Flux-line-cutting effects at the critical current of cylindrical type-II superconductors

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(Received 10 November 1989)

Our critical-state model, which includes the effects of both flux-line cutting and flux pinning, is applied to a current-carrying cylinder in an axial magnetic field. The time evolution of the internal magnetic-field distribution is computed for currents just slightly above the critical current at which a longitudinal electric field first appears. According to the theory, when an axial magnetic field is nonzero, a unique magnetic-field distribution is ultimately produced. On the other hand, when the axial magnetic field is zero, the final magnetic-field distribution, longitudinal magnetic moment, and critical current all depend upon the sample's magnetic history.

I. INTRODUCTION

A great amount of both experimental and theoretical work has been devoted to the problem of the internal magnetic-field distribution just above the critical current of a type-II superconductor subjected to a parallel applied magnetic field.[1–48] In a recent series of papers we have developed a macroscopic theory, the general critical-state model,[16–19,37–39] to attack a related problem, that of the response of a type-II superconducting slab in a parallel magnetic field that varies in both magnitude and direction. This model incorporates flux-line cutting[49,50] into the usual critical-state theory describing flux pinning.[50–52] Flux-line cutting (intersection and cross joining of adjacent nonparallel vortices) provides a mechanism by which the magnitude B of the magnetic-flux density is reduced inside the sample. It also allows $\boldsymbol{J}$ and $\boldsymbol{E}$ components parallel to the local magnetic field.

Several workers[10,30,31,53–61] have found that under some regimes, when vortices are forced to tilt, they somehow escape from the sample, although the magnetic pressure or Lorentz force in the surface region is directed inwards, leading to a quasi-steady-state B value which is smaller than the magnitude of the internal magnetic induction B at the beginning of the experiment. According to our theory, flux-line cutting consumes B. This reduction of B is a natural consequence of Faraday's law, which when written as Eq. (1) of Ref. 37 states that when flux-line cutting occurs, B is not conserved. From the empirical models suggested in Refs. 10, 30, 31, and 53–61, one can show that in the active regions of the sample, those where fluxoids are moving and tilting, both the induced current density $\boldsymbol{J}$ and the electric field $\boldsymbol{E}$ have components $J_{\parallel}$ and $E_{\parallel}$ parallel to the local magnetic induction $\boldsymbol{B}$. This parallel component of $\boldsymbol{E}$ can be understood in terms of flux-line cutting. Recently, it has been shown that for a sufficiently large current density $J_{c\parallel}$ parallel to the vortices, instabilities of the vortex array[3–6,13,20] can occur which lead to flux-line cutting, thereby generating a component[2,14,15] $E_{\parallel}$ parallel to $\boldsymbol{B}$.

In this paper we use our critical-state theory to show how to calculate the unique time-independent macroscopic-magnetic-field distribution $\boldsymbol{B}$ just above the critical current of a type-II superconducting cylinder in a parallel applied magnetic field. We also calculate $\boldsymbol{B}$ in the absence of the applied field and demonstrate that the internal $\boldsymbol{B}$ distribution and the critical-current density depend upon the specimen's magnetic history, resulting in larger critical currents for larger remanent longitudinal magnetic moments.

II. THE GENERAL CRITICAL-STATE MODEL

We consider a high-κ, irreversible type-II superconducting infinite cylinder of radius R. Applied to the surface at $r=R$ is a parallel magnetic field $\boldsymbol{B}_a=B_a\hat{\mathbf{z}}$, where $B_a=|\boldsymbol{B}_a|$. A current I is applied to the cylinder, such that the azimuthal field on the surface is $H_\phi(R)=I/2\pi R$. We assume that to good approximation $\boldsymbol{B}=\mu_0\boldsymbol{H}$ inside the sample, and we neglect any surface barriers against flux entry or exit. We further assume that in steady state, the magnetic induction $\boldsymbol{B}$ inside the sample is independent of time and depends only on the coordinate r. Thus $\boldsymbol{B}(r)=B(r)\hat{\boldsymbol{\alpha}}(r)$, where $B=|\boldsymbol{B}|$ and

$$\hat{\boldsymbol{\alpha}}(r)=\hat{\boldsymbol{\phi}}\sin\alpha+\hat{\mathbf{z}}\cos\alpha \; . \tag{1}$$

The surface boundary condition is $\boldsymbol{B}(R)=\boldsymbol{B}_s=B_s\hat{\boldsymbol{\alpha}}_s$, where

$$B_s=(B_a^2+B_\phi^2)^{1/2}=[B_a^2+(\mu_0 I/2\pi R)^2]^{1/2} \tag{2}$$

and $\hat{\boldsymbol{\alpha}}_s$ is the direction of $\boldsymbol{B}$ at the surface; see Fig. 1.

Resolving the current density $\boldsymbol{J}$ and the electric field $\boldsymbol{E}$ into their components parallel and perpendicular to the local $\boldsymbol{B}$ (i.e., writing $\boldsymbol{J}=J_{\parallel}\hat{\boldsymbol{\alpha}}+J_{\perp}\hat{\boldsymbol{\beta}}$, and $\boldsymbol{E}=E_{\parallel}\hat{\boldsymbol{\alpha}}+E_{\perp}\hat{\boldsymbol{\beta}}$, where $\hat{\boldsymbol{\beta}}=\hat{\boldsymbol{\alpha}}\times\hat{\mathbf{r}}$), we obtain from Ampere's law

$$J_{\parallel}=\mu_0^{-1}B\left[\frac{\partial\alpha}{\partial r}+\frac{\sin\alpha\cos\alpha}{r}\right] , \tag{3}$$

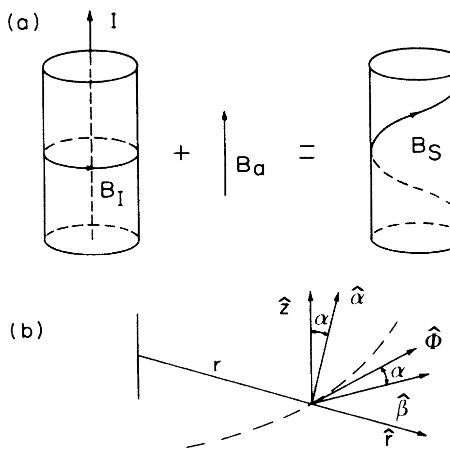

FIG. 1. (a) Experiment considered, and (b) local coordinates used in the calculations. Note that $\hat{\alpha}$ and $\hat{\beta}$ are perpendicular to $\hat{r}$.

$$J_{\perp} = -\mu_0^{-1}\left[\frac{\partial B}{\partial r} + B\frac{\sin^2\alpha}{r}\right] . \tag{4}$$

From Faraday's law, $\nabla \times E = \partial B / \partial t$, we obtain

$$\frac{\partial B}{\partial t} = -\frac{\partial E_{\perp}}{\partial r} - E_{\perp}\frac{\cos^2\alpha}{r} - E_{\parallel}\left[\frac{\partial \alpha}{\alpha r} + \frac{\sin\alpha\cos\alpha}{r}\right] , \tag{5}$$

$$\frac{\partial \alpha}{\partial t} = \frac{1}{B}\left[\frac{\partial E_{\parallel}}{\partial r} + E_{\parallel}\frac{\sin^2\alpha}{r} - E_{\perp}\left[\frac{\partial \alpha}{\partial r} - \frac{\sin\alpha\cos\alpha}{r}\right]\right] . \tag{6}$$

In steady state the left-hand side of Eqs. (5) and (6) are zero, and $\nabla \times E = 0$ dictates that $E = E_0\hat{z}$, where E_0 is a constant independent of r.

The general critical-state model[16–19,37–39] states that metastable stationary distributions of B, in which $E_{\perp} = 0$, occur only where the magnitude of $J_{\perp}$ is smaller than $J_{c\perp}(B)$, the transverse critical-current density at the threshold for depinning of the vortex array. Similarly, metastable distributions of α, in which $E_{\parallel} = 0$, occur only where the magnitude of $J_{\parallel}$ is smaller than $J_{c\parallel}(B)$, the parallel critical-current density at the threshold for flux-line cutting in the vortex array. We assume that the pinning and flux-cutting properties are isotropic, such that $J_{c\perp}$ and $J_{c\parallel}$ depend upon the magnitude of B but not upon their orientation.

In analogy with the usual critical-state model, we assume that the electric field behaves as

$$E_{\perp} = \rho_{\perp}(J_{\perp} \mp J_{c\perp}) \tag{7}$$

when $E_{\perp} \gtrless 0$ and

$$E_{\parallel} = \rho_{\parallel}(J_{\parallel} \mp J_{c\parallel}) \tag{8}$$

when $E_{\parallel} \gtrless 0$, where $\rho_{\perp}$ and $\rho_{\parallel}$ are the effective flux-flow and flux-cutting resistivities of the material. Since we assume here that $|E_{\perp}| \ll \rho_{\perp}J_{c\perp}$ and $|E_{\parallel}| \ll \rho_{\parallel}J_{c\parallel}$, the computed B fields are independent of $\rho_{\perp}$ and $\rho_{\parallel}$.

III. RESULTS IN LONGITUDINAL FIELD

In this section we use the preceding formalism to calculate the field distribution inside the cylinder. For simplicity we take $J_{c\perp}$ and $J_{c\parallel}$ as constants independent of B. We consider first the time evolution from a state in which the applied current is zero but the external parallel magnetic field B_a is different from zero, to a state in which the external field remains unchanged but a current exceeding the critical current $I_c(B)$ is suddenly applied. We assume that the magnitude of B_a is between B_{c1} and B_{c2} such that initially the B distribution inside the cylinder is uniform and numerically equal to B_a. Using Eqs. (5) and (6) we can calculate, with the aid of Eqs. (3), (4), (7), and (8), the time development of the B and α profiles. Figure 2 illustrates this behavior for the case in which the initial state is nonmagnetic,[39] $B = 1$ T and $\alpha = 0$ in the whole sample. We consider $R = 1.5$ mm, $J_{c\perp} = J_{c\parallel} = 4 \times 10^3$ A/cm^2, and

$$J_z \begin{cases} = 0, \quad t < 0 , & \text{(9a)} \\ = 6.4 \times 10^3 \text{A/cm}^2, \quad t > 0 . & \text{(9b)} \end{cases}$$

This plot exhibits several zones in which different phenomena occur depending upon different regimes inside the sample. For example curves (2b) correspond to the case in which only the fluxoids near the surface (0.65 mm $\le r \le$ 1.5 mm) have had time to start tilting. In this regime there are three regions: An O zone ($0 \le r \le 0.65$ mm) in which neither flux-line cutting nor transport occur; a cutting (C) zone (0.65 mm $< r \le 1.395$ mm) in which flux-line cutting occurs but flux transport does not; and a cutting and transport (CT) zone (1.395 mm $< r \le 1.5$ mm) in which both flux-line cutting and flux transport occur, such that flux transport replenishes the

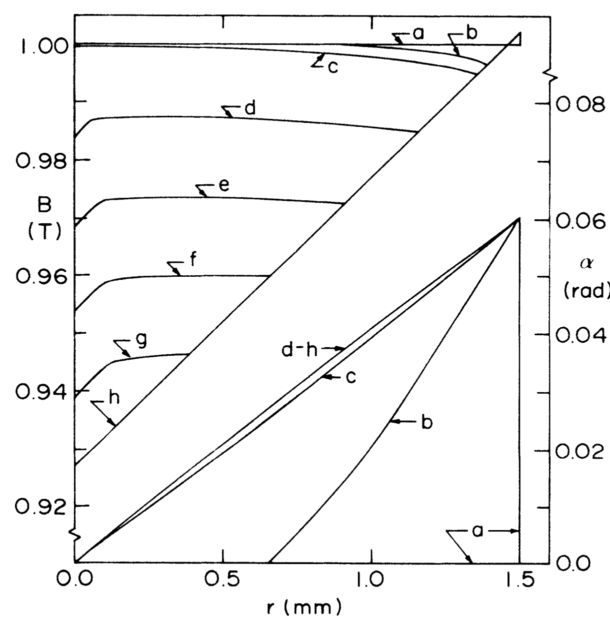

FIG. 2. Time development of the B and α profiles as discussed in Sec. III starting from a nonmagnetic initial state.

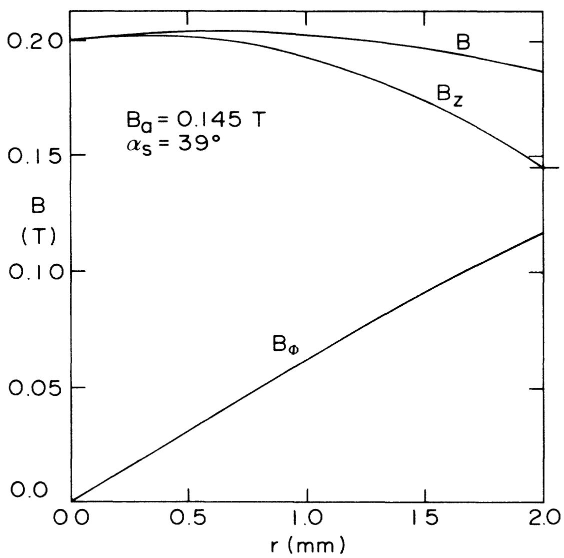

FIG. 3. Internal magnetic-field distribution at the critical current. In this case $B_a=0.145$ T and $J=J_c=9.367\times10^3$ A/cm^2.

B that flux-line cutting consumes. Curves (2e) show another case with two CT zones and a C zone in between. Curves (2h) correspond to the final metastable state exhibiting a unique CT zone where flux-line cutting is continuously occurring and consuming B but transport from the surface is continuously replenishing it.

When the applied magnetic field B_a is different from zero and the current slightly exceeds the critical current $I_c(B)$, $\boldsymbol{B}$ evolves into a unique critical-state distribution given by the solutions of

$$\frac{\partial B}{\partial r}=\mu_0 J_{c\perp}(\boldsymbol{B})-B\frac{\sin^2\alpha}{r} , \tag{10}$$

$$\frac{\partial \alpha}{\partial r}=\frac{\mu_0 J_{c\parallel}(\boldsymbol{B})}{B}-\frac{\sin\alpha\cos\alpha}{r} . \tag{11}$$

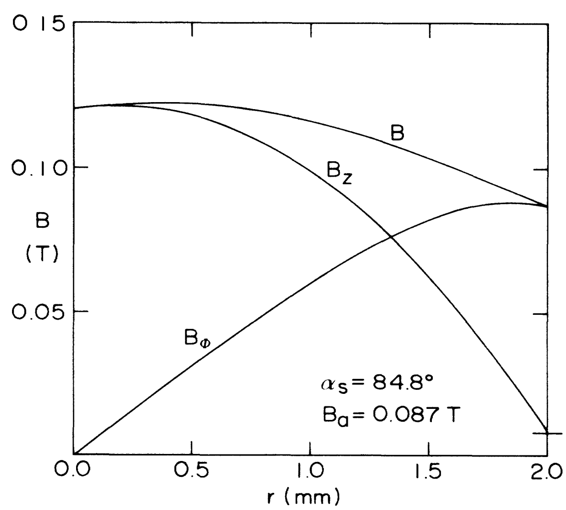

FIG. 4. Same as Fig. 3 but $B_a=0.087$ T and $J=J_c=6.924\times10^3$ A/cm^2.

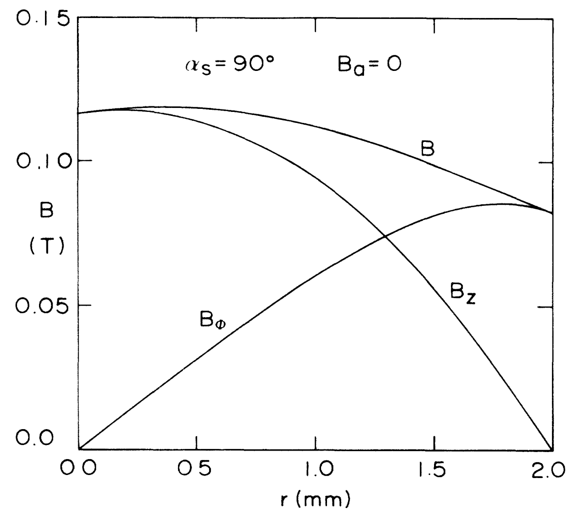

FIG. 5. Internal magnetic-field distribution produced when B_a is reduced from $B_a>0$ to $B_a=0$ with $I=I_c$. In this case $J=J_c=6.591\times10^3$ A/cm^2.

These equations easily can be solved numerically. Figure 3 shows the internal $\boldsymbol{B}$ distribution for a sample with $J_{c\perp}=10^3$ A/cm^2, $J_{c\parallel}=10^4$ A/cm^2 at $I_c(\boldsymbol{B})$; in this case $B_a=0.145$ T. Figure 4 shows the same characteristics but at $B_a=0.087$ T.

When $B_a=0$, the field distribution depends upon the specimen's magnetic history. Figure 5 shows the distribution which arises when B_a is reduced from $B_a>0$ to $B_a=0$ with $I=I_c(B)$. Figure 6 shows two different dis-

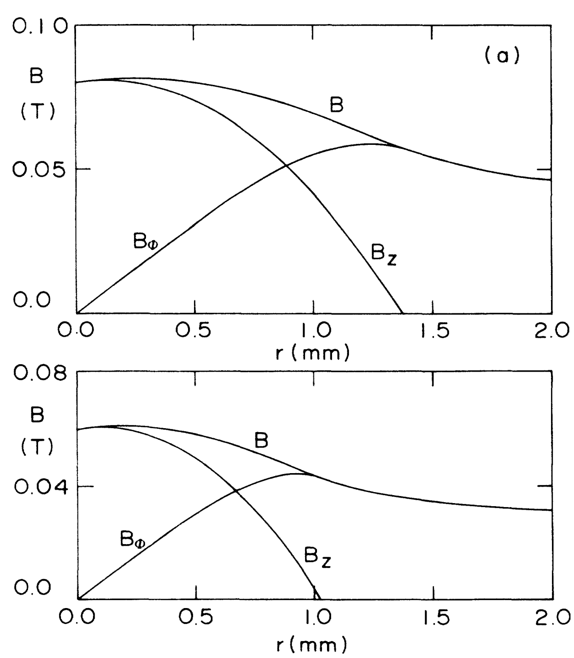

FIG. 6. Two final field distributions in zero applied field, which depend upon the specimen's magnetic history. (a) $J=J_c=3.646\times10^3$ A/cm^2. (b) $J=J_c=2.488\times10^3$ A/cm^2.

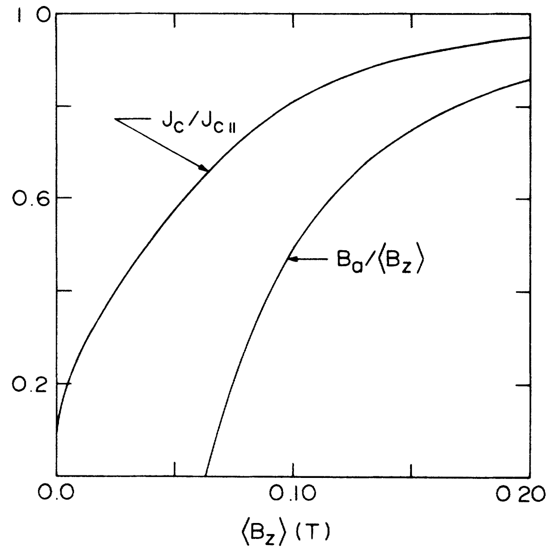

FIG. 7. Critical current density vs applied field. In zero applied field J_c depends upon the remanent longitudinal magnetic moment.

tributions for $B_a=0$ at $I_c(B)$. They correspond to different values of $B_\phi(r=R)$ which itself depends upon I.

From Fig. 7, a plot of the average critical current density

$$J_c = \langle J_z \rangle = J_{c\parallel}\langle \cos\alpha \rangle + J_{c\perp}\langle \sin\alpha \rangle \ , \tag{12}$$

it can be seen that when $B_a=0$, since $\mu_0\langle M_z \rangle = \langle B_z \rangle - \mu_0 H_a$, J_c depends upon the remanent longitudinal magnetic moment. For these calculations we have taken $J_{c\perp}=0.1J_{c\parallel}=$const., independent of B.

IV. SUMMARY AND DISCUSSION

In this paper we have applied our general critical-state theory for calculating the internal magnetic-field distribution for a type-II superconducting cylinder at the critical current when an external magnetic field is applied. We also calculated the behavior when the applied field is zero. We found, as in the experiments of Walmsley,[50] that above the critical current in "zero" longitudinal field, "spontaneous" longitudinal magnetic moments appear, and that the longitudinal voltage level at a fixed value of the current depends upon the magnitude of the magnetic moment. A larger magnitude of the magnetic moment yields a smaller voltage for a given above-critical current. In summary, spontaneous longitudinal moments appear, and the bigger the moment, the bigger the corresponding critical current.

The general critical-state theory, which includes the effects of both flux-line cutting and flux pinning, predicts a unique internal magnetic-field distribution at the critical current in an applied longitudinal field. In zero applied field it predicts a magnetic-history dependence of the final magnetic-field distribution, longitudinal magnetic moment, and critical-current density. Those results agree with experiments of Walmsley.[50]

ACKNOWLEDGMENTS

Ames Laboratory is operated for the U.S. Department of Energy by Iowa State University under Contract No. W-7405-Eng-82. This work was supported in part by the Director for Energy Research, Office of Basic Energy Sciences.

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