Charge fluctuations: Spin fluctuations and superconductivity in a CuO_2 sheet

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A $U_d = \infty$ Anderson lattice model with a tight-binding band structure is used to model the CuO_2 sheets of the high-T_c oxides. The model is treated within a strong-coupling slave-boson method treating spin and charge fluctuations on equal footing. At the mean-field level the normal state is a Fermi liquid at finite doping with a transition to a charge-transfer insulator at half-filling for a large enough charge-transfer gap. Charge fluctuations give rise to a repulsive quasiparticle interaction, but spin fluctuations give rise to an antiferromagnetic exchange interaction and it is shown that this leads to strong-coupling s-wave pairing for physically reasonable values of the charge-transfer gap.

I. INTRODUCTION

In recent years new and unconventional methods for treating strongly correlated electron systems have been developed, initiated in part by the discovery of the mixed-valence heavy-fermion compounds and more recently by the discovery of the high-temperature superconducting oxides. Various "slave-boson" approaches have been introduced. Following the early work of Barnes,[1] Coleman,[2] and Read and Newns[3] showed how the single-occupancy constraint of the infinite-U Anderson model (thought to contain the essential physics of the single-impurity Kondo problem) could be implemented by introducing an auxiliary spinless Bose field (slave boson) to describe the empty (full) state of the rare-earth impurity. Derived thermodynamic properties, when compared with Bethe ansatz results, were exact in the limit of large quasiparticle degeneracy N and could be systematically improved upon by $1/N$ expansion. Transport properties for which few exact results are available could also be obtained[4] within the same technique. In the corresponding lattice model, slave-boson-mediated interactions have been shown to lead to an instability of the Fermi-liquid phase with a transition to a superconducting ground state with d-wave symmetry at leading order, $O(1/N)$, in the residual quasiparticle interaction.[5,6] Later it was shown that a complete description of the physics of the Kondo lattice problem could only be achieved with the inclusion of spin fluctuations;[7] these effects although formally of order $(1/N)^2$ could be large and in particular could lead to a superconducting instability in both d and p channels. Indeed the latter possibility had been suggested by several authors[8–10] drawing on the analogy with ^{3}He in many respects the archetypal heavy-fermion system. In this context ferromagnetic spin fluctuations are known to drive a p-wave instability and dominate the physics in the low-temperature regime.[11–13]

Until recently the Gutzwiller[14] variational approach was the only microscopic basis of the Fermi-liquid description of ^{3}He. An important advance in technique was made when Kotliar and Ruckenstein[15] (KR) formulated a slave-boson approach to the Hubbard model in which spin degrees of freedom were incorporated explicitly and treated on an equal footing with the charge degrees of freedom. It was shown that the mean-field theory of this model is equivalent to the Gutzwiller approximation[14] to the Gutzwiller ansatz. This then provides a means of extending the Gutzwiller variational scheme to finite temperature as well as a systematic way of improving on the classical approximations involved in the Gutzwiller approximation. Calculations beyond the mean-field approximation of KR have been reported in both weak- (Ref. 16) and strong- (Ref. 17) coupling limits. However, as the KR formalism in its original form was not spin rotationally invariant (SRI), attempts to calculate the $T^3 \ln T$ contribution to the specific heat of ^{3}He, arising from fluctuations about the mean field, within a Hubbard lattice gas model were off by a factor of 3 due to the neglect of transverse spin fluctuations. The slave-boson approach of KR has recently been developed in a manifestly SRI form[18,19] and used to study the Hubbard model with isotropic band structure at half-filling. The $T^3 \ln T$ spin-fluctuation contribution to the specific heat determined within this formulation reduces at weak coupling to the result obtained from paramagnon theory.[20]

In a different context, one- or two-band extended Hubbard models which contain in the Hamiltonian an antiferromagnetic spin-exchange term, often referred to as t-J models, have been used by many authors[21–28] to discuss the physics of the high-T_c CuO_2 superconductors. In particular in Ref. 24 it was shown that, even within mean-field theory, such a formulation has the appealing feature of allowing a transition to a non-Fermi liquid phase away from half-filling. On the other hand as noted, it has been shown in a strong-coupling calculation that hybridization in the Anderson lattice model generates a Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction between local f moments via the polarization of conduction electrons, albeit at two-loop order.[7] Similar considerations in the present context would lead to the superexchange J between copper moments via the oxygen orbitals. Therefore the procedure of adding exchange terms to the description by hand is unsatisfactory within a consistent loop or $(1/N)$ expansion about the saddle point. This becomes even more apparent, as we shall see, when charge and spin fluctuations are treated on an equal footing, generating spin exchange at one-loop order.

These mean-field theories have predicted spin-exchange-mediated superconductivity with s-wave symmetry, although the mixed $s+id$ state discovered by Kotliar[25] and, equivalently, the flux phase of Affleck and Marston[26] were later shown to be the stable solutions at half-filling. Indeed in a model constrained to no double occupancy on the Cu sites, $U=\infty$, and no interorbital correlations, spin exchange would appear to be the only way of producing s-wave pairing. The constraint fluctuations originating in the local no-double-occupancy constraint generally produce a repulsive interaction even in the extended s-wave channel, irrespective of the details of the band structure and Fermi surface.[5,6,29,30]

In this paper we consider the Anderson lattice Hamiltonian (ALH) with a qualitatively accurate band structure appropriate for Cu-O planes in which a nearest-neighbor hopping is assumed, and all other overlap integrals viewed as negligible.[31] The Coulomb repulsion on the Cu sites, U_d, is clearly the largest energy in the problem[31] and can be safely set to infinity, following a procedure commonly used in the treatment of heavy-fermion systems. This excludes the possibility of double-hole occupation on Cu sites. The on-site repulsions on O sites, as well as intersite Coulomb interactions will be neglected in what follows. This version of the ALH has been considered previously by several authors;[30,32] however, the strong-coupling formalism used in these discussions totally neglects spin fluctuations at leading order in the quasiparticle interaction. Within this formulation of the model the system undergoes a metal–charge-transfer-insulator transition at half-filling for large enough charge-transfer gap, a feature which does not seem to depend on the intrinsic oxygen bandwidth.[33] By considering coupling constants in various symmetry channels it was found[30] that over a wide range of values for the charge-transfer gap and in the doping regime 0.05–0.20 only the B_{2g}-symmetric d-wave channel was attractive, as conjectured in Ref. 32. This situation is similar to that found in the one-band Hubbard model, considered in a similar context in Ref. 29.

It is our purpose here to study the ALH within a slave-boson technique that treats charge and spin fluctuations on an equal footing and in a spin rotationally invariant manner. We believe that, within a necessarily low-order fluctuation calculation, this provides a better treatment of the CuO_2 system than the earlier asymmetric approaches previously described.[30,32] Since the Cu-O overlap is another large parameter in the problem,[31] the dynamics of the model is assumed to be dominated by the Cu-O hybridization. We will show that within the present strong-coupling approach this generates, at the Gaussian level, not only a hard-core charge-fluctuation-mediated interaction between the quasiparticles of the theory but also an antiferromagnetic exchange interaction. As a consequence we have within the model the possibility of a spin-fluctuation-mediated attractive interaction in the spin singlet state, and hence a superconducting ground state with s-wave symmetry. We find in fact, a strong-coupling attractive interaction in the s-wave channel with A_{1g} symmetry, whereas the attractive interaction in the d-wave channel with B_{1g} symmetry is weak coupling, due to the weak dispersion of the quasiparticle interaction, and hence does not compete in energy. The Fermi-liquid parameters F_0^a and F_1^a are positive and hence in accordance with general theorems[34] both spin- and charge-density waves can propagate.

II. MODEL AND FORMALISM

To be specific, we will assume that the $U_d=\infty$ ALH with a qualitatively accurate band structure, appropriate for a square lattice, provides a relevant description of the CuO_2 sheet. In the hole representation the Hamiltonian is given by

$$H=\sum_{j\sigma} E_d d_{j\sigma}^{\dagger} d_{j\sigma}+\sum_{i\sigma} E_p p_{i\sigma}^{\dagger} p_{i\sigma} +2V\sum_{k}\gamma(\mathbf{k})[d_{\mathbf{k},\sigma}^{\dagger} p_{\mathbf{k}\sigma}+\text{H.c.}]\ , \tag{2.1}$$

when the additional operator constraint,

$$\sum_{\sigma} d_{j,\sigma}^{\dagger} d_{j,\sigma}\lesssim 1\ , \tag{2.2}$$

is imposed at all sites at all times. Here $d_{j,\sigma}^{\dagger}$ creates a hole on a Cu $3d_{x^2-y^2}$ orbital, while $p_{i,\sigma}^{\dagger}$ creates a hole in an O $2p_x$ or O $2p_y$ orbital. E_d and E_p are the site energies of the $3d_{x^2-y^2}$ orbital and the degenerate O $2p_x$ and O $2p_y$ orbitals, respectively; V is the Cu-O hopping matrix element and

$$\gamma(\mathbf{k})=[\sin^2(k_x/2)+\sin^2(k_y/2)]^{1/2}$$

when lengths are measured in units of the Cu-Cu bond length. The charge-transfer gap we referred to in the Introduction is defined within this model as $2\Delta\equiv E_p-E_d$.

To treat the nonholonomic constraint, Eq. (2.2), by field theoretical techniques we slave-bosonize the Hamiltonian, Eq. (2.1), in a spin rotationally invariant manner.[18,19] We define two boson fields e_j and $s_{j;\sigma\sigma'}$ in the following way:

$$|Oj\rangle=e_j^{\dagger}|\text{vac}\rangle \tag{2.3a}$$

and

$$|\sigma j\rangle=\sum_{\sigma'} s_{j;\sigma\sigma'}^{\dagger} d_{j,\sigma'}^{\dagger}|\text{vac}\rangle\ . \tag{2.3b}$$

Here $|\text{vac}\rangle$ denotes a site with no Cu orbitals, $e^{\dagger}$ creates a $3d^{10}$ state on the Cu orbital, whereas the composite operator in Eq. (2.3b) creates the $3d^9$ state on the Cu orbital. To preserve SRI, $s_{j;\sigma\sigma'}$ must be a 2×2 matrix,[18,19] which is conveniently expanded in Pauli matrices as follows:

$$s_{j,\alpha\beta}=\sum_{\mu=0}^{3}(s_{j,\mu}/\sqrt{2})\tau_{\mu;\alpha\beta}\ ; \tag{2.4}$$

here $\vec{\tau}_0$ is the 2×2 identity matrix. The normalization of the $s_{j,\mu}$ fields has been chosen so that

$$\text{Tr}(\vec{s}_j^{\dagger}\vec{s}_j)=\sum_{\mu=0}^{3} s_{j,\mu}^{\dagger} s_{j,\mu}\ ;$$

from now on we adopt a notation in which repeated μ indices are summed over. Constraints must be imposed on the system to enforce conservation of spin and charge and to ensure that the Cu sites can never be doubly occupied.[15–19] These constraints are given by

$$Q_j = e_j^\dagger e_j + s_{j,\mu}^\dagger s_{j,\mu} = 1 \ , \tag{2.5a}$$

or equivalently, in the $U_d = \infty$ case,

$$Q_j = e_j^\dagger e_j + \sum_\sigma d_{j,\sigma}^\dagger d_{j,\sigma} = 1 \ , \tag{2.5b}$$

and in addition

$$L_{0j} = \sum_\sigma d_{j\sigma}^\dagger d_{j\sigma} - (s_{j,0}^\dagger s_{j,0} + \mathbf{s}_j^\dagger \cdot \mathbf{s}_j) = 0 \tag{2.6}$$

and

$$\mathbf{L}_j = \sum_{\sigma\sigma'} d_{j,\sigma}^\dagger \tau_{\sigma\sigma'} d_{j,\sigma'} - [s_{j,0}^\dagger \mathbf{s}_j + \mathbf{s}_j^\dagger s_{j,0} + i(\mathbf{s}_j^\dagger \times \mathbf{s}_j)] = 0 \ . \tag{2.7}$$

The constraints are implemented in the usual way[3] by coupling the operators Q_j and $L_{j,\mu}$ to Lagrange multipliers and redefining the Hamiltonian as

$$H \rightarrow H + i \sum_j \lambda_j^{(1)} (Q_j - 1) + i \sum_j \lambda_{j,\mu}^{(2)} L_{j,\mu} \ ; \tag{2.8}$$

as both Q_j and $L_{j,\mu}$ commute with the Hamiltonian, the constraints are rigorously enforced at all times. When considering fluctuations around the mean-field theory, to be described below, it is more convenient for computational purposes to use the form of the constraint given in Eq. (2.5a). This form involves fewer couplings between Lagrange multipliers and fermion fields thereby reducing the number of contributions to the boson polarizabilities.

Using the representation Eq. (2.3) for the physical states together with the redefined Hamiltonian, Eq. (2.8), the grand canonical partition function for the system can be written as an imaginary time functional integral[3]

$$Z = \int Ds_\mu^\dagger Ds_\mu De^\dagger De \, D\lambda^{(1)} D\lambda_\mu^{(2)} \exp\left[- \int_0^\beta \mathcal{L}(\tau) \, d\tau \right] , \tag{2.9}$$

where β is the inverse temperature and the Lagrangian $\mathcal{L}(\tau)$ in the absence of an external magnetic field is given by

$$\mathcal{L}(\tau) = \mathcal{L}^B(\tau) + \mathcal{L}_{\text{eff}}^F(\tau) \ , \tag{2.10}$$

where the pure boson contribution to the Lagrangian, $\mathcal{L}^B(\tau)$, is given by

$$\mathcal{L}^B(\tau) = \sum_j e_j^\dagger \frac{\partial}{\partial\tau} e_j + \sum_j s_{j,\mu}^\dagger \left[\frac{\partial}{\partial\tau} + E_d + i\lambda_j^{(1)} - i\lambda_{j,0}^{(2)} \right] s_{j,\mu} + i \sum_j \lambda_j^{(1)} (e_j^\dagger e_j - 1) - i \sum_j \lambda_j^{(2)} [s_{j,0}^\dagger \mathbf{s}_j + \mathbf{s}_j^\dagger s_{j,0} + i(\mathbf{s}_j^\dagger \times \mathbf{s}_j)] \ . \tag{2.11}$$

The contribution to the Lagrangian from the fermion sector $\mathcal{L}_{\text{eff}}^F(\tau)$ is obtained when the fermion degrees of freedom are integrated out and is defined by

$$\exp\left[- \int_0^\beta d\tau \mathcal{L}_{\text{eff}}^F(\tau) \right] = \int Dd^\dagger Dd \, Dp^\dagger Dp \exp\left[- \int_0^\beta d\tau \mathcal{L}^F(\tau) \right] , \tag{2.12}$$

where

$$\mathcal{L}^F(\tau) = \sum_{j\sigma\sigma'} d_{j,\sigma}^\dagger \left[\left[\frac{\partial}{\partial\tau} - \mu + i\lambda_{j,0}^{(2)} \right] \delta_{\sigma,\sigma'} + i\boldsymbol{\lambda}_j^{(2)} \cdot \boldsymbol{\tau}_{\sigma\sigma'} \right] d_{j,\sigma'} + \sum_{i,\sigma} p_{i,\sigma}^\dagger \left[\frac{\partial}{\partial\tau} + E_p - \mu \right] p_{i,\sigma} + \sum_{\mathbf{k},\mathbf{q},\sigma,\sigma'} 2V\gamma(\mathbf{k}) (p_{\mathbf{k},\sigma}^\dagger z_{\mathbf{q},\sigma\sigma'} d_{\mathbf{k}+\mathbf{q},\sigma'} + \text{H.c.}) \ , \tag{2.13}$$

and the field $\vec{z}_j$ is defined by

$$z_{j;\sigma\sigma'} = s_{j;\sigma\sigma'} e_j^\dagger \ . \tag{2.14}$$

The functional integral over $\mathcal{L}$, Eq. (2.9), is a faithful representation of the partition function provided a gauge is fixed[3] to avoid multiple counting of gauge equivalent field configurations, e.g., $\partial\lambda_j^{(1)}/\partial\tau = 0$, $\partial\lambda_{j,\mu}^{(2)}/\partial\tau = 0$, the "Cartesian gauge." The fields e_j and $s_{j,0}$ transform as scalars under spin rotations, whereas the fields $\mathbf{s}_j$ transform as a vector, hence within the present context they are naturally interpreted as charge constraint, charge and spin fluctuations, respectively. The former have been shown to actually represent the removal of degrees of freedom.[35] The charge and spin fluctuations have the opposite property and are thus additional degrees of freedom in the system.[16,18,19] Correspondingly the Lagrange multipliers $\lambda_j^{(1)}$ and $\lambda_{j,0}^{(2)}$ may be viewed as local chemical potentials fixing the occupation number on the Cu sites appropriately, whereas the $\boldsymbol{\lambda}_j^{(2)}$ are to be regarded as local magnetic fields fixing the direction of spin polarization.

It has been shown[15] that if double-hole occupancy on Cu sites is allowed, use of a matrix field $z_{j;\sigma\sigma'}$, which reduces to Eq. (2.14) in the $U_d = \infty$ limit, leads to spurious results in the weak-coupling limit, when the kinetic energy of the system must remain unrenormalized from

its free-particle value. We find that even in the present context of the $U_d=\infty$ ALH, a renormalization of Eq. (2.14) is necessary to obtain meaningful results for thermodynamic properties when Gaussian fluctuations about mean-field theory are included; similar conclusions have been reached by others.[36,37] The problem of obtaining the correct weak-coupling limit was partially resolved at the mean-field level by KR who chose a specific renormalization of the $z_{j;\sigma\sigma'}$ such that the free-particle result was regained, a procedure which has recently been generalized to the SRI case[18,19] as

$$\vec{z}_j=[(1-f_j^\dagger f_j)\vec{\tau}_0-\vec{s}_j^\dagger\vec{s}_j]^{-1/2}(e_j^\dagger\vec{s}_j+(\vec{s}_j f_j^\dagger)\times[(1-e_j^\dagger e_j)\vec{\tau}_0-\vec{s}_j^\dagger\vec{s}_j]^{-1/2} . \tag{2.15}$$

Here $f_j^\dagger$ creates a doubly occupied Cu orbital and $\vec{s}_j$ is the time-reversed version of the matrix field $\vec{s}$. All choices of renormalization of the $\vec{z}$ fields compatible with the rigorous constraints, Eqs. (2.5) and (2.6) are equivalent if no approximations are made. In the strong-coupling limit considered here $f_j^\dagger$ can be set equal to zero and Eq. (2.15) reduces to

$$\vec{z}_j=(e_j^\dagger e_j\vec{\tau}_0+\vec{s}_j^\dagger\vec{s}_j)^{-1/2}e_j\vec{s}_j(\vec{s}_j^\dagger\vec{s}_j)^{-1/2} , \tag{2.16}$$

which will be used in the following.

In the mean-field approximation all boson fields are taken to be independent of τ and the Cu-site index. It is then possible by a global SU(2) transformation, together with a corresponding global rotation of the spinors $d_{j,\sigma}$ and $p_{j,\sigma}$ to diagonalize the matrix field $\vec{z}_j$ simultaneously at all sites. Hence at the mean-field level the SRI formalism is equivalent to KR. We will now consider the model, which has a perfectly nested Fermi surface at half-filling, only in a doping regime sufficiently far away from half-filling that the ground state is known to be paramagnetic. Then in the saddle-point approximation the effective action per unit cell in the paramagnetic phase is given by

$$S_{\rm MF}/\beta N_s=s_0^2(E_d+\lambda^{(1)}-\lambda_0^{(2)})+\lambda^{(1)}(e^2-1)-\frac{2}{\beta N_s}\sum_{\mathbf{k},\omega_n}\ln\{[i\omega_n-(E_{\mathbf{k}}^{(-)}-\mu)][i\omega_n-(E_{\mathbf{k}}^{(+)}-\mu)]\} , \tag{2.17}$$

where N_s is the number of unit cells and $\omega_n=(2n+1)\pi/\beta$ is a Fermionic Matsubara frequency. The energies of the upper and lower ($\alpha=\pm1$) quasiparticle bands are given by

$$E_{\mathbf{k}}^{(\alpha)}=\tfrac{1}{2}\{E_p+\lambda_0^{(2)}/+\alpha[(E_p-\lambda_0^{(2)})^2+16V^2\gamma^2(\mathbf{k})z^2]^{1/2}\} , \tag{2.18}$$

where $z^2=e^2/[e^2+(s_0/\sqrt{2})^2]$ and $\lambda_0^{(2)}$ plays the role of the renormalized d-level energy. Note that the top of the lower quasiparticle band is located at $\lambda_0^{(2)}$ and the bottom of the upper one at E_p when $E_p>\lambda_0^{(2)}$. The c numbers s_0, $\lambda^{(1)}$, e, and $\lambda_0^{(2)}$ are determined self consistently by extremizing the effective action $S_{\rm MF}$; in addition the chemical potential is positioned in accordance with Luttinger's theorem. In the low-temperature regime $k_BT\ll E_p-\lambda_0^{(2)}$ it can be seen by performing the Matsubara sum in Eq. (2.17) that the term involving $E_k^{(+)}$ does not contribute to the effective action. The system is therefore described by an effective one-band Hamiltonian,

$$H_{\rm eff}=E_0+\sum_{\mathbf{k},\sigma}(E_{\mathbf{k}}^{(-)}-\mu)C_{\mathbf{k},\sigma}^{(-)\dagger}C_{\mathbf{k},\sigma}^{(-)} ; \tag{2.19}$$

here

$$E_0=N_s[s_0^2(E_d+\lambda^{(1)}-\lambda_0^{(2)})+\lambda^{(1)}(e^2-1)] .$$

The quasiparticle operators $C_{\mathbf{k},\sigma}^{(-)}$ ($C_{\mathbf{k},\sigma}^{(+)}$), pertaining to the upper (lower) quasiparticle bands with dispersion $E_{\mathbf{k}}^{(-)}$ ($E_{\mathbf{k}}^{(+)}$) are related to $p_{\mathbf{k},\sigma}$ and $d_{\mathbf{k},\sigma}$ in the following way:

$$p_{\mathbf{k},\sigma}=v_{\mathbf{k}}^{(-)}C_{\mathbf{k},\sigma}^{(-)}+v_{\mathbf{k},\sigma}^{(+)}C_{\mathbf{k},\sigma}^{(+)} ,$$
$$d_{\mathbf{k},\sigma}=u_{\mathbf{k}}^{(-)}C_{\mathbf{k}\sigma}^{(-)}+u_{\mathbf{k}}^{(+)}C_{\mathbf{k},\sigma}^{+} , \tag{2.20}$$

and the coherence factors u and v are given by

$$u_{\mathbf{k}}^{(-)^2}=v_{\mathbf{k}}^{(+)^2}=\frac{4V^2z^2\gamma^2(\mathbf{k})}{(E_{\mathbf{k}}^{(-)}-\lambda_0^{(2)})^2+4V^2z^2\gamma^2(\mathbf{k})} ,$$
$$u_{\mathbf{k}}^{(+)^2}=v_{\mathbf{k}}^{(-)^2}=1-u_{\mathbf{k}}^{(-)^2} . \tag{2.21}$$

Mean-field theories of this type have been studied extensively.[17–19,30,32,33,37,38] Here we only summarize the main features. In the paramagnetic phase at half-filling there is a transition from a metal to a charge-transfer insulator[30,32,33] and a corresponding breakdown of Fermi-liquid theory above a critical charge-transfer gap Δ_c. For the particular model we consider here, and within the KR scheme, for $V=1.6$ eV, $2\Delta_c\cong9.8$ eV. When compared with the actual values appropriate for the La-Cu-O compounds, for example, this is a rather large value, as expected, since the charge-transfer gap should include the interorbital Coulomb interaction U_{pd} (Ref. 39) neglected here. The KR formulation also provides a good starting point for discussion of the antiferromagnetism of the CuO_2 planes. It has been shown that, within mean-field theory, the ground state at half-filling is an antiferromagnet, itinerant or localized depending on Δ, with antiferromagnetic gap $\Delta_{\rm AF}\cong\Delta$.[37] It has been suggested that the antiferromagnetic state is unstable to metallization at very low levels of doping,[38] although this point needs further study. In general, away from half-filling the system is metallic with mass enhancement (m^*/m) depending on the size of the charge-transfer gap. Whereas doping produces rapid metallization of the insulating state close to the transition point,[32,33,38] for $\Delta<\Delta_c$ (m^*/m) is essentially doping independent. Further, at the moderate doping levels, e.g., at $x_h\cong0.10$ that

we will consider here the system is well into the paramagnetic phase and physical quantities are insensitive to the size of the oxide gap.

Our main purpose here is to study the effective interaction between quasiparticles and possible superconducting instabilities of the CuO_2 model with physical parameters in a range thought to be approximate to the high-T_c oxides. To this end, we consider the model in a doping regime well into the paramagnetic phase; the solutions of the stationary point conditions are necessary input for the study of fluctuations to which we now turn.

III. FLUCTUATIONS AND THE EFFECTIVE QUASIPARTICLE INTERACTION

The system defined by Eqs. (2.9)–(2.13) can be treated directly, but to study low-frequency effects it is more convenient to perform a Read-Newns gauge transformation[3] separating the amplitude and phase of the boson fields e_j and $s_{j,\mu}$

$$e_j \rightarrow e_j e^{i\theta_j}, \quad s_{j,\mu} \rightarrow s_{j,\mu} e^{i\chi_{j,\mu}} . \tag{3.1}$$

The Lagrangian Eq. (2.9) can then be expressed in terms of real valued Bose fields by promoting the Lagrange multipliers to gauge fields in the following way:

$$\begin{aligned} \lambda_j^{(1)} &\rightarrow \lambda_j^{(1)}(\tau) = \lambda_j^{(1)} - \frac{\partial \theta_j}{\partial \tau} , \\ \lambda_{j,\mu}^{(2)} &\rightarrow \lambda_{j,\mu}(\tau) = \lambda_{j,\mu}^{(2)} - \delta_{\mu,0} \frac{\partial \theta_j}{\partial \tau} - \frac{\partial \chi_{j,\mu}}{\partial \tau} . \end{aligned} \tag{3.2}$$

Then $\mathcal{L}^B$ and $\mathcal{L}^F$, Eqs. (2.11) and (2.13), become

$$\begin{aligned} \mathcal{L}^B(\tau) = & \sum_{j,\mu} s_{j,\mu}(E_d + i\lambda_j^{(1)} - i\lambda_{j,0}^{(2)}) s_{j,\mu} \\ & + i \sum_j \lambda_j^{(1)}(e_j^2 - 1) - 2i \sum_j s_{j,0} \boldsymbol{\lambda}_j^{(2)} \cdot \mathbf{s}_j , \end{aligned} \tag{3.3}$$

and

$$\begin{aligned} \mathcal{L}^F(\tau) = & \sum_{j;\sigma\sigma'} d_{j\sigma}^\dagger \left[\left(\frac{\partial}{\partial \tau} - \mu + i\lambda_{j,0}^{(2)} \right) \delta_{\sigma,\sigma'} + i\boldsymbol{\lambda}_j^{(2)} \cdot \boldsymbol{\tau}_{\sigma\sigma} \right] d_{j,\sigma'} + \sum_{i\sigma} p_{i,\sigma}^\dagger \left(\frac{\partial}{\partial \tau} + E_p - \mu \right) p_{i,\sigma} \\ & + \sum_{\substack{\mathbf{k},\mathbf{q} \\ \sigma\sigma'}} 2V\gamma(\mathbf{k}) (p_{\mathbf{k}}^\dagger z_{\mathbf{q};\sigma\sigma'} d_{\mathbf{k}+\mathbf{q},\sigma'} + d_{\mathbf{k}+\mathbf{q},\sigma'}^\dagger z_{\mathbf{q};\sigma'\sigma} p_{\mathbf{k}\sigma}) . \end{aligned} \tag{3.4}$$

In Eq. (3.3) the terms involving the time derivatives of the amplitudes e_j and $s_{j,\mu}$ have disappeared due to the periodicity in τ of the boson fields.

The fermion fields are now integrated out in the usual way contributing the term $\mathcal{L}_{\text{eff}}^F(\tau)$ to the effective Lagrangian which is now a functional of the fluctuating Bose fields only. The inverse boson propagator is a 10×10 matrix. However, at the Gaussian level it is block diagonal comprised of a 4×4 matrix representing the charge fluctuations and a 6×6 matrix representing spin fluctuations. This is a consequence of the fact that the Pauli matrices are traceless, therefore a trace over a single Pauli matrix vanishes precluding any coupling between charge and spin fluctuations at one-loop order. Consequently, the inverse boson propagator is given by

$$\overleftrightarrow{D}^{-1} = \begin{bmatrix} \overleftrightarrow{D}_{\text{CF}}^{-1} & 0 \\ 0 & \overleftrightarrow{D}_{\text{SF}}^{-1} \end{bmatrix} ; \tag{3.5}$$

here $\overleftrightarrow{D}_{\text{CF}}^{-1}$ and $\overleftrightarrow{D}_{\text{SF}}^{-1}$ are the inverse propagators for charge and spin fluctuations, respectively and are determined from distinct Dyson equations

$$\overleftrightarrow{D}_{\text{CF}}^{-1} = (\overleftrightarrow{D}_{\text{CF}}^{o})^{-1} - \overleftrightarrow{\Sigma}_{\text{CF}} , \tag{3.6}$$

$$\overleftrightarrow{D}_{\text{SF}}^{-1} = (\overleftrightarrow{D}_{\text{SF}}^{0})^{-1} - \overleftrightarrow{\Sigma}_{\text{SF}} . \tag{3.7}$$

The bare boson propagators, which can be read off from Eq. (3.3), are given by

$$(\overleftrightarrow{D}_{\text{SF}}^{0})^{-1} = 2 \begin{bmatrix} (E_d + \lambda^{(1)} - \lambda_0^{(2)}) \overleftrightarrow{1} , & -is_0 \overleftrightarrow{1} \\ -is_0 \overleftrightarrow{1} , & 0 \end{bmatrix} , \tag{3.8}$$

where $\overleftrightarrow{1}$ is the 3×3 unit matrix. Similarly

$$(\overleftrightarrow{D}_{\text{CF}}^{0})^{-1} = 2 \begin{bmatrix} \lambda^{(1)} & 0 & 0 & ie \\ 0 & (E_d + \lambda^{(1)} - \lambda_0^{(2)}) & -is_0 & is_0 \\ 0 & -is_0 & 0 & 0 \\ ie & is_0 & 0 & 0 \end{bmatrix} . \tag{3.9}$$

The leading-order contribution to the boson self-energy $\overleftrightarrow{\Sigma}$ is obtained from the expansion of $\mathcal{L}_{\text{eff}}^F$ to lowest, quadratic order in the fluctuating fields. These terms are of two distinct types, shown in Fig. 1; either a particle-hole bubble generated at second order from the simple three-point vertex, one boson and two fermions, or the Hartree-like closed loop generated from the four-point vertices, two bosons and two fermions, which are now possible in the presence of the composite operators $z_{j,\sigma\sigma'}$. All contributions to the self-energy matrices $\overleftrightarrow{\Sigma}_{\text{CF}}$ and $\overleftrightarrow{\Sigma}_{\text{SF}}$ are given in Appendix A.

By inverting the resulting matrices Eqs. (3.6) and (3.7) the propagators governing the boson fluctuations are obtained. These fluctuations give rise to residual quasiparticle interactions, and the resulting effective Hamiltonian for the system can be written

$$H_{\text{eff}} = H_{\text{eff}}^{(0)} + H_{\text{eff}}^{(1)} , \tag{3.10}$$

where $H_{\text{eff}}^{(0)}$ is given in Eq. (2.19) and

$$\begin{aligned} H_{\text{eff}}^{(1)} = & \tfrac{1}{2} \sum_{\substack{\mathbf{k},\mathbf{k}'\mathbf{q} \\ \alpha,\beta,\gamma,\delta}} [\Gamma^s(\mathbf{q}) \delta_{\alpha\beta}\delta_{\gamma\delta} + \Gamma^a(\mathbf{q}) \boldsymbol{\sigma}_{\alpha\beta} \cdot \boldsymbol{\sigma}_{\gamma\delta}] \\ & \times C_{\mathbf{k}+\mathbf{q},\alpha}^{(-)\dagger} C_{\mathbf{k}'-\mathbf{q},\gamma}^{(-)\dagger} C_{\mathbf{k}',\delta}^{(-)} C_{\mathbf{k},\beta}^{(-)} . \end{aligned} \tag{3.11}$$

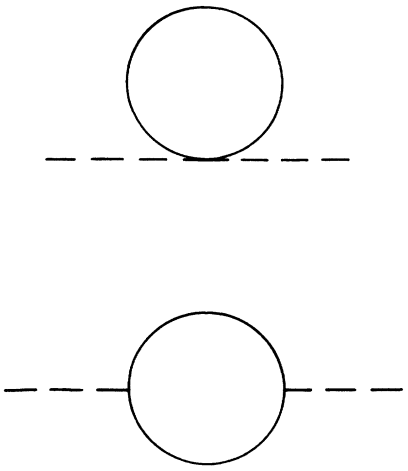

FIG. 1. Generic boson self-energy diagrams at one-loop order.

In Eq. (3.10) the solutions to the stationary-point conditions appearing in $H_{\text{eff}}^{(0)}$ should be one-loop corrected.

Analytic expressions for Γ^s and Γ^a involving one-boson exchange are given in Appendix B. The spin-symmetric and spin-antisymmetric parts of the quasiparticle interaction, Γ^s and Γ^a, are shown as a function of $\mathbf{q}$ along two different symmetry directions in the Brillouin zone in Figs. 2 and 3, respectively. Γ^s is positive in sign as expected; it represents the hard-core interaction originating in the local no-double-occupancy constraint on the copper sites. The important point to be made here is that Γ^a is also positive and hence the spin-boson fluctuations give rise to an antiferromagnetic exchange interaction between the quasiparticles. In the limit $\Delta \gg V$, where we can made contact with canonical perturbation theory[40] (CPT), Γ^a (as well as Γ^s) is only weakly dispersive, and

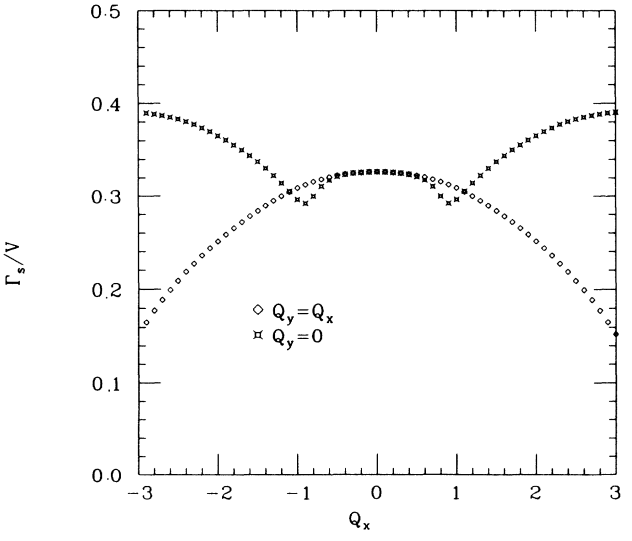

FIG. 2. Spin-symmetric part of the quasiparticle interaction resulting from one-boson exchange, along two symmetry directions (1,1) and (0,1) with bare parameters taken to be $V=1.4$ eV, $x_h=0.10$, and $2\Delta=4.0$ eV.

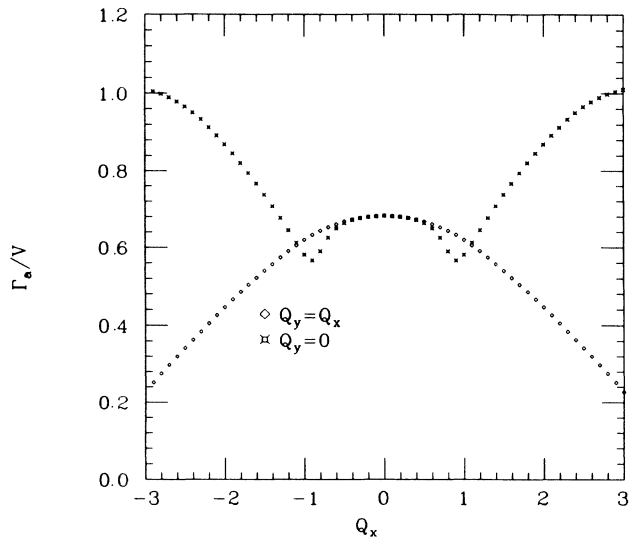

FIG. 3. Spin-antisymmetric part of the quasiparticle interaction resulting from one-boson exchange, along two symmetry directions (1,1) and (0,1) with bare parameters $V=1.4$ eV, $x_h=0.10$, and $2\Delta=4.0$ eV.

therefore, under these conditions the spin-spin interaction is short range in real space and will be dominated by antiferromagnetic Kondo scattering, in agreement with the findings of Zaanen Olés within CPT.[40] In the limit of interest in the high-T_c oxides, however, $\Delta \cong V$, the effects of Kondo scattering, with a bare energy scale of order V^2/Δ and superexchange with, in this model, a bare coupling of order $(V^2/\Delta)(V^2/\Delta^2)$, cannot be disentangled easily. Nevertheless, as can be seen from Figs. 2 and 3, the result is an effective interaction with contributions from charge fluctuations and spin fluctuations that are of the same order and functional form. As remarked recently,[41] such a relationship between spin and charge polarizabilities seems to be required to account for normal-state data.

A quasiparticle spin triplet sees the interaction potential

$$V_t=\tfrac{1}{2}[\Gamma^s(\mathbf{k}-\mathbf{k}')+\Gamma^a(\mathbf{k}-\mathbf{k}')-(\mathbf{k}\rightarrow-\mathbf{k})]\ , \tag{3.12}$$

whereas the singlet sees

$$V_s=\tfrac{1}{2}[\Gamma^s(\mathbf{k}-\mathbf{k}')-3\Gamma^a(\mathbf{k}-\mathbf{k}')+(k\rightarrow-k)]\ . \tag{3.13}$$

In view of the remarks made above, depending on the details of the system, Eq. (3.13) could lead therefore to an attractive interaction between quasiparticles in a singlet state and hence the possibility of pairing in a relative angular momentum s state. Γ^s and Γ^a are screened interactions and the boson self-energy corrections involved contain both inter- and intraband terms. We have checked and found that when interband contributions are included only minor quantitative changes in the effective interactions result. Therefore we have an *a posteriori* demonstration that the physics is governed by an effective one-band quasiparticle Hamiltonian. In the large Δ limit, $\Delta \gg V$, the quasiparticles carry p charge and d spin. When $\Delta \sim V$ the d-charge degrees of freedom are no

longer frozen in and a description in terms of a mixed valence compound is more appropriate, as noted elsewhere.[40]

IV. SUPERCONDUCTING INSTABILITIES

We now turn to consider the possible superconducting instabilities of the model, and look at the behavior of the coupling constants in various symmetry channels both as a function of doping x_h and charge-transfer gap Δ. As in the Eliashberg theory of electron-phonon coupling, we introduce Fermi-surface-averaged spectral weights of the two-particle irreducible vertex $\Gamma(\mathbf{q},\omega)$

$$F^i(\omega)\equiv\frac{1}{\pi}\langle \mathrm{Im}\Gamma(\mathbf{q},\omega)\rangle^i_{\mathrm{FS}} . \tag{4.1}$$

The average $\langle\ \rangle^i_{\mathrm{FS}}$ is defined by

$$\langle A\rangle^i_{\mathrm{FS}}=\frac{[1/(2\pi)^d]\int(dS/|\mathbf{v}_k|)\int(dS'/|\mathbf{v}_{k'}|)g_i(\mathbf{k})A(\mathbf{k},\mathbf{k}')g_i(\mathbf{k}')}{\int(dS/|\mathbf{v}_k|)g_i^2(\mathbf{k})} , \tag{4.2}$$

where $\int dS$ is an integration over the Fermi surface, $(1/(2\pi)^d)|\mathbf{v}_k|^{-1}$ the density of states, and g_i a Fermi surface harmonic;[42] as we model Cu-O sheets cubic harmonics are used. The superconducting transition temperature can be determined from these spectral weights. Here, however, we only consider the one moment of $F^i(\omega)$

$$\lambda_i\equiv\int_0^\infty d\omega\frac{F^i(\omega)}{\omega}=\langle \mathrm{Re}\Gamma(\mathbf{q},0)\rangle^i_{\mathrm{FS}} , \tag{4.3}$$

which measures the strength of the pairing interaction between quasiparticles in a given symmetry channel; hence only the zero-frequency limit of the irreducible vertex is needed; then

$$\begin{aligned}&\Gamma^{\mathrm{triplet}}(\mathbf{q},\omega=0)\equiv\Gamma^{\mathrm{triplet}}(\mathbf{q})=V_t ,\\ &\Gamma^{\mathrm{singlet}}(\mathbf{q},\omega=0)=\Gamma^{\mathrm{singlet}}(\mathbf{q})=V_s ,\end{aligned} \tag{4.4}$$

where V_t and V_s are defined in Eqs. (3.12) and (3.13), respectively. We use a convention such that negative λ_i corresponds to an attractive interaction.

We have determined the λ_i by computing $\Gamma(\mathbf{q})$ numerically with parameters appropriate for La_2CuO_4 and considered its projection into different symmetry channels, for scattering on the Fermi surface. The results are shown in Figs. 4 and 5. For all dopings considered, and over a wide range of charge-transfer gaps the d-wave channel with B_{1g} symmetry

$$g_{x^2-y^2}=\cos k_x-\cos k_y , \tag{4.5}$$

and the extended s-wave channel A_{1g}

$$g_{x^2+y^2}=\cos k_x+\cos k_y , \tag{4.6}$$

together with the global s wave were found to be attractive. The B_{2g} d-wave channel

$$g_{xy}=\sin k_x\sin k_y , \tag{4.7}$$

as well as the p-wave channels were found to be repulsive. The weak variation of the coupling constants on doping,

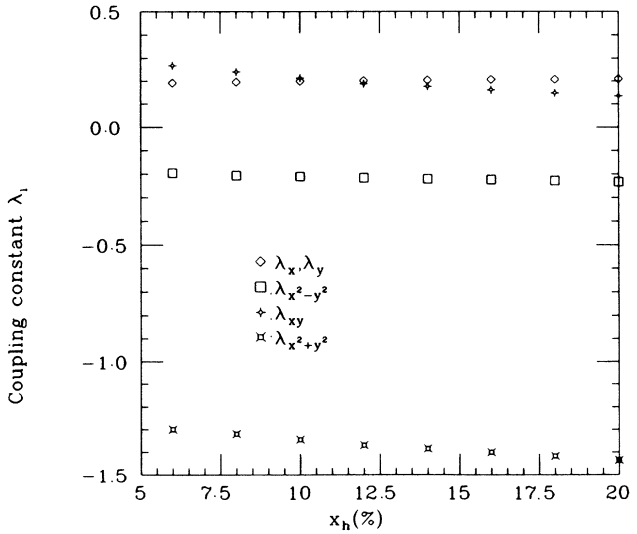

FIG. 4. λ_i, as defined in the text, as a function of doping x_h, with $V=1.4$ eV, and $2\Delta=4.0$ eV.

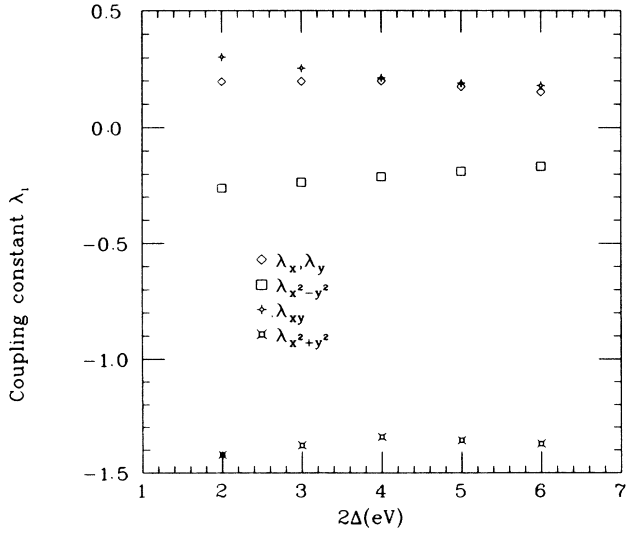

FIG. 5. λ_i, as defined in the text, as a function of charge-transfer gap 2Δ, for $x_h=0.10$, and $V=1.4$ eV.

found in Fig. 4, and on the charge-transfer gap found in Fig. 5, is to be expected. In the first instance at the value of $E_p - E_d = 4.0$ eV chosen, which is appropriate for the La-Cu-O compounds, Δ is much less than the critical value Δ_c of the model; as a result, the coupling constants are computed for a system well into the metallic regime where the doping dependence of physical parameters is known to be weak. In the second instance, for $x_h \cong 0.10$, the model is again well into the paramagnetic metallic regime where physical quantities are not expected to be sensitive to the value of the oxide gap. As can be seen from both Figs. 4 and 5, the coupling constant in both the global and extended s-wave channels lies in the strong-coupling regime for the charge-transfer gaps we have considered. The present calculation does not distinguish between these two possibilities. On the other hand, because of the weak dispersion of the quasiparticle interaction the B_{1g} and all other higher angular momentum channels are weak coupling and therefore do not compete in energy. Within the present model we are therefore led to predict a superconducting order parameter that has either global or A_{1g} s-wave symmetry, consistent with observation.[43] Inclusion of next nearest neighbor, O-O hopping, would resolve this degeneracy but would not change the conclusion that the s wave would be the dominant attractive channel.

V. CONCLUSION

In conclusion we have treated the $U_d = \infty$ ALH with a strong-coupling method that treats charge and spin fluctuations on an equal footing. Fluctuations of the boson fields generate an attractive interaction between quasiparticles in a spin singlet s state. We believe, given the experimental evidence for the existence of strong on-site Coulomb interactions on the copper orbitals together with the omnipresent antiferromagnetic spin fluctuations, that this provides a better framework for studying the physics of the CuO_2 systems than either weak-coupling methods or strong-coupling approaches that neglect spin fluctuations.

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APPENDIX A

The contributions to the boson self energies at the Gaussian level involve two types of diagram. A particle-hole polarizability and a Hartree-like closed loop. In the paramagnetic phase there are only three distinct particle-hole bubbles, which we list:

$$\Pi_{ss}(\mathbf{q},i\omega_\nu) = -\frac{8V^2}{\beta}\sum_{\mathbf{k},\omega_n}[\gamma^2(\mathbf{k}+\mathbf{q})G^{pp}_{\omega_n+\omega_\nu}(\mathbf{k}+\mathbf{q})G^{dd}_{\omega_n}(\mathbf{k}) + \gamma^2(\mathbf{k})G^{dd}_{\omega_n+\omega_\nu}(\mathbf{k}+\mathbf{q})G^{pp}_{\omega_n}(\mathbf{k}) + 2\gamma(\mathbf{k})\gamma(\mathbf{k}+\mathbf{q})G^{pd}_{\omega_n+\omega_\nu}(\mathbf{k}+\mathbf{q})G^{pd}_{\omega_n}(\mathbf{k})] , \tag{A1}$$

$$\Pi_{\lambda s}(\mathbf{q},i\omega_\nu) = -\frac{4V}{\beta}\sum_{\mathbf{k},\omega_n}[\gamma(\mathbf{k}+\mathbf{q})G^{pd}_{\omega_n+\omega_\nu}(\mathbf{k}+\mathbf{q})G^{dd}_{\omega_n}(\mathbf{k}) + \gamma(\mathbf{k})G^{pd}_{\omega_n}(\mathbf{k})G^{dd}_{\omega_n+\omega_\nu}(\mathbf{k}+\mathbf{q})] = \Pi_{s\lambda} , \tag{A2}$$

and

$$\Pi_{\lambda\lambda}(\mathbf{q},i\omega_\nu) = -\frac{2}{\beta}\sum_{\mathbf{k},\omega_n} G^{dd}_{\omega_n}(\mathbf{k})G^{pp}_{\omega_n+\omega_\nu}(\mathbf{k}+\mathbf{q}) . \tag{A3}$$

Here $\omega_n = (2n+1)\pi/\beta$ and $\omega_\nu = 2m\pi/\beta$ are odd and even Matsubara frequencies, respectively. The Green's functions G^{pp}, G^{dd}, and G^{pd} are the mean-field p electron, d electron, and mixed propagators, respectively, defined by

$$G^{pp}(\mathbf{k},i\omega_n) = \frac{v^{(-)^2}_{\mathbf{k}}}{i\omega_n - E^{(-)}_{\mathbf{k}}} + \frac{v^{(+)^2}_{\mathbf{k}}}{i\omega_n - E^{(+)}_{\mathbf{k}}} , \tag{A4}$$

$$G^{pd}(\mathbf{k},i\omega_n) = \frac{u^{(-)}_{\mathbf{k}}v^{(-)}_{\mathbf{k}}}{i\omega_n - E^{(-)}_{\mathbf{k}}} + \frac{u^{(+)}_{\mathbf{k}}v^{(+)}_{\mathbf{k}}}{i\omega_n - E^{(+)}_{\mathbf{k}}} , \tag{A5}$$

$$G^{dd}(\mathbf{k},i\omega_n) = \frac{u^{(-)^2}_{\mathbf{k}}}{i\omega_n - E^{(-)}_{\mathbf{k}}} + \frac{u^{(+)^2}_{\mathbf{k}}}{i\omega_n - E^{(+)}_{\mathbf{k}}} . \tag{A6}$$

The sum over Matsubara frequencies appearing in Eqs. (A1)–(A3) is easily evaluated to give

$$\Pi_{ss}(\mathbf{q},i\omega_\nu) = 16V^2\sum_{\mathbf{k}}\{\tfrac{1}{2}[\gamma(\mathbf{k}+\mathbf{q})v^{(-)}_{\mathbf{k}+\mathbf{q}}u^{(-)}_{\mathbf{k}} + \gamma(\mathbf{k})u^{(-)}_{\mathbf{k}+\mathbf{q}}v^{(-)}_{\mathbf{k}}]^2R_{11}(\mathbf{k},\mathbf{q},i\omega_\nu) + [\gamma(\mathbf{k}+\mathbf{q})v^{(-)}_{\mathbf{k}}v^{(-)}_{\mathbf{k}+\mathbf{q}} - \gamma(\mathbf{k})u^{(-)}_{\mathbf{k}}u^{(-)}_{\mathbf{k}+\mathbf{q}}]^2R_{12}(\mathbf{k},\mathbf{q},i\omega_\nu)\} , \tag{A7}$$

$$\Pi_{s\lambda}(\mathbf{q},i\omega_\nu)=-16zV^2\sum_{\mathbf{k}}\left[\frac{\gamma^2(\mathbf{k}+\mathbf{q})}{E_{\mathbf{k}+\mathbf{q}}}u_{\mathbf{k}}^{(-)^2}R_{11}(\mathbf{k},\mathbf{q},i\omega_\nu)+\left[\frac{\gamma^2(\mathbf{k}+\mathbf{q})}{E_{\mathbf{k}+\mathbf{q}}}v_{\mathbf{k}}^{(-)^2}-\frac{\gamma^2(\mathbf{k})}{E_{\mathbf{k}}}u_{\mathbf{k}+\mathbf{q}}^{(-)^2}\right]R_{12}(\mathbf{k},\mathbf{q},i\omega_\nu)\right], \tag{A8}$$

and

$$\Pi_{\lambda\lambda}=-2\sum_{\mathbf{k}}[u_{\mathbf{k}+\mathbf{q}}^{(-)^2}u_{\mathbf{k}}^{(-)^2}R_{11}(\mathbf{k},\mathbf{q},i\omega_\nu)+2u_{\mathbf{k}+\mathbf{q}}^{(-)^2}v_{\mathbf{k}}^{(-)^2}R_{12}(\mathbf{k},\mathbf{q},i\omega_\nu)]\ . \tag{A9}$$

Here

$$R_{11}(\mathbf{k},\mathbf{q},i\omega_\nu)=-\frac{f(E_{\mathbf{k}+\mathbf{q}}^{(-)})-f(E_{\mathbf{k}}^{(-)})}{E_{\mathbf{k}+\mathbf{q}}^{(-)}-E_{\mathbf{k}}^{(-)}-i\omega_\nu}\ , \tag{A10}$$

$$R_{12}(\mathbf{k},\mathbf{q},i\omega_\nu)=-\tfrac{1}{2}[f(E_{\mathbf{k}+\mathbf{q}}^{(-)})-f(E_{\mathbf{k}}^{(+)})]\left[\frac{1}{E_{\mathbf{k}+\mathbf{q}}^{(-)}-E_{\mathbf{k}}^{(+)}-i\omega_\nu}+\frac{1}{E_{\mathbf{k}+\mathbf{q}}^{(-)}-E_{\mathbf{k}}^{+}+i\omega_\nu}\right], \tag{A11}$$

and

$$E_{\mathbf{k}}=[(E_p-\lambda_0^{(2)})^2+16V^2\gamma^2(\mathbf{k})z^2]^{1/2}\ , \tag{A12}$$

all other symbols have been defined in the text. The Hartree-like diagrams are all proportional to

$$H=-\frac{8V}{\beta}\sum_{\mathbf{k}\omega_n}\gamma(\mathbf{k})G^{pd}(\mathbf{k},\omega_n)=-8V\sum_{k}\gamma(\mathbf{k})u_{\mathbf{k}}^{(-)}v_{\mathbf{k}}^{(-)}f(E_{\mathbf{k}}^{(-)})\ . \tag{A13}$$

With these definitions the boson self energies in the spin sector can be written as

$$\vec{\vec{\Sigma}}_{\rm SF}=\vec{\vec{H}}_{\rm SF}+\vec{\vec{\Pi}}_{\rm SF}\ , \tag{A14}$$

where

$$\vec{\vec{H}}_{\rm SF}=\begin{bmatrix}\eta_{\rm ss}H\vec{\vec{1}} & 0\vec{\vec{1}}\\ 0\vec{\vec{1}} & 0\vec{\vec{1}}\end{bmatrix}, \tag{A15}$$

and

$$\vec{\vec{\Pi}}_{\rm SF}=\begin{bmatrix}\omega_s^2\Pi_{ss}\vec{\vec{1}} & i\omega_{\rm s}\Pi_{s\lambda}\vec{\vec{1}}\\ i\omega_{\rm s}\Pi_{\lambda s}\vec{\vec{1}} & i^2\Pi_{\lambda\lambda}\vec{\vec{1}}\end{bmatrix}; \tag{A16}$$

here $\vec{\vec{1}}$ is the 3×3 unit matrix,

$$\omega_{\rm s}=\frac{e}{s_0}\left[\frac{1-s_0/\sqrt{2}}{[e^2+(s_0/\sqrt{2})^2]^{1/2}}+\frac{(s_0/\sqrt{2})^2}{e^2+(s_0/\sqrt{2})^2}\right], \tag{A17}$$

and

$$\eta_{\rm ss}=\eta_{\rm ss}^{(1)}+\eta_{\rm ss}^{(2)}\ , \tag{A18}$$

where

$$\eta_{\rm ss}^{(1)}=\frac{1}{2}\left[\frac{e}{e^2+(s_0/\sqrt{2})^2}\left[\frac{3(s_0/\sqrt{2})^2}{[e^2+(s_0/\sqrt{2})^2]^{1/2}}-1\right]+\frac{e}{(s_0/\sqrt{2})[e^2+(s_0/\sqrt{2})^2]^{1/2}}[3(s_0/\sqrt{2})-1]\right], \tag{A19}$$

and

$$\eta_{\rm ss}^{(2)}=\frac{-e}{s_0/\sqrt{2}}\frac{1}{[e^2+(s_0/\sqrt{2})^2]^{1/2}}\left[1-\frac{s_0/\sqrt{2}}{[e^2+(s_0/\sqrt{2})^2]^{1/2}}(1-s_0/\sqrt{2})\right]. \tag{A20}$$

The self-energy in the charge fluctuation sector can be expressed in a similar way as

$$\vec{\vec{\Sigma}}_{\rm CF}=\vec{\vec{H}}_{\rm CF}+\vec{\vec{\Pi}}_{\rm CF}\ , \tag{A21}$$

where

$$\vec{\vec{\Pi}}_{\mathrm{CF}}=\begin{bmatrix} \omega_e^2\Pi_{ss} & \omega_e\omega_{s_0}\Pi_{ss} & i\omega_e\Pi_{s\lambda} & 0 \\ \omega_e\omega_{s_0}\Pi_{ss} & \omega_{s_0^2}\Pi_{ss} & i\omega_{s_0}\Pi_{s\lambda} & 0 \\ i\omega_e\Pi_{\lambda s} & i\omega_{s_0}\Pi_{\lambda s} & i^2\Pi_{\lambda\lambda} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{A22}$$

and

$$\vec{\vec{H}}_{\mathrm{CF}}=\begin{bmatrix} \vec{\vec{H}}_{\mathrm{es}} & 0\vec{\vec{1}} \\ 0\vec{\vec{1}} & 0\vec{\vec{1}} \end{bmatrix}, \tag{A23}$$

where

$$\vec{\vec{H}}_{es}=\begin{bmatrix} \eta_{ee}H & \eta_{es_0}H \\ \eta_{s_0e}H & \eta_{s_0s_0}H \end{bmatrix}. \tag{A24}$$

In this case $\vec{\vec{1}}$ is a 2×2 unit matrix and

$$\omega_e=\frac{1}{[e^2+(s_0/\sqrt{2})^2]^{1/2}}-\frac{e^2}{e^2+(s_0/\sqrt{2})^2}, \tag{A25}$$

$$\omega_{s_0}=\frac{e}{s_0}\left[\frac{1-s_0/\sqrt{2}}{[e^2+(s_0/\sqrt{2})^2]^{1/2}}-\frac{(s_0/\sqrt{2})^2}{e^2+(s_0/\sqrt{2})^2}\right], \tag{A26}$$

$$\eta_{ee}=\frac{3e}{e^2+(s_0/\sqrt{2})^2}\left[\frac{e^2}{[e^2+(s_0/\sqrt{2})^2]^{1/2}}-1\right], \tag{A27}$$

$$\eta_{es_0}=\eta_{es_0}^{(1)}+\eta_{es_0}^{(2)}, \tag{A28}$$

where

$$\eta_{es_0}^{(1)}=\frac{1}{s_0[e^2+(s_0/\sqrt{2})^2]^{1/2}}\left[1+\frac{3e^2(s_0/\sqrt{2})^2}{e^2+(s_0/\sqrt{2})^2}\right] \tag{A29}$$

and

$$\eta_{es_0}^{(2)}=-\frac{1}{\sqrt{2}}\left[\frac{1}{[e^2+(s_0/\sqrt{2})^2]^{1/2}}+\frac{1}{s_0/\sqrt{2}}-\frac{e^2}{e^2+(s_0/\sqrt{2})^2}\right]; \tag{A30}$$

finally,

$$\eta_{s_0s_0}=\eta_{s_0s_0}^{(1)}+\eta_{s_0s_0}^{(2)}, \tag{A31}$$

where

$$\eta_{s_0s_0}^{(1)}=\eta_{\mathrm{ss}}^{(1)} \tag{A32}$$

and

$$\eta_{s_0s_0}^{(2)}=-\frac{e}{(s_0/\sqrt{2})}\frac{1}{[e^2+(s_0/\sqrt{2})^2]^{1/2}}\left[1+\frac{(s_0/\sqrt{2})(1-s_0/\sqrt{2})}{[e^2+(s_0/\sqrt{2})^2]^{1/2}}\right]. \tag{A33}$$

APPENDIX B

The effective quasiparticle coupling Γ was defined in Eq. (3.11) of the text

$$\Gamma(\mathbf{q})=\Gamma^s(\mathbf{q})\delta_{\alpha\beta}\delta_{\gamma\delta}+\Gamma^a(\mathbf{q})\boldsymbol{\sigma}_{\alpha\beta}\cdot\boldsymbol{\sigma}_{\gamma\delta}. \tag{B1}$$

The analytic form of the spin antisymmetric part Γ^a originating in the fluctuations of the vector bosons is

$$\Gamma^a(\mathbf{q})=-[\omega_s^2\Gamma_{ss}D_{ss}+i^2\Gamma_{\lambda^{(2)}\lambda^{(2)}}D_{\lambda^{(2)}\lambda^{(2)}}+i\omega_s(\Gamma_{s\lambda^{(2)}}D_{s\lambda^{(2)}}+\Gamma_{\lambda^{(2)}s}D_{\lambda^{(2)}s^{(2)}})]. \tag{B2}$$

The boson propagators are obtained by inverting Eq. (3.7). The resultant expressions are quite simple in the paramagnetic phase:

$$D_{s_1 s_1}=\frac{D^{-1}_{\lambda_1^{(2)}\lambda_1^{(2)}}}{\tilde{D}}=D_{s_2 s_2}=D_{s_3 s_3}\ , \tag{B3}$$

$$D_{\lambda_1^{(2)}\lambda_1^{(2)}}=\frac{D^{-1}_{s_1 s_1}}{\tilde{D}}=D_{\lambda_2^{(2)}\lambda_2^{(2)}}=D_{\lambda_3^{(2)}\lambda_3^{(2)}}\ , \tag{B4}$$

$$D_{s_1\lambda_1^{(2)}}=-\frac{D^{-1}_{\lambda_1^{(2)}s_1}}{\tilde{D}}=D_{s_2\lambda_2^{(2)}}=D_{s_3\lambda_3^{(2)}}\ , \tag{B5}$$

where the determinant

$$\tilde{D}=D^{-1}_{s_1 s_1}D^{-1}_{\lambda_1^{(2)}\lambda_1^{(2)}}-(D^{-1}_{s_1\lambda_1^{(2)}})^2\ . \tag{B6}$$

The vertex functions Γ_{ss}, $\Gamma_{\lambda^{(2)}s}$, and $\Gamma_{\lambda^{(2)}\lambda^{(2)}}$ are given by

$$\Gamma_{ss}=4V^2[\gamma(\mathbf{k}_1)v^{(-)}_{\mathbf{k}_1}u^{(-)}_{\mathbf{k}_1+\mathbf{q}}+\gamma_{\mathbf{k}_1+\mathbf{q}}v^{(-)}_{\mathbf{k}_1+\mathbf{q}}u^{(-)}_{\mathbf{k}_1+\mathbf{q}}u^{(-)}_{\mathbf{k}_1}][\gamma(\mathbf{k}_2-\mathbf{q})v^{(-)}_{\mathbf{k}_2-\mathbf{q}}u^{(-)}_{\mathbf{k}_2}+\gamma(\mathbf{k}_2)u^{(-)}_{\mathbf{k}_2-\mathbf{q}}v^{(-)}_{\mathbf{k}_2}]\ , \tag{B7}$$

$$\Gamma_{\lambda^{(2)}\lambda^{(2)}}=u^{(-)}_{\mathbf{k}_1}u^{(-)}_{\mathbf{k}_2}u^{(-)}_{\mathbf{k}_1+\mathbf{q}}u^{(-)}_{\mathbf{k}_2-\mathbf{q}}\ , \tag{B8}$$

$$\Gamma_{s\lambda^{(2)}}=2Vu^{(-)}_{\mathbf{k}_2}u^{(-)}_{\mathbf{k}_2-\mathbf{q}}[\gamma(\mathbf{k}_1)v^{(-)}_{\mathbf{k}_1}u^{(-)}_{\mathbf{k}_1+\mathbf{q}}+\gamma(\mathbf{k}_1+\mathbf{q})v^{(-)}_{\mathbf{k}_1+\mathbf{q}}u^{(-)}_{\mathbf{k}_1}]\ . \tag{B9}$$

Similarly, the spinsymmetric part Γ^s, originating in the fluctuations of the scalar bosons, is found to be given by

$$\begin{aligned}\Gamma^s=-[&\omega_e^2\Gamma_{ee}D_{ee}+\omega_{s_0}^2\Gamma_{s_0 s_0}D_{s_0 s_0}\\&+i^2\Gamma_{\lambda_0^{(2)}\lambda_0^{(2)}}D_{\lambda_0^{(2)}\lambda_0^{(2)}}+i\omega_{s_0}(\Gamma_{s_0\lambda_0^{(2)}}D_{s_0\lambda_0^{(2)}}+\Gamma_{\lambda_0^{(2)}s_0}D_{\lambda_0^{(2)}s_0})\\&+i\omega_e(\Gamma_{e\lambda_0^{(2)}}D_{e\lambda_0^{(2)}}+\Gamma_{\lambda_0^{(2)}e}D_{\lambda_0^{(2)}e})+2\omega_e\omega_{s_0}\Gamma_{es_0}D_{es_0}]\ .\end{aligned} \tag{B10}$$

Here

$$\Gamma_{es_0}=\Gamma_{ee}=\Gamma_{s_0 s_0}=\Gamma_{ss},\quad \Gamma_{e\lambda_0^{(2)}}=\Gamma_{s_0\lambda_0^{(2)}}=\Gamma_{s\tilde{\lambda}^2}\ , \tag{B11}$$

and the boson propagator matrix elements are obtained on inverting Eq. (3.6).

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