortex-solid phase is a nearly perfect Abrikosov lattice in the weak-pinning limit,¹⁻⁴ or a "glasslike" state.⁵ Finally, it is not known whether the vortex-solid-vortex-liquid "phase transition" is first-order, second-order, or merely a gradual crossover. Recent controversial experimental results and interpretations⁶⁻¹² have added more complications to this issue.

In this paper, we consider an extreme type-II superconductor ($\kappa \gg 1$) with a high critical temperature (T_{c0}). We focus on the clean-limit approximation, and only discuss the thermal effects on vortex motion. This approach has the advantage of leaving out additional complications, such as the Lorentz force on flux lines in the presence of external currents. Using the elastic theory proposed in Refs. 1-3, we show that large thermal fluctuations and finite-size effects may result in anisotropic two-dimensional melting below the mean-field upper critical field $H_{c2}(T)$. Therefore vortex motion may occur well below $H_{c2}(T)$ in thin samples, unless strong pinning mechanisms are present.

We first consider an upper critical field $H_{c2}(T_M) = H_{c2}(0)(1 - T_M/T_{c0})^{2\nu}$, where T_{c0} is the zero-field superconducting transition temperature, and ν is the static exponent which describes the temperature dependence of the superconducting coherence length ($\xi_s \sim |T - T_{c0}|^{-\nu}$). In general, $\nu = \frac{1}{2}$ in the mean-field approximation, and

$\nu \geq \frac{2}{3}$ for a disordered three-dimensional (3D) XY model.¹³ Since the exact value of ν is still unknown, we only limit the following discussions to the mean-field approximation. We also note that the notations $T_M(H)$ and $H_{c2}(T)$ both refer to the same mean-field, three-dimensional-melting phase-transition boundary in the (H, T) phase diagram.

Generally, the vortex-solid melting transition in a type-II superconductor occurs if $\mu(T \rightarrow T_M) \approx 0$,^{1-4,6} where μ is the shear modulus of the flux-line lattice (FLL). In this context, a vortex correlation length ξ_v is introduced,¹ which may be broken into two components: the longitudinal correlation length $\xi_{v,\parallel}$ (along the magnetic field) and the transverse correlation length $\xi_{v,\perp}$ (perpendicular to the magnetic field). The correlation length ξ_v describes the translational correlation of flux lines in the displacement-field space, and is defined (analogous to the definition for superfluids¹⁴) via the correlation function C_G ,¹

$$C_G(\mathbf{r}_\perp, z) \equiv \langle \exp\{i\mathbf{G} \cdot [\mathbf{u}(\mathbf{r}_\perp, z) - \mathbf{u}(0,0)]\} \rangle,$$

$$C_G(\mathbf{r}_\perp, z \rightarrow \infty) = \exp(-\frac{1}{2} G^2 \langle u^2 \rangle) (1 + \xi_{v,\parallel}/z), \quad (1)$$

$$C_G(r_\perp \rightarrow \infty, z) = \exp(-\frac{1}{2} G^2 \langle u^2 \rangle) (1 + \xi_{v,\perp}/r_\perp),$$

where \mathbf{G} is a reciprocal FLL vector, and \mathbf{u} is the flux-line (FL) displacement field.

From elastic theory and the definition in Eq. (1), the correlation lengths $\xi_{v,\parallel}$ and $\xi_{v,\perp}$ for a finite sample thickness $2l_c$ are related to the shear and tilt moduli μ and K by the following expressions:^{1,15}

$$\frac{\xi_{v,\parallel}}{z} \approx \lim_{z \rightarrow \infty} \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{\cos(Q_n z)}{4l_c} e^{iq_\perp r_\perp} G_i G_j \left[\frac{k_B T}{\mu q_\perp^2 + K q_z^2} P_{ij}^T(q_\perp) + \frac{k_B T}{(2\mu + \lambda) q_\perp^2 + K q_z^2} P_{ij}^L(q_\perp) \right],$$

$$\frac{\xi_{v,\perp}}{r_\perp} \approx \lim_{r_\perp \rightarrow \infty} \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{\cos(Q_n z)}{4l_c} e^{iq_\perp r_\perp} G_i G_j \left[\frac{k_B T}{\mu q_\perp^2 + K q_z^2} P_{ij}^T(q_\perp) + \frac{k_B T}{(2\mu + \lambda) q_\perp^2 + K q_z^2} P_{ij}^L(q_\perp) \right], \quad (2)$$

where $n = \text{integer}$, $Q_n = n\pi/2l_c$, $i, j = x, y$, $P_{ij}^T(q_\perp)$ and $P_{ij}^L(q_\perp)$ are two-dimensional transverse and longitudinal projection operators,^{1,3,4} μ , $(2\mu + \lambda)$, and K are the shear, compression, and tilt moduli, respectively, and the sum over n, i, j is understood. The shear modulus μ is nearly independent of wave vector, and decreases with increasing temperature via the following temperature dependence:¹⁵

$$\begin{aligned} \mu &\approx 7 \times 10^{-3} \left(\frac{H_{c2}}{\kappa} \right)^2 (1 - H/H_{c2})^2 \text{ for } \frac{1}{2} H_{c2} \ll H < H_{c2}, \\ &\approx \frac{1}{4} \frac{H\Phi_0}{(4\pi\lambda_L)^2} \text{ for } H_{c1} < H < \frac{1}{2} H_{c2}, \end{aligned} \quad (3)$$

where λ_L is the London penetration depth. The compression and tilt moduli $(2\mu + \lambda)$ and K are q dependent, and therefore their nonlocal expressions should be considered in calculating Eq. (2). For $\kappa \gg 1$, $M_3 \gg M_1$, $\mathbf{q} = \mathbf{G} + \mathbf{k}$,

$$\begin{aligned} \xi_{v,\parallel}(H, T, \kappa) &\approx \frac{k_B T G_\delta^2}{8\pi} \left(\frac{1}{\mu} + \frac{1}{2\mu + \lambda} \right) \approx \frac{k_B T G_\delta^2}{4\pi\mu} = \left(\frac{600k_B}{\Phi_0 H_{c2}^2(0)} \right) \left[\frac{\kappa^2 T H}{[(1 - T/T_{c0})^{2\nu} - H/H_{c2}(0)]^2} \right], \\ \xi_{v,\perp}(H, T, \kappa) &\approx \frac{k_B T G_\delta^2}{8\pi} \left[\frac{1}{\sqrt{\mu K}} + \frac{1}{[(2\mu + \lambda)K]^{1/2}} \right] \\ &\approx \frac{k_B T G_\delta^2}{4\pi\sqrt{\mu K}} = \left[\frac{220k_B}{\Phi_0 [H_{c2}(0)]^{3/2}} \right] \left[\frac{\kappa^2 T [H(M_3/M_1)]^{1/2}}{[(1 - T/T_{c0})^{2\nu} - H/H_{c2}(0)]^{3/2}} \right]. \end{aligned} \quad (5)$$

Note that both $\xi_{v,\parallel}$ and $\xi_{v,\perp}$ diverge at $T \rightarrow T_M(H)$, because $[1 - T_M(H)/T_{c0}]^{2\nu} = H/H_{c2}(0)$. Thus Eq. (5) is consistent with the assumption of a second-order phase transition at $T_M(H)$. We also note that the correlation lengths in Eqs. (2) and (5) are only correct if there are no dislocations.

Instead of using the Lindemann criterion of melting for an infinite system,¹⁻⁴ we show below that there may be a dimensional crossover temperature $T_X(H)$ below $T_M(H)$, due to the finite sample dimensions.¹⁵ Consider an external magnetic field applied along the c axis of a slab high-temperature superconductor with $l_{ab} \gg l_c$, where l_{ab} and l_c being the averaged ab -plane radius and the half-thickness of the sample, respectively. Since $\xi_{v,\parallel} \gg \xi_{v,\perp}$ and $l_c \ll l_{ab}$, there is a finite temperature interval, $T_X(H) < T < T_M(H)$, in which the conditions $\xi_{v,\parallel} \geq l_c$ and $\xi_{v,\perp} < l_{a,b}$ may be satisfied. Here $T_X(H)$ is defined as the temperature where $\xi_{v,\parallel} = l_c$ for a given field H . We suggest that a continuous 2D melting transition may take place within the temperature interval $T_X(H) < T < T_M(H)$, and that the vortex phase in the temperature range $T_X(H) < T < T_M(H)$ can be described by a "quasi-two-dimensionally ordered vortex liquid."

More explicitly, we find from Eq. (5) that the condition for the onset of a 2D melting ($\xi_{v,\parallel} \geq l_c$) may be written as follows:

$$\begin{aligned} \mathcal{F}(T) &\equiv \left(1 - \frac{T}{T_{c0}} \right)^{2\nu} \leq \mathcal{G}(H, T, \kappa, l_c) \\ &\equiv \left[\frac{H}{H_{c2}(0)} \right] + \left[\frac{A}{H_{c2}(0)} \frac{\sqrt{TH}\kappa}{\sqrt{l_c}} \right], \end{aligned} \quad (6)$$

where $A \equiv (600k_B/\Phi_0)^{1/2}$ in cgs units. Note that $T_X(H)$

and $\frac{1}{2} H_{c2}(T) \ll H < H_{c2}(T)$, we find $(|\mathbf{k}|/G_0) > [(1 - h)/2h\kappa^2] \ll 1$ for most $|\mathbf{k}|$ values, which implies that the nonlocality is satisfied.²⁻⁴ Here $G_0 = 2\pi\sqrt{2H}/3^{1/4} \times \sqrt{\Phi_0}$ is the smallest nonzero reciprocal-lattice vector, and Φ_0 is the flux quantum. Thus, $(2\mu + \lambda) \sim \mu$, and

$$\begin{aligned} K(|\mathbf{k}| \neq 0) &= K_0 \frac{(1-h)}{2h\kappa^2} \left(\frac{M_1}{M_3} \right) \\ &\approx \frac{H^2}{8\pi\kappa^2} \frac{[H_{c2}(T) - H]}{H} \left(\frac{M_1}{M_3} \right), \end{aligned} \quad (4)$$

where $K_0 \equiv K(|\mathbf{k}| = 0) = H^2/(4\pi)$ is the tilt modulus in local elastic theory,² $h \equiv H/H_{c2}(T)$, and M_1, M_3 are the electronic masses for the ab plane and the c axis, respectively.

From Eqs. (2)-(4), $\xi_{v,\parallel}(H, T)$ and $\xi_{v,\perp}(H, T)$ becomes (in cgs units):

occurs at $\mathcal{F}(T_X) = \mathcal{G}(T_X, H, \kappa, l_c)$. The physical significances of $\mathcal{F}(T)$ and $\mathcal{G}(H, T, \kappa, l_c)$ can be manifested by rewriting Eq. (6) into the following expression,

$$H_{c2}(T) \leq H + \Delta H, \quad \Delta H \equiv A\kappa\sqrt{TH/l_c}. \quad (7)$$

The effect of a finite sample thickness on the melting tran-

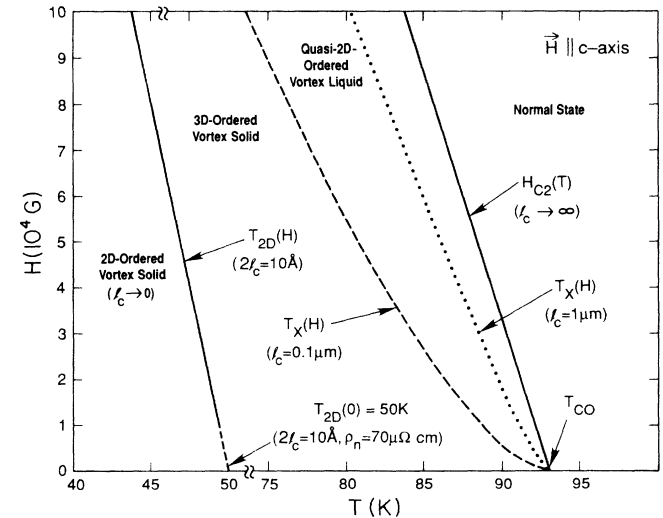


FIG. 1. A proposed mixed-state vortex phase diagram of high-temperature superconductors is shown for $H \parallel c$ axis of the superconductor. The dotted and dashed lines of $T_X(H)$ correspond to sample half-thicknesses $l_c = 1$ and $0.1 \mu\text{m}$, respectively. The two solid lines denote the mean-field upper critical field $H_{c2}(T_M)$ in the thick-sample limit ($l_c \rightarrow \infty$), and the 2D melting transition $T_{2D}(H)$ in the thin-sample limit ($l_c \ll \lambda$), respectively.

sition may therefore be considered as adding an effective magnetic field ΔH to the applied field, such that a 2D melting transition begins as soon as the “total” magnetic field $[H + \Delta H]$ exceeds the upper critical field $H_{c2}(T)$. We emphasize that in the first-order approximation, $T_X(H)$ is not sensitive to the effective-mass anisotropy, because $\xi_{v,\parallel}$ is mostly determined by μ , as shown in Eq. (5).

In Fig. 1, we obtain the magnetic-field dependence of $T_X(H)$ in the mean-field limit ($\nu = \frac{1}{2}$) by calculating the solutions for $\mathcal{F}(T) = \mathcal{G}(H, T)$ of Eq. (6), for samples with half-thicknesses $l_c = 1$ and $0.1 \mu\text{m}$, and parameters $H_{c2}(0) = 10^6 \text{ G}$, $T_{c0} = 93 \text{ K}$, and $\kappa = 200$. From Eq. (6) it is obvious that the values of $T_X(H)$ are sensitive to material properties such as T_{c0} , ν , κ , and $H_{c2}(0)$. We note from Eqs. (6) and (7) that $T_X(H)$ in a thinner sample is consistently lower than that of a thicker sample. Furthermore, the temperature difference $\Delta T(H) \equiv [T_M(H) - T_X(H)]$ increases with increasing magnetic fields and

$$\left[1 - \frac{T_{X'}}{T_{c0}} \right]^{2\nu} = \left[\frac{H}{H_{c2}(0)} \right] + \left[\frac{A'}{H_{c2}(0)} \left[\frac{(T_{X'})^2 H \kappa^4}{l_{ab}^2} \left(\frac{M_3}{M_1} \right) \right] \right]^{1/3}, \quad (8)$$

where $A' \equiv (220k_B/\Phi_0)^{2/3} \approx 2.8 \times 10^{-5} \text{ (cmG/K)}^{2/3}$, in cgs units. We find that for a typical $\text{YBa}_2\text{Cu}_3\text{O}_7$ single crystal with $l_{ab} \approx 0.5 \text{ mm}$, the difference of the crossover temperature $T_{X'}(H)$ from the bulk melting transition temperature $T_M(H)$ is about $\Delta T' \equiv T_M - T_{X'} < 0.1 \text{ K}$ for $H \leq 10 \text{ T}$ and $(M_3/M_1) = 100$, much smaller than that along the c axis. Therefore the vortex-solid melting transition for a slab high-temperature superconductor with $H \parallel c$ axis is generally determined by the longitudinal crossover temperature $T_X(H)$, unless the mass anisotropy is much greater than the geometric anisotropy. That is, unless the condition $(M_3/M_1)(l_c/l_{ab})^2 \gg (A/A')^3 \times (\sqrt{l_c H})/(\sqrt{T} \kappa)$ is satisfied.

For comparison, we remark that in Ref. 3 the FLL melting has been studied by using the Lindemann criterion in an infinite system. The FLL-melting temperature thus obtained is found to be significantly reduced by a large electronic mass anisotropy, in contrast to our finding for $\frac{1}{2} H_{c2}(T) \ll H < H_{c2}(T)$ and a finite sample thickness. This difference may be due to much larger mean-square FL displacements $\langle u^2 \rangle$ for an infinite system ($l_c \rightarrow \infty$). However, a more general description of the FLL melting theory for a finite-size sample in an intermediate magnetic field ($H_{c1} < H < H_{c2}$) is still to be explored.

In the thick sample limit ($l_c \rightarrow \infty$), we note that $\mathcal{G}(H, T) \rightarrow \{H/H_{c2}(0)\}$ from Eq. (6), and therefore $T_X(H) = T_M(H)$, indicating an ideal three-dimensional

decreasing sample thicknesses.

These theoretical predictions qualitatively agree with recent experimental results by various groups:^{8–12} (i) We find that the values of $T_X(H)$, determined by the temperature of onset resistivity, generally decrease faster with increasing field in $\text{YBa}_2\text{Cu}_3\text{O}_7$ films and thin single crystals than those in thick single crystals.^{8,10–12} (ii) An anomalous change of slope in the resistivity versus temperature plot at a constant field ($H_{c1} \ll H < H_{c2}$) occurs above the onset of resistivity;^{10,12} i.e., above $T_X(H)$. We attribute the temperature where the anomaly occurs to $T_M(H)$, and find that $\Delta T(H)$ indeed increases with increasing H , consistent with our predictions.

In addition to the finite-size effect imposed by the thickness of a sample, there is also a transverse crossover temperature $T_{X'}(H)$ in the ab plane due to the finite sample dimension. We can estimate $T_{X'}$ by assigning $\xi_{v,\perp} = l_{ab}$ and by using Eqs. (4) and (5):

melting. On the other hand, $T_X \rightarrow T_{2D}$ in the thin-film limit ($l_c \ll \Lambda$, $\Lambda \equiv \lambda_B^2/l_c$ denotes the effective penetration depth in 2D), where the 2D-melting temperature T_{2D} is implicitly determined by the universal condition:¹⁶

$$\lim_{T \rightarrow T_{2D}} \left[\frac{1}{\mu_{2D}(T)} + \frac{1}{\mu_{2D}(T) + \lambda_{2D}(T)} \right] = \frac{a_0^2}{4\pi k_B T_{2D}}, \quad (9)$$

where $a_0 \equiv 1.075 \sqrt{\Phi_0/H}$ is the Abrikosov FLL constant, and μ_{2D} , λ_{2D} are the Lamé constants of the two-dimensional FLL. We may obtain quantitative estimates of the 2D-melting temperature as a function of the magnetic field from the two-dimensional FLL melting theory in thin-film superconductors.¹⁷ According to Ref. 17, the zero-field 2D-melting temperature for a thin film with sheet resistance R_n is

$$T_{2D}(0) \approx T_{c0} \left[1 + \frac{3.8}{A_1} \left(\frac{R_n}{R_c} \right) \right]^{-1}, \quad (10)$$

where $R_c \equiv \hbar/e^2 = 4.12 \text{ k}\Omega/\text{square}$, and $0.4 < A_1 \leq 0.75$ is a constant.¹⁷ Typically, $T_{c0} = 93 \text{ K}$ and the normal-state resistivity $\rho_n \approx 70 \mu\Omega \text{ cm}$ for $\text{YBa}_2\text{Cu}_3\text{O}_7$ single crystals. Assuming a sample thickness $2l_c = 10 \text{ \AA}$, and $A_1 = 0.75$,¹⁷ we obtain $T_{2D}(0) \approx 50 \text{ K}$.

The 2D-melting temperature $T_{2D}(H)$ in a magnetic field can be obtained by rewriting Eq. (9) into the following equation (in cgs units):

$$k_B T_{2D}(H) = \frac{0.353 A_2}{8\pi\sqrt{3}} \left[\frac{H_{c2}(T_{2D}) - H}{H_{c2}(T_{2D})} \right]^2 \frac{\Phi_0^2}{4\pi^2 \Lambda(T_{2D})}, \\ \approx \frac{0.353 A_2 \Phi_0^2 l_c}{32\pi^3 \sqrt{3}} \left[1 - \frac{H}{H_{c2}(0)(1 - T_{2D}(H)/T_{c0})} \right]^2 [\lambda_B(0)]^{-2} \left[1 - \left(\frac{T_{2D}(H)}{T_{c0}} \right)^4 \right], \quad (11)$$

where $A_2 \sim 1$ is a constant,¹⁶ and a two-fluid model has been assumed in Eq. (11) so that $\lambda_B(T) = \lambda_B(0)[1 - (T/T_{c0})^4]^{-1/2}$. We thus obtain an estimate of the 2D-melting phase boundary $T_{2D}(H)$ for a thin-film $\text{YBa}_2\text{Cu}_3\text{O}_7$ with $2l_c = 10 \text{ \AA}$ and $\lambda_B(0) \sim 10^3 \text{ \AA}$, as shown in Fig. 1.

The vortex phases are therefore summarized as follows: (i) A 3D-ordered vortex-solid phase for $T < T_X(H)$, if l_c is in the bulk limit; or a 2D-ordered vortex-solid phase for $T < T_{2D}(H)$, if $l_c \ll \Lambda$. (ii) A quasi-2D-ordered vortex-liquid phase for $T_X(H) < T < T_M(H)$, which continuously melts with increasing temperature. (iii) The normal state for $T > T_M(H)$. Thus $T_X(H)$ is bound by two limits: $T_M(H)$ if $l_c \rightarrow \infty$, and $T_{2D}(H)$ if $l_c \ll \Lambda$.

Since accurate predictions of $T_M(H)$ and $T_X(H)$ depend on the material properties, only the principles of obtaining $T_X(H)$, rather than the absolute values of $T_X(H)$ estimated here, should be taken seriously. We also note that our semiempirical calculations are based on a mean-field, clean-limit approximation. Obviously the material parameters T_{c0} , ν , κ , $H_{c1}(0)$, and $H_{c2}(0)$ may be very sensitive to the presence of pinning defects and dislocations. It is reasonable to expect modifications for $\xi_{v,\parallel}$ and $\xi_{v,\perp}$ in the strong pinning or extremely dirty limit, so that the crossover temperature $T_X(H)$ may be quite different from our clean-limit approximation. In addition, we note that $T_X(H)$ may be reduced further if $\nu > \frac{1}{2}$ [see Eq. (6)]. Thus, we expect more significant finite-size effects in the XY critical regime, where $\nu \approx \frac{2}{3}$.¹³

Finally, we note that the anisotropic, continuous 2D-melting phenomenon is much more significant in high-temperature superconductors than in conventional type-II superconductors. As shown in Eqs. (6) and (7), the magnitude of the deviation of $T_X(H)$ from $H_{c2}(T)$ is $\Delta H \propto \kappa \sqrt{TH}/l_c$. Consequently, the values of ΔH in high-temperature superconductors are generally 1–2 orders of magnitude greater than those in conventional superconductors, due to higher transition temperatures, higher upper critical fields, and larger κ values in the former case.

In summary, we have shown that large thermal fluctuations and finite-size effects of high-temperature superconductors may result in an anisotropic, the continuous two-dimensional vortex-solid melting below the mean-field $H_{c2}(T)$. The onset temperature $T_X(H)$ of the continuous 2D melting is sensitive to the sample thickness (l_c) and other material parameters, such as T_{c0} , ν , κ , and $H_{c2}(0)$. In the limit of $\frac{1}{2}H_{c2} \ll H < H_{c2}(T)$ and for given material properties, $T_X(H)$ coincides with the mean-field upper critical field [$H_{c2}(T_M)$] if $l_c \rightarrow \infty$; $T_X(H)$ decreases with decreasing sample thickness l_c , and approaches the two-dimensional melting temperature $T_{2D}(H)$ if $l_c \ll \Lambda$.

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