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# Ballistic quasiparticle propagation and symmetry of the superconducting order parameter

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We suggest the study of the ballistic propagation of quasiparticles in anisotropic superconductors as a means to identify the symmetry of the superconducting order parameter. Quasiparticles injected by a point contact could be detected by an array of tunneling junctions. The resulting pattern could be interpreted as a real-space image of the wave-vector dependence of the superconducting gap.

## INTRODUCTION

The problem of identifying the symmetry of the order parameter of an anisotropic superconductor, i.e., a superconductor with an energy gap $\Delta_{\mathbf{k}}$ that vanishes on lines or points on the Fermi surface, has not found a satisfactory solution yet. Although the presence of excitations at arbitrarily low energies completely changes the low-temperature properties of these superconductors as compared to ordinary superconductors, experimental determination of the specific heat and various transport coefficients have not led to a clear identification of an anisotropic superconducting order parameter: the interpretation of the experiments is hampered by the influence of impurities,[1] possible existence of order-parameter domains,[2] and a variety of other effects (for a list of experiments proposed so far and their problems see Ref. 3). Since the *existence* of anisotropic superconductivity has been made very probable by the observation of two superconducting transitions in the heavy-fermion compounds $U_{1-x}Th_xBe_{13}$ (Ref. 4) and $UPt_3$,[5] new experiments are necessary that allow us to study the *structure* of the order parameter. We propose to use a point contact to inject low-energy quasiparticles into the anisotropic superconductor. If their kinetic energy $E$ is less than the maximum of the gap parameter $\Delta_{\mathbf{k}}$ on the Fermi surface, the propagation of quasiparticles is possible only in directions $\mathbf{k}$ where $E > \Delta_{\mathbf{k}}$. This allows for the determination of the zeros in $\Delta_{\mathbf{k}}$.

## THE EXPERIMENT

An *anisotropic* superconductor is characterized by a single-particle spectrum $E_{\mathbf{k}} = (\xi_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2)^{1/2}$, where $\xi_{\mathbf{k}} = k^2/2m - \mu$ is the kinetic energy measured from the Fermi energy (we have assumed a spherical Fermi surface for simplicity) and $\Delta_{\mathbf{k}}$ is the $\mathbf{k}$-dependent gap in the spectrum. The superconductor is called *unconventional* if the order parameter belongs to a symmetry which is different from the trivial even-parity $\Gamma_1$ symmetry.[6] In particular, unconventional superconductors often have anisotropic gaps.

A junction consisting of a normal conductor and an anisotropic superconductor will behave very differently as a function of the intrinsic reflection coefficient $R$ of the interface. If the interface is ideal, i.e., if there are no interface potentials or Fermi velocity discontinuities, an electron incoming with wave vector $\mathbf{k}$ from the normal side will be subject to *Andreev reflection*[7] if its kinetic energy is less than $\Delta_{\mathbf{k}}$. That means that a Cooper pair will be transmitted into the superconductor whereas a hole is reflected in the direction of the incoming electron (retroreflection).

If, on the other hand, the interface is strongly nonideal (as in the case of a normal point contact on a superconductor), an incoming electron will be specularly reflected with a high probability, and we obtain a normal-superconductor (NS) tunneling junction that will not transmit any current for low voltages $V$ (i.e., $eV < \Delta_{\mathbf{k}}$ corresponding to low energies of the incoming particles). For voltages $V$ such that $eV > \Delta_{\mathbf{k}}$ a quasiparticle current will be transmitted. In Fig. 1 these two extreme cases are shown in terms of Andreev reflection and transmission coefficients for a given direction $\mathbf{k}$.[3] Reflection and transmission coefficients and differential conductivity are related as follows: If a voltage $V$ is applied across the interface, the current on the normal side (that we assume to have an energy-independent density of states) can be written as

$$I_{\mathbf{k}} \sim \int_{-\infty}^{\infty} dE\,[f(E-eV)-f(E)]\mathcal{T}(E,\mathbf{k}) = \tfrac{1}{2}\int_0^{\infty} dE \left[\tanh\left(\frac{E+eV}{2T}\right) - \tanh\left(\frac{E-eV}{2T}\right)\right]\mathcal{T}(E,\mathbf{k})\,. \tag{1}$$

Here, $I_{\mathbf{k}}$ is the quasiparticle *charge* current (note that a Bogolyubov quasiparticle carries a current of $ev_F$ regardless of its group velocity; see, e.g., Ref. 8) in the direction $\mathbf{k}$ if a voltage $V$ is applied, $f$ is the Fermi function, and $\mathcal{T}(E,\mathbf{k})$ is a generalized transmission coefficient which depends on the experimental situation under consideration:

$$\mathcal{T}(E,\mathbf{k}) = 1 + R_A(E,\mathbf{k}) - R_N(E,\mathbf{k}) \tag{2}$$

 

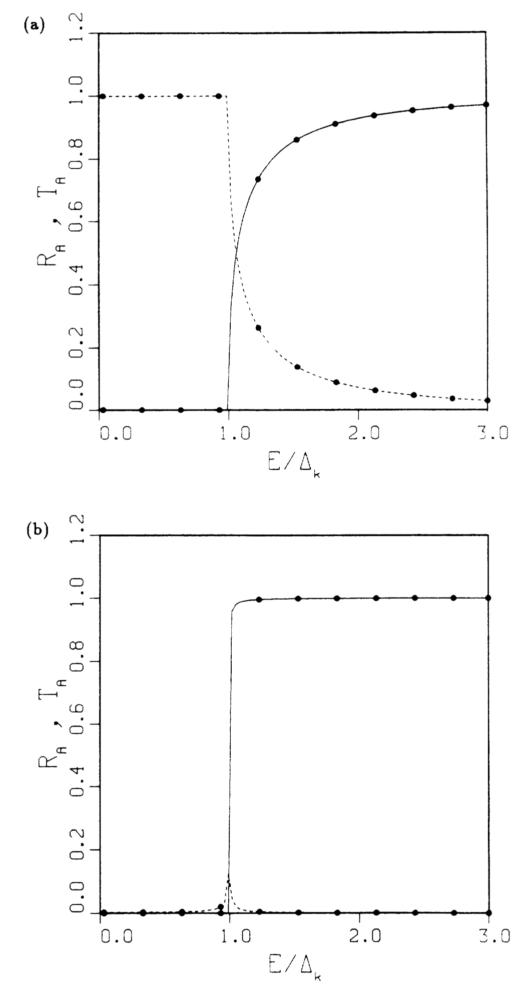


FIG. 1. Andreev reflection coefficient $R_A$ (dashed line) and transmission coefficient $T_A$ (solid line) vs quasiparticle energy $E$ for an NS interface. The energy $E$ is measured in units of $\Delta_{\mathbf{k}}$, the gap for quasiparticle excitations with wave vector **k**. The intrinsic reflection coefficient $R$ was chosen to simulate (a) an ideal contact with $R=0$ and (b) a tunneling junction with $R=0.9$.

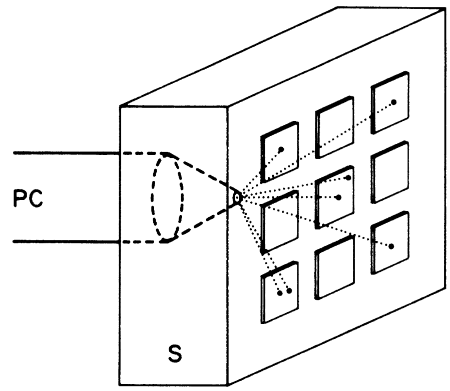


FIG. 2. Experimental configuration is proposed: Electrons are injected by the point contact PC into one side of the superconductor $S$ and detected by an array of SN junctions on the opposite side. We have assumed a point contact diameter $\sim 0.1$ $\mu$m (Ref. 9), a sample thickness of several $\mu$m, and an SN-junction area of the order of 1 $\mu m^2$ per junction.

if we are interested in the total current through the interface, with $R_A$ and $R_N$ denoting the Andreev and normal reflection coefficients, respectively. Note that Andreev reflection increases the current since the reflected holes carry positive charge.

Since Eq. (1) predicts a quasiparticle current depending on direction, we propose to analyze this angular dependence with an array of NS junctions as counterelectrodes to the injecting point contact. This is illustrated in Fig. 2 where we have assumed a point-contact diameter $\sim 0.1$ $\mu$m,[9] a sample thickness of several $\mu$m, and a junction area of the order of 1 $\mu m$.[2]

Since any scattering mechanism is detrimental to this experiment, the mean free path for quasiparticles in the sample has to be larger than (or at least of the order of) the sample thickness. Possible mechanisms include electron-phonon scattering, pair recombination, both of which are negligible at low temperatures, as well as impurity scattering. The quasiparticle mean free path in an-

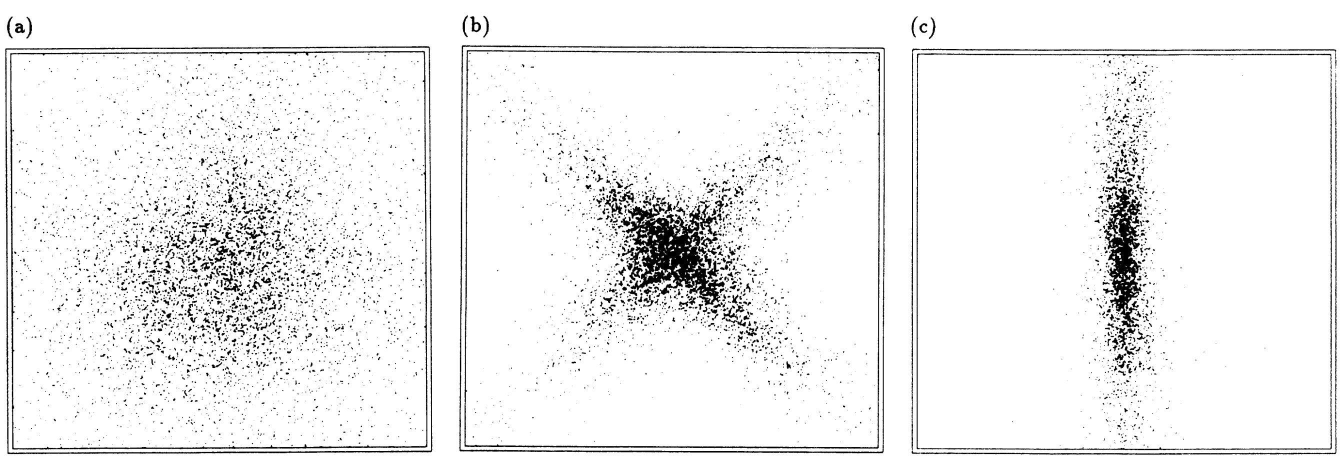


FIG. 3. Results of Monte Carlo simulations showing the density of quasiparticles arriving at the sample surface opposite to the point contact. Each of the squares shows an area of $2d\times 2d$, where $d$ is the sample thickness. (a) $\Delta=0$, (b) $\Delta_{\mathbf{k}}=k_x^2-k_y^2$, and (c) $\Delta_{\mathbf{k}}=k_x(k_y+ik_z)$.

isotropic superconductors with impurities depends very much on the scattering strength of the impurities and on the type of the order parameter.[1,10,11] We will assume that samples can be prepared that are sufficiently clean. They also have to be single crystalline: This is a necessary condition to prevent a multidomain structure of the order parameter.[2]

To illustrate the current distribution, we have evaluated Eq. (1) using a Monte Carlo procedure. We assumed isotropic injection with $k_z > 0$ by the point contact, $R = 0.1$ for the intrinsic reflection coefficient, and show the density of quasiparticles arriving on the opposite side of the superconductor, i.e., the $xy$ plane. In Fig. 3 we show three typical results at a temperature of $0.1T_C$ and a junction voltage of $eV = 0.3\Delta_{max}$: (a) illustrates the case of an isotropic $s$-wave superconductor; no structure is apparent aside from the increased density of points in the center of the diagram due to purely geometrical effects (isotropic distribution projected on a plane). In (b), a $d$-wave superconductor with $\Delta_{\mathbf{k}} = k_x^2 - k_y^2$ has been chosen. The quasiparticles cannot propagate in the directions where the gap is maximal, only along the "channels" where $|k_x| \sim |k_y|$ so that a clear pattern is obtained. In (c) we show the results for yet another $d$-wave superconductor: here, $\Delta_{\mathbf{k}} = k_x(k_y + ik_z)$ so that the quasiparticle current should be largest for the detectors arranged along the $y$ axis.

In conclusion, we have described a method of probing the directional dependence of the superconducting order parameter by transmission of quasiparticles through a single-crystal sample. The detection of the quasiparticles by an array of SN tunnel junctions provides a real-space image of $\Delta_{\mathbf{k}}$ and allows one to identify the type of the order parameter.

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