Kosterlitz-Thouless mechanism of two-dimensional superconductivity

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The possibility of a nonzero T_c superconducting phase transition in a purely two-dimensional system is discussed. We present a parity-invariant model that exhibits perfect diamagnetism and superconductivity due to the fact that the electromagnetic $U_E(1)$ group is realized in the Kosterlitz-Thouless (KT) mode in the vacuum. The superconducting phase transition is of the KT type, and the transition temperature is calculated through the parameters of the model. The connection with the anyon superconductivity mechanism is discussed.

The conventional explanation of superconductivity phenomenon is based on the spontaneous breakdown of the continuous $U_E(1)$ symmetry of electric charge.[1] Most of the CuO high-T_c superconductors, however, have a highly anisotropic, almost two-dimensional structure.[2] Hohenberg, Mermin and Wagner, and Coleman[3] proved that in purely two-dimensional (2D) systems the critical temperature of a phase transition that involves spontaneous breaking of continuous symmetry is strictly zero. Therefore, to get T_c of order 100 K one has to introduce a weak interplane coupling.[4] This strategy underlies most theories of high-T_c superconductivity.

There exists, however, the possibility of a nonzero T_c phase transition in two dimensions (with *zero* interplane coupling) via the Kosterlitz-Thouless (KT) mechanism.[5] There is a growing body of experimental data that points to the KT nature of the superconducting phase transition in CuO materials.[6] In such transitions no continuous symmetry is broken (the Mermin-Wagner-Coleman theorem does not apply) and the conventional explanation of superconductivity cannot be exploited. On closer inspection it becomes clear that the essential feature of spontaneous breaking of the $U_E(1)$ symmetry, necessary to explain perfect diamagnetism, is the appearance of a massless Nambu-Goldstone (NG) boson.[1] After coupling to electromagnetism this boson "mixes" with the photon and generates the photon's mass via the Anderson-Higgs mechanism.

The KT phase is associated with a different type of $U_E(1)$ symmetry realization. The symmetry is not broken but the vacuum is nevertheless not a singlet. As a result, a massless mode (which we shall call a KT boson) appears. It has quantum numbers identical to those of the NG boson. This massless mode can produce equally well perfect diamagnetism (the Meissner effect).

In this Brief Report we construct a class of 2D models exhibiting the above mechanism for superconductivity. They are based on our previous observation that in some (2+1)-dimensional theories of QED_3 type, global symmetries are realized in the KT mode.[7] Abelian gauge theories with several fermion species arise as a continuum limit of microscopic Hubbard-type models[8] which encourages us to consider this class of theories as possibly relevant to high-T_c superconductivity.

For the sake of simplicity we concentrate on Lorentz-invariant theories although all the arguments go through in the nonrelativistic case.[9] Let us consider the following simple Lagrangian (not coupled yet to electromagnetic field):

$$\mathcal{L}_0=\bar{\psi}(i\partial\!\!\!/-m\tau_3)\psi-gS_\mu\bar{\psi}\gamma_\mu\tau_3\psi-\tfrac{1}{4}G_{\mu\nu}^2\,. \tag{1}$$

Here ψ_a, $a=1,2$ is a doublet of two-component complex (Dirac) spinors. The matrix $\tau_3=\text{diag}(1,-1)$ acts on the flavor index a. S_μ is a vector field and $G_{\mu\nu}\equiv\partial_\mu S_\nu-\partial_\nu S_\mu$ is its field strength.

The Lagrangian Eq. (1) for any value of m is invariant under parity, global $U_E(1)$ transformations, and local U(1) gauge transformation $\psi(x)\rightarrow\exp[ig\lambda(x)\tau_3]\psi(x)$, $S_\mu\rightarrow S_\mu-\partial_\mu\lambda$, which we shall call chiral rotation. The corresponding Noether currents are $J_\mu=\bar{\psi}\gamma_\mu\psi$ and $I_\mu=\bar{\psi}\gamma_\mu\tau_3\psi$. The field ψ_a (not very far from the phase-transition region) describes fermionic excitations. The field S_μ should be thought of as describing a bosonic excitation which arises from interactions on the microscopic level.

The Hubbard-Stratonovich field a_μ is usually introduced in the mean-field treatment of the Hubbard-Heisenberg model.[8] However, it cannot be identified with our S_μ. First, the gauge invariance associated with a_μ is present only in the strong coupling limit of the Hubbard model at half-filling. Second, a_μ couples to electromagnetic current $J_\mu=\bar{\psi}\gamma_\mu\psi$, whereas S_μ couples to the chiral current $I_\mu=\bar{\psi}\gamma_\mu\tau_3\psi$. Third, a_μ is a compact variable unlike S_μ.

The main result of Ref. 7 in the context of the present model is that the $U_E(1)$ symmetry is implemented in the KT mode and the S_μ field represents the corresponding KT boson. This follows from the observation that quantum effects generate a nonzero matrix element of J_μ between the vacuum and a state containing one quantum of the transversal component of S_μ.

$$\langle 0|J_\mu|S\rangle=\frac{g}{4\pi}\frac{k_\mu}{|\mathbf{k}|^{1/2}}\,. \tag{2}$$

This expression is valid to all orders in perturbation theory.

The symmetry, however, is not broken spontaneously since all symmetry-violating amplitudes vanish. As a consequence, there appears a pole at zero momentum in the current-current correlator, indicating superconductivity

$$\langle TJ_\mu(k)J_\nu(-k)\rangle = \frac{g^2}{(4\pi)^2}\left[\frac{k_\mu k_\nu}{k^2} - g_{\mu\nu}\right]. \quad (3)$$

The vacuum is a singlet of both parity and chiral rotations.

The exchange of massless S_μ quanta generates a long-range logarithmic confining potential between fermions. Importantly, S_μ couples to the chiral current rather than to the electromagnetic one. Therefore all finite energy excitations are chiral singlets, although they may carry electric charge. In particular, the spectrum does not contain fermions. Obviously, there will be mesonic bound states in the following channels:

$$X_+ = \epsilon^{ab}\psi_a^\dagger\psi_b^\dagger,\ X_- = \epsilon^{ab}\psi_a\psi_b,\ Y^a = \psi_a^\dagger\psi_a. \quad (4)$$

The Y boson is a neutral parity doublet, whereas the $X\pm$ are charged parity singlets, carrying electric charges 2 and -2, respectively. As the temperature increases thermal fluctuations cause deconfinement of fermions. Kosterlitz and Thouless showed[5] that this confinement-deconfinement phase transition occurs at the temperature

$$T_c \approx \frac{g^2}{8\pi}. \quad (5)$$

This expression is valid when the fermions are infinitely heavy. For finite mass the critical temperature is lower. The first-order correction is

$$T_c = \frac{g^2}{8\pi}(1 - ae^{-8\pi m/g^2}), \quad (6)$$

where a is a positive constant of order unity.

Above this temperature there is a plasma of free chiral charges in which S_μ excitations are screened. The zero-momentum pole in the correlator of electric currents disappears and the theory describes a normal metal.

Let us now couple the system to external electromagnetic fields. To illustrate the basic mechanism of perfect diamagnetism we consider first the standard coupling to the two-dimensional electromagnetic potential A_μ^*,

$$\mathcal{L} = \mathcal{L}_0 + eJ_\mu A_\mu^* - \tfrac{1}{4}F^{*2}. \quad (7)$$

Integrating over fermions we obtain the effective Lagrangian for A_μ^* and S_μ. The quadratic part contains the mixing terms

$$\mathcal{L}_{\text{eff}}[A^*,S] = -\tfrac{1}{4}G^2 - \tfrac{1}{4}F^{*2} + \frac{e^*g}{4\pi}\epsilon^{\mu\nu\lambda}A_\mu^*\partial_\nu S_\lambda. \quad (8)$$

The following linear transformation

$$A_+^* = \frac{1}{\sqrt{2}}(A^* + S),\quad A_-^* = \frac{1}{\sqrt{2}}(A^* - S), \quad (9)$$

diagonalizes it,

$$\mathcal{L}_{\text{eff}}[A_+^*, A_-^*] = -\tfrac{1}{4}F_+^{*2} - \tfrac{1}{4}F_-^{*2} + \frac{\mu^*}{4}\epsilon^{\mu\nu\rho}(A_{+\mu}^*F_{+\nu\rho}^* - A_{-\mu}^*F_{-\nu\rho}^*), \quad (10)$$

where $\mu^* = e^*g/4\pi$ is the Chern-Simons topological mass.[10] Note the opposite signs of topological masses of $A_{+\mu}^*$ and $A_{-\mu}^*$. This is due to the fact that parity is not spontaneously broken: A_+^* and A_-^* constitute a parity doublet. The propagator of electromagnetic vector potential A_μ^* is

$$\langle A_\mu^*(k)A_\nu^*(-k)\rangle = \frac{1}{k^2 - \mu^{*2}}\left[g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right]. \quad (11)$$

The photon acquires a *parity-conserving* (nontopological) mass. This mechanism of photon mass generation will be called the vector Anderson-Higgs mechanism. The field S_μ also acquires mass μ. In particular this means that logarithmic potential is cut off on scales larger than $1/\mu$. Although the KT phase transition is smeared out, the qualitative behavior of the system below T_c is still the same.[11]

The field $F_{\mu\nu}^*$ $(\mu,\nu = 0,1,2)$ cannot describe the B_z, E_x, E_y components of the real 3D electromagnetic field for several obvious reasons. The coupling e^* appearing in the Lagrangian Eq. (7) has the dimension of the square root of *mass*, while electric charge e is dimensionless. Also the potential between two charges e^* confined to the plane is logarithmic rather than $1/r$. This unphysical coupling would confine electric charges, thus neutralizing any charge carriers.

A proper way to describe the coupling of the 3D electromagnetic field to charges and currents confined to a plane is by the action

$$S = \int d^2x\,dt\left[\mathcal{L}_0 + eA_\mu J_\mu - F_{\mu\nu}\frac{1}{\sqrt{\partial^2}}F^{\mu\nu}\right]. \quad (12)$$

Here $F_{\mu\nu}$ $(\mu = 0,1,2)$ are 3D physical magnetic field B_z and in-plane components of electric field E_x, E_y. The Lagrangian Eq. (12) is derived in the following way. Assuming that charges and currents are confined to the xy plane at all times, one can find a unique solution of the 3D Maxwell equations for $E_z(x,y,z)$, $B_x(x,y,z)$, $B_y(x,y,z)$ as well as for E_x, E_y, and B_z outside the plane. This is then substituted into the 3D Maxwell action and the integration over the z coordinate is performed. One can easily see that this action leads to a $1/r$ potential in the static case. The derivation of Eq. (12) is given elsewhere.[12] Now performing the steps that led from Eq. (8) to Eq. (11) we obtain

$$\langle A_\mu(k)A_\nu(-k)\rangle = \frac{1}{|k| - \mu}\left[g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right], \quad (13)$$

where $\mu = e^2g^2/(4\pi)^2$ is the physical photon mass. The penetration depth is therefore

$$\lambda = \frac{4\pi}{\alpha g^2}, \quad (14)$$

where α is the fine-structure constant.

For the sake of simplicity we took in Eq. (12) the light velocity and the velocity of the S_μ field waves to be equal. This is of course not a realistic assumption but it does not qualitatively change the picture. Restoring the velocities v and c in the Lagrangian Eq. (12) we obtain

$$\mathcal{L}=\frac{1}{2}(\mathcal{E}^2-\mathcal{B}^2)+\frac{1}{2}\left[E^i\frac{1}{\sqrt{\partial^2}}E^i-B\frac{1}{\sqrt{\partial^2}}B\right]+\psi^\dagger\left[i\frac{\partial}{\partial t}-iv\alpha_i\partial_i-mv^2\right]$$
$$\times\psi-g(\psi^\dagger\tau_3\psi S_0-\psi^\dagger\alpha_i\tau_3\psi S_i)-e(\psi^\dagger\psi A_0-\frac{v}{c}\psi^\dagger\alpha_i\psi A_i)\,,$$

where $\mathcal{B}=\partial_1 S_2-\partial_2 S_1$, $\mathcal{E}_i=-(1/v)\dot{S}_i+\partial_i S_0$ and $B=\partial_1 A_2-\partial_2 A_1$, $E_i=-(1/c)\dot{A}_i+\partial_i A_0$. The penetration depth is determined from the transversal part of the correlator of A_i:

$$\langle A_i A_j\rangle=\frac{c}{(c^2k^2-\omega^2)^{1/2}-(eg/4\pi)^2}\left[\delta^{ij}-\frac{k_ik_j}{k^2}\right]-\frac{c\omega^2}{c(c^2k^2-\omega^2)^{3/2}-(eg/4\pi)^2(v^2k^2-\omega^2)}\frac{k_ik_j}{k^2}\,.$$

The penetration depth remains unchanged $\lambda=4\pi c/\alpha g^2$ and independent of v as in the usual 3D metal superconductors.

The continuum model we discussed should be considered as a continuum limit of a microscopic theory. Several of the key ingredients of the continuum model presented above [Eq. (1)] are present in the conventional description of superfluidity in ^{4}He thin films.[13] In this system the analog of S_μ is the third sound wave. It minimally couples to the *current of vortices in superfluid* described as singularities of the phase of a scalar order parameter. This is analogous to our chiral current I_μ. The particle number is analogous to the electric charge density J_0. The obvious differences are that vortex variables were not introduced explicitly and that the field ψ carries both chiral charge I (vorticity) and electric charge J (particle number), whereas in ^{4}He vortices do not carry the particle number charge.

Similar description applies also to thin superconducting metal films.[14] In this case the logarithmic potential binds magnetic vortices and antivortices in pairs. The chiral charge of Eq. (7) corresponds to the density of magnetic vortices. This can be seen by calculating induced chiral charge in an external magnetic field. It is given by[15]

$$I_\mu=\frac{e^2}{4\pi}\tilde{F}_\mu\,.$$

Again, contrary to the standard description, our "vortices" are charged.

It is interesting to note that in the model of superconductivity we considered, the chiral charge I_0 is always induced in the external magnetic field and therefore can be identified with magnetic flux. Any immediate generalization, like introducing several fermionic species, shares this basic feature. This follows from the fact that the matrix element of electric current $\langle 0|J_\mu|S\rangle$ in Eq. (2) and the induced vacuum chiral current in the external magnetic field are given by the same diagram.[7,15]

The mass term in the Lagrangian Eq. (1) need not be put in by hand. Instead one can start with a four-fermion interaction of the form $(\bar{\psi}\tau_3\psi)^2$. This term preserves the discrete Z_2 symmetry $\psi_1\rightarrow\psi_2$ which is present in the lattice models. Nonperturbatively four-Fermi terms are not irrelevant in 2+1 dimensions.[16] The mass is then generated dynamically via spontaneous breaking of the Z_2 symmetry. Since the symmetry is discrete, the symmetry restoring phase transition even in 2D occurs at finite temperature.[17] Another possibility is that the mass is generated dynamically due to the gauge interaction itself. Analytical[18] and numerical[19] calculations suggest that for sufficiently small number of flavors ($N<8$) parity conserving mass is generated dynamically.[20]

Note that, in order to have the KT-type phase transition into a superconducting state, two ingredients are generally required. First, there should exist a zero mode to cause perfect diamagnetism. Second, there should be logarithmic potential between "vortices." In the model Eq. (1) both arise naturally as a result of KT realization of the $U_E(1)$ symmetry.

As the last point, we comment on the relation between the KT mechanism and the anyon picture of superconductivity.[21,22] Anyons can be described by the coupling of a fermion field to the so-called statistical gauge field a_μ. The action of a_μ contains just the Chern-Simons term $(\gamma/8\pi)\epsilon_{\mu\nu\lambda}a_\mu f_{\nu\lambda}$. For integer values of statistical parameter $\gamma=n$ the current-current correlator acquires a pole at zero momentum and anyon gas becomes a superconductor. In particular, the theory with $n=2$ describes semions. In this case the theory has an SU(2) global symmetry, which may mirror the SU(2) spin symmetry of the Hubbard model. In Ref. 7 we observed that, at the same value of statistical parameter, the $U_E(1)$ symmetry is implemented in the KT mode. This supports the conjecture of Ref. 22 that an anyon superconductor does not possess an order parameter. Note, however, the important phenomenological difference between Eq. (1) and an anyonic model of high T_c. Our model conserves parity in all phases, whereas the anyon mechanism leads to observable T- and P-breaking effects.[23]

To conclude, we presented a KT model of two-dimensional superconductivity leading to finite T_c. The role of an interplane coupling is just to shift by small amount the values obtained in a 2D theory. It would be interesting to verify our conclusions beyond perturbation theory (for example by Monte Carlo simulation) and to construct a detailed microscopic model leading to a theory of this type in the continuum limit.

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